Practical Lesson № 1, 2. Calculation of Double Integrals in Polar System of Coordinates (13.03.2020, 20.03.2020)

Class work	**Answers**
To pass to polar system coordinates ρ *and* φ *(*$x=\rho\cos\varphi,\ y=\rho\sin\varphi$ *in the double integral* $\iint\limits_D f(x,y)dxdy$ *and find the limits of the integration*	
1. D is a circle: 1) $x^2+y^2\le R^2$; 2) $x^2+y^2\le ax$; 3) $x^2+y^2\le by$.	1) $\int\limits_0^{2\pi}d\varphi\int\limits_0^R f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho$; 2) $\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\int\limits_0^{a\cos\varphi} f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho$; 3) $\int\limits_0^{\pi}d\varphi\int\limits_0^{b\sin\varphi} f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho$.
2. D is common part of the following domains $x^2+y^2\le ax$ и $x^2+y^2\le by$.	$\int\limits_0^{arctg\frac{a}{b}}d\varphi\int\limits_0^{b\sin\varphi} f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho+$ $+\int\limits_{arctg\frac{a}{b}}^{\frac{\pi}{2}}d\varphi\int\limits_0^{a\cos\varphi} f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho$.
3. D is domain given by the following inequalities: $x\ge 0$, $y\ge 0$, $(x^2+y^2)^3\le 4a^2x^2y^2$.	$\int\limits_0^{\frac{\pi}{2}}d\varphi\int\limits_0^{a\sin 2\varphi} f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho$.
2.To calculate double integral by passing to polar system coordinate	
4. $\int\limits_0^R dx\int\limits_0^{\sqrt{R^2-x^2}}\ln(1+x^2+y^2)dy$.	$\frac{\pi}{4}\left[(1+R^2)\ln(1+R^2)-R^2\right]$.

5. $\iint\limits_D \arctan\frac{y}{x}dxdy$, where D is part of the rang: $x^2+y^2\geq 1,\ x^2+y^2\leq 9$, $y\geq\frac{x}{\sqrt{3}},\ y\leq x\sqrt{3}$.	$\frac{\pi^2}{6}$.
3. Using the double integrals calculate the solid volumes bounded by given surfaces (It is assumed that all parameters included in condition of the problems are positive values)	
6. Cylinder $x^2+y^2=4$ and planes $z=0$ and $z=x+y+10$.	40π.
7. Cylinder $x^2+y^2=R^2$, paraboloid $Rz=2R^2+x^2+y^2$ and plane $z=0$.	$\frac{5}{2}\pi R^3$.
8. Cylinders $x^2+y^2=x$ and $x^2+y^2=2x$, paraboloid $z=x^2+y^2$ and planes $x+y=0$, $x-y=0$ and plane $z=0$.	$\frac{15}{8}\left(\frac{3\pi}{8}+1\right)$.
Homework	**Answers**
1. To find integration limits in double integrals $\iint\limits_D f(x,y)dxdy$ *passing to polar coordinates* ρ *and* φ *(*$x=\rho\cos\varphi,\ y=\rho\sin\varphi$*)*	
1. D is domain bounded by circles – $x^2+y^2=4x$, $x^2+y^2=8x$ and straight lines $y=x$ and $y=2x$.	$\int\limits_{\frac{\pi}{4}}^{arctg\,2}d\varphi\int\limits_{4\cos\varphi}^{8\cos\varphi}f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho$.
2. D is inner part of the right lobe of the Bernoulli lemniscate $\left(x^2+y^2\right)^2=a^2\left(x^2-y^2\right)$.	$\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}}d\varphi\int\limits_{0}^{a\sqrt{\cos 2\varphi}}f(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho$.

