Line in plane. Practice.

Task. Coordinate of the triangle vertices are given. Find:

1) equation of the triangle sides;
2) the equation of the altitude dropped from the vertex A.
3) the equation of the middle line of the triangle parallel to the side CB.

A(7; 9) B(-2; 0) C(-3; 2)

Solution.

Let's find equations of the sides as straight line passing through 2 points.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

AB: $\frac{x-7}{-2-7} = \frac{y-9}{-9}$ $\qquad \frac{x-7}{-9} = \frac{y-9}{-9}$

$y-9 = x-7$

$\underline{y = x+2.}$

$BC: \frac{x+2}{-3+2} = \frac{y}{2}$

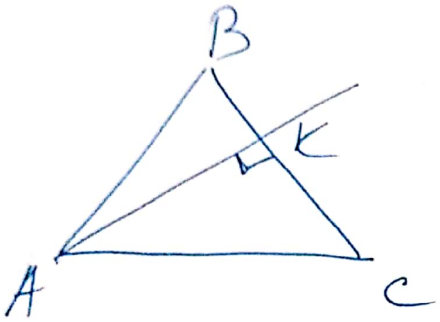
$\frac{x+2}{-1} = \frac{y}{2} \Rightarrow y = -2x-2$

$AC: \frac{x+3}{10} = \frac{y-2}{7} \Rightarrow 7(x+3) = 10(y-2)$

$y-2 = \frac{7}{10}(x+3)$

$y = \frac{7}{10}x + \frac{21}{10} + 2$

$y = \frac{7}{10}x + \frac{41}{10}.$



2) Let's find altitude of $\triangle ABC$ dropped from vertix A.

AK is $\perp$ to BC $\Rightarrow$ their slopes are

$k_{AK} = -\frac{1}{k_{BC}}$ $\quad k_{BC} = -2$

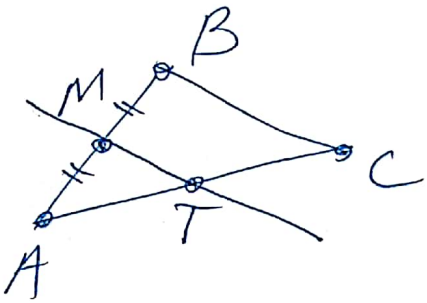
$k_{AK} = \frac{1}{2}$

This line passes through point A so we can present equation as

$y - y_0 = k(x - x_0)$ $\quad A(7;9)$

$y - 9 = \frac{1}{2}(x-7) \Rightarrow y = \frac{1}{2}x - \frac{7}{2} + 9 = \frac{1}{2}x + \frac{11}{2}$

3). middle line comes through point which is middle between points A and B.



$$x_M = \frac{x_A + x_B}{2} \qquad y_M = \frac{y_A + y_B}{2}$$

$$x_M = \frac{7-2}{2} = \frac{5}{2} \qquad y_M = \frac{9+0}{2} = \frac{9}{2}$$

This line // to the line BC ser their slopes are equal

$$MT \parallel BC \Rightarrow k_{MT} = k_{BC}$$

Practice. Analytical geometry
Curves of the second order.

Common equation is

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0.$$

The second order curves are:
circle, ellipse, hyperbole,
parabole, pair of straight lines, etc

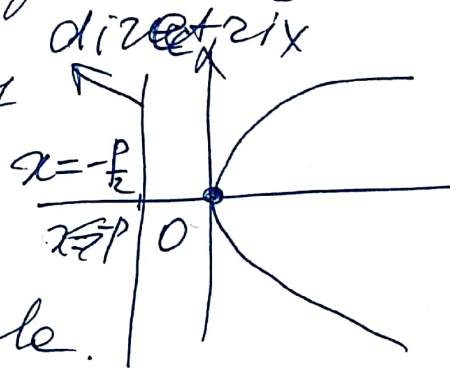
Circle. Canonical equation is

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

where (x_0, y_0) - the origin
R - radius

Parabola Canonical equations

$$y^2 = 2px \quad \text{or} \quad x^2 = 2py$$



Point O is called vertex of parabole.

If vertex is point (x_0, y_0) and directr is parallel to one of axes then