Distance learning materials for students

E-118іа.е, E-318іб.е, E-118іл.е, E-618іб.е, MIT-203.8і

Practical Lesson № 5, 6. Calculation of Triple Integral in Cylindrical and Spherical Systems of Coordinates. Geometrical Applications of the Triple Integrals (10.04.2020, 17.04.2020)

<table>
<tr><th>Classwork</th><th>Answers</th></tr>
<tr><td colspan="2">1. Find integration limits in triple integrals $\iiint\limits_{\Omega} f(x,y,z)dxdydz$ passing to cylindrical coordinates ρ, φ, z
($x=\rho\cos\varphi,\ y=\rho\sin\varphi,\ z=z$) or spherical ones ρ, θ, φ
($x=\rho\cos\varphi\sin\theta,\ y=\rho\sin\varphi\sin\theta,\ z=\rho\cos\theta$)</td></tr>
<tr><td>5.1. Ω is domain bounded by cylinder $x^2+y^2=2x$, plane $z=0$ and paraboloid $z=x^2+y^2$.</td><td>$\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int\limits_{0}^{2\cos\varphi} \rho d\rho \int\limits_{0}^{\rho^2} f(\rho\cos\varphi, \rho\sin\varphi, z)dz$.</td></tr>
<tr><td>5.2. Ω is part of the sphere $x^2+y^2+z^2\le R^2$,
lying inside cylinder
$(x^2+y^2)^2=R^2(x^2-y^2)$ $(x\ge 0)$.</td><td>$\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\varphi \int\limits_{0}^{R\sqrt{\cos 2\varphi}} \rho d\rho \int\limits_{-\sqrt{R^2-\rho^2}}^{\sqrt{R^2-\rho^2}} f(\rho\cos\varphi, \rho\sin\varphi, z)dz$.</td></tr>
<tr><td>5.3. Ω is common part of two spheres $x^2+y^2+z^2\le R^2$ и $x^2+y^2+(z-R)^2\le R^2$.</td><td>$\int\limits_{0}^{2\pi} d\phi \int\limits_{0}^{R\frac{\sqrt{3}}{2}} \rho d\rho \int\limits_{R-\sqrt{R^2-\rho^2}}^{\sqrt{R^2-\rho^2}} f(\rho\cos\phi, \rho\sin\phi, z)dz$.</td></tr>
<tr><td colspan="2">2. Using the triple integrals to calculate solid volume bounded by given surfaces (It is assumed that all parameters included in condition of the problems are positive values)</td></tr>
</table>

6.1. Paraboloid $(x-1)^2+y^2=z$ and plane $2x+z=2$.	$\frac{\pi}{2}$. Projection of the solid on plane Oxy is a circle.
6.2. Sphere $x^2+y^2+z^2=4$ and paraboloid $x^2+y^2=3z$.	$\frac{19}{6}\pi$ and $\frac{15}{2}\pi$. To pass to cylinder coordinates.
6.3. Paraboloid $z=16-x^2-y^2$ and cone $z=6\sqrt{x^2+y^2}$.	24π.
6.4. $x^2+y^2+z^2=1$, $x^2+y^2+z^2=16$, $z^2=x^2+y^2$, $x=0$, $y=0$, $z=0$ $(x\ge 0,\ y\ge 0,\ z\ge 0)$.	$\frac{21\left(2-\sqrt{2}\right)}{4}\pi$. To pass to spherical coordinates
Homework	**Answers**
1. Find integration limits in triple integrals $\iiint\limits_{\Omega} f(x,y,z)dxdydz$ *passing to cylindrical coordinates* ρ, φ, z ($x=\rho\cos\varphi,\ y=\rho\sin\varphi,\ z=z$) *or spherical ones* ρ, θ, φ ($x=\rho\cos\varphi\sin\theta,\ y=\rho\sin\varphi\sin\theta,\ z=\rho\cos\theta$)	
5.4. Ω is domain lying in the first octant and bounded by cylinder $x^2+y^2=R^2$ and planes $z=0$, $z=1$, $y=x$ and $y=x\sqrt{3}$.	$\int_0^1 dz\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\varphi\int_0^R f(\rho\cos\varphi,\rho\sin\varphi,z)\rho d\rho$.
5.5. Ω is part of the sphere $$x^2+y^2+z^2\le R^2,$$ lying in the first octant.	

Answers $\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta \int_0^{\frac{\pi}{2}} d\varphi \int_0^R f(\rho\cos\varphi\,\sin\theta, \rho\sin\varphi\,\sin\theta, \rho\cos\theta)\rho^2 d\rho$.
2. Using the triple integrals to calculate solid volume bounded by given surfaces (It is assumed that all parameters included in condition of the problems are positive values)

6.5. Sphere $x^2 + y^2 + z^2 = R^2$ and paraboloid $x^2 + y^2 = R(R - 2z)$ $(z \geq 0)$.	$\frac{5}{12}\pi R^3$.
6.6. Sphere $x^2 + y^2 + z^2 = 64$ and paraboloid $12z = x^2 + y^2$ (take part of the sphere lying inside cone).	$\frac{608}{3}\pi$.
6.7. Paraboloid $z = 28 - x^2 - y^2$ and cone $z = 12\sqrt{x^2 + y^2}$.	40π .

Practical Lesson № 7. Geometrical and Physical Applications of the Triple Integrals (24.04.2020)

Classwork	Answers
7.1. Calculate mass of the solid bounded by straight circle cylinder with radius R and with height *H*, if its density at any point is equal to distance square from the point to center of the cylinder base.	$\frac{\pi R^2 H}{6}\left(3R^2 + 2H^2\right)$

7.2. Calculate the moment of inertia relative to the origin of the homogeneous solid (γ=1), bounded by the paraboloid $z = x^2 + y^2$ and plane $z = 4$	$Tip: I_0 = \iiint\limits_V (x^2 + y^2 + z^2) dV$ $\frac{224\pi}{3}$
7.3. Calculate the center of gravity of the homogeneous solid (γ=1), bounded by half of sphere $x^2 + y^2 + z^2 = R^2$, $z \geq 0$.	$C(0, 0, \frac{3}{8}R)$
Homework	**Answers**
7.4. Calculate the mass of the inhomogeneous solid bounded by the sphere $x^2 + y^2 + z^2 = 9$ and straight circle cylinder $x^2 + y^2 = 4$, if density distribution function $\gamma = z$.	$\frac{88\pi}{5}$.
7.5. Calculate the mass of the inhomogeneous solid bounded by the sphere $x^2 + y^2 + z^2 = 16$, the cone $x^2 + y^2 = 3z^2$ and surfaces $x \geq 0, y \geq 0, z \geq 0$ if density distribution function $\gamma = z$.	12π.
7.6. Calculate the mass of the inhomogeneous solid bounded by the sphere $x^2 + y^2 + z^2 = R^2$, and surface $z \geq 0$, if density distribution function $\gamma = z$.	$\frac{R^4\pi}{4}$.