Distance learning materials for students

E-118ia.e, E-318ib.e, E-118iл.e, E-618iб.e, MIT-203.8i

Lecture 1 (20.03.2020)

1. The Element of Area in Curvilinear Coordinates

Take in plane uOv the elementary rectangular with vertices $Q_1(u,v)$, $Q_2(u+\Delta u,v)$, $Q_3(u+\Delta u,v+\Delta v)$, and $Q_4(u,v+\Delta v)$ Fig. 3.13, and consider also the curvilinear quadrangle corresponding to it in plane xOy. Two pairs of the coordinate lines, Fig. 3.14, generate this quadrangle. Formulas of transformation are in the form $x=g(u,v)$, $y=h(u,v)$, then coordinates of its vertices will be defined as

$$P_1(g(u,v),h(u,v)), P_2(g(u+\Delta u,v),h(u+\Delta u,v)),$$

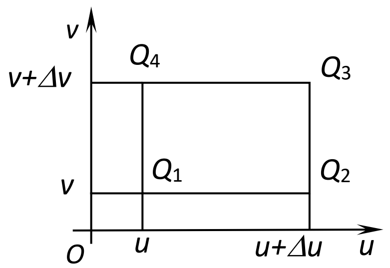

Fig. 3.13

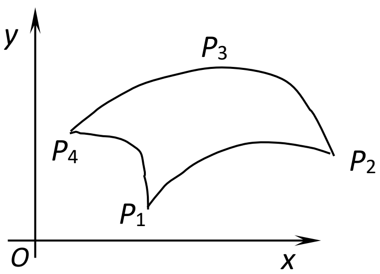

Fig. 3.14

$P_3(g(u+\Delta u,v+\Delta v),h(u+\Delta u,v+\Delta v))$ and $P_4(g(u,v+\Delta v),h(u,v+\Delta v))$. Denote $\delta=\sqrt{(\Delta u)^2+(\Delta v)^2}$. Then supposing that the functions $g(u,v)$ and $h(u,v)$ have continuous partial derivative $\frac{\partial g}{\partial u}$, $\frac{\partial g}{\partial v}$, $\frac{\partial h}{\partial u}$ and $\frac{\partial h}{\partial v}$ in domain $\overline{G}$, we can change up to value $o(\delta)$ the points P_1, P_2, P_3 and P_4 by points P_1', P_2', P_3', P_4' with following coordinates:

$$P_1'(g(u,v),h(u,v)),$$

$$P_2'\left(g(u,v)+\frac{\partial g}{\partial u}\Delta u, h(u,v)+\frac{\partial h}{\partial u}\Delta u\right),$$

$$P_3'\left(g(u,v)+\frac{\partial g}{\partial u}\Delta u+\frac{\partial g}{\partial v}\Delta v, h(u,v)+\frac{\partial h}{\partial u}\Delta u+\frac{\partial h}{\partial v}\Delta v\right),$$

$$P_4'\left(g(u,v)+\frac{\partial g}{\partial v}\Delta v, h(u,v)+\frac{\partial h}{\partial v}\Delta v\right)^{*)}.$$

Since $\overrightarrow{P_1'P_2'}=\left\{\frac{\partial g}{\partial u}\Delta u,\ \frac{\partial h}{\partial u}\Delta u\right\}$ and $\overrightarrow{P_4'P_3'}=\left\{\frac{\partial g}{\partial u}\Delta u,\ \frac{\partial h}{\partial u}\Delta u\right\}$, then vectors $P_4'P_3'$ and $P_{1.}'P_2'$ will be equal, $\overrightarrow{P_4'P_3'}=\overrightarrow{P_1'P_2'}$. Analogously $\overrightarrow{P_1'P_4'}=\overrightarrow{P_2'P_3'}$ and hence figure $P_1'P_2'P_3'P_4'$ is parallelogram. Its area may be approximately calculated as $|[\overrightarrow{P_1'P_2'},\overrightarrow{P_2'P_3'}]|$. But $\overrightarrow{P_2'P_3'}=\left\{\frac{\partial g}{\partial v}\Delta v, \frac{\partial h}{\partial v}\Delta v\right\}$ and consequently

