

Dear Students,

*I ask you to complete the individual task below on the topic "Differential calculus of functions of many variables." Send, please, the completed task to my email address **until April 10, 2020.:***

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This topic will be estimated based on the results of this assignment

If you have any questions, then you can ask me also through my E-mail

Individual task to topic" Differential calculus for function with several variables"

Task 1.

2.1.1. Show that the function $u = \sin x + (\sin y - \sin x)^2$, satisfies the following equation

$$\frac{\partial u}{\partial y} \cos x + \frac{\partial u}{\partial x} \cos y = \cos x \cos y$$

2.1.2. Show that the function $z = \frac{y}{(x^2 - y^2)}$, satisfies the following equation

$$\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} = \frac{z}{y^2}$$

2.1.3. Show that the function $z = y + (x^2 - y^2)$ satisfies the following equation

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x$$

2.1.4. Show that the function $z = xy + x \cos \frac{y}{x}$ satisfies the following equation

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z + xy$$

2.1.5. Show that the function $z = y(x^2 - y^2) + x \cos \frac{y}{x}$ satisfies the following equation

$$\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} = \frac{z}{y^2}$$

2.1.6. Show that the function $z = 2xy$ satisfies the following equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

2.1.7. Show that the function $z = x^2 - y^2$ satisfies the following equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

2.1.8. Show that the function $z = 2xy$ satisfies the following equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

2.1.9. Show that the function $z = x^3 - 3xy^2$ satisfies the following equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

2.1.10. Show that the function $z = 3x^2y - y^3$ satisfies the following equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

2.1.11. Show that the function $z = \ln(x^2 + y^2 + 5)$ satisfies the following equation

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{4xy}{(x^2 + y^2 + 5)^2}$$

2.1.12. Show that the function $z = \arcsin \frac{x}{y}$ satisfies the following equation

$$\frac{x}{y^2} \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

2.1.13. $z = x + y + \cos(x/y)$. Prove that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = x + y$.

2.1.14. $z = \frac{1}{2}(x^2 + y^2) + (x - y)$. Prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = x + y$.

2.1.15. $z = xe^{\frac{x}{y}} - x^2 - y^2$. Prove that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z - x^2 - y^2$.

2.1.16. $z = y \cos(x^2 - y^2)$. Prove that $y^2 \frac{\partial z}{\partial x} + xy \frac{\partial z}{\partial y} = xz$.

2.1.17. $z = x \ln\left(\frac{y}{x^2}\right)$. Prove that $x \frac{\partial z}{\partial x} + 2y \frac{\partial z}{\partial y} = z$.

2.1.18. $z = \frac{y^2}{3x} + 2xy$. Prove that $x^2 \frac{\partial z}{\partial x} - xy \frac{\partial z}{\partial y} + y^2 = 0$.

- 2.1.19. $z = y \cos(x - y)$. Prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{x}{y}$.
- 2.1.20. $z = x^2 \ln y$, where $x = \frac{u}{v}$, $y = 3u - 2v$. Find $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$.
- 2.1.21. $z = \frac{x + y}{(x^2 - y^2)}$. Prove that $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = z$.
- 2.1.22. $u = \frac{yz}{x}$, where $x = e^t$, $y = \ln t$, $z = t^2 - 1$. Find $\frac{dz}{dt}$.
- 2.1.23. $z = \arctan(xy)$, where $y = e^x$. Find $\frac{dz}{dx}$.
- 2.1.24. $u = \arcsin \frac{x}{z}$, where $z = \sqrt{x^2 + 1}$. Find $\frac{\partial u}{\partial x}$ and $\frac{du}{dx}$.
- 2.1.25. $z = \arctg \frac{y}{x}$, where $x = e^{2t+1}$, $y = e^{2t-1}$. Find $\frac{dz}{dt}$.
- 2.1.26. $u = xyz$, where $x = t^2 + 1$, $y = \ln t$, $z = \tan t$. Find $\frac{du}{dt}$.
- 2.1.27. $u = \frac{z}{\sqrt{x^2 + y^2}}$, where $x = R \cos t$, $y = R \sin t$, $z = H$. Find $\frac{du}{dt}$.
- 2.1.28. $z = \ln \sin \frac{x}{\sqrt{y}}$, where $x = 3t^2$, $y = \sqrt{t^2 + 1}$. Find $\frac{dz}{dt}$.
- 2.1.29. $z = \ln(x^2 + y^2)$, where $y = \frac{1}{3}x^3 + x$. Find $\frac{\partial z}{\partial x}$ and $\frac{dz}{dx}$.
- 2.1.30. $z = \arctan \frac{x + y}{1 - xy}$, where $y = \cos x$. Find $\frac{dz}{dx}$.

