

**Distance Learning. Higher Mathematics.**  
**ASSIGNMENTS.**  
**Period: 3 April 2020 – 24 April 2020**

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## The control tasks to the chapter 7

**Task 1.** Calculate the definite integrals:

$$7.1.1. \int_{-2}^0 (x+2)^2 \cos 3x dx; \quad 7.1.2. \int_0^1 \frac{x+1}{x^2+6x+9} dx; \quad 7.1.3. \int_0^4 x^2 \sqrt{16-x^2} dx;$$

$$7.1.4. \int_0^2 \frac{x}{1+x+x^2} dx; \quad 7.1.5. \int_0^{2\pi} (3-7x^2) \cos 2x dx; \quad 7.1.6. \int_0^{\frac{\pi}{2}} x^2 \sin x dx;$$

$$7.1.7. \int_0^3 (x^2-3x) \sin 2x dx; \quad 7.1.8. \int_0^{\frac{\pi}{4}} \frac{dx}{1+2\sin^2 x}; \quad 7.1.9. \int_1^4 x \ln x dx;$$

$$7.1.10. \int_0^1 x \sqrt{1-x^2} dx; \quad 7.1.11. \int_1^3 \frac{\ln^2 x}{\sqrt[3]{x^2}} dx; \quad 7.1.12. \int_1^2 x \ln(1+x) dx;$$

$$7.1.13. \int_1^{e^2} \frac{\ln^2 x}{\sqrt{x}} dx; \quad 7.1.14. \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1+\cos^2 x} dx; \quad 7.1.15. \int_1^e \sqrt{x} \ln^2 x dx;$$

$$7.1.16. \int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x dx; \quad 7.1.17. \int_2^3 (x-1)^2 \ln(x-1) dx; \quad 7.1.18. \int_0^{\pi} \cos^2 x dx;$$

$$7.1.19. \int_{-1}^0 (x+2) \ln^2(x+2) dx; \quad 7.1.20. \int_0^4 \frac{\sqrt{x}+1}{\sqrt{x}+2} dx; \quad 7.1.21. \int_{-1}^1 x^2 e^{\frac{-x}{2}} dx;$$

$$7.1.22. \int_{-2}^0 x^3 \sqrt{x^2+9} dx; \quad 7.1.23. \int_0^1 x^2 e^{3x} dx; \quad 7.1.24. \int_{\frac{\sqrt{2}}{2}}^1 \frac{\sqrt{1-x^2}}{x^2} dx;$$

$$7.1.25. \int_0^1 \frac{4 \operatorname{arctg} x - x}{1+x^2} dx; \quad 7.1.26. \int_0^1 \frac{x}{x^2+3x+2} dx; \quad 7.1.27. \int_0^{\frac{\pi}{4}} \frac{4-7 \operatorname{tg} x}{2+3 \operatorname{tg} x} dx;$$

$$7.1.28. \int_0^1 x e^{-2x} dx; \quad 7.1.29. \int_1^e \frac{x^2 + \ln x^2}{x} dx; \quad 7.1.30. \int_0^{\pi} x \sin x dx;$$

**Task 2.** Calculate the definite integrals.

$$7.2.1. \int_0^1 \frac{(x^2 + 1)dx}{(x^3 + 3x + 1)^3}$$

$$7.2.2. \int_0^2 \frac{x^3}{x^2 + 4} dx$$

$$7.2.3. \int_0^{\pi/4} \frac{2 \cos x + 3 \sin x}{(3 \cos x - 2 \sin x)^3} dx$$

$$7.2.4. \int_{\sqrt{3}}^{\sqrt{8}} \frac{x - 2/x}{\sqrt{1 + x^2}} dx$$

$$7.2.5. \int_0^{\sqrt{3}} \frac{2 \operatorname{arctg} x + x}{1 + x^2} dx$$

$$7.2.6. \int_0^1 \frac{x^3}{x^2 + 1} dx$$

$$7.2.7. \int_1^3 \frac{1 - \sqrt{x}}{\sqrt{x}(x + 1)} dx$$

$$7.2.8. \int_1^e \frac{1 + \ln x}{x} dx$$

$$7.2.9. \int_1^e \frac{x^2 + \ln x^2}{x} dx$$

$$7.2.10. \int_0^1 \frac{x^3}{(x^2 + 1)^2} dx$$

$$7.2.11. \int_{e+1}^{e^2+1} \frac{4 + \ln(x-1)}{x-1} dx$$

$$7.2.12. \int_0^1 \frac{3 \operatorname{arctg} x + x}{x^2 + 1} dx$$

$$7.2.13. \int_{\pi}^{2\pi} \frac{x + \cos x}{x^2 + 2 \sin x} dx$$

$$7.2.14. \int_0^{1/2} \frac{8x + \operatorname{arctg} 2x}{4x^2 + 1} dx$$

$$7.2.15. \int_0^1 \frac{x dx}{x^4 + 1}$$

$$7.2.16. \int_{\sqrt{3}}^{\sqrt{8}} \frac{x - 1/x}{\sqrt{1 + x^2}} dx$$

$$7.2.17. \int_0^{\sin^{-1} 1} \frac{(\arcsin x)^2 - 2}{\sqrt{1 - x^2}} dx$$

$$7.2.18. \int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x \sqrt{1 + x^2}}$$