$$|[\overrightarrow{P_1'P_2'},\overrightarrow{P_2'P_3'}]|=$$

$$=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial g}{\partial u}\Delta u & \frac{\partial h}{\partial u}\Delta u & 0 \\ \frac{\partial g}{\partial v}\Delta v & \frac{\partial h}{\partial v}\Delta v & 0 \end{vmatrix} = \vec{k}\begin{vmatrix} \frac{\partial g}{\partial u}\Delta u & \frac{\partial h}{\partial u}\Delta u \\ \frac{\partial g}{\partial v}\Delta v & \frac{\partial h}{\partial v}\Delta v \end{vmatrix} = \begin{vmatrix} \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \\ \frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} \end{vmatrix}\Delta u\Delta v\vec{k}.$$

The determinant

$$J(u,v)=\begin{vmatrix} \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \\ \frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} \end{vmatrix}$$

or another form is called **Jacob's determinant** or **Jacobian** of the functions $g(u,v)$ and $h(u,v)$ and denoted by $\frac{\partial(g,h)}{\partial(u,v)}$.

So

$$|[\overrightarrow{P_1'P_2'},\overrightarrow{P_2'P_3'}]|=J(u,v)\Delta u\Delta v\vec{k}.$$

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*) Here and lower it is supposed that the partial derivatives $\frac{\partial g}{\partial u},\frac{\partial h}{\partial u},\frac{\partial g}{\partial v}$ and $\frac{\partial h}{\partial v}$ are evaluated at the point (u,v).

Thus

$$S_{P_1'P_2'P_3'P_4'} = |J(u,v)|\Delta u \Delta v. \qquad (3.13)$$

From this it follows that the area of quadrangle $P_1'P_2'P_3'P_4'$ up value $o(\delta^2)$ is equal to $|J(u,v)|\Delta u \Delta v$. Therefore the value $|J(u,v)|dudv$ is taken as element of area in curvilinear coordinates.

In particular for polar coordinates we have

$$x = \rho\cos\varphi,\ y = \rho\sin\varphi,$$

and hence

$$J(\rho,\varphi) = \begin{vmatrix} \dfrac{\partial x}{\partial \rho} & \dfrac{\partial x}{\partial \varphi} \\ \dfrac{\partial y}{\partial \rho} & \dfrac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -\rho\sin\varphi \\ \sin\varphi & \rho\cos\varphi \end{vmatrix} = \rho.$$

Since $\rho \geq 0$ then element of area in polar coordinates will be equal to

$$ds = \rho d\rho d\varphi. \qquad (3.14)$$

Consequently the figure $P_1P_2P_3P_4$ up smallest value of higher order we can consider as rectangular with sides $P_1P_2 = \Delta\rho$ and $P_1P_4 = \rho\Delta\varphi$ (Fig. 3.15).

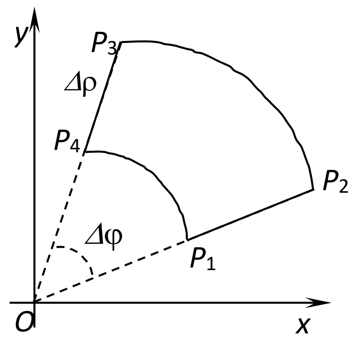

Fig. 3.15

2. Change of the Variables in the Double Integrals

Let the integral $\iint\limits_D f(x,y)dxdy = I$ be given over the domain D. Suppose that the domain D is transformed by formulas (3.10) in domain G of plane uOv. We suppose that this mapping is one-to-one correspondence. Divide the domain G into subdomains $\Delta G_1, \Delta G_2, .., \Delta G_n$ with areas $\Delta\sigma_1, \Delta\sigma_2, ..., \Delta\sigma_n$ by an arbitrary way. Then the domain D will be divided into subdomains $\Delta D_1, \Delta D_2, .., \Delta D_n$ by associated lines with areas $\Delta s_1, \Delta s_2, .., \Delta s_n$ (Fig. 3.16). In each subdomain ΔD_k take an arbitrary point and form integral sum

