Practice_ 13.03. 2020

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1.19. Practice Lesson №4. Extrema of a Function

Classwork	Answers
Find the stationary points of the following functions:	
4.1. $z = 2x^3 + xy^2 + x^2 + y^2$.	$(0,0), (-5/3,0), (-1,2), (-1,-2)$.
4.2. $z = xy(a - x - y)$.	$(0,0)$, $(0,a)$, $(a,0)$, $(a/3, a/3)$.
4.3. $z = \sin x + \sin y + \cos(x+y)$ $\left(0 \le x \le \frac{\pi}{4},\ 0 \le y \le \frac{\pi}{4}\right)$.	$(\pi/6, \pi/6)$.
4.4. $z = y\sqrt{1+x} + x\sqrt{1+y}$.	$(-2/3, -2/3)$
4.5. $u = 3\ln x + 2\ln y + 5\ln z + \ln(22 - x - y - z)$.	$(6,4,10)$.
4.6. Find the stationary points of the function $z(x,y)$ defined by implicitly: $2x^2 + 2y^2 + z^2 + 8xz - z + 8 = 0$.	$(-2,0)$, $(16/7,0)$, every point will be stationary point only for one of branches of the function.
4.7*. Find point of extrema of the given function $z = 2xy - 3x^2 - 2z^2 + 10$.	$(0,0)$. **Note.** To prove that stationary point is maximum point it is enough to present the function in the following form: $z = 10 - (x-y)^2 - 2x^2 - y^2$.
4.8. Find the points of extrema of the	$(-1,1)$.

function $z = x^2 + xy + y^2 + x - y + 1$.	
4.9. To prove that function $z = x^4 + y^4 - 2x^2 - 4xy - 2y^2$ has minimum at the points: $x = \sqrt{2}$, $y = \sqrt{2}$ and $x = -\sqrt{2}$, $y = -\sqrt{2}$.	
4.10. Find stationary points of the function $z = x^3 y^2 (12 - x - y)$, if it is known that $x > 0$, $y > 0$. Investigate the character of these points.	The point $(6,4)$ is point of maximum.
Homework	**Answers**
Find the stationary points of the following functions:	
4.11. $z = e^{2x}(x + y^2 + 2y)$.	$(1/2, -1)$.
4.12. $z = (2ax - x^2)(2by - y^2)$.	$(0,0)$, $(0,2b)$, (a,b), $(2a,0)$, $(2a,2b)$.
4.13. $z = \dfrac{a + bx + cy}{\sqrt{1 + x^2 + y^2}}$.	$(b/a\,, c/a)$.
4.14. $u = 2x^2 + y^2 + 2z - xy - xz$	$(2,1,7)$.
4.15. Let the function z be given implicitly: $5x^2 + 5y^2 + 5z^2 - 2xy - 2xz - 2yz - 72 = 0$. Find its stationary points.	$(1,1)$, $(-1,-1)$.
4.16. Find the extrema points of the function $z = 4(x - y) - x^2 - y^2$.	$(2,-2)$.
4.17. To prove that the function $z = x^2 + xy + y^2 + \dfrac{a^3}{x} + \dfrac{a^2}{y}$ has minimum at the point $x = y = \dfrac{a}{\sqrt[3]{3}}$.	

4.18. To prove that the function $z = x^3 + y^2 - 6xy - 39x + 18y + 20$ has minimum at the point $x = 5,\ y = 6$.	
4.19. Find the stationary points of the function $z = x^3 + y^3 - 3xy$ and investigate their for extremum.	The function has no extrema at the point $(0,0)$. The point $(1,1)$ is minimum point.