$$7.2.19. \int_0^1 \frac{x dx}{\sqrt{1 + x^2} + x^4}$$

$$7.2.20. \int_{\sqrt{2}}^2 \frac{dx}{x \sqrt{x^2 - 1}}$$

$$7.2.21. \int_1^4 \frac{1/(2\sqrt{x}) + 1}{(\sqrt{x} + x)^2} dx$$

$$7.2.22. \int_0^{\sqrt{3}} \frac{2x + \operatorname{arctg}^3 x}{1 + x^2} dx$$

$$7.2.23. \int_{\sqrt{2}}^{\sqrt{3}} \frac{x dx}{\sqrt{x^4 - x^2 - 1}}$$

$$7.2.24. \int_{\pi/4}^{\pi/2} \frac{\sin x + x \cos x}{(x \sin x)^2} dx$$

$$7.2.25. \int_{\pi}^{2\pi} \frac{1 - \cos x}{(x - \sin x)^2} dx$$

$$7.2.26. \int_{-1}^0 \frac{\operatorname{tg}^2(x+1)}{\cos^2(x+1)} dx$$

$$7.2.27. \int_2^9 \frac{xdx}{\sqrt[3]{x-1}}$$

$$7.2.28. \int_0^{\pi/4} \operatorname{tg} x \cdot \ln \cos x dx$$

$$7.2.29. \int_0^{\pi/4} \frac{\sin x - \cos x}{(\cos x + \sin x)^4} dx$$

$$7.2.30. \int_0^1 \frac{x^3 + x}{x^4 + 1} dx$$

**Task 3.** Calculate the definite integrals.

$$7.3.1. \int_0^{\pi/4} \frac{dx}{\cos x(1 + \cos x)}$$

$$7.3.2. \int_0^{\pi/2} \frac{\sin x dx}{3 + 2 \sin x}$$

$$7.3.3. \int_0^{\pi/2} \frac{\sin x dx}{2 + \cos x}$$

$$7.3.4. \int_0^{\pi/4} \frac{dx}{(1 + \sin x + \cos x)^2}$$

$$7.3.5. \int_{\pi/2}^{2 \operatorname{arctg} 2} \frac{dx}{\sin x(1 + \sin x)}$$

$$7.3.6. \int_0^{2\pi/3} \frac{\cos^2 x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.7. \int_0^{\pi/2} \frac{\sin^2 x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.8. \int_{-\pi/2}^0 \frac{\sin x dx}{(1 + \cos x - \sin x)^2}$$

$$7.3.9. \int_{-2\pi/3}^0 \frac{\cos^2 x dx}{(1 + \cos x - \sin x)^2}$$

$$7.3.10. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \sin x)^2}$$

$$7.3.11. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.12. \int_0^{\pi/2} \frac{\cos x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.13. \int_0^{2 \operatorname{arctg}(1/2)} \frac{(1 - \sin x) dx}{\cos x(1 + \cos x)}$$

$$7.3.14. \int_{-2\pi/3}^0 \frac{\cos x dx}{(1 + \cos x - \sin x)}$$

$$7.3.15. \int_{-\pi/2}^0 \frac{\cos x dx}{(1 + \cos x - \sin x)^2}$$

$$7.3.16. \int_0^{2 \operatorname{arctg}(1/3)} \frac{\cos x dx}{(1 + \cos x)(1 - \sin x)}$$

$$7.3.17. \int_0^2 \frac{\cos x dx}{(1 + \cos x + \sin x)}$$

$$7.3.18. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \cos x + \sin x)}$$

$$7.3.19. \int_{\pi/3}^{\pi/2} \frac{\cos x dx}{(1 - \cos x + \sin x)}$$

$$7.3.20. \int_0^{\pi/2} \frac{\sin x dx}{5 + 4 \sin x}$$

$$7.3.21. \int_{2\operatorname{arctg}(1/3)}^{2\operatorname{arctg}(1/2)} \frac{dx}{\sin x(1 - \sin x)}$$

$$7.3.22. \int_0^{\pi/2} \frac{\cos x - \sin x}{(1 + \sin x)^2} dx$$

$$7.3.23. \int_{\pi/2}^{2\operatorname{arctg} 2} \frac{dx}{\sin^2 x(1 + \cos x)}$$

$$7.3.24. \int_{\pi/2}^{2\operatorname{arctg} 2} \frac{dx}{\sin^2 x(1 - \cos x)}$$

$$7.3.25. \int_{2\operatorname{arctg}(1/2)}^{\pi/2} \frac{\cos x dx}{(1 - \cos x)^3}$$

$$7.3.26. \int_0^{\pi/2} \frac{\cos x - \sin x}{(1 + \cos x)^2} dx$$

$$7.3.27. \int_{2\operatorname{arctg}(1/2)}^{\pi/2} \frac{dx}{(1 + \sin x - \cos x)^2}$$

$$7.3.28. \int_0^{2\pi/3} \frac{1 + \sin x}{(1 + \cos x + \sin x)} dx$$

$$7.3.29. \int_0^{2\operatorname{arctg}(1/2)} \frac{1 + \sin x}{(1 - \sin x)^2} dx$$

$$7.3.30. \int_0^{\pi/2} \frac{1 + \cos x}{(1 + \sin x + \cos x)} dx$$

**Task 4.** Calculate the definite integrals.

$$7.4.1. \int_0^{\pi/4} \frac{2\operatorname{tg}^2 x - \operatorname{tg} x + 2}{3 - \operatorname{tg} x} dx$$

$$7.4.2. \int_0^{\arccos \sqrt{2/3}} \frac{(2 + \operatorname{tg} x) dx}{\sin^2 x + 2 \cos^2 x - 3}$$

