$$f'_x,\ \frac{\partial f}{\partial x},\ f'_y,\ \frac{\partial f}{\partial y}$$

are used. So the definition of partial derivatives can be formulated in the following way: f'_x is the derivative of the function $f(x,y)$ with respect to the variable x, provided the variable y is constant; f'_y is the derivative with to y calculated on the assumption that x is constant.

1.6. Practice Lesson № 1.
Evaluation of the Partial Derivatives

Classwork	Answers
Find the partial derivatives of the given functions with respect to each independent variables (x, y, z, u, v are variables):	
1.1. $z = x^3y - y^3x$.	$\frac{\partial z}{\partial x} = 3x^2y - y^3$; $\frac{\partial z}{\partial y} = x^3 - 3y^2x$.
1.2. $z = \frac{x^3 + y^3}{x^2 + y^2}$.	$\frac{\partial z}{\partial x} = \frac{x^4 + 3x^2y^2 - 2xy^3}{\left(x^2 + y^2\right)^2}$; $\frac{\partial z}{\partial y} = \frac{y^4 + 3x^2y^2 - 2x^3y}{\left(x^2 + y^2\right)^2}$.
1.3. $z = x\sqrt{y} + \frac{y}{\sqrt[3]{x}}$.	$\frac{\partial z}{\partial x} = \sqrt{y} - \frac{y}{3\sqrt[3]{x^4}}$; $\frac{\partial z}{\partial y} = \frac{x}{2\sqrt{y}} + \frac{1}{\sqrt[3]{x}}$.
1.4. $z = arctg\frac{x}{y}$.	$\frac{\partial z}{\partial x} = \frac{y}{x^2 + y^2}$; $\frac{\partial z}{\partial y} = -\frac{x}{x^2 + y^2}$.
1.5. $z = x^y$.	$\frac{\partial z}{\partial x} = yx^{y-1}$; $\frac{\partial z}{\partial y} = x^y \ln x$.
1.6. $z = \ln\frac{\sqrt{x^2 + y^2} - x}{\sqrt{x^2 + y^2} + x}$.	$\frac{\partial z}{\partial x} = -\frac{2}{\sqrt{x^2 + y^2}}$; $\frac{\partial z}{\partial y} = \frac{2x}{y\sqrt{x^2 + y^2}}$.

1.7. $z=\ln tg\frac{x}{y}$.	$\frac{\partial z}{\partial x}=\frac{2}{y\sin\frac{2x}{y}}$; $\frac{\partial z}{\partial y}=-\frac{2x}{y^2\sin\frac{2x}{y}}$.
1.8. $z=\ln(x+\ln y)$.	$\frac{\partial z}{\partial x}=\frac{1}{x+\ln y}$; $\frac{\partial z}{\partial y}=\frac{1}{y(x+\ln y)}$.
1.9. $z=\sin\frac{x}{y}\cos\frac{y}{x}$.	$\frac{\partial z}{\partial x}=\frac{1}{y}\cos\frac{x}{y}\cos\frac{y}{x}+\frac{y}{x^2}\sin\frac{x}{y}\sin\frac{y}{x}$; $\frac{\partial z}{\partial y}=-\frac{x}{y^2}\cos\frac{x}{y}\cos\frac{y}{x}-\frac{1}{x}\sin\frac{x}{y}\sin\frac{y}{x}$.
1.10. $z=(1+xy)^y$.	$\frac{\partial z}{\partial x}=y^2(1+xy)^{y-1}$; $\frac{\partial z}{\partial y}=xy(1+xy)^{y-1}+(1+xy)^y\ln(1+xy)$.
1.11. $z=x^{xy}$.	$\frac{\partial z}{\partial x}=x^{x^y}x^{y-1}(y\ln x+1)$; $\frac{\partial z}{\partial y}=x^y x^{x^y}\ln^2 x$.
1.12. $u=xy+yz+zx$.	$\frac{\partial u}{\partial x}=y+z$; $\frac{\partial u}{\partial y}=x+z$; $\frac{\partial u}{\partial z}=x+y$.
1.13. $u=x^3+yz^2+3yx-x+z$.	$\frac{\partial u}{\partial x}=3x^2+3y-1$; $\frac{\partial u}{\partial y}=z^2+3x$; $\frac{\partial u}{\partial z}=2yz+1$.
1.14. $u=e^{x(x^2+y^2+z^2)}$.	$\frac{\partial u}{\partial x}=(3x^2+y^2+z^2)e^{x(x^2+y^2+z^2)}$; $\frac{\partial u}{\partial y}=2xye^{x(x^2+y^2+z^2)}$; $\frac{\partial u}{\partial z}=2xze^{x(x^2+y^2+z^2)}$.
1.15. $u=\ln(x+y+z)$.	$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}=\frac{\partial u}{\partial z}=\frac{1}{x+y+z}$.
1.16. $u=x^{y^z}$.	$\frac{\partial u}{\partial x}=y^z x^{y^z-1}$; $\frac{\partial u}{\partial y}=zy^{z-1}x^{y^z}\ln x$;

	$\frac{\partial u}{\partial z} = x^{y^z} \cdot \ln x \cdot y^z \cdot \ln y$.
1.17. $z = 2\sqrt{\frac{1-\sqrt{xy}}{1+\sqrt{xy}}}$.	$\frac{\partial z}{\partial x} = -\frac{y}{\left(1+\sqrt{xy}\right)\sqrt{xy - x^2y^2}}$; $\frac{\partial z}{\partial y} = -\frac{x}{\left(1+\sqrt{xy}\right)\sqrt{xy - x^2y^2}}$.
1.18. $z = \sqrt{1-\left(\frac{x+y}{xy}\right)^2} +$ $+ \arcsin\frac{x+y}{xy}$.	$\frac{\partial z}{\partial x} = -\frac{1}{x^2}\sqrt{\frac{xy-x-y}{xy+x+y}}$; $\frac{\partial z}{\partial y} = -\frac{1}{y^2}\sqrt{\frac{xy-x-y}{xy+x+y}}$.
Homework	**Answers**
Find the partial derivatives of the given functions with respect to each independent variables (x, y, z, u, v are variables):	
1.19. $z = \left(5x^2y - y^3 + 7\right)^3$.	$\frac{\partial z}{\partial x} = 3\left(5x^2y - y^3 - 7\right)^2 10xy$; $\frac{\partial z}{\partial y} = 3\left(5x^2y - y^3 + 7\right)^2\left(5x^2 - 3y^2\right)$.
1.20. $z = \ln\left(x + \sqrt{x^2+y^2}\right)$.	$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{x^2+y^2}}$; $\frac{\partial z}{\partial y} = \frac{y}{x^2+y^2+x\sqrt{x^2+y^2}}$.