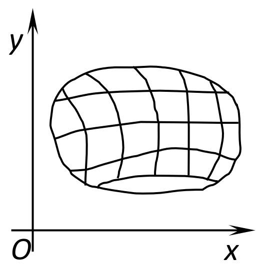

Fig. 3.16

$$\tilde{S} = \sum_{k=1}^{n} f(x_k,\, y_k)\Delta s_k \,.$$

On base of formula (3.13)

$$\Delta s_k = |J(\xi_k,\, \eta_k)|\Delta\sigma_k,$$

where $(\xi_k,\, \eta_k)$ is some point of domain ΔG_k. Let us take as the point (x_k, y_k) such point of the domain ΔD_k, which is transformed exactly at the point $(\xi_k,\, \eta_k)$ (since the integral exists then a point (x_k, y_k) in integral sum may be chosen by any way). In other words, we suppose

$$x_k = g(\xi_k, \eta_k),\ y_k = h(\xi_k, \eta_k).$$

Then the integral sum takes the form:

$$\tilde{S} = \sum_{k=1}^{n} f(g(\xi_k,\, \eta_k), h(\xi_k,\, \eta_k))|J(\xi_k,\, \eta_k)|\Delta\sigma_k \,.$$

Assuming that the function $f(x, y)$ is continuous in the domain D and passing to limit we get

$$I = \iint_D f(g(u,v), h(u,v))|J(u,v)|d\sigma,$$

that is

$$\iint_D f(x, y)dxdy = \iint_G f(g(u,v), h(u,v))|J(u,v)|d\sigma^{*)}. \qquad (3.15)$$

This is formula of change of the variables in the double integrals. In particular case for polar coordinates this formula is presented as

$$\iint_D f(x, y)dxdy = \iint_G f(\rho\cos\varphi, \rho\sin\varphi)\rho d\rho d\varphi. \qquad (3.16)$$

Example 1. Calculate the volume of the solid bounded by plane $z = 0$ and surfaces $(x-1)^2 + y^2 = 1$ and $z = x^2 + y^2$ (Fig. 3.17).

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*) It may be shown that formula (3.15) is also valid in case if imagination (3.11) of the domain G on domain D is one to one correspondence only inner points of these domains.

Solution. We have

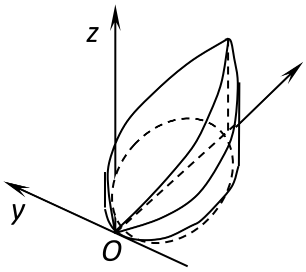

Fig. 3.17

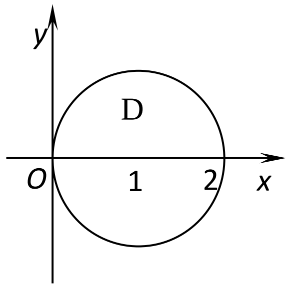

Fig. 3.18

$$V = \iint_D \left(x^2 + y^2\right) dxdy .$$

Let us pass to polar coordinates. In the given case

$$f(\rho\cos\varphi, \rho\sin\varphi)\rho = \left(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi\right)\rho = \rho^3,$$

therefore on base of (3.16) we get

$$V = \iint_G \rho^3 d\rho d\varphi .$$

Rewrite the equation of the domain boundary D (Fig. 3.18) in the form

$$x^2 + y^2 = 2x .$$

In polar coordinates this equation takes the following form

$$\rho^2 = 2\rho\cos\varphi .$$

Or

$$\rho = 2\cos\varphi.$$

Hence on base of (3.16)

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\int_0^{2\cos\varphi} \rho^3 d\rho \right) d\varphi = \frac{1}{4} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \rho^4 \Bigg|_0^{2\cos\varphi} d\varphi = 4\frac{1}{4} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4\varphi d\varphi =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos 2\varphi)^2 d\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1 + 2\cos 2\varphi + \frac{1}{2} + \frac{1}{2}\cos 4\varphi \right) d\varphi = \frac{3}{2}\pi .$$

Note. While solving the concrete problems the domain G may be not shown on figure but limits of integration for every variables ρ and φ we can determine using the kind of domain projection D.