$$7.4.3. \int_{\pi/4}^{\operatorname{arctg} 3} \frac{4\operatorname{tg} x - 3}{4 \cos^2 x + 1 - \sin 2x} dx$$

$$7.4.4. \int_{\arccos 4/\sqrt{17}}^{\pi/4} \frac{(1 + 2\operatorname{ctg} x) dx}{(2 \sin x + \cos x)^2}$$

$$7.4.5. \int_{-\operatorname{arctg} 1/3}^0 \frac{(3\operatorname{tg} x + 1) dx}{1 + 2 \sin 2x - 5 \cos 2x}$$

$$7.4.6. \int_{\arcsin 1/\sqrt{37}}^{\pi/4} \frac{\operatorname{tg} x dx}{3 \sin 2x + 5 \cos^2 x}$$

$$7.4.7. \int_0^{\operatorname{arctg} 1/3} \frac{(8 + \operatorname{tg} x) dx}{9 \sin^2 x + \cos^2 x}$$

$$7.4.8. \int_{\pi/4}^{\operatorname{arctg} 3} \frac{dx}{(3\operatorname{tg} x + 5) \sin 2x}$$

$$7.4.9. \int_0^{\arccos 1/\sqrt{7}} \frac{(3 + 2\operatorname{tg} x) dx}{\sin^2 x + 3 \cos^2 x - 1}$$

$$7.4.10. \int_0^{\arccos 1/\sqrt{6}} \frac{(3\operatorname{tg}^2 x - 1) dx}{\operatorname{tg}^2 x + 5}$$

$$7.4.11. \int_{\arccos 1/\sqrt{10}}^{\arccos 1/\sqrt{26}} \frac{dx}{(6 + 5\operatorname{tg} x) \sin 2x}$$

$$7.4.12. \int_{-\arcsin 2/\sqrt{5}}^{\pi/4} \frac{(2 - \operatorname{tg} x) dx}{(\sin x + 3 \cos x)^2}$$

$$7.4.13. \int_{\pi/4}^{\arccos 1/\sqrt{26}} \frac{dx}{(6 - \operatorname{tg} x) \sin 2x}$$

$$7.4.14. \int_{-\arccos 1/\sqrt{5}}^0 \frac{(7 - 3\operatorname{tg} x) dx}{\operatorname{tg} x + 3}$$

$$7.4.15. \int_{\pi/4}^{\arcsin 2/\sqrt{5}} \frac{(4\operatorname{tg}x - 5)dx}{4\cos^2 x - \sin 2x + 1}$$

$$7.4.16. \int_{-\arccos 1/\sqrt{10}}^0 \frac{(3\operatorname{tg}^2 x - 50)dx}{2\operatorname{tg}x + 7}$$

$$7.4.17. \int_0^{\pi/4} \frac{(3 + \operatorname{tg}x)dx}{(\sin x + 2\cos x)^2}$$

$$7.4.18. \int_0^{\operatorname{arctg} 2/3} \frac{(4 + \operatorname{tg}x)dx}{5\sin^2 x + 3\cos^2 x}$$

$$7.4.19. \int_0^{\operatorname{arctg} 3} \frac{(2 + \operatorname{tg}x)dx}{\sin^2 x + 9\cos^2 x}$$

$$7.4.20. \int_{\pi/4}^{\arccos 1/\sqrt{3}} \frac{\operatorname{tg}x dx}{\sin^2 x - 5\cos^2 x + 4}$$

$$7.4.21. \int_{\pi/4}^{\arcsin \sqrt{2/3}} \frac{\operatorname{tg}x dx}{7\sin^2 x + \cos^2 x}$$

$$7.4.22. \int_0^{\pi/4} \frac{(2 - 3\operatorname{tg}x)dx}{2 + 3\operatorname{tg}x}$$

$$7.4.23. \int_0^{\arcsin 3/\sqrt{10}} \frac{(2\operatorname{tg}x - 5)dx}{(4\cos x - \sin x)^2}$$

$$7.4.24. \int_0^{\arcsin \sqrt{7/8}} \frac{\sin^2 x dx}{2 + 3\cos 2x}$$

$$7.4.25. \int_0^{\pi/4} \frac{(3\operatorname{tg}x + 2)dx}{\sin 2x + 5}$$

$$7.4.26. \int_{\arcsin 2/\sqrt{5}}^{\arcsin 3/\sqrt{10}} \frac{(2 + \operatorname{tg}x)dx}{(5 - \operatorname{tg}x)\sin 2x}$$

$$7.4.27. \int_0^{\arcsin \sqrt{3/7}} \frac{\operatorname{tg}x^2 dx}{3\sin^2 x + 4\cos^2 x - 7}$$

$$7.4.28. \int_0^{\operatorname{arctg} 2} \frac{(11 - \operatorname{tg}x)dx}{3\sin^2 x + \cos^2 x}$$

$$7.4.29. \int_0^{\pi/4} \frac{\sin^2 x dx}{3\cos 2x - 4}$$

$$7.4.30. \int_{\pi/4}^{\operatorname{arctg} 3} \frac{(1 + \operatorname{ctg}x)dx}{(\sin x + 2\cos x)^2}$$

