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Practice Lesson № 5. Geometrical Applications of the Differential Calculus for Function with Two Variables

Classwork	Answers
Geometrical applications	
5.1. Form an equations of the tangent line and normal plane to given curve: $x = at$, $y = \frac{1}{2}at^2$, $z = \frac{1}{3}at^3$ at the point $(6a, 18a, 72a)$.	$x - 6a = \frac{y - 18a}{6} = \frac{z - 72a}{36}$; $x + 6y + 36z = 2706a$.
5.2. Form an equations of the tangent line and normal plane to given curve: $y^2 + z^2 = 25$, $x^2 + y^2 = 10$ at the point $(1, 3, 4)$.	$\frac{x-1}{12} = \frac{y-3}{-4} = \frac{z-4}{3}$; $12x - 4y + 3z - 12 = 0$.
5.3. On line $r\{\cos t, \sin t, e^t\}$ find the point, at which the tangent line will be parallel to the plane $\sqrt{3}x + y - 4 = 0$.	$r_0\left\{\frac{\sqrt{3}}{2}, \frac{1}{2}, e^{\frac{\pi}{6}}\right\}$.
5.4. Form equations of tangent planes and normal to the surface $z = 2x^2 - 4y^2$ at the point $(2, 1, 4)$.	$8x - 8y - z = 4$; $\frac{x-2}{8} = \frac{y-1}{-8} = \frac{z-4}{-1}$.
5.5. Form equations of tangent planes	$x - y + 2z - \frac{\pi}{2} = 0$;

and normal to the surface $z=\mathrm{arctg}\,\frac{y}{x}$ at the point $\left(1, 1, \frac{\pi}{4}\right)$.	$\frac{x-1}{1}=\frac{y-1}{-1}=\frac{z-\frac{\pi}{2}}{2}$.
5.6. Form equations of tangent planes and normal to the surface $x^3+y^3+z^3+xyz-6=0$ at the point $(1, 2, -1)$.	$x+11y+5z-18=0$; $\frac{x-1}{1}=\frac{y-2}{11}=\frac{z+1}{5}$.
5.7. Form an equation of the tangent plane to surface $z=xy$, if this plane is perpendicular to straight line $\frac{x+2}{2}=\frac{y+2}{2}=\frac{z-1}{-1}$.	$2x+y-z=2$.
5.8. To surface $x^2-y^2-3z=0$ draw tangent plane passing through point $A(0, 0, -1)$ and parallel to straight line $\frac{x}{2}=\frac{y}{1}=\frac{z}{2}$.	$4x-2y-3z=3$.
Homework	**Answers**
5.9. Form equations of the tangent line and normal plane to given curve $x=t-\sin t$, $y=1-\cos t$, $z=4\sin\frac{t}{2}$ at the point $(\pi/2-1, 1, 2\sqrt{2})$.	$\frac{x-\frac{\pi}{2}+1}{1}=\frac{y-1}{1}=\frac{z-2\sqrt{2}}{\sqrt{2}}$; $x+y+\sqrt{2}z=\frac{\pi}{2}+4$.
5.10. Form equations of the tangent line and normal plane to given curve $2x^2+3y^2+z^2=47$, $x^2+2y^2=z$ at	$\frac{x+2}{27}=\frac{y-1}{28}=\frac{z-6}{4}$; $27x+28y+4z+2=0$.

the point $(-2, 1, 6)$.	
5.11. Form equations of tangent planes and normal to the surface $z = xy$ at the point $(1, 1, 1)$.	$x + y - z - 1 = 0$; $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{-1}$.
5.12. Form equations of tangent planes and normal to the surface $z = \sqrt{x^2 + y^2} - xy$ at the point $(3, 4, -7)$.	$17x + 11y + 5z = 60$; $\frac{x-3}{17} = \frac{y-4}{11} = \frac{z+7}{5}$.
5.13. To ellipsoid $x^2 + 2y^2 + z^2 = 1$ draw the tangent plane, which is parallel to plane $x - y + 2z = 0$.	$x - y + 2z = \sqrt{\frac{11}{2}}$ and $x - y + 2z = -\sqrt{\frac{11}{2}}$.
5.14. Show that the surfaces $x + 2y - \ln z + 4 = 0$ and $x^2 - xy - 8x + z + 5 = 0$ are touching to each other (they have the common tangent plane) at the point $(2, -3, 1)$.	