GROUP I-219ia, КН-219 ae,219iae,219,iбe,219iве

2.8. Practice Lesson № 7.

Gradient and Directional Derivatives

Classwork	Answers
Derivative with respect to given direction. Gradient	
7.1. 1) $z = x^2 + y^2$. Find grad z at the point $(3, 2)$. 2) $z = \sqrt{4 + x^2 + y^2}$. Find grad z at the point $(2, 1)$. 3) $z = \operatorname{arctg}\frac{y}{x}$. Find grad z at the point (x_0, y_0).	1) $6i + 4j$; 2) $\frac{1}{3}(2i + j)$; 3) $\frac{-y_0 i + x_0 j}{x_0^2 + y_0^2}$.
7.2. 1) Find the point at which gradient of the function $z = \ln\left(x + \frac{1}{y}\right)$ is the following $\bar{i} - \frac{16}{9}\bar{j}$.	1) $\left(-\frac{1}{3}, \frac{3}{4}\right)$; $\left(\frac{7}{3}, -\frac{3}{4}\right)$.
7.3. 1) Find derivative of the function $z = x^2 - 3x^2y + 3xy^2 + 1$ at the point $M(3, 1)$ with respect to direction from this point to point $(6, 5)$. 2) Find derivative of the function $z = x^2y^2 - xy^3 - 3y - 1$ at the point $(2, 1)$ with respect to direction from this point to the origin.	1) 0; 2) $-\sqrt{5}$.
7.4. Find derivative of the function $z = \ln(x + y)$ at the point $(1, 2)$, lying on	$\frac{\sqrt{2}}{3}$.

parabola $y^2 = 4x$ with respect to direction of this parabola.	
7.5. Find derivative of the function $u = x^2y^2 + z^3 - xyz$ at the point $M(1, 1, 2)$ with respect to direction, generating with coordinate axes the angles 60°, 45°, 60° relatively.	5.
7.6. To prove that a derivative of the function $u = f(x, y, z)$ with respect to direction of its gradient is equal to modulus of the gradient.	
Homework	**Answers**
Derivative with respect to given direction. Gradient	
7.7. 1) $z = \arcsin\frac{x}{x+y}$. Find an angle between gradients of this function at the points $(1,1)$ и $(3,4)$. 2) The functions $z = \sqrt{x^2 + y^2}$ and $z = x - 3y + \sqrt{3xy}$ are given. Find an angle between gradients of these functions at the point $(3,4)$.	1) $\cos\alpha \approx 0{,}99$, $\alpha = 8^\circ$; 2) $\cos\alpha \approx -1{,}99$, $\alpha \approx 101^\circ 30'$.
7.8. Find the points at which modulus of the gradient of the function $z = (x^2 + y^2)^{\frac{3}{2}}$ are equal to 2.	2) Points lying on the circle $x^2 + y^2 = \frac{2}{3}$.
7.9. 1) Find derivative of the function $z = \arctan xy$ at the point $(1,1)$ with	1) $\frac{\sqrt{2}}{2}$;

respect to direction of bisector of the first coordinate angle. 2) Find derivative of the function $z=\ln\left(e^x+e^y\right)$ at the origin with respect to direction of the ray, generating the angle α with abscissa.	2) $\dfrac{\cos\alpha+\sin\alpha}{2}$.
7.10. Find derivative of the function $z=\arctan\dfrac{y}{x}$ at the point $\left(\dfrac{1}{2},\dfrac{\sqrt{3}}{2}\right)$, lying on the circle $x^2+y^2-2x=0$ with respect to direction of this circle.	$\dfrac{1}{2}$.
7.11. Find derivative of the function $u=xyz$ at the point $A(5,1,2)$ with respect to direction from this point to point $B(9,4,14)$.	$\dfrac{98}{13}$.
7.12. Find derivative of the function $u=\dfrac{1}{r}$, где $r^2=x^2+y^2+z^2$ with respect to direction of its gradient.	$\dfrac{1}{r^2}$.

2.9. Variants of the Control Tasks for Topic "Differential Calculus of the Function with Several Variables"

CARD № 1

Task 1. Find equation of the tangent plane and normal straight line to given surface S at the point $M_0(x_0,y_0,z_0)$

$$S:\ x^2+y^2+z^2+6z-4x+8=0,\qquad M_0(2,1,-1).$$

Task 2. Verify, does the function u satisfy the given equation?

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0,\ u=\arctan\frac{x}{y}.$$

Task 3. Investigate for extreme the following function

$$z=y\sqrt{x}-2y^2-x+14y$$

Task 4. Find the greatest and the smallest values of the function $z=z(x,y)$ in domain $\overline{D}$, bounded by given curves:

$$z=3x+y-xy,\ \overline{D}:\ y=x,\ y=4,\ x=0.$$

Task 5. Find derivative of the function $\varphi=\sqrt{xy}+\sqrt{4-z^2}$ at the point $M_0(1,1,2)$ with respect to direction $\bar{l}=-2\bar{i}+2\bar{j}-\bar{k}$.

CARD № 2

Task 1. Find equation of the tangent plane and normal straight line to given surface S at the point $M_0(x_0,y_0,z_0)$

$$S:\ x^2+z^2-4y^2=-2xy,\quad M_0(-2,1,2);$$

Task 2. Verify, does the function u satisfy the given equation?