Task 2.

Investigate the following functions for an extreme:

- 2.2.1. $z = x^3 y^2 (6 - x - y)$;
- 2.2.2. $z = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$;
- 2.2.3. $z = \sin x + \sin y + \sin(x + y)$, ($0 \leq x \leq \pi/2$, $0 \leq y \leq \pi/2$);
- 2.2.4. $z = \sin x \sin y \sin(x + y)$, ($0 \leq x \leq \pi$, $0 \leq y \leq \pi$);
- 2.2.5. $z = x^2 + xy + y^2 - 2x - y$;
- 2.2.6. $z = e^{-x^2 - y^2} (2x^2 + y^2)$;

- 2.2.7. $z = x^4 + y^4 - 2x^2 + 4xy - 2y^2$;
- 2.2.8. $z = 1 + 6x - x^2 - xy - y^2$;
- 2.2.9. $z = x^2 - xy + y^2 - 2x + y$;
- 2.2.10. $z = x^3 - 3xy + y^3$;
- 2.2.11. $z = 3x^2 - x^3 + 3y^2 + 4y$;
- 2.2.12. $z = xy + \frac{50}{x} + \frac{20}{y} \quad (x > 0, y > 0)$;
- 2.2.13. $z = e^{2x+3y}(8x^2 - 6xy + 3y^2)$;
- 2.2.14. $z = e^{x^2-y}(5 - 2x + y)$;
- 2.2.15. $z = (5x + 7y - 25)e^{-(x^2+xy+y^2)}$;
- 2.2.16. $z = x^2 + xy + y^2 - 4\ln x - 10\ln y$;
- 2.2.17. $z = \sin x + \cos y + \cos(x - y), (0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{2})$;
- 2.2.18. $z = x - 2y + \ln \sqrt{x^2 + y^2} + 3 \arctan \frac{y}{x}$;
- 2.2.19. $z = xy \ln(x^2 + y^2)$;
- 2.6.20. $z = xy \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}, (a > 0, b > 0)$;
- 2.2.21. $z = x^2 + y^2 - 2\ln x - 18\ln y, (x > 0, y > 0)$;
- 2.2.22. $z = x^3 + 3xy^2 - 15x - 12y$;
- 2.2.23. $z = 2x^3 + 2y^3 - 36xy + 430$;
- 2.2.24. $z = x^2 + xy + y^2 - 2x - y$;
- 2.2.25. $z = 3x^2 - x^3 + 3y^2 + 4y$;
- 2.2.26. $z = e^{-x^2-y^2}(2x^2 + y^2)$;
- 2.2.27. $z = x^4 + y^4 - 2x^2 + 4xy - 2y^2$;
- 2.2.28. $z = x^3 - 3xy + y^3$;
- 2.2.29. $z = 3x^2 - x^3 + 3y^2 + 4y$;
- 2.2.30. $z = e^{x^2-y}(5 - 2x + y)$

2.3.22. $z = x + y$, D: $x^2 + y^2 = 1; x \geq 0; y \geq 0$

2.3.23. $z = x^2 + y^2$, D: if $\frac{x}{2} + \frac{y}{3} = 1; x \geq 0; y \geq 0$

2.3.24. $z = \cos^2 x + \cos^2 y$, D: $x - y = \pi/4; x \leq 0; y \leq 0$

2.3.25. $z = x^2 + y^2 - 2y - 2x - 8$, D: $x + y = 1; x = 0; y = 0;$

Task 4.

