

Practic 2 (Linear Algebra)

Topic. The Method by Gauss

Example. Solve the system of equations using the method by Gauss

$$\begin{cases} x_1 + 2x_2 - 3x_3 + 4x_4 = 7 \\ 2x_1 + 4x_2 + 5x_3 - x_4 = 2 \\ 5x_1 + 10x_2 + 7x_3 + 2x_4 = 11 \end{cases} .$$

Solution. The direct way:

$$\begin{aligned} \tilde{A} &= \left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 7 \\ 2 & 4 & 5 & -1 & 2 \\ 5 & 10 & 7 & 2 & 11 \end{array} \right) \sim \begin{matrix} e_3 + e_1(-5) \rightarrow e_3 \\ e_2 + e_1(-2) \rightarrow e_2 \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 7 \\ 0 & 0 & 11 & -9 & -12 \\ 0 & 0 & 22 & -18 & -24 \end{array} \right) \sim \\ &\sim \begin{matrix} e_3 / (-2) \\ e_3 + e_2 \rightarrow e_3 \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 7 \\ 0 & 0 & 11 & -9 & -12 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 7 \\ 0 & 0 & 11 & -9 & -12 \\ 0 & 0 & 11 & -9 & -12 \end{array} \right) \end{aligned}$$

The initial system is equivalent to the following system of equations

$$\begin{cases} x_1 + 2x_2 - 3x_3 + 4x_4 = 7 \\ 11x_3 - 9x_4 = -12 \end{cases} .$$

The backward way:

$$11x_3 = -12 + 9x_4, x_3 = \frac{3}{11}(-4 + 3x_4),$$

$$x_1 = -2x_2 + \frac{9}{11}(-4 + 3x_4) - 4x_4 + 7 = -2x_2 - \frac{17}{11}x_4 + \frac{41}{11};$$

x_2, x_4 are free unknowns, x_1, x_3 are basic ones.

$$\text{The general solution: } X = \begin{pmatrix} -2x_2 - \frac{17}{11}x_4 + \frac{41}{11} \\ x_2 \\ \frac{9}{11}x_4 - \frac{12}{11} \\ x_4 \end{pmatrix} = x_2 \cdot \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \cdot \begin{pmatrix} -\frac{17}{11} \\ 0 \\ \frac{9}{11} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{41}{11} \\ 0 \\ -\frac{12}{11} \\ 0 \end{pmatrix}$$

Since x_2 and x_4 are free variables they could be designated as $x_2 = C_1$; $x_4 = C_2$.

Solve systems:

$$1. \begin{cases} 2x_1 + x_2 - x_3 - 3x_4 = 2; \\ 4x_1 + x_3 - 7x_4 = 3; \\ 2x_2 - 3x_3 + x_4 = 1; \\ 2x_1 + 3x_2 - 4x_3 - 2x_4 = 3. \end{cases} \quad 2. \begin{cases} 2x_1 - 3x_2 + 4x_3 - x_4 = 1; \\ x_1 + x_2 + x_3 = 0; \\ 5x_1 - 2x_2 + x_3 - x_4 = 0; \end{cases} \quad 3. \begin{cases} 2x_1 + x_2 + x_3 + 3x_4 = 3; \\ x_1 + 2x_2 - x_3 + 2x_4 = -3; \\ x_1 + 3x_2 - x_4 = 0; \\ 3x_1 + 4x_2 + 2x_3 = 6. \end{cases}$$