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0,\ u=\frac{y}{x}.$$

Task 3. Investigate for extreme the following function

$$z=x^3+8y^3-6xy+5.$$

Task 4. Find the greatest and the smallest values of the function $z=z(x,y)$ in domain $\overline{D}$, bounded by given curves:

$$z=xy-x-2y,\ \overline{D}:\ x=3,\ y=x,\ y=0.$$

Task 5. Find derivative of the function $\varphi=\ln(x^2+2y)$ at the point $A(4,-4)$ of the parabola $y^2=4x$ with respect to the direction of this curve.

CARD № 3

Task 1. Find equation of the tangent plane and normal straight line to given surface S at the point $M_0(x_0, y_0, z_0)$

$$\text{S: } x^2 + y^2 + z^2 - xy + 3z = 7, \quad M_0(1,2,1).$$

Task 2. Verify, does the function u satisfy the given equation?

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\left(x^3 - y^3\right),\ u = \ln\frac{y}{x} + x^3 - y^3.$$

Task 3. Investigate for extreme the following function

$$z = 2x^3 + 2y^3 - 6xy + 5.$$

Task 4. Find the greatest and the smallest values of the function $z = z(x, y)$ in domain $\overline{D}$, bounded by given curves:

$$z = x^2 + 2xy - y^2 - 4x,\ \overline{D}:\ x - y + 1 = 0,\ x = 3,\ y = 0.$$

Task 5. Find derivative of the function $\varphi = \ln\left(x^2 + 4xy + 3y^2\right)$ at the point $A(2,0)$ with respect to the direction of the vector collinear to the bisector of the 1^{st} coordinate angle.

CARD № 4

Task 1. Find equation of the tangent plane and normal straight line to given surface S at the point $M_0(x_0, y_0, z_0)$

$$\text{S: } x^2 + y^2 + z^2 + 6y + 4x = 8, \quad M_0(-1,1,2).$$

Task 2. Verify, does the function u satisfy the given equation?

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0,\ u = \ln\left(x^2 + (y+1)^2\right).$$

Task 3. Investigate for extreme the following function

$$z = 2xy - 3x^2 - 2y^2 + 10.$$

Task 4. Find the greatest and the smallest values of the function $z = z(x, y)$ in domain $\overline{D}$, bounded by given curves:

$$z = x^2 + y^2 - 2x - 2y + 8,\ \overline{D}:\ x = 0,\ y = 0,\ x + y - 1 = 0.$$

Task 5. Find derivative of the function $\varphi = 3xz - \dfrac{z}{x}$ at the point $A(-1,2,3)$ with respect to the direction of $\vec{\ell} = \vec{i} - \vec{j} + \sqrt{2}\vec{k}$.

CARD № 5

Task 1. Find equation of the tangent plane and normal straight line to given surface S at the point $M_0(x_0, y_0, z_0)$

$$S:\ 2x^2 - y^2 + z^2 - 4z + y = 13,\ M_0(2,1,-1).$$

Task 2. Verify, does the function u satisfy the given equation?

$$y\frac{\partial^2 u}{\partial x \partial y} = (1 + y\ln x)\frac{\partial u}{\partial x}, \qquad u = x^y.$$

Task 3. Investigate for extreme the following function

$$z = xy(6 - x - y).$$

Task 4. Find the greatest and the smallest values of the function $z = z(x, y)$ in domain $\overline{D}$, bounded by given curves:

$$z = 2x^3 - xy^2 + y^2,\ \overline{D}:\ x = 0,\ x = 1,\ y = 0,\ y = 6.$$

Task 5. Find derivative of the function

$$\varphi = 2\sqrt{y - z} + y\arctan x$$

at the point $A(1,4,-5)$ with respect to the direction of $\vec{\ell} = 4\vec{i} - 3\vec{j}$.

CARD № 6

Task 1. Find equation of the tangent plane and normal straight line to given surface S at the point $M_0(x_0, y_0, z_0)$

$$S:\ z = x^2 + y^2 - 4xy + 3x - 15, \quad M_0(-1,3,4).$$

Task 2. Verify, does the function u satisfy the given equation?

$$x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} = 0,\ u = e^{xy}.$$

Task 3. Investigate for extreme the following function

$$z = (x-1)^2 + 2y^2.$$

Task 4. Find the greatest and the smallest values of the function $z = z(x, y)$ in domain $\overline{D}$, bounded by given curves:

$$z = x^2 - 2y^2 + 4xy - 6x - 1,\ \overline{D}:\ x = 0,\ y = 0,\ x + y - 3 = 0.$$

Task 5. Find derivative of the function $\varphi = 2x^2 - \ln(z^2 + y^2)$ at the point $A(2,-1,1)$ with respect to the direction of $\vec{\ell} = \{0;-1;0\}$.

CARD № 7

Task 1. Find equation of the tangent plane and normal straight line to given surface S at the point $M_0(x_0, y_0, z_0)$

$$S:\ z = x^2 + 2y^2 + 4xy - 5y - 10,\ M_0(-7,1,8).$$

Task 2. Verify, does the function u satisfy the given equation?

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ u = \operatorname{arctg} \frac{y}{x}.$$

Task 3. Investigate for extreme the following function

$$z = (x-5)^2 + y^2 + 1.$$

Task 4. Find the greatest and the smallest values of the function $z = z(x, y)$ in domain $\overline{D}$, bounded by given curves:

$z = 3x^2 + 3y^2 - 2x - 2y + 2,\ \overline{D}:\ x = 0,\ y = 0,\ x + y - 1 = 0.$

Task 5. Find derivative of the function $\varphi = \cos(x - 2y) + 4yz^2$ at the point $A(\pi/2, \pi/4, 1)$ with respect to the direction of $\vec{\ell} = \{2; -1; 2\}$