2.4.1. Write equations of a tangent line and a normal plane for the given curves at the given point $x = t - \sin t$, $y = t - \cos t$, $z = 4 \sin t / 2$ at $t = \pi / 2$;

2.4.2. On the curve $x = \cos t$, $y = \sin t$, $z = e^t$ find the point, at which the tangent line is parallel to the plane $\sqrt{3}x + y - 4 = 0$.

2.4.3. On the curve $x = \frac{t^4}{4}$, $y = \frac{t^3}{3}$, $z = \frac{t^2}{2}$ find the points at which the tangent to this curve is parallel to the plane $x + 3y + 2z - 10 = 0$.

For the given surfaces 2.4.4–2.4.11 find the equations of the tangent planes and normal line at the given points:

2.4.4. $x^2 + y^2 + z^2 = 16z$ at the point $(8\cos\alpha, 8\sin\alpha, 8)$;

2.4.5. $2\sqrt{\frac{x}{z}} + 2\sqrt{\frac{y}{z}} = 8$ at the point $(2, 2, 1)$;

2.4.6. $x^2yz + 2x^2z - 3xyz + 2 = 0$ at the point $(1, 0, -1)$;

2.4.7. $z = e^{x\cos y}$ at the point $(1, \pi, 1/e)$;

2.4.8. $4 + \sqrt{x^2 + y^2 + z^2} = x + y + z$ at the point $(2, 3, 6)$;

2.4.9. $z = \arctan \frac{y}{x}$ at the point $(1, 1, \pi/4)$;

2.4.10. $z = \frac{x^3 - 6xy + y^3}{4}$ at the point $(2, 2, -2)$.

2.4.11. $z^2 + 4z + x^2 = 0$ at the points of intersection with the axis OZ.

2.4.12. For the surface $x^2 - z^2 - 2x + 6y = 4$ find the equation of the normal parallel to the straight line $\frac{x+2}{1} = \frac{y}{3} = \frac{z+1}{4}$.

2.4.13. For the surface $z = 4x - xy + y^2$ find the equation of the tangent plane, parallel to the plane $4x + y + z - 2z + 9 = 0$.

2.4.14. Find the tangent planes to the surface $x^2 + 2y^2 + 3z^2 = 21$ parallel to the plane $x + 4y + 6z = 0$.

2.4.15. On the surface $x^2 + y^2 - z^2 - 2x$ find the points at which the tangent planes are parallel to the coordinate planes.

2.4.16. Define the tangent plane to the surface $x^2 + 4y^2 + z^2 = 36$ parallel to the plane $x + y - z = 0$.

2.4.17. Find at which point the tangent plane to the surface $z = 4 - x^2 - y^2$ is parallel to the plane $2x + 2y + z = 0$. Find the equation of this tangent plane.

2.4.18. Find the tangent plane to the surface $x^2 - y^2 - 3z = 0$ passing through the point $A(0, 0, -1)$ parallel to the curve $\frac{x}{2} = y = \frac{z}{2}$.

2.4.19. On the surface $x^2 + y^2 + z^2 - 6y + 4z = 12$ find the point, at which the tangent planes are parallel to the coordinate planes.

2.4.20. For the surface $z = xy$ find the equation of the tangent plane perpendicular to the straight line $\frac{x+2}{2} = \frac{y+2}{2} = \frac{z-1}{-1}$.

