

Individual Tasks Analytical Geometry

Task 1. Find equation of plane passing through the points M_1 , M_2 , M_3 and distance from the point M_0 to this plane.

1.1. $M_1(-3, 4, -7)$, $M_2(1, 5, -4)$, $M_3(-5, -2, 0)$, $M_0(-12, 7, -1)$.

1.2. $M_1(-1, 2, -3)$, $M_2(4, -1, 0)$, $M_3(2, 1, -2)$, $M_0(1, -6, -5)$.

1.3. $M_1(-3, -1, 1)$, $M_2(-9, 1, -2)$, $M_3(3, -5, 4)$, $M_0(-7, 0, -1)$.

1.4. $M_1(1, -1, 1)$, $M_2(-2, 0, 3)$, $M_3(2, 1, -1)$, $M_0(-2, 4, 2)$.

1.5. $M_1(1, 2, 0)$, $M_2(1, -1, 2)$, $M_3(0, 1, -1)$, $M_0(2, -1, 4)$.

1.6. $M_1(1, 0, 2)$, $M_2(1, 2, -1)$, $M_3(2, -2, 1)$, $M_0(-5, -9, 1)$.

1.7. $M_1(1, 2, -3)$, $M_2(1, 0, 1)$, $M_3(-2, -1, 6)$, $M_0(3, -2, -9)$.

1.8. $M_1(3, 10, -1)$, $M_2(-2, 3, -5)$, $M_3(-6, 0, -3)$, $M_0(-6, 7, -10)$.

1.9. $M_1(-1, 2, 4)$, $M_2(-1, -2, -4)$, $M_3(3, 0, -1)$, $M_0(-2, 3, 5)$.

1.10. $M_1(0, -3, 1)$, $M_2(-4, 1, 2)$, $M_3(2, -1, 5)$, $M_0(-3, 4, -5)$.

1.11. $M_1(1, 3, 0)$, $M_2(4, -1, 2)$, $M_3(3, 0, 1)$, $M_0(4, 3, 0)$.

1.12. $M_1(-2, -1, -1)$, $M_2(0, 3, 2)$, $M_3(3, 1, -4)$, $M_0(-21, 20, -16)$.

1.13. $M_1(-3, -5, 6)$, $M_2(2, 1, -4)$, $M_3(0, -3, -1)$, $M_0(3, 6, 68)$.

1.14. $M_1(2, -4, -3)$, $M_2(5, -6, 0)$, $M_3(-1, 3, -3)$, $M_0(2, -10, 8)$.

1.15. $M_1(1, -1, 2)$, $M_2(2, 1, 2)$, $M_3(1, 1, 4)$, $M_0(-3, 2, 7)$.

1.16. $M_1(1, 3, 6)$, $M_2(2, 2, 1)$, $M_3(-1, 0, 1)$, $M_0(5, -4, 5)$.

1.17. $M_1(-4, 2, 6)$, $M_2(2, -3, 0)$, $M_3(-10, 5, 8)$, $M_0(-12, 1, 8)$.

1.18. $M_1(7, 2, 4)$, $M_2(7, -1, -2)$, $M_3(-5, -2, -1)$, $M_0(10, 1, 8)$.

1.19. $M_1(2, 1, 4)$, $M_2(3, 5, -2)$, $M_3(-7, -3, 2)$, $M_0(-3, 1, 8)$.

1.20. $M_1(-1, -5, 2)$, $M_2(-6, 0, -3)$, $M_3(3, 6, -3)$, $M_0(10, -8, -7)$.

1.21. $M_1(0, -1, -1), M_2(-2, 3, 5), M_3(1, -5, -9), M_0(-4, -13, 6)$.

1.22. $M_1(5, 2, 0), M_2(2, 5, 0), M_3(1, 2, 4), M_0(-3, -6, -8)$.

1.23. $M_1(2, -1, -2), M_2(1, 2, 1), M_3(5, 0, -6), M_0(14, -3, 7)$.

1.24. $M_1(-2, 0, -4), M_2(-1, 7, 1), M_3(4, -8, -4), M_0(-6, 5, 5)$.