<table>
<tr><td colspan="2">2. To calculate the given double integrals passing to polar coordinates:</td></tr>
<tr><td>3. $\iint\limits_D \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}dxdy$. Domain D is defined by inequalities $x^2+y^2 \le 1$, $x \ge 0$, $y \ge 0$.</td><td>$\frac{\pi(\pi-2)}{8}$.</td></tr>
<tr><td>4. $\iint\limits_D \sqrt{R^2-x^2-y^2}dxdy$, where D is a circle $x^2+y^2 \le Rx$.</td><td>$\frac{R^3}{3}\left(\pi-\frac{4}{3}\right)$.</td></tr>
<tr><td colspan="2">3. Using the double integrals calculate the solid volumes bounded by given surfaces (It is assumed that all parameters included in condition of the problems are positive values)</td></tr>
<tr><td>5. Cylinder $x^2+y^2=2x$, planes $2x-z=0$ and $4x-z=0$.</td><td>2π.</td></tr>
<tr><td>6. Cylinder $x^2+y^2=2ax$, paraboloid $z=\frac{x^2+y^2}{a}$ and plane $z=0$.</td><td>$\frac{3}{2}\pi a^3$.</td></tr>
<tr><td>7. Hyperbolic paraboloid $z=\frac{xy}{a}$, cylinder $x^2+y^2=ax$ and plane $z=0$ ($x \ge 0$, $y \ge 0$).*</td><td>$\frac{a^3}{24}$.</td></tr>
<tr><td>8. Cylinders $x^2+y^2=2x$, $x^2+y^2=2y$ and planes $z=x+2y$ and $z=0$.</td><td>$\frac{3}{2}\left(\frac{\pi}{2}-1\right)$.</td></tr>
</table>

Distance learning materials for students

E-118ia.e, E-318ib.e, E-118iл.e, E-618iб.e, MIT-203.8i

Practical Lesson № 3, 4. Calculation of the Triple Integrals in Cartesian System Coordinates (27.03.2020, 03.04.2020)

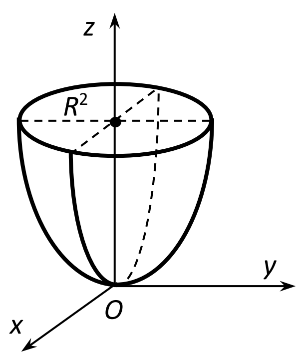
<table>
<tr><th>Classwork</th><th>Answers</th></tr>
<tr><td colspan="2">1. Calculate the triple integrals and choose the true answer</td></tr>
<tr><td>1. $I=\int_{-1}^{1}xdx\int_{0}^{2}dy\int_{a}^{0}dz$;

2. $I=\int_{a}^{0}dx\int_{0}^{2}dz\int_{-1}^{0}dy$.</td><td>$a)\ I=0$;
$b)\ I=(1\cdot 1\cdot(-a))=-a$;
$c)\ I=((-a)\cdot 2\cdot 1)=-2a$;
$d)\ I=(a\cdot 1\cdot 1)=a$.</td></tr>
<tr><td>3. $I=\int_{2}^{3}dx\int_{-a}^{a}zdz\int_{5}^{6}dy$.

4. $I=\int_{-1}^{0}dx\int_{0}^{1}dy\int_{0}^{a}dz$.</td><td>$a)\ I=0$;
$b)\ I=-a$;
$c)\ I=a$;
$d)\ I=2a$.</td></tr>
<tr><td colspan="2">2. Restore the domain of integration</td></tr>
<tr><td>Find the domain G of the integration taking into account the limits of integration:

5. $I=\int_{-R}^{R}dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}dy\int_{x^2+y^2}^{R^2}f(x,y,z)dz$.

6. $I=\int_{-R}^{R}dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}dy\int_{-R^2}^{-x^2-y^2}f(x,y,z)dz$.</td><td>$a)$ G is a part of the rotation paraboloid $z=x^2+y^2$, bounded by plane $z=R^2$.


</td></tr>
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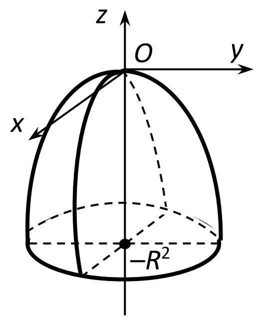
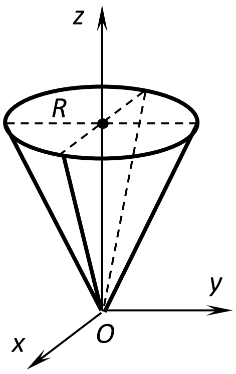
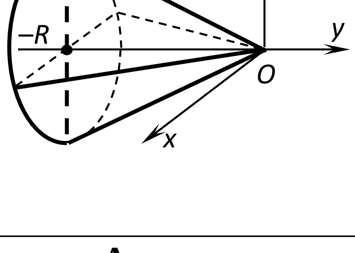
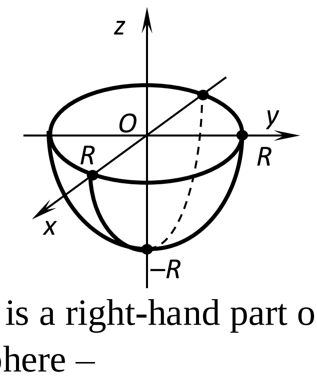
	b) G is part of the rotation paraboloid $z=-x^2-y^2$ bounded by plane $z=-R^2$. 
7. $I=\int\limits_{-R}^{R}dx\int\limits_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}dy\int\limits_{\sqrt{x^2+y^2}}^{R}f(x,\ y,\ z)dz$. **8.** $I=\int\limits_{-R}^{R}dx\int\limits_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}dz\int\limits_{-R}^{-\sqrt{x^2+y^2}}f(x,\ y,\ z)dy$.	*a*) G is upper part of the cone $x^2+y^2=z^2$ bounded by plane $z=R$.  *b*) G is the left-hand part of the cone $x^2+z^2=y^2$ bounded by plane $z=-R$. 
Homework	**Answers**
1. Calculate the triple integrals and choose the true answer	