**Task 5.** Calculate the definite integrals.

$$7.5.1. \int_2^4 \frac{\sqrt{x^2 - 4}}{x^4} dx$$

$$7.5.2. \int_3^6 \frac{\sqrt{x^2 - 9}}{x^4} dx$$

$$7.5.3. \int_0^2 \frac{dx}{(16 - x^2)^{3/2}}$$

$$7.5.4. \int_{-3}^3 x^2 \sqrt{9 - x^2} dx$$

$$7.5.5. \int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2 - 2}}{x^4} dx$$

$$7.5.6. \int_0^3 \sqrt{9 - x^2} dx$$

$$7.5.7. \int_0^{9/2} \frac{x^2}{\sqrt{81 - x^2}} dx$$

$$7.5.8. \int_0^2 \frac{dx}{(4 + x^2)^{3/2}}$$

$$7.5.9. \int_0^2 \frac{x^2}{\sqrt{16-x^2}} dx$$

$$7.5.11. \int_1^2 \frac{\sqrt{x^2-1}}{x^4} dx$$

$$7.5.13. \int_0^2 x^2 \sqrt{4-x^2} dx$$

$$7.5.15. \int_0^3 \frac{dx}{(9+x^2)^{3/2}}$$

$$7.5.17. \int_0^{1/\sqrt{2}} \frac{x^4}{\sqrt{(1-x^2)^3}} dx$$

$$7.5.19. \int_0^{\sqrt{3}} \sqrt{3-x^2} dx$$

$$7.5.21. \int_0^7 x^2 \sqrt{49-x^2} dx$$

$$7.5.23. \int_0^{2/\sqrt{2}} \frac{x^4 dx}{(16-x^2)\sqrt{16-x^2}}$$

$$7.5.25. \int_0^2 \frac{x^4}{\sqrt{(8-x^2)^3}} dx$$

$$7.5.27. \int_0^{3/2} \frac{x^2}{\sqrt{9-x^2}} dx$$

$$7.5.29. \int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx$$

$$7.5.10. \int_0^{\sqrt{3}} \frac{dx}{(4-x^2)^{3/2}}$$

$$7.5.12. \int_0^3 \frac{dx}{(9+x^2)^{3/2}}$$

$$7.5.14. \int_0^4 \sqrt{16-x^2} dx$$

$$7.5.16. \int_0^{\sqrt{5}/2} \frac{dx}{(5-x^2)^{3/2}}$$

$$7.5.18. \int_0^1 \frac{x^4 dx}{(2-x^2)^{3/2}}$$

$$7.5.20. \int_0^{\sqrt{2}} x^2 \sqrt{2-x^2} dx$$

$$7.5.22. \int_0^{4\sqrt{3}} \frac{dx}{(64-x^2)^{3/2}}$$

$$7.5.24. \int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1+x^2)^3}}$$

$$7.5.26. \int_0^1 \sqrt{4-x^2} dx$$

$$7.5.28. \int_0^{\sqrt{2}/2} \frac{dx}{(1-x^2)\sqrt{1-x^2}}$$

$$7.5.30. \int_0^{\sqrt{2}} \frac{x^4 dx}{(4-x^2)^{3/2}}$$

**Task 6.** Calculate areas of the figures, restricted by the following curves.

$$7.6.1. x = \frac{1}{y\sqrt{1 + \ln y}}, \quad y = e^3, \quad y = 1, \quad x = 0;$$

$$7.6.2. y = \frac{1}{x^2} e^{1/x}, \quad y = 0, \quad x = 1, \quad x = 2;$$

$$7.6.3. y = x^2 \sqrt{16 - x^2}, \quad (0 \leq x \leq 4), \quad y = 0;$$

$$7.6.4. x = \sqrt{4 - y^2}, \quad x = 0, \quad y = 0, \quad y = 1;$$

$$7.6.5. y = x^2 \cos x, \quad x = 0, \quad y = 0, \quad x = \pi/2;$$

$$7.6.6. x = 4 - (y - 1)^2, \quad x = y^2 - 4y + 3;$$

$$7.6.7. y = x\sqrt{9 - x^2}, \quad (0 \leq x \leq 3), \quad y = 0;$$

$$7.6.8. y = \sin x \cos^2 x, \quad (0 \leq x \leq \pi/2), \quad y = 0;$$

$$7.6.9. y = x^2 \sqrt{4 - x^2}, \quad (0 \leq x \leq 2), \quad y = 0;$$

$$7.6.10. y = \sqrt{e^x - 1}, \quad x = \ln 2, \quad y = 0;$$

$$7.6.11. y = \arccos x, \quad x = 0, \quad y = 0;$$

$$7.6.12. y = 2x - x^2 + 3, \quad y = x^2 - 4x + 3;$$

$$7.6.13. x = \arccos y, \quad x = 0, \quad y = 0;$$

$$7.6.14. y = x^2 \sqrt{8 - x^2}, \quad (0 \leq x \leq 2\sqrt{2}), \quad y = 0;$$

$$7.6.15. y = x\sqrt{1 - x^2}, \quad (0 \leq x \leq 1), \quad y = 0;$$

$$7.6.16. y = \frac{1}{1 + \cos x}, \quad x = -\pi/2, \quad x = \pi/2, \quad y = 0;$$