2.4.21. Write equations of a tangent line and a normal plane for the given curves at the given point $x = t - \sin t$, $y = t - \cos t$, $z = 4 \sin t / 2$ at $t = \pi / 2$;

2.4.22. On the curve $x = \cos t$, $y = \sin t$, $z = e^t$ find the point, at which the tangent line is parallel to the plane $\sqrt{3}x + y - 4 = 0$.

2.4.23. On the curve $x = \frac{t^4}{4}$, $y = \frac{t^3}{3}$, $z = \frac{t^2}{2}$ find the points at which the tangent to this curve is parallel to the plane $x + 3y + 2z - 10 = 0$.

2.4.24. For the surface $x^2 - z^2 - 2x + 6y = 4$ find the equation of the normal parallel to the straight line $\frac{x+2}{1} = \frac{y}{3} = \frac{z+1}{4}$.

2.4.25. For the surface $z = 4x - xy + y^2$ find the equation of the tangent plane, parallel to the plane $4x + y + z - 2z + 9 = 0$.

2.4.26. Find the tangent planes to the surface $x^2 + 2y^2 + 3z^2 = 21$ parallel to the plane $x + 4y + 6z = 0$.

2.4.27. On the surface $x^2 + y^2 - z^2 - 2x$ find the points at which the tangent planes are parallel to the coordinate planes.

2.4.28. Define the tangent plane to the surface $x^2 + 4y^2 + z^2 = 36$ parallel to the plane $x + y - z = 0$.

2.4.29. For the given surfaces $2\frac{x}{z} + 2\frac{y}{z} = 8$ find the equations of the tangent planes and normal line at the point $(2, 2, 1)$;

2.4.30. For the given surfaces $x^2yz + 2x^2z - 3xyz + 2 = 0$ find the equations of the tangent planes and normal line at the point $(1, 0, -1)$;

Task 5.

Find the gradient of the function $u = F(x, y, z)$ at the point $A(x_0, y_0, z_0)$:

2.5.1. $u = -x^2 + y^2 + z^2 - 6y + 4z - 12$, $A(2, 8, 0)$;

2.5.2. $u = x^2 - y^2 - 3z$, $A(-3, 0, 3)$;

2.5.3. $u = x^2 + y^2 - 3z^2 + 2y$, $A(0, -3, 1)$;

2.5.4. $u = 5x^2 + 2y^2 - 3z^2 + 2y$, $A(0, -3, 1)$;

2.5.5. $u = x^2 - y^2 + z$, $A(1, 2, 5)$;

2.5.6. $u = \frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{8}$, $A(4, 3, 4)$;

2.5.7. $u = 4x^2 - 2y^2 + 3z^3 + 2y$, $A(1, 2, 0)$;

2.5.8. $u = 3xyz - z^3$, $A(0, 2, -2)$;

2.5.9. $u = z - \frac{x^2}{2} + y^2$, $A(2, -1, 1)$;

2.5.10. $u = z - x^2 - y^2 - xy - x + y - 1$, $A(2, -2, 9)$;

2.5.11. $u = z - x^3 - y^3 + 3xy$, $A(1, 1, -1)$;

2.5.12. $u = 4x^2 - y^2 + z^2 - 16$, $A(-2, 1, -1)$;

2.5.13. $u = x^2 - 2y^2 + 2z^2 - 1$, $A(1, -1, 1)$;