3. Physical Application of the Double Integral

3.1. Calculation of the Mass of an Inhomogeneous Plate

It was shown (formula 3.3), that mass of the plate, that fills domain $\overline{D}$ on the plane xOy and has density $\rho(x, y)$, is expressed by formula

$$m = \iint_D \rho(x,\ y)dxdy.$$

3.2. Calculation of the Inertia Moment of the Plate

The moment of inertia *I* of a material point *M* of mass *m* relatively to some point *O* is defined as product of mass *m* by the square of its distance *r* from the point O:

$$I_0 = mr^2.$$

The moment of inertia of a material points system $m_1, m_2, \ldots, m_n$ relatively to *O* is the sum of moments of inertia of the individual points of the system:

$$I_0 = \sum_{i=1}^{i=n} m_i r_i^{\ 2}.$$

Let us determine the moment of inertia of material plate, filling in the domain $\overline{D}$ and the density of which is given by function $\rho(x,\ y)$.

Divide this plate into elementary parts ΔD_k, where $k = 1, 2, \ldots, n$ (Fig. 3.20). The moment of inertia of the domain ΔD_k relatively to point *O* is approximately equal to

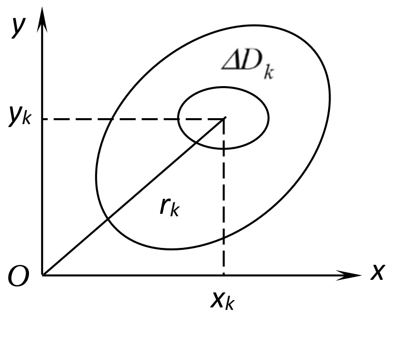

Fig. 3.20

$$(\Delta I_0)_k = r_k^2 \Delta m_k = (x_k^2 + y_k^2)\rho(x_k, y_k)\Delta\sigma_k.$$

And hence the moment of all plate will be approximately equal to integral sum

$$I_0 \approx \sum_{k=1}^{k=n} (\Delta I_0)_k = \sum_{k=1}^{k=n} (x_k^2 + y_k^2)\rho(x_k, y_k)\Delta\sigma_k.$$

Passing to limit as the diameter of each elementary subdomains approaches zero we get the exact value for moment of inertia of the given plate:

$$I_0 = \iint\limits_D \left(x^2 + y^2\right)\rho(x, y)dxdy.$$

If the plate is homogeneous, that is $\rho(x, y) \equiv \rho \equiv const$ then we obtain:

$$I_0 = \rho \iint\limits_D \left(x^2 + y^2\right)dxdy.$$

It is obviously that the moments of inertia relatively to the axis Ox and Oy will be equal respectively:

$$I_x = \iint\limits_D y^2 \rho(x, y)dxdy, \; I_y = \iint\limits_D x^2 \rho(x, y)dxdy.$$

Whence in particular it follows that

$$I_0 = I_x + I_y.$$

3. The Coordinates of the Gravity Center of the Material Plate

Divide the plate into parts $\Delta D_1, \Delta D_2, ..., \Delta D_n$. In each subdomains ΔD_k choose an arbitrary point $M_k(x_k, y_k)$. Considering the plate as discrete model of n material points $M_1, M_2, ..., M_n$ with masses $\Delta m_1 = \rho(x_1, y_1)\Delta\sigma_1$, $\Delta m_2 = \rho(x_2, y_2)\Delta\sigma_2$, …, $\Delta m_n = \rho(x_n, y_n)\Delta\sigma_n$ we obtain abscissa of the gravity center:

$$x_c \approx \frac{\sum\limits_{k=1}^{n} x_k \Delta m_k}{\sum\limits_{k=1}^{n} \Delta m_k} = \frac{\sum\limits_{k=1}^{n} x_k \rho(x_k, y_k)\Delta\sigma_k}{\sum\limits_{k=1}^{n} \rho(x_k, y_k)\Delta\sigma_k},$$

then passing to limit when at $\lambda \to 0$ we obtain the exact formula

$$x_c = \frac{\iint\limits_D x\rho(x, y)dxdy}{\iint\limits_D \rho(x, y)dxdy}$$

and similarly for ordinate we have:

$$y_c = \frac{\iint\limits_D y\rho(x,y)dxdy}{\iint\limits_D \rho(x,y)dxdy}.$$

If the plate is homogeneous, then reducing by number ρ the both fractions we get:

$$x_c = \frac{\iint\limits_D xdxdy}{S}, \; y_c = \frac{\iint\limits_D ydxdy}{S},$$

where S is area of the plate.