1.25. $M_1(14, 4, 5), M_2(-5, -3, 2), M_3(-2, -6, -3), M_0(-1, -8, 7)$.

1.26. $M_1(1, 2, 0), M_2(3, 0, -3), M_3(5, 2, 6), M_0(-13, -8, 16)$.

1.27. $M_1(2, -1, 2), M_2(1, 2, -1), M_3(3, 2, 1), M_0(-5, 3, 7)$.

1.28. $M_1(1, 1, 2), M_2(-1, 1, 3), M_3(2, -2, 4), M_0(2, 3, 8)$.

1.29. $M_1(2, 3, 1), M_2(4, 1, -2), M_3(6, 3, 7), M_0(-5, -4, 8)$.

1.30. $M_1(1, 1, -1), M_2(2, 3, 1), M_3(3, 2, 1), M_0(-3, -7, 6)$.

Task 2. Find canonical equations of the given straight line.

2.1.
$$\begin{cases} 2x + y + z - 2 = 0 \\ 2x - y - 3z + 6 = 0 \end{cases}$$

2.2.
$$\begin{cases} x - 3y + 2z + 2 = 0 \\ x + 3y + z + 14 = 0 \end{cases}$$

2.3.
$$\begin{cases} x - 2y + z - 4 = 0 \\ 2x + 2y - z - 8 = 0 \end{cases}$$

2.4.
$$\begin{cases} x + y + z - 2 = 0 \\ x - y - 2z + 2 = 0 \end{cases}$$

2.5.
$$\begin{cases} 2x + 3y + z + 6 = 0 \\ x - 3y - 2z + 3 = 0 \end{cases}$$

2.6.
$$\begin{cases} 3x + y - z - 6 = 0 \\ 3x - y + 2z = 0 \end{cases}$$

2.7.
$$\begin{cases} x + 5y + 2z + 11 = 0 \\ x - y - z - 1 = 0 \end{cases}$$

2.8.
$$\begin{cases} 3x + 4y - 2z + 1 = 0 \\ 2x - 4y + 3z + 4 = 0 \end{cases}$$

2.9.
$$\begin{cases} 5x + y - 3z + 4 = 0 \\ x - y + 2z + 2 = 0 \end{cases}$$

2.10.
$$\begin{cases} x - y - z - 2 = 0 \\ x - 2y + z + 4 = 0 \end{cases}$$

2.11.
$$\begin{cases} 4x + y - 3z + 2 = 0 \\ 2x - y + z - 8 = 0 \end{cases}$$

2.12.
$$\begin{cases} 3x + 3y - 2z - 1 = 0 \\ 2x - 3y + z + 6 = 0 \end{cases}$$

2.13.
$$\begin{cases} 6x - 7y - 4z - 2 = 0 \\ x + 7y - z - 5 = 0 \end{cases}$$

2.14.
$$\begin{cases} 8x - y - 3z - 1 = 0 \\ x + y + z + 10 = 0 \end{cases}$$

$$2.15. \begin{cases} 6x - 5y - 4z + 8 = 0 \\ 6x + 5y + 3z + 4 = 0 \end{cases}$$

$$2.16. \begin{cases} x + 5y - z - 5 = 0 \\ 2x - 5y + 2z + 5 = 0 \end{cases}$$

$$2.17. \begin{cases} 2x - 3y + z + 6 = 0 \\ x - 3y - 2z + 3 = 0 \end{cases}$$

$$2.18. \begin{cases} 5x + y + 2z + 4 = 0 \\ x - y - 3z + 2 = 0 \end{cases}$$

$$2.19. \begin{cases} 4x + y + z + 2 = 0 \\ 2x - y - 3z - 8 = 0 \end{cases}$$

$$2.20. \begin{cases} 2x + y - 3z - 2 = 0 \\ 2x - y + z + 6 = 0 \end{cases}$$

$$2.21. \begin{cases} x + y - 2z - 2 = 0 \\ x - y + z + 2 = 0 \end{cases}$$

$$2.22. \begin{cases} x + 5y - z + 11 = 0 \\ x - y + 2z - 1 = 0 \end{cases}$$

