Since $\Delta > 0$ and $A = a_{11} = \frac{1}{e} > 0$ at the point $(-1,0)$ the function attains a minimum equal to $u_{\min}\Big|_{\substack{x=-1\\y=0}} = -\frac{1}{e}$.

Note. Let the function $f(M) = f(x_1, x_2, ..., x_n)$ be m times differentiable in some neighborhood of the point $M_0(x_1, x_2, ..., x_n)$ and all partial derivatives of the m-th order be continuous at this point, moreover $df(M_0) = 0$, $d^2 f(M_0) = d^3 f(M_0) = ... = d^{m-1} f(M_0) = 0$, $d^m f(M_0) \neq 0$.

Then, if m is odd, then the point M_0 will not be the point of extremum; otherwise if m is even, the function $f(x, y)$ has an extremum at the point M_0: a local maximum, if $d^m f(M_0) < 0$, and a local minimum, if $d^m f(M_0) > 0$.

1.19. Practice Lesson №4. Extrema of a Function

Classwork	**Answers**
Find the stationary points of the following functions:	
4.1. $z = 2x^3 + xy^2 + x^2 + y^2$.	$(0,0), (-5/3,0), (-1,2), (-1,-2)$.
4.2. $z = xy(a - x - y)$.	$(0,0), (0,a), (a,0), (a/3, a/3)$.
4.3. $z = \sin x + \sin y + \cos(x + y)$ $\left(0 \le x \le \frac{\pi}{4}, 0 \le y \le \frac{\pi}{4}\right)$.	$(\pi/6, \pi/6)$.
4.4. $z = y\sqrt{1+x} + x\sqrt{1+y}$.	$(-2/3, -2/3)$

4.5. $u = 3\ln x + 2\ln y + 5\ln z + \ln(22 - x - y - z)$.	(6,4,10).
4.6. Find the stationary points of the function $z(x, y)$ defined by implicitly: $2x^2 + 2y^2 + z^2 + 8xz - z + 8 = 0$.	$(-2,0)$, $(16/7,0)$, every point will be stationary point only for one of branches of the function.
4.7*. Find point of extrema of the given function $z = 2xy - 3x^2 - 2z^2 + 10$.	(0,0). **Note.** To prove that stationary point is maximum point it is enough to present the function in the following form: $z = 10 - (x - y)^2 - 2x^2 - y^2$.
4.8. Find the points of extrema of the function $z = x^2 + xy + y^2 + x - y + 1$.	$(-1,1)$.
4.9. To prove that function $z = x^4 + y^4 - 2x^2 - 4xy - 2y^2$ has minimum at the points: $x = \sqrt{2}$, $y = \sqrt{2}$ and $x = -\sqrt{2}$, $y = -\sqrt{2}$.	
4.10. Find stationary points of the function $z = x^3 y^2 (12 - x - y)$, if it is known that $x > 0$, $y > 0$. Investigate the character of these points.	The point (6,4) is point of maximum.
Homework	**Answers**
Find the stationary points of the following functions:	
4.11. $z = e^{2x}(x + y^2 + 2y)$.	$(1/2,-1)$.

4.12. $z = (2ax - x^2)(2by - y^2)$.	(0,0), (0,2b), (a,b), (2a,0), (2a,2b).
4.13. $z = \dfrac{a + bx + cy}{\sqrt{1 + x^2 + y^2}}$.	$(b/a, c/a)$.
4.14. $u = 2x^2 + y^2 + 2z - xy - xz$	(2,1,7).
4.15. Let the function z be given implicitly: $5x^2 + 5y^2 + 5z^2 - 2xy - 2xz - 2yz - 72 = 0$. Find its stationary points.	(1,1), $(-1,-1)$.
4.16. Find the extrema points of the function $z = 4(x - y) - x^2 - y^2$.	$(2,-2)$.
4.17. To prove that the function $z = x^2 + xy + y^2 + \dfrac{a^3}{x} + \dfrac{a^2}{y}$ has minimum at the point $x = y = \dfrac{a}{\sqrt[3]{3}}$.	
4.18. To prove that the function $z = x^3 + y^2 - 6xy - 39x + 18y + 20$ has minimum at the point $x = 5$, $y = 6$.	
4.19. Find the stationary points of the function $z = x^3 + y^3 - 3xy$ and investigate their for extremum.	The function has no extrema at the point (0,0). The point (1,1) is minimum point.

2.5. Practice Lesson № 6. Problems Connected with Finding Conditional Extremum

Classwork	**Answers**
The greatest and the smallest values of the function	
6.1. Find the greatest and the smallest values of the function $z = x^2 - y^2$ in circle $z = x^2 + y^2 \le 4$.	The greatest and the smallest values of the function attain on boundary of the domain; the greatest: $z = 4$ at points $(2,0)$ and $(-2,0)$; the smallest: $z = -4$ at points $(0,2)$ and $(0,-2)$. The stationary point $(0,0)$ is not point of extreme.
6.2. Find the greatest and the smallest values of the function $z = x^2 y(4 - x - y)$ in triangle bounded by lines $x = 0$, $y = 0$, $x + y = 6$.	The greatest value is $z = 4$ at the stationary point $(2,1)$ (this point is maximum point). The smallest value of the function is $z = -64$ at the point $(4,2)$ on the border of the domain.
6.3. On the plane Oxy find the point the sum of squares of the distances of this point to straight lines $x = 0$, $y = 0$, $x + 2y - 16 = 0$ would be the smallest.	$\left(\frac{8}{5}, \frac{16}{5}\right)$.
Conditional extremum	
6.4. The following points $A(4, 0, 4)$, $B(4, 4, 4)$, $C(4, 4, 0)$ are given. On surface of the sphere $x^2 + y^2 + z^2 = 4$ find such point S, that volume of pyramid $SABC$ would be: a) the greatest, b) the smallest. Check out the answer by elementary-geometric way.	a) $(-2, 0, 0)$; b) $(2, 0, 0)$.