$$7.6.17. y = \sin 2x \cos^3 x, \quad (0 \leq x \leq \pi/2), \quad y = 0;$$

$$7.6.18. x = 4 - y^2, \quad x = y^2 - 2y;$$

$$7.6.19. y = 4 - x^2, \quad y = x^2 - 2x;$$

$$7.6.20. y = \sqrt{4 - x^2}, \quad (0 \leq x \leq 1), \quad y = 0;$$

$$7.6.21. y = \sin x \cos^2 x, \quad (0 \leq x \leq \pi/2), \quad y = 0;$$



$$7.6.22. y = 2x - x^2 + 3, \quad y = x^2 - 4x + 3;$$

$$7.6.23. y = (x - 2)^3, \quad y = 4x - 8;$$

$$7.6.24. y = (x + 1)^2, \quad y^2 = x + 1;$$

$$7.6.25. y = x\sqrt{49 - x^2}, \quad (0 \leq x \leq 7), \quad y = 0;$$

$$7.6.26. y = x \operatorname{arctg} x, \quad x = \sqrt{3}, \quad y = 0;$$

$$7.6.27. x = \sqrt{e^y - 1}, \quad x = 0, \quad y = \ln 2;$$

$$7.6.28. y = \frac{x}{1 + \sqrt{x}}, \quad x = 1, \quad y = 0;$$

$$7.6.29. x = (y - 2)^3, \quad x = 4y - 8;$$

$$7.6.30. y = \frac{x}{(x^2 + 1)^2}, \quad x = 1, \quad y = 0;$$

**Task 7.** Calculate areas of the figures, restricted by the curves, given by equations in parametric form.

$$7.7.1. \begin{cases} x = 4\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \end{cases} \\ x = 2 \quad (x \geq 2)$$

$$7.7.3. \begin{cases} x = 3 \cos t \\ y = 8 \sin t \end{cases} \\ y = 4\sqrt{3} \quad (y \geq 4\sqrt{3})$$

$$7.7.5. \begin{cases} x = 8(t - \sin t) \\ y = 8(1 - \cos t) \end{cases} \\ y = 12, \quad (0 < x < 16\pi, y \geq 4\sqrt{3})$$

$$7.7.7. \begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \end{cases} \\ y = 2, \quad (0 < x < 4\pi, y \geq 2)$$

$$7.7.2. \begin{cases} x = 4(t - \sin t) \\ y = 4(1 - \cos t) \end{cases} \\ y = 6, \quad (0 < x < 8\pi, y \geq 6)$$

$$7.7.4. \begin{cases} x = 32 \cos^3 t \\ y = 3 \sin^3 t \end{cases} \\ x = 12\sqrt{3} \quad (x \geq 12\sqrt{3})$$

$$7.7.6. \begin{cases} x = 2\sqrt{2} \cos t \\ y = 5\sqrt{2} \sin t \end{cases} \\ y = 5 \quad (y \geq 5)$$

$$7.7.8. \begin{cases} x = 9 \cos t \\ y = 4 \sin t \end{cases} \\ y = 2 \quad (y \geq 2)$$

$$7.7.9. \begin{cases} x = 24 \cos^3 t \\ y = 2 \sin^3 t \end{cases}$$

$$x = 9\sqrt{3} \quad (x \geq 9\sqrt{3})$$

$$7.7.11. \begin{cases} x = 8 \cos^3 t \\ y = 8 \sin^3 t \end{cases}$$

$$x = 1 \quad (x \geq 1)$$

$$7.7.13. \begin{cases} x = 2\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \end{cases}$$

$$x = 1 \quad (x \geq 1)$$

$$7.7.15. \begin{cases} x = 6(t - \sin t) \\ y = 6(1 - \cos t) \end{cases}$$

$$y = 6, (0 < x < 12\pi, y \geq 6)$$

$$7.7.17. \begin{cases} x = 2\sqrt{2} \cos t \\ y = 3\sqrt{2} \sin t \end{cases}$$

$$y = 3 \quad (y \geq 3)$$

$$7.7.19. \begin{cases} x = 16 \cos^3 t \\ y = \sin^3 t \end{cases}$$

$$x = 6\sqrt{3} \quad (x \geq 6\sqrt{3})$$

$$7.7.21. \begin{cases} x = 4(t - \sin t) \\ y = 4(1 - \cos t) \end{cases}$$

$$y = 4, (0 < x < 8\pi, y \geq 4)$$

$$7.7.23. \begin{cases} x = 10(t - \sin t) \\ y = 10(1 - \cos t) \end{cases}$$

$$y = 15, (0 < x < 20\pi, y \geq 15)$$

$$7.7.25. \begin{cases} x = 3 \cos t \\ y = 8 \sin t \end{cases}$$

$$y = 4 \quad (y \geq 4)$$

$$7.7.10. \begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$$

$$y = 1, (0 < x < 2\pi, y \geq 1)$$

$$7.7.12. \begin{cases} x = \sqrt{2} \cos t \\ y = 4\sqrt{2} \sin t \end{cases}$$

$$y = 4 \quad (y \geq 4)$$

$$7.7.14. \begin{cases} x = 6 \cos t \\ y = 4 \sin t \end{cases}$$

$$y = 2\sqrt{3} \quad (y \geq 2\sqrt{3})$$

$$7.7.16. \begin{cases} x = 32 \cos^3 t \\ y = \sin^3 t \end{cases}$$

$$x = 4 \quad (x \geq 4)$$

$$7.7.18. \begin{cases} x = 3(t - \sin t) \\ y = 3(1 - \cos t) \end{cases}$$

$$y = 3, (0 < x < 6\pi, y \geq 3)$$

$$7.7.20. \begin{cases} x = 2 \cos t \\ y = 6 \sin t \end{cases}$$

$$y = 3 \quad (y \geq 3)$$

$$7.7.22. \begin{cases} x = 4\sqrt{2} \cos^3 t \\ y = 2\sqrt{2} \sin^3 t \end{cases}$$