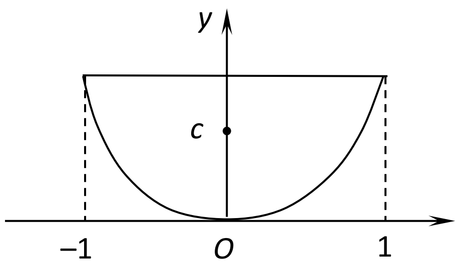

Fig. 3.21

Example. Find the center of gravity of the homogeneous figure, bounded by lines $y = x^2$ and $y = 1$ (Fig. 3.21).

Solution. It is clear that $x_c = 0$. Further

$$\iint\limits_D yds = \int\limits_{-1}^{1}\left(\int\limits_{x^2}^{1} ydy\right)dx = \frac{1}{2}\int\limits_{-1}^{1} y^2 \Big|_{x^2}^{1} dx = \frac{1}{2}\int\limits_{-1}^{1}\left(1 - x^4\right)dx =$$

$$\int\limits_{0}^{1}\left(1 - x^4\right)dx = \left(x - \frac{x^5}{5}\right)\Bigg|_0^1 = 1 - \frac{1}{5} = \frac{4}{5};$$

$$S = \int\limits_{-1}^{1}\left(1 - x^2\right)dx = 2\int\limits_{0}^{1}\left(1 - x^2\right)dx = 2\left(x - \frac{x^3}{3}\right)\Bigg|_0^1 = 2\left(1 - \frac{1}{3}\right) = \frac{4}{3};$$

hence
$$y_c = \frac{4}{5} \div \frac{4}{3} = \frac{3}{5}.$$

Lecture 2 (03.04.2020)

1. Definition and Properties of the Triple Integral

A triple integral is natural generalization of the theory of the double integrals on the 3-dimension space.

Let a function $f(x, y, z)$ be given in closed domain D of **the 3 - dimension space R³**. Divide the domain D by some surfaces on n subdomains $\Delta D_1, \Delta D_2, ..., \Delta D_n$ with volumes $\Delta V_1, \Delta V_2, ..., \Delta V_n$. In each subdomains ΔD_k we take an arbitrary point $M_k(x_k, y_k, z_k)$ and calculate the value $f(x_k, y_k, z_k)$. Let us form the sum

$$\tilde{S} = \sum f(x_k, y_k, z_k)\Delta V_k .$$

It is called integral sum of function $f(x, y, z)$ in domain D.

The maximum diameter among all diameters of subdomains ΔD_1, $\Delta D_2, \ldots$, ΔD_n denote by λ. Let λ approaches zero $\lambda \to 0$. It means that the domain D will be divided infinitely and each subdomain ΔD_k will contracts to appropriate point M_k. If there exists limit $\lim_{\lambda \to 0}(\tilde{S}_\lambda)$, which independents on ways of partition of the domain D and choice of the points then this limit is called **the triple integral of function $f(x, y, z)$ over the domain D** and denoted by

$$\iiint_D f(x, y, z)dv.$$

So by definition we have

$$\iiint_D f(x, y, z)dV = \lim_{\lambda \to \infty} \sum_{k=1}^{n} f(x_k, y_k, z_k)\Delta V_k .$$

Namely from this definition it follows full analogy between the double and triple integrals and in particular their properties.

For example, instead of equality

$$\iint_D ds = S_D$$

now we will have

$$\iiint_D dV = V_D,$$

where V_D is volume of domain D.

Exactly as for definite integral we can establish the following theorem.

Theorem. If function $f(x,y,z)$ is continuous in domain D, then integral $\iiint_V f(x,y,z)dV$ will exist.(Without proof).

The triple integral has not geometrical sense, but it has physical sense. Let function is non-negative $f(x,y,z) \geq 0$ for all $(x,y,z) \in D$. Then this function $f(x,y,z)$ may be considered as density of substances in domain D. In this case the value $f(x_k, y_k, z_k)\Delta V_k$ is approximately equal to mass of substance in domain ΔD_k. Adding these masses and passing to limit as $\lambda \to 0$, we get that integral $\iiint_D f(x,y,z)dv$ is mass of substance in over domain D.