- 2.5.14. $u = x^2 + 2y^2 - 2z^2 - 1$, $A(-1,1,-1)$;
- 2.5.15. $u = 4x^2 - y^2 + z^2 - 16$, $A(-2,1,-1)$;
- 2.5.16. $u = x^2 + y^2 - 4z^2$, $A(2,-2,-\sqrt{2})$;
- 2.5.17. $u = x^2 - y^2 + z^2 - 4$, $A(1,1,-2)$;
- 2.5.18. $u = x^2 + y^2 + z - 4$, $A(-1,-1,2)$;
- 2.5.19. $u = x^2 - y^2 - z^2$, $A(-2,1,-\sqrt{3})$;
- 2.5.20. $u = -x^2 + 4y^2 - z^2 - 4$, $A(-2,-2,4)$;
- 2.5.21. $u = x^2 + 9y^2 - 6x + z^2 - 4z + 4$, $A(3,-1,2)$;
- 2.5.22. $u = x^2 + y^2 - 2z - 10$, $A(2,2,-1)$;
- 2.5.23. $u = x^2 + 4y^2 - z^2 - 4$, $A(0,-1,0)$;
- 2.5.24. $u = x^2 + y^2 - 4z - 9$, $A(2,-1,-1)$;
- 2.5.25. $u = x^2 - y^2 - z^2$, $A(-3,2,-\sqrt{5})$;
- 2.5.26. $u = x^2 - y^2 - z^2 + 6y - 4z + 12$, $A(2,8,0)$;
- 2.5.27. $u = x^2 - y^2 - 3z$, $A(-3,0,3)$;
- 2.5.28. $u = x^2 + y^2 - 3z^2 - 2y$, $A(0,3,1)$;
- 2.5.29. $u = x^2 + y^2 - z$, $A(-1,2,5)$;
- 2.5.30. $u = \frac{x^2}{16} - \frac{y^2}{3} + \frac{z^2}{8}$, $A(4,3,4)$.

Task 6

Find the derivative of the function $\varphi(x, y, z)$ with respect to the given direction:

- 2.6.1. $\varphi = xyz$ with respect to the direction \vec{AB} , where $A(1,-1,1)$, $B(2,3,1)$;
- 2.6.2. $\varphi = (x^2 + y^2 + z^2)^{3/2}$ at the point $A(1,1,1)$ with respect to $\vec{\ell} = 2\vec{j} - \vec{k}$;
- 2.6.3. $\varphi = \arctg(xy)$ at the point $A(1,1)$ with respect to the direction of the bisector of the first coordinate angle;
- 2.6.4. $\varphi = \ln(e^x + e^y)$ at the point $A(0,0)$ with respect to the direction of a ray forming the angle $\pi/6$ with the axis OX ;

2.6.5. $\varphi = \ln(xy + yz + xz)$ at the point $A(0,1,1)$ of the circle $\begin{cases} x = \cos t \\ y = \sin t \\ z = 1 \end{cases}$ with respect to the

direction of this circle.

2.6.6. $\varphi = \operatorname{arctg} \frac{x}{y}$ at the point $A(1, -\sqrt{3})$ of the circle $x^2 + y^2 = 4x$ with respect to the

direction of this circle;

2.6.7. $\varphi = x^2 + y^2$ at the point $A(-1,1)$ of the circle $x^2 + y^2 = 2$ with respect to the direction of this circle;

2.6.8. $\varphi = xz^2 + 2yz$ at the point $A(1,0,2)$ of the circle $\begin{cases} x^2 + y^2 - 2x + 2y + 1 = 0 \\ z = 6 \end{cases}$ with

respect to the direction of this circle;

2.6.9. $\varphi = \ln(x^2 + 2y)$ at the point $A(4,-4)$ of the parabola $y^2 = 4x$ with respect to the direction of this curve;

2.6.10. $\varphi = y^2 + 2yx$ at the point $A(0,-2)$ of the ellipse $x^2 + \frac{y^2}{4} = 1$ with respect to the direction of this curve;

2.6.11. $\varphi = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ at the point $A(x_0, y_0, z_0)$ with respect to the direction of the radius-vector of this point;

2.6.12. $\varphi = \ln(x^2 + 4xy + 3y^2)$ at the point $A(2,0)$ with respect to the direction of the vector collinear to the bisector of the 1st coordinate angle;

2.6.13. $\varphi = 3xz - \frac{z}{x}$ at the point $A(-1,2,3)$ with respect to the direction of $\vec{\ell} = \vec{i} - \vec{j} + \sqrt{2}\vec{k}$;