$$x = 2 \quad (x \geq 2)$$

$$7.7.24. \begin{cases} x = 8 \cos^3 t \\ y = 4 \sin^3 t \end{cases}$$

$$x = 3\sqrt{3} \quad (x \geq 3\sqrt{3})$$

$$7.7.26. \begin{cases} x = 6(t - \sin t) \\ y = 6(1 - \cos t) \end{cases}$$

$$y = 9, (0 < x < 12\pi, y \geq 9)$$

$$7.7.27. \begin{cases} x = 8\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \end{cases}$$

$$x = 4 \quad (x \geq 4)$$

$$7.7.29. \begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \end{cases}$$

$$y = 3, (0 < x < 4\pi, y \geq 3)$$

$$7.7.28. \begin{cases} x = 6 \cos t \\ y = 2 \sin t \end{cases}$$

$$y = \sqrt{3} \quad (y \geq \sqrt{3})$$

$$7.7.30. \begin{cases} x = 16 \cos^3 t \\ y = 2 \sin^3 t \end{cases}$$

$$x = 2 \quad (x \geq 2)$$

**Task 8.** Calculate areas of the figures, restricted by the following curves, given by equations in polar coordinates.

$$7.8.1. \rho = \sin 3\varphi$$

$$7.8.3. \rho = \cos 2\varphi$$

$$\rho = \sin \varphi, \rho = \sqrt{2} \cos(\varphi - \pi/4)$$

$$7.8.5. \left( 0 \leq \varphi \leq \frac{3}{4}\pi \right)$$

$$\rho = \sqrt{2} \cos(\varphi - \pi/4),$$

$$7.8.7. \rho = \sqrt{2} \sin(\varphi - \pi/4),$$

$$\left( \frac{\pi}{4} \leq \varphi \leq \frac{3}{4}\pi \right)$$

$$7.8.9. \rho = \sin \varphi, \rho = 2 \sin \varphi$$

$$7.8.11. \rho = 2 \cos 4\varphi$$

$$7.8.13. \rho = 4 \sin 4\varphi$$

$$7.8.15. \rho = 4 \sin \varphi, \rho = 2 \sin \varphi$$

$$7.8.17. \rho = 3 \cos 6\varphi$$

$$7.8.19. \rho = \frac{5}{2} \cos \varphi, \rho = \frac{3}{2} \cos \varphi$$

$$7.8.21. \rho = 1 + \sqrt{3} \cos \varphi$$

$$7.8.23. \rho = 2 \cos \varphi, \rho = 5 \cos \varphi$$

$$7.8.2. \rho = \cos 3\varphi$$

$$7.8.4. \rho = 4 \sin 3\varphi, \rho = 2, (\rho \geq 2)$$

$$7.8.6. \rho = \frac{1}{2} + \sin \varphi$$

$$7.8.8. \rho = \frac{1}{2} + \cos \varphi$$

$$7.8.10. \rho = \frac{5}{2} \sin \varphi, \rho = \frac{3}{2} \sin \varphi$$

$$7.8.12. \rho = \cos \varphi, \rho = 3 \cos \varphi$$

$$7.8.14. \rho = \cos \varphi - \sin \varphi$$

$$7.8.16. \rho = 12 \sin \varphi, \rho = 25 \sin \varphi$$

$$7.8.18. \rho = \cos \varphi + \sin \varphi$$

$$7.8.20. \rho = 2 \sin 6\varphi$$

$$7.8.22. \rho = 1 + \sqrt{2} \sin \varphi$$

$$\rho = \sin \varphi, \rho = \cos \varphi$$

$$7.8.24. \left( 0 \leq \varphi \leq \frac{\pi}{2} \right)$$

$$7.8.25. \rho = 6 \cos 3\varphi, \rho = 3, (\rho \geq 3)$$

$$7.8.27. \rho = 2 \cos 3\varphi, \rho = 1, (\rho \geq 1)$$

$$\rho = 2\sqrt{3} \sin \varphi, \rho = 2 \cos \varphi$$

$$7.8.29. \left( 0 \leq \varphi \leq \frac{\pi}{2} \right)$$

$$7.8.25. \rho = 6 \cos 3\varphi, \rho = 3, (\rho \geq 3)$$

$$7.8.27. \rho = 2 \cos 3\varphi, \rho = 1, (\rho \geq 1)$$

$$\rho = 2\sqrt{3} \sin \varphi, \rho = 2 \cos \varphi$$

$$7.8.29. \left( 0 \leq \varphi \leq \frac{\pi}{2} \right)$$

$$\rho = \cos \varphi,$$

$$7.8.26. \rho = \sqrt{2} \cos(\varphi - \pi/4),$$

$$\left( -\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{2} \right)$$

$$7.8.28. \rho = 6 \sin 3\varphi, \rho = 3, (\rho \geq 3)$$

$$\rho = \sqrt{3} \cos \varphi, \rho = \sin \varphi$$

$$7.8.30. \left( 0 \leq \varphi \leq \frac{\pi}{2} \right)$$

$$\rho = \cos \varphi,$$

$$7.8.26. \rho = \sqrt{2} \cos(\varphi - \pi/4),$$