2. Calculation of the Triple Integral in the Cartesian Coordinates System

Let domain V is regular in direction *Oz* axis and $z = g_1(x,y)$ and $z = g_2(x,y)$ be equations of the lower and upper boundaries of the domain D (Fig. 4.1). It means that any straight line parallel to *Oz* cuts the boundary domain at no more than two points. And B is 2-dimension domain obtained as result of projection of the solid V on the plane xOy (Fig. 4.2). Suppose that $y = h_1(x)$ and $y = h_2(x)$ [*)] $(a \leq x \leq b)$ are equations of the lower and upper boundaries of the domain D. Then analogously to the double integral we can prove the following formula for calculation of the triple integral by threefold iterated integral:

[*)] There are supposed that functions $g_1(x,y)$, $g_2(x,y)$, $h_1(x)$, $h_2(x)$ are single-valued.

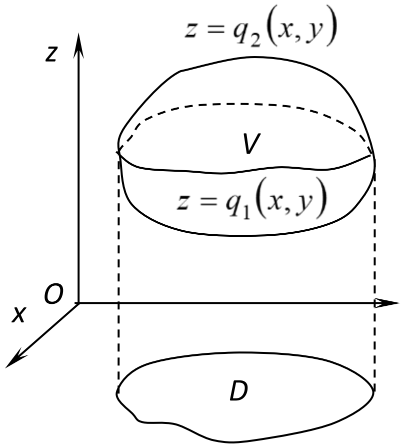

Fig. 4.1

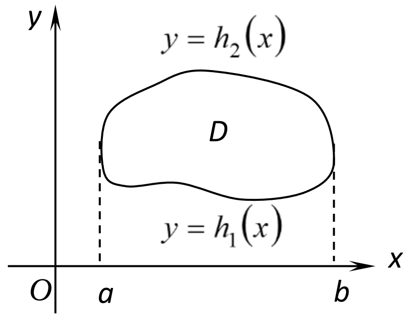

Fig. 4.2.

$$\iiint f(x,y,z)dV = \int_a^b \left\{ \int_{h_1(x)}^{h_2(x)} \left[\int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)dz \right] dy \right\} dx. \qquad (4.1)$$

Note, that the order of integration may be changed. The value $dv = dxdydz$ is called element of volume in Cartesian coordinates.

Rule of Finding Limits of Integration

We take the following steps to reduce a triple integral to an iterated one.

1. Divide the domain into regular subdomains in the direction Oz, if it is necessary, that is if some line parallel to *z*-axis has more than two common points with boundary of the solid *V*.

2. Fix arbitrary x and y inside domain *D*, which is projection of

the solid *V* on plane *xOy*. Let a line parallel to *z*-axis cut the boundary of the given solid *V* at two points with coordinates $z_1 = g_1(x,y)$ and $z_2 = g_2(x,y)$. The expressions $g_1(x,y)$ and $g_2(x,y)$ should be taken as the limits of integration with respect to *z*. So we obtain that

$$\iiint_V f(x,y,z)dxdydz = \iint_D dxdy \int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)dz.$$

3. The domain of definition D of the function of x, y (obtained after integration with respect to z) is the projection of the given domain V on the xOy-plane. After