2.6.14. $\varphi = \sqrt{xy} + \sqrt{4 - z^2}$ at the point $A(-1,-1,0)$ with respect to the direction of $\vec{\ell} = 2\vec{i} - 2\vec{j} + \vec{k}$;

2.6.15. $\varphi = x^3 + \sqrt{4y^2 + z^2}$ at the point $A(-2,2,-3)$ with respect to the direction of $\vec{\ell} = -3\vec{i} + 4\vec{k}$;

2.4.16. $\varphi = 2\sqrt{y-z} + y \operatorname{arctg} x$ at the point $A(1,4,-5)$ with respect to the direction of $\vec{\ell} = 4\vec{i} - 3\vec{j}$;

2.6.17. $\varphi = 2x^2 - \ln(z^2 + y^2)$ at the point $A(2,-1,1)$ with respect to the direction of $\vec{\ell} = \{0; -1; 0\}$;

2.6.18. $\varphi = \cos(x - 2y) + 4yz^2$ at the point $A(\pi/2, \pi/4, 1)$ with respect to the direction of $\vec{\ell} = \{2; -1; 2\}$;

2.6.19. $\varphi = y^2 + \operatorname{arctg}(x - z)$ at the point $A(0,-2,-1)$ with respect to the direction of $\vec{\ell} = \{2; -6; -3\}$;

- 2.6.20. $\varphi = xy^2 + \sqrt{yx^2 - z^3}$ at the point $A(-1,1,-2)$ with respect to the direction of $\vec{\ell} = 8\vec{i} - 4\vec{j} - 8\vec{k}$;
- 2.6.21. $\varphi = x^2y^2z + \ln(z+1)$ at the point $A(4,-3,0)$ with respect to the direction of $\vec{\ell} = 2\vec{i} + \vec{k}$;
- 2.6.22. $\varphi = yz + \frac{z}{x}$ at the point $A(-1,3,4)$ with respect to the direction of $\vec{\ell} = \vec{i} - \vec{k}$;
- 2.6.23. $\varphi = \sqrt{xy} + \sqrt{yz} + \sqrt{xz}$ at the point $A(-1,-1,-1)$ with respect to the direction of $\vec{\ell} = \vec{i} + \vec{j} + \vec{k}$;
- 2.6.24. $\varphi = 3\sqrt{x-y} + 2\sqrt{z-x}$ at the point $A(1,-3,5)$ with respect to the direction of $\vec{\ell} = -\vec{i} + 2\vec{k}$;
- 2.6.25. $\varphi = x^2 - 5\text{arcctg}(y+2z)$ at the point $A(1,1,0)$ with respect to the direction of $\vec{\ell} = 2\vec{i} + \vec{j} - 2\vec{k}$.
- 2.6.26. $\varphi = x^2 + y^2 - 3x + 2y$ with respect to the direction of the vector \vec{AB} , where $A(0,0,0), B(3,4,0)$;
- 2.6.27. $\varphi = x^2z - 2xyz + z^2$ at the point $A(3,1,1)$ with respect to the direction of $\vec{\ell} = \vec{i} + \sqrt{2}\vec{j} + \vec{k}$;
- 2.6.28. $\varphi = xyz$ at the point $A(5,1,-8)$ with respect to the direction to this point from the point $B(9,4,4)$;
- 2.6.29. $\varphi = \sqrt{xy} + \sqrt{4-z^2}$ at the point $A(1,1,2)$ with respect to the direction of $\vec{\ell} = -2\vec{i} + 2\vec{j} - \vec{k}$;
- 2.6.30. $\varphi = 5x^2yz - 7xy^2z + 5xyz^2$ at the point $A(1,1,1)$ with respect to the direction of $\vec{\ell} = 8\vec{i} - 4\vec{j} + 8\vec{k}$;