Practice Analytical Geometry (1)

Straight Line in Space.

Problem 1. Find canonical equation of the given straight line.

$$\begin{cases} 3x+3y+z-1=0 \\ 2x-3y-2z+6=0 \end{cases}$$

Solution. (see example in lecture 10-11)

Let's present normal vectors of given planes: plane 1 $\vec{n_1}(3;3,1)$

plane 2 $\vec{n_2}(2;-3,-2)$

So $\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 3 & 3 & 1 \\ 2 & -3 & -2 \end{vmatrix} = i(-1)^2 \begin{vmatrix} 3 & 1 \\ -3 & -2 \end{vmatrix} +$

$+j(-1)^3 \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} + k(-1)^4 \begin{vmatrix} 3 & 3 \\ 2 & -3 \end{vmatrix} =$

$= -3i + 8\vec{j} - 15\vec{k}$

Then as direction vector $\vec{l}$ we can take

$\vec{l} = (3;-8,15)$.

TO find point

let's ~~take~~ $z=0$.

As point M_0 belongs to both planes then

$$\begin{cases} 3x+3y-1=0 \\ 2x-3y+6=0 \end{cases} \Big| + \Rightarrow \quad \begin{aligned} 5x &= -5 \\ x_0 &= -1 \end{aligned}$$

Put to any equation $-3+3y-1=0$

$$3y=4$$

$$y_0=\frac{4}{3}$$

$$M_0\left(-1;\ \frac{4}{3};\ 0\right)$$

Then canonical equation is

$$\cancel{A(x-x_0)+B(y-y_0)+C(z-z_0)=0}$$

$$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$$

$$\frac{x+1}{3}=\frac{y-\frac{4}{3}}{-8}=\frac{z}{15}$$

Problem 2. Find point of intersection of the given straight line and plane

$$\frac{x-5}{-1}=\frac{y+3}{5}=\frac{z-1}{2} \qquad 3x+7y-5z-11=0$$

straight line plane

Solution. Let's present parametrical equations of given line

$$\frac{x-5}{-1}=\frac{y+3}{5}=\frac{z-1}{2}=t$$

$$\left.\begin{array}{l}\frac{x-5}{-1}=t\\[6pt] \frac{y+3}{5}=t\\[6pt] \frac{z-1}{2}=t\end{array}\right\} \Rightarrow \begin{cases}x=-t+5\\ y=5t-3\\ z=2t+1\end{cases}$$

put here

Then as point is common ~~we~~ put obtained ~~equat~~ expressions to equation of plane

$$3(-t+5)+7(5t-3)-5(2t+1)-11=0.$$

$$-3t+15+35t-21-10t-5-11=0.$$

$$22t-22=0$$

$$t=1$$

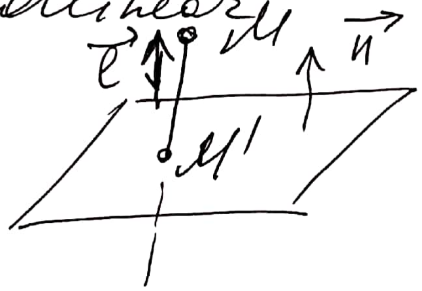
$$M(4;2,3)$$

Problem 3. Find projection of point M on the given plane

$M(3;3;3)$ $\quad 8x+6y+8z-25=0$

Selection.

Let's find equation of straight line that is perpendicular to plane and passes through point M. As direction vector $\vec{l}$ for this line we can take normal vector of plane because they are collinear. M' is projection!



$$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$$

$$\frac{x-3}{8}=\frac{y-3}{6}=\frac{z-3}{8}$$

$\vec{n}(8,6,8)$

Then we find point of intersection of obtained line and plane. (as in previous example).

$$\left.\begin{aligned}\frac{x-3}{8}&=t\\ \frac{y-3}{6}&=t\\ \frac{z-3}{8}&=t\end{aligned}\right\} \Rightarrow \begin{aligned}x&=8t+3\\ y&=6t+3\\ z&=8t+3.\end{aligned} \Rightarrow$$

$$8(8t+3)+6(6t+3)+8(8t+3)-25=0.$$

$$64t + 24 + 36t + 18 + 64t + 24 - 25 = 0.$$

$$164t + 41 = 0$$

$$t = -\frac{1}{4}$$

So point of intersection which is projection of M on the given plane has coordinates

$$x = 8 \cdot \left(-\frac{1}{4}\right) + 3 = -2 + 3 = 1$$

$$y = 6 \cdot \left(-\frac{1}{4}\right) + 3 = -\frac{3}{2} + 3 = \frac{3}{2}$$

$$z = 1.$$

$$\underline{M'\left(1; \frac{3}{2}; 1\right)}$$

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