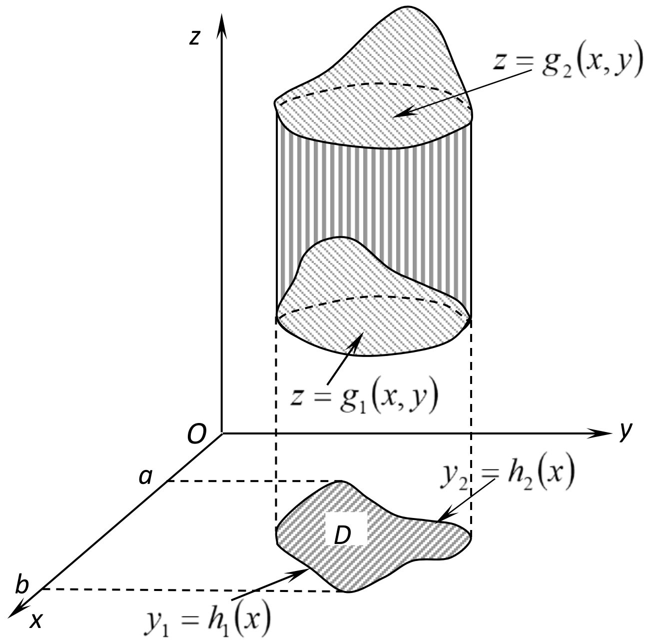

Fig. 4.3

calculation of the integral $\int\limits_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)dz$, where the variables x and y are constants we go to the double integral over domain D. The rule of finding limits of integration for the double integral is known. So we get

$$\iiint\limits_V f(x,y,z)dxdydz = \int\limits_a^b dx \int\limits_{h_1(x)}^{h_2(x)} dy \int\limits_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)dz.$$

Example 1. Find the limits of the triple integral of a function f taken over the sphere $x^2 + y^2 + z^2 = a^2$. (Fig. 4.4)

Solution. This solid is regular in direction *Oz* and its projection on the plane *xOy* is

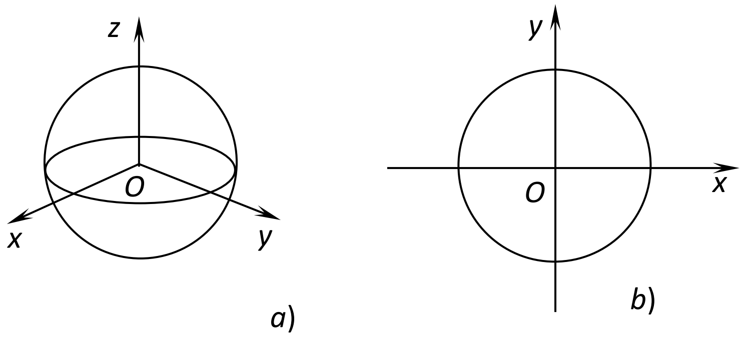

Fig. 4.4

circle. Thus

$$\iiint f(x,y,z)dxdydz = \int_{-a}^{a} dx \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy \int_{-\sqrt{(a^2-x^2-y^2)}}^{\sqrt{(a^2-x^2-y^2)}} f(x,y,z)dz .$$

Example 2. Calculate a mass of solid bounded by planes $x=0$, $y=0$, $x+y=1$ and cones $z=\sqrt{x^2+y^2}$ and $z=2\sqrt{x^2+y^2}$, if its density at each

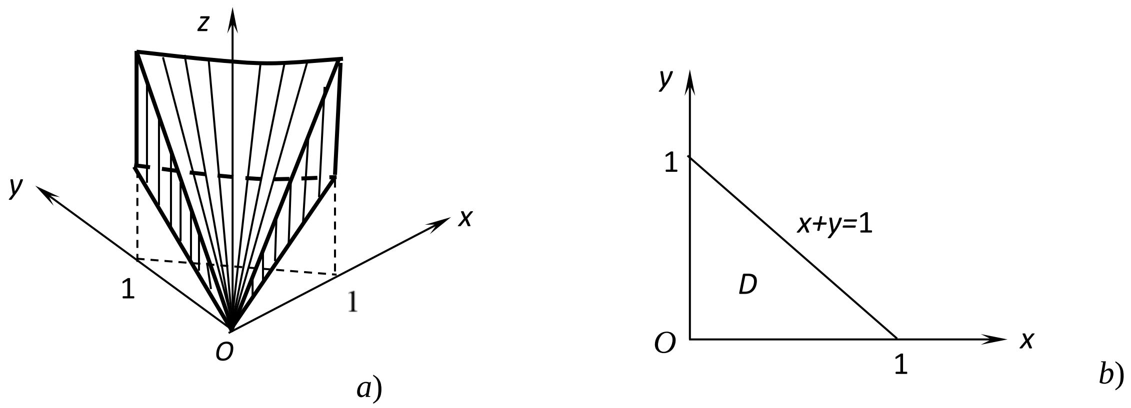

Fig. 4.5

point is equal $\rho(x,y,z)=xyz$ (Fig 4.5).