1. $I=\int\limits_0^a xdx\int\limits_0^x ydy\int\limits_0^y zdz$. **2.** $I=\int\limits_0^a dx\int\limits_0^x ydy\int\limits_0^y zdz$.	$a)\ I=\dfrac{a^4}{24}$; $b)\ I=\dfrac{a^4}{8}$; $c)\ I=\dfrac{a^4}{48}$; $d)\ I=\dfrac{a^5}{40}$.
3. $I=\int\limits_0^a dx\int\limits_0^x dy\int\limits_0^{xy} dz$. **4.** $I=\int\limits_0^a xdx\int\limits_0^x ydy\int\limits_0^{xy} zdz$.	$a)\ I=\dfrac{a^4}{8}$; $b)\ I=\dfrac{a^5}{10}$; $c)\ I=\dfrac{a^6}{12}$; $d)\ I=\dfrac{a^7}{14}$.

2. Restore domain of integration	
Find the domain G of the integration taking into account the limits of integration: **5.** $I=\int\limits_{-R}^{R} dx \int\limits_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy \int\limits_{-\sqrt{R^2-x^2-y^2}}^{0} f(x,\ y,\ z)dz$. **6.** $I=\int\limits_{-R}^{R} dx \int\limits_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dz \int\limits_{0}^{\sqrt{R^2-x^2-y^2}} f(x,\ y,\ z)dy$.	$a)$ G is an upper part of the sphere $x^2+y^2+z^2=R^2$, which is bounded by plane $z=0$.  $b)$ G is a right-hand part of the sphere –

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<tr>
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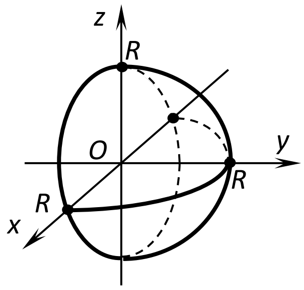
<td>$x^2 + y^2 + z^2 = R^2$, which is bounded by plane $y = 0$.
</td>
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7.

$$I = \int_{-\sqrt{3}R}^{\sqrt{3}R} dx \int_{-\sqrt{3R^2-x^2}}^{\sqrt{3R^2-x^2}} dy \int_{\sqrt{R^2+x^2+y^2}}^{2R} f(x, y, z) dz.$$

8.

$$I = \int_{-\sqrt{3}R}^{\sqrt{3}R} dx \int_{-\sqrt{3R^2-x^2}}^{\sqrt{3R^2-x^2}} dy \int_{-2R}^{-\sqrt{R^2+x^2+y^2}} f(x, y, z) dz$$

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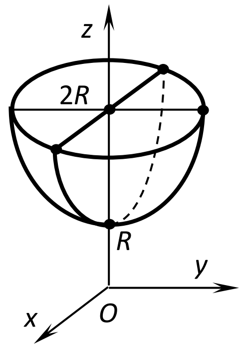
a) G is an upper part of the



hyperboloid

$x^2 + y^2 - z^2 = -R^2$,

which is bounded by plane

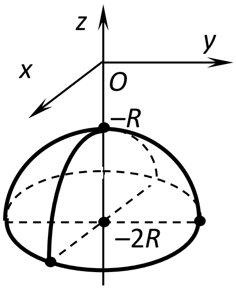
$z = 2R$.

b) G is a lower part of the hyperboloid

$x^2 + y^2 - z^2 = -R^2$,

which is bounded by plane,

$z = -2R$.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