Practice Lesson_01_04_20

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1.4. Practice Lesson № 1. Integration of the Equations with Separable Variables

Classwork	**Answers**
Find the general solution of the equations:	
1.1. $(xy^2+x)dx+(y-x^2y)dy=0$.	$1+y^2=C(1-x^2)$.
1.2. $yy'=\frac{1-2x}{y}$.	$y=\sqrt[3]{C+3x-3x^2}$.
1.3. $xy'+y=y^2$.	$Cx=\frac{y-1}{y}$.
1.4. $y'=10^{x+y}$.	$10^x+10^{-y}=C$.
1.5. $y'=\cos(x-y)$, (put $u=x-y$).	$x+\cot\frac{x-y}{2}=C$.
Solve the initial-value problem:	
1.6. $y'\sin x=y\ln y$, $y\left(\frac{\pi}{2}\right)=e$.	$y=e^{\tan\frac{x}{2}}$.
1.7. $\sin y\cos x dy=\cos y\sin x dx$, $y(0)=\frac{\pi}{4}$.	$\cos x=\sqrt{2}\cos y$.
Homework	**Answers**
Find the general solution of the equations:	
1. 8. $xyy'=1-x^2$.	$x^2+y^2=\ln Cx^2$.
1.9. $y'\tan(x)-y=a$.	$y=C\sin x-a$.
1.10. $y'+\sqrt{\frac{1-y^2}{1-x^2}}=0$.	$x\sqrt{1-y^2}+y\sqrt{1-x^2}=C$.

1.11. $e^{-s}\left(1+\frac{ds}{dt}\right)=1.$	$e^{t}=C\left(1-e^{-S}\right).$
1.12. $y'+\sin\frac{x+y}{2}=\sin\frac{x-y}{2}.$	$\ln\left\lvert\tan\frac{y}{4}\right\rvert=C-2\sin\frac{x}{2}.$
1.13. $y'=3x-2y+5.$	$4y-6x-7=Ce^{-2x}.$
1.14. $y'\sqrt{1+x+y}=x+y-1.$	$x+C=2u+\frac{2}{3}\ln\lvert u-1\rvert-$ $-\frac{8}{3}\ln(u+2),\ u=\sqrt{1+x+y}.$
Solve the initial-value problem:	
1.15. $y'=\frac{1+y^2}{1+x^2},y(0)=1.$	$y=\frac{1+x}{1-x};$
1.16. $y\text{-}xy'=b\left(1+x^2y'\right),\ y(1)=1.$	$y=\frac{b+x}{1+bx}.$
Additional Tasks	
1.17. Find the line passing through the point $(2,3)$ and having the property that interval of its any tangent enclosed between the coordinate axes is bisected at the point of tangency.	Hyperbole $xy=6$.
1.18. Find the line passing through the point $(2,0)$ and having the property that interval of tangent enclosed between the point of tangency and ordinate axis has constant length which is equal to two.	Tractrix $y=\sqrt{4-x^2}+$ $+2\ln\left\lvert\frac{2-\sqrt{4-x^2}}{x}\right\rvert.$

1.6. Practice Lesson №2. Integration of the Homogeneous Differential Equations in Euler's Sense

Classwork	Answers

Find the general solution of the equations:	
2.1. $y' = \frac{y^2}{x^2} - 2$.	$y - 2x = Cx^3(y+x)$.
2.2. $xdy - ydx = ydy$.	$\ln\lvert y\rvert + \frac{x}{y} = C$.
2.3. $y' = \frac{x}{y} + \frac{y}{x}$.	$y = \pm x\sqrt{2\ln\lvert Cx\rvert}$.
2.4. $y' = e^{\frac{y}{x}} + \frac{y}{x}$.	$\ln\lvert Cx\rvert = -e^{-\frac{y}{x}}$.
2.5. $(3y^2 + 3xy + x^2)dx = (x^2 + 2xy)dy$.	$(x+y)^2 = Cx^3 e^{-\frac{x}{x+y}}$.
2.6. $y' = \frac{3x - 4y - 2}{3x - 4y - 3}$.	$x - y + C = \ln\lvert 3x - 4y + 1\rvert$.
2.7. $y' = \frac{2x - y + 1}{x - 2y + 1}$.	$x^2 - xy + y^2 + x - y = C$.
Solve the initial-value problem:	
2.8. $(xy' - y)\arctan\frac{y}{x} = x$, $y(1) = 0$	$\sqrt{x^2 + y^2} = e^{\frac{y}{x}\arctan\frac{y}{x}}$
2.9. $y' = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$, $y(1) = -1$	$y = -x$
Homework	**Answers**
Find the general solution of the equations:	
2.10. $y' = \frac{x+y}{x-y}$	$\arctan\frac{y}{x} = \ln C\sqrt{x^2 + y^2}$
2.11. $y' = \frac{2xy}{x^2 - y^2}$	$x^2 + y^2 = Cy$
2.12. $xy' - y = \sqrt{x^2 + y^2}$	$x^2 = C^2 + 2Cy$
2.13. $xy' = y\ln\frac{y}{x}$	$y = xe^{1+Cx}$

2.14. $(x+y+1)dx=(2x+2y-1)dy$	$x-2y+\ln\|x+y\|=C$
2.15. $y'=\dfrac{2y-x-5}{2x-y+4}$	$(x+y-1)^2=C(x-y+3)$
Solve the initial-value problem:	
2.16. $(y^2-3x^2)dy+2xydx=0,\ y(0)=1$	$y^3=y^2-x^2$
2.17. $y\left(\dfrac{dy}{dx}\right)^2+2x\dfrac{dy}{dx}-y=0,\ y(0)=\sqrt{5}$	$y^2=5\pm 2\sqrt{5}x$
Additional Tasks	
2.18. Find the line if square of the length of the segment cut off by any tangent on the ordinate axis is the product of the coordinates of the tangency point.	$x=Ce^{\pm 2\sqrt{\frac{y}{x}}}$
2.19. What surface of revolution is the mirror of projector, if after reflection the light rays from a point source are directed by a parallel beam?	Form of the paraboloid of revolution.