Solution. Using the physical sense of the triple integral we get $m=\iiint_V xyzdv$.

By virtue of formula (4.1) we have

$$m=\int_0^1\left[\int_0^{1-x}\left(\int_{\sqrt{x^2+y^2}}^{2\sqrt{x^2+y^2}} xyzdz\right)dy\right]dx=\int_0^1\left[x\int_0^{1-x} y\left(\int_{\sqrt{x^2+y^2}}^{2\sqrt{x^2+y^2}} zdz\right)dy\right]dx=$$

$$=\frac{1}{2}\int_0^1\left(x\int_0^{1-x} yz^2\Big|_{\sqrt{x^2+y^2}}^{2\sqrt{x^2+y^2}}dy\right)dx=\frac{1}{2}\int_0^1\left[x\int_0^{1-x} y3\left(x^2+y^2\right)dy\right]dx=$$

$$=\frac{3}{2}\int_0^1\left[x\int_0^{1-x}\left(x^2y+y^3\right)dy\right]dx=\frac{3}{2}\int_0^1 x\left(\frac{x^2y^2}{2}+\frac{y^4}{4}\right)\Bigg|_0^{1-x}dx=$$

$$=\frac{3}{8}\int_0^1 x\left[2x^2(1-x)^2+(1-x)^4\right]dx=$$

$$=\frac{3}{8}\int_0^1\left(2x^3-4x^4+2x^5+x-4x^2+6x^3-4x^4+x^5\right)dx=$$

$$=\frac{3}{8}\int_0^1\left(8x^3-8x^4+3x^5+x-4x^2\right)dx=$$

$$=\frac{3}{8}\left(2x^4-\frac{8}{5}x^5+\frac{1}{2}x^6+\frac{1}{2}x^2-\frac{4}{3}x^3\right)\Bigg|_0^1=\frac{3}{8}\left(2-\frac{8}{5}+\frac{1}{2}+\frac{1}{2}-\frac{4}{3}\right)=\frac{1}{40}.$$

It is clear that the triple integrals may be applied to calculation of the moments of inertia and coordinates of gravity center also.

Example 2. Calculate mass of the solid bounded by the cylinder $x^2=2y$ and planes $z=0,\ 2y+z=2$, if at each point its volumes density is numerically equal to z-coordinate of its point.

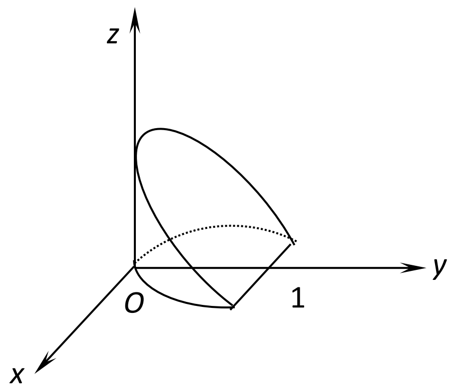

Fig. 4.6

Solution. The cylindrical solid (Fig. 4.6) is bounded from above by the plane $z=2-2y$. This plane cuts the plane $z=0$ on the line $y=1$. Mass of the solid filling up the domain V is calculated with help of the triple integral:

$m=\iiint\limits_V \delta(x,y,z)dxdydz$, where $\delta(x,y,z,)$ is volumes density. In our case $\delta(x,y,z)=z$ and

$$m=\iiint\limits_V zdxdydz=\int_0^1 dy\int_{-\sqrt{2y}}^{\sqrt{2y}}dx\int_0^{2-2y}zdz=2\int_0^1(1-y)^2dy\int_{-\sqrt{2y}}^{\sqrt{2y}}dx=$$

$$=4\int_0^1(1-y)^2\sqrt{2y}dy=4\sqrt{2}\int_0^1\left(y^{\frac{1}{2}}-2y^{\frac{3}{2}}+y^{\frac{5}{2}}\right)dy=$$

$$=4\sqrt{2}\left(\frac{2}{3}y^{\frac{3}{2}}-\frac{4}{5}y^{\frac{5}{2}}+\frac{2}{7}y^{\frac{7}{2}}\right)\Bigg|_0^1=\frac{64\sqrt{2}}{105}.$$