Practice. Analytical geometry
Curves of the second order.

Common equation is

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0.$$

The second order curves are:
circle, ellipse, hyperbole,
parabole, pair of straight lines, etc

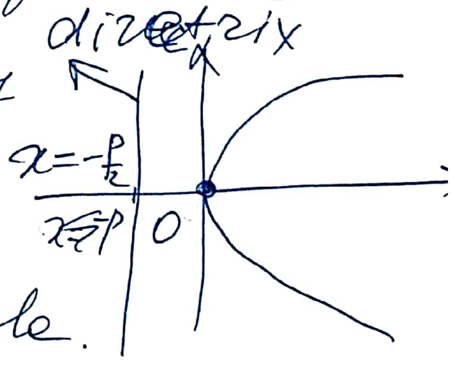
Circle. Canonical equation is

$$(x-x_0)^2 + (y-y_0)^2 = R^2$$

where (x_0, y_0) - the origin
R - radius

Parabole Canonical equations

$$y^2 = 2px \quad \text{or} \quad x^2 = 2py$$



Point O is called vertex
of parabole.

If vertex is point (x_0, y_0) and directrix
is parallel to one of axes then

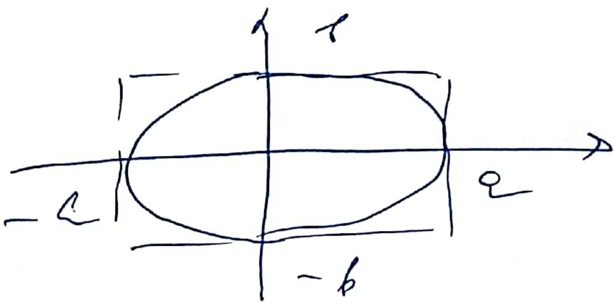
canonical equation has form

$$(y-y_0)^2 = 2p(x-x_0) \quad \text{or} \quad (x-x_0)^2 = 2p(y-y_0)$$

Ellipse Canonical equation

$$\frac{x}{a^2} + \frac{y}{b^2} = 1$$

$a > b \Rightarrow$ a is called a major semi-axis
b is called a minor semi-axis



If center of ellipse is in the point (x_0, y_0) but ellipse axes are parallel to coordinate axes then canonical equation has form

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1.$$

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The transverse axis is of lenghth 2a. and is distance between vertices of the hyperbola. The conjugate axis is of length 2b. and is perpendicular to the transverse axis.

If center of hyperbola is in the point (x_0, y_0) and hyperbola axes are parallel to coordinate axes than canonical equation of this hyperbola has form

$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$$

or

$$-\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1.$$

2) x^2-4y^2 …

Task. Reduse the equation of the curve to the canonical form and plot graph.

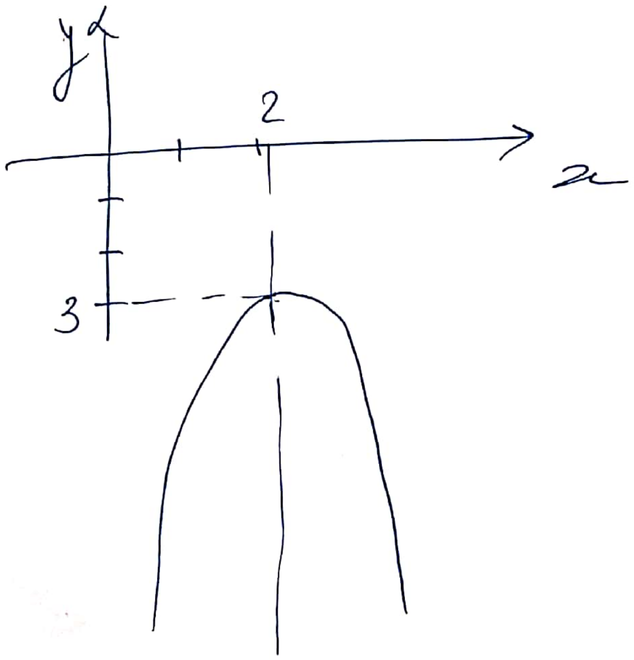
$$y=-2x^2+8x-5.$$

$$y=-2\left(x^2-4x+\frac{5}{2}\right)=-2\left(x-4x+4-4+\frac{5}{2}\right)=$$

$$=-2\left((x-2)^2-\frac{3}{2}\right)=-2(x-2)^2+3.$$

$$-\frac{1}{2}(y-3)=(x-2)^2$$ canonical form

Vertex $O(2;3)$



2) $x^2 - 4y^2 + 6x + 16y - 11 = 0$

$$(x^2 + 6x + 9 - 9) - 4(y^2 - 4y + 4 - 4) = 11$$

$$(x-3)^2 - 9 - 4(y-2)^2 + 16 = 11$$

$$(x-3)^2 - 4(y-2)^2 = 4$$

$$\frac{(x-3)^2}{\underbrace{4}_{a^2}} - \frac{(y-2)^2}{\underbrace{1}_{b^2}} = 1$$

$a = 2$
$b = 1$