

$$\vec{a} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 1 & -1 & 2 \end{vmatrix} = \vec{i}(4) - \vec{j} \cdot 0 + \vec{k}(-2).$$

$$\vec{a} = (4; 0; -2) \quad \text{or} \quad \vec{a} = (2; 0; -1).$$

The equation of tangent line has the following form:

$$\frac{x-1}{2} = \frac{y-1}{0} = \frac{z-2}{-1}.$$

Zero in dominator means that the tangent line is perpendicular to y -axis, because projection of direction vector on this axes is equal to zero.

2.3. Practice Lesson № 5. Geometrical Applications of the Differential Calculus for Function with Two Variables

Classwork	Answers
<i>Geometrical applications</i>	
<p>5.1. Form an equations of the tangent line and normal plane to given curve: $x = at$, $y = \frac{1}{2}at^2$, $z = \frac{1}{3}at^3$ at the point $(6a, 18a, 72a)$.</p>	$x - 6a = \frac{y - 18a}{6} = \frac{z - 72a}{36};$ $x + 6y + 36z = 2706a.$
<p>5.2. Form an equations of the tangent line and normal plane to given curve: $y^2 + z^2 = 25$, $x^2 + y^2 = 10$ at the point $(1, 3, 4)$.</p>	$\frac{x-1}{12} = \frac{y-3}{-4} = \frac{z-4}{3};$ $12x - 4y + 3z - 12 = 0.$

<p>5.3. On line $r\{\cos t, \sin t, e^t\}$ find the point, at which the tangent line will be parallel to the plane $\sqrt{3}x + y - 4 = 0$.</p>	$r_0 \left\{ \frac{\sqrt{3}}{2}, \frac{1}{2}, e^{\frac{\pi}{6}} \right\}.$
<p>5.4. Form equations of tangent planes and normal to the surface $z = 2x^2 - 4y^2$ at the point $(2, 1, 4)$.</p>	$8x - 8y - z = 4;$ $\frac{x-2}{8} = \frac{y-1}{-8} = \frac{z-4}{-1}.$
<p>5.5. Form equations of tangent planes and normal to the surface $z = \operatorname{arctg} \frac{y}{x}$ at the point $\left(1, 1, \frac{\pi}{4}\right)$.</p>	$x - y + 2z - \frac{\pi}{2} = 0;$ $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z-\frac{\pi}{4}}{2}.$
<p>5.6. Form equations of tangent planes and normal to the surface $x^3 + y^3 + z^3 + xyz - 6 = 0$ at the point $(1, 2, -1)$.</p>	$x + 11y + 5z - 18 = 0;$ $\frac{x-1}{1} = \frac{y-2}{11} = \frac{z+1}{5}.$
<p>5.7. Form an equation of the tangent plane to surface $z = xy$, if this plane is perpendicular to straight line $\frac{x+2}{2} = \frac{y+2}{2} = \frac{z-1}{-1}$.</p>	$2x + y - z = 2.$
<p>5.8. To surface $x^2 - y^2 - 3z = 0$ draw tangent plane passing through point $A(0, 0, -1)$ and parallel to straight line $\frac{x}{2} = \frac{y}{1} = \frac{z}{2}$.</p>	$4x - 2y - 3z = 3.$

Homework**Answers**

5.9. Form equations of the tangent line and normal plane to given curve

$$x = t - \sin t, \quad y = 1 - \cos t, \quad z = 4 \sin \frac{t}{2}$$

at the point $(\pi/2 - 1, 1, 2\sqrt{2})$.

$$\frac{x - \frac{\pi}{2} + 1}{1} = \frac{y - 1}{1} = \frac{z - 2\sqrt{2}}{\sqrt{2}};$$

$$x + y + \sqrt{2}z = \frac{\pi}{2} + 4.$$

5.10. Form equations of the tangent line and normal plane to given curve

$$2x^2 + 3y^2 + z^2 = 47, \quad x^2 + 2y^2 = z$$

at the point $(-2, 1, 6)$.

$$\frac{x + 2}{27} = \frac{y - 1}{28} = \frac{z - 6}{4};$$

$$27x + 28y + 4z + 2 = 0.$$

5.11. Form equations of tangent planes and normal to the surface

$$z = xy \text{ at the point } (1, 1, 1).$$

$$x + y - z - 1 = 0;$$

$$\frac{x - 1}{1} = \frac{y - 1}{1} = \frac{z - 1}{-1}.$$

5.12. Form equations of tangent planes and normal to the surface

$$z = \sqrt{x^2 + y^2} - xy \text{ at the point } (3, 4, -7).$$

$$17x + 11y + 5z = 60;$$

$$\frac{x - 3}{17} = \frac{y - 4}{11} = \frac{z + 7}{5}.$$

5.13. To ellipsoid $x^2 + 2y^2 + z^2 = 1$

draw the tangent plane, which is

parallel to plane $x - y + 2z = 0$.

$$x - y + 2z = \sqrt{\frac{11}{2}} \text{ and}$$

$$x - y + 2z = -\sqrt{\frac{11}{2}}.$$

5.14. Show that the surfaces

$$x + 2y - \ln z + 4 = 0 \text{ and}$$

$$x^2 - xy - 8x + z + 5 = 0$$

are touching to each other (they have the common tangent plane) at the point $(2, -3, 1)$.