$$\left( -\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{2} \right)$$

$$7.8.28. \rho = 6 \sin 3\varphi, \rho = 3, (\rho \geq 3)$$

$$\rho = \sqrt{3} \cos \varphi, \rho = \sin \varphi$$

$$7.8.30. \left( 0 \leq \varphi \leq \frac{\pi}{2} \right)$$

**Task 9.** Calculate the length of arcs of the following curves, given by equations in rectangular coordinates.

$$7.9.1. y = \ln \frac{5}{2x}, \quad \sqrt{3} \leq x \leq \sqrt{8}$$

$$7.9.2. y = e^x + 6, \quad \ln \sqrt{8} \leq x \leq \ln \sqrt{15}$$

$$7.9.3. y = 1 - \ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$$

$$7.9.4.$$

$$y = \sqrt{1-x^2} + \arcsin x, \quad 0 \leq x \leq \frac{7}{9}$$

$$7.9.5. y = \ln(x^2 - 1), \quad 2 \leq x \leq 3$$

$$7.9.6. y = -\ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$$

$$7.9.7. y = \arcsin x - \sqrt{1-x^2},$$

$$0 \leq x \leq \frac{15}{16}$$

$$7.9.8. y = 2 + \operatorname{ch} x, \quad 0 \leq x \leq 1$$

$$7.9.9. y = \ln(1-x^2), \quad 0 \leq x \leq \frac{1}{4}$$

$$7.9.10. y = 2 + \arcsin \sqrt{x} + \sqrt{x-x^2},$$

$$\frac{1}{4} \leq x \leq 1$$

$$7.9.11. y = 1 - \ln(x^2 - 1), \quad 3 \leq x \leq 4 \quad 7.9.12. y = (1 - e^x - e^{-x})/2, \quad 0 \leq x \leq 3$$

$$7.9.13. y = 4 + \arccos \sqrt{x} - \sqrt{x - x^2}, \quad 0 \leq x \leq \frac{1}{2} \quad 7.9.14. y = e^x + e, \quad \ln \sqrt{3} \leq x \leq \ln \sqrt{15}$$

$$7.9.15. y = \ln \frac{7}{4} - \ln x, \quad \sqrt{3} \leq x \leq \sqrt{8} \quad 7.9.16. y = 2 + \ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$$

$$7.9.17. y = (e^{2x} + e^{-2x} + 3)/4, \quad 0 \leq x \leq 2 \quad 7.9.18. y = 3 + \operatorname{ch} x, \quad 0 \leq x \leq 1$$

$$7.9.19. y = 1 + \arcsin x - \sqrt{1 - x^2}, \quad 0 \leq x \leq \frac{3}{4} \quad 7.9.20. y = e^x + 7, \quad \ln \sqrt{8} \leq x \leq \ln \sqrt{24}$$

$$7.9.21. y = \ln \sin x, \quad \frac{\pi}{3} \leq x \leq \frac{\pi}{2} \quad 7.9.22. y = -\arccos x + \sqrt{1 - x^2} + 2, \quad 0 \leq x \leq \frac{9}{16}$$

$$7.9.23. y = -\arccos \sqrt{x} - \sqrt{x - x^2} + 5, \quad \frac{1}{9} \leq x \leq 1 \quad 7.9.24. y = 1 - \ln \sin x, \quad \frac{\pi}{3} \leq x \leq \frac{\pi}{2}$$

$$7.9.25. y = e^x - 2, \quad \ln \sqrt{3} \leq x \leq \ln \sqrt{8} \quad 7.9.26. y = \ln x, \quad \sqrt{3} \leq x \leq \sqrt{15}$$

$$7.9.27. y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2 \quad 7.9.28. y = 6 - e^x, \quad \ln \sqrt{8} \leq x \leq \ln \sqrt{15}$$

$$7.9.29. y = 3 + (e^x + e^{-x})/2, \quad 0 \leq x \leq 2 \quad 7.9.30. y = 1 - \ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$$

**Task 10.** Calculate the length of arcs of the following curves, given by equations in polar coordinates.

$$7.10.1. \rho = 2(1 - \sin \varphi), \quad -\frac{\pi}{4} \leq \varphi \leq 0 \quad 7.10.2.$$

$$\rho = 3(1 + \sin \varphi), \quad 0 \leq \varphi \leq \frac{\pi}{2}$$

$$7.10.3. \rho = 4(1 - \cos \varphi), \quad -\frac{\pi}{2} \leq \varphi \leq -\frac{\pi}{4} \quad 7.10.4. \rho = 12e^{12\varphi/5}, \quad 0 \leq \varphi \leq \frac{\pi}{3}$$

$$7.10.5. \rho = 4e^{4\varphi/3}, \quad \frac{\pi}{4} \leq \varphi \leq \frac{\pi}{2} \quad 7.10.6. \rho = 3(1 - \cos \varphi), \quad -\frac{\pi}{4} \leq \varphi \leq 0$$

