# Practice Analytical Geometry.

## Plane.

**Problem 1.** Find equation of plane with normal vector $\vec{n}(-1;4;2)$ and $M_0(1;0;4)$

**Solution**

General equation of plane is

$$Ax+By+Cz+D=0.$$

Let's use formula

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0.$$

where $\vec{n}(A,B,C)$

$M_0(x_0,y_0,z_0)$

So.

$$-1(x-1)+4(y-0)+2(z-4)=0$$

$$-x+1+4y+2z-8=0$$

$$-x+4y+2z-7=0.$$

$$\underline{x-4y-2z+7=0}$$

**Problem 2.** Find equation of plane passing through the points $M_1, M_2, M_3$, where $M_1(1;1;-1)$, $M_2(2;3,1)$, $M_3(3;2;1)$.

Solution.

To find equation of the plane let's use formula of plane passing through 3 points

current coordinates

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} = 0$$

where $x_1, y_1 \ldots$ are coordinates of points.

In our case

$$\begin{array}{llll} & x_0 & y_0 & z_0 \\ M_1( & 1; & 1, & -1) \\ & x_1 & y_1 & z_1 \\ M_2( & 2; & 3, & 1) \\ & x_2 & y_2 & z_2 \\ M_3( & 3, & 2, & 1) \end{array}$$

$$\begin{vmatrix} x-1 & y-1 & z+1 \\ 2-1 & 3-1 & 1-(-1) \\ 3-2 & 2-1 & 1-(-1) \end{vmatrix} = 0$$

Let's expand along the $1^{st}$ row.

$$\begin{vmatrix} x-1 & y-1 & z+1 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = (x-1)\cdot(-1)^2 \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + (y-1)(-1)^3 \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} +$$

$$+ (z-1)(-1)^4 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = (x-1)\cdot 2 + (z-1)(-1) =$$

$$= 2x - 2 - z + 1 = 0.$$

$$\underline{2x - z - 1 = 0}$$

Problem 3. Find distance from the point M plane.

plane: $2x - z - 1 = 0$

point $M_0$ : $M(1; 2; -2)$

Solution

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} =$$

$$= \frac{|2 \cdot 1 - 1 \cdot (-2) - 1|}{\sqrt{2^2 + (-1)^2}} = \frac{|2+2-1|}{\sqrt{5}} = \frac{3}{\sqrt{5}}.$$