$$\begin{array}{ll}
7.10.7. \rho = 21(1 + \sin \varphi), & -\frac{\pi}{3} \leq \varphi \leq \frac{\pi}{3} \\
7.10.8. \rho = 1 - \sin \varphi, & 0 \leq \varphi \leq \frac{\pi}{2} \\
7.10.9. \rho = 7e^{7\varphi/12}, & 0 \leq \varphi \leq \frac{\pi}{4} \\
7.10.10. \rho = \sqrt{3}e^{2\varphi}, & -\frac{\pi}{2} \leq \varphi \leq -\frac{\pi}{4} \\
7.10.11. \rho = \sqrt{2}e^\varphi, & 0 \leq \varphi \leq \frac{\pi}{2} \\
7.10.12. \rho = 6e^{12\varphi/5}, & 0 \leq \varphi \leq \frac{\pi}{4} \\
7.10.13. \rho = 8(1 - \cos \varphi), & -\frac{2\pi}{3} \leq \varphi \leq 0 \\
7.10.14. \rho = 3 \cos \varphi, & 0 \leq \varphi \leq \frac{\pi}{6} \\
7.10.15. \rho = 6 \sin \varphi, & 0 \leq \varphi \leq \frac{\pi}{2} \\
7.10.16. \rho = 11\varphi, & 0 \leq \varphi \leq \frac{8}{7} \\
7.10.17. \rho = 3e^{3\varphi}, & -\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{4} \\
7.10.18. \rho = 7e^{7\varphi/3}, & 0 \leq \varphi \leq \frac{\pi}{3} \\
7.10.19. \rho = 2 \sin \varphi, & 0 \leq \varphi \leq \frac{2\pi}{3} \\
7.10.20. \rho = 3\varphi, & 0 \leq \varphi \leq \frac{4}{7} \\
7.10.21. \rho = 5(1 - \cos \varphi), & -\frac{\pi}{3} \leq \varphi \leq 0 \\
7.10.22. \rho = a \sin^3 \frac{\varphi}{3}, & 0 \leq \varphi \leq \frac{\pi}{2} \\
7.10.23. \rho = a\varphi, & 0 \leq \varphi \leq 2\pi \\
7.10.24. \rho = a(1 + \cos \varphi), & 0 \leq \varphi \leq 2\pi \\
7.10.25. \rho = \frac{1}{\varphi}, & \frac{1}{2} \leq \varphi \leq 2 \\
7.10.26. \rho = a \sec^2 \frac{\varphi}{2}, & 0 \leq \varphi \leq \frac{\pi}{2} \\
7.10.27. \rho = 4\varphi, & 0 \leq \varphi \leq \frac{3}{4} \\
7.10.28. \rho = 6 \cos \varphi, & 0 \leq \varphi \leq \frac{\pi}{4} \\
7.10.29. \rho = \sqrt{3}e^{2\varphi}, & -\frac{\pi}{3} \leq \varphi \leq \frac{\pi}{3} \\
7.10.30. \rho = 2e^{4\varphi/3}, & -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}
\end{array}$$

**Task 11.** Calculate the volume of the figures, restricted by the following surfaces.

$$7.11.1. \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{64} = 1, \quad z = 0, \quad z = 4$$

$$7.11.2. \frac{x^2}{25} + \frac{y^2}{9} - z^2 = 1, \quad z = 0, z = 2$$

$$7.11.3. \frac{x^2}{27} + y^2 = 1, z = \frac{y}{\sqrt{3}}, z = 0, (y \geq 0)$$

$$7.11.4. z = x^2 + 9y^2, \quad z = 6$$

$$7.11.5. z = x^2 + 3y^2, \quad z = 1$$

$$7.11.6. \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{196} = 1, \quad z = 0, \quad z = 6$$

$$7.11.7. \frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{25} = 1, \quad z = 0, \quad z = 3$$

$$7.11.8. \frac{x^2}{9} + y^2 = 1, \quad z = y, \quad z = 0, \quad (y \geq 0)$$

$$7.11.9. z = x^2 + 3y^2, \quad z = 1$$

$$7.11.10. \frac{x^2}{9} + \frac{y^2}{4} - z^2 = 1, \quad z = 0, \quad z = 3$$

$$7.11.11. \frac{x^2}{25} + \frac{y^2}{9} - \frac{z^2}{100} = -1, \quad z = 20$$

$$7.11.12. z = x^2 + 9y^2, \quad z = 4$$

$$7.11.13. \frac{x^2}{27} + \frac{y^2}{25} = 1, \quad z = \frac{y}{\sqrt{3}}, \quad z = 0, \quad (y \geq 0)$$

$$7.11.14. x^2 + \frac{y^2}{4} - z^2 = 1, \quad z = 0, \quad z = 3$$

$$7.11.15. \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{25} = 1, \quad z = -2, \quad z = 3$$

$$7.11.16. z = 7x^2 + 4y^2, \quad z = 3$$

$$7.11.17. \frac{x^2}{49} + \frac{y^2}{25} - z^2 = 1, \quad z = 0, \quad z = 7$$

$$7.11.18. \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{9} = 1, \quad z = 2, \quad z = 0$$

$$7.11.19. z = 9x^2 + y^2, \quad z = 3$$

$$7.11.20. \frac{x^2}{9} + \frac{y^2}{16} - \frac{z^2}{64} = -1, \quad z = 16$$

$$7.11.21. z = x^2 + 9y^2, \quad z = 4$$

$$7.11.22. \frac{x^2}{81} + \frac{y^2}{25} - z^2 = 1, \quad z = 2, \quad z = -1$$

$$7.11.23. x^2 + y^2 = 4, \quad z = y, \quad z = 0, \quad (y \geq 0)$$

$$7.11.24. \frac{x^2}{4} + y^2 - z^2 = 1, \quad z = 3, \quad z = -1$$

$$7.11.25. \frac{x^2}{24} + \frac{y^2}{9} + \frac{z^2}{16} = 1, \quad z = 3, \quad z = 0$$

$$7.11.26. \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{36} = -1, \quad z = 7$$

$$7.11.27. \frac{x^2}{3} + \frac{y^2}{16} = 1, \quad z = y\sqrt{3}, \quad z = 0, \quad (y \geq 0)$$

$$7.11.28. \quad z = 2x^2 + 7y^2, \quad z = 7$$

$$7.11.29. \quad x = 2y^2 + 18z^2, \quad x = 3$$

$$7.11.30. \quad y = x^2 + 9z^2, \quad y = 2$$