$$2.23. \begin{cases} x - y + z - 2 = 0 \\ x - 2y - z + 4 = 0 \end{cases}$$

$$2.24. \begin{cases} 6x - 7y - z - 2 = 0 \\ x + 7y - 4z - 5 = 0 \end{cases}$$

$$2.25. \begin{cases} x + 5y + 2z - 5 = 0 \\ 2x - 5y - z + 5 = 0 \end{cases}$$

$$2.26. \begin{cases} x - 3y + z + 2 = 0 \\ x + 3y + 2z + 14 = 0 \end{cases}$$

$$2.27. \begin{cases} 2x + 3y - 2z + 6 = 0 \\ x - 3y + z + 3 = 0 \end{cases}$$

$$2.28. \begin{cases} 3x + 4y + 3z + 1 = 0 \\ 2x - 4y - 2z + 4 = 0 \end{cases}$$

$$2.29. \begin{cases} 3x + 3y + z - 1 = 0 \\ 2x - 3y - 2z + 6 = 0 \end{cases}$$

$$2.30. \begin{cases} 6x - 5y + 3z + 8 = 0 \\ 6x + 5y - 4z + 4 = 0 \end{cases}$$

Task 3. Find point of intersection of the given straight line and plane.

$$3.1. \frac{x-2}{-1} = \frac{y-3}{-1} = \frac{z+1}{4}, \quad x + 2y + 3z - 14 = 0.$$

$$3.2. \frac{x+1}{3} = \frac{y-3}{-4} = \frac{z+1}{5}, \quad x + 2y - 5z + 20 = 0.$$

$$3.3. \frac{x-1}{-1} = \frac{y+5}{4} = \frac{z-1}{2}, \quad x - 3y + 7z - 24 = 0.$$

$$3.4. \frac{x-1}{1} = \frac{y}{0} = \frac{z+3}{2}, \quad 2x - y + 4z = 0.$$

$$3.5. \frac{x-5}{1} = \frac{y-3}{-1} = \frac{z-2}{0}, \quad 3x + y - 5z - 12 = 0.$$

$$3.6. \frac{x+1}{-3} = \frac{y+2}{2} = \frac{z-3}{-2}, \quad x + 3y - 5z + 9 = 0.$$

$$3.7. \frac{x-1}{-2} = \frac{y-2}{1} = \frac{z+1}{-1}, \quad x-2y+5z+17=0.$$

$$3.8. \frac{x-1}{2} = \frac{y-2}{0} = \frac{z-4}{1}, \quad x-2y+4z-19=0.$$

$$3.9. \frac{x+2}{-1} = \frac{y-1}{1} = \frac{z+4}{-1}, \quad 2x-y+3z+23=0.$$

$$3.10. \frac{x+2}{1} = \frac{y-2}{0} = \frac{z+3}{0}, \quad 2x-3y-5z-7=0.$$

$$3.11. \frac{x-1}{2} = \frac{y-1}{-1} = \frac{z+2}{3}, \quad 4x+2y-z-11=0.$$

$$3.12. \frac{x-1}{1} = \frac{y+1}{0} = \frac{z-1}{-1}, \quad 3x-2y-4z-8=0.$$

$$3.13. \frac{x+2}{-1} = \frac{y-1}{1} = \frac{z+3}{2}, \quad x+2y-z-2=0.$$

$$3.14. \frac{x+3}{1} = \frac{y-2}{-5} = \frac{z+2}{3}, \quad 5x-y+4z+3=0.$$

$$3.15. \frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-4}{3}, \quad x+3y+5z-42=0.$$

$$3.16. \frac{x-3}{-1} = \frac{y-4}{5} = \frac{z-4}{2}, \quad 7x+y+4z-47=0.$$

$$3.17. \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-1}{5}, \quad 2x+3y+7z-52=0.$$

$$3.18. \frac{x-3}{2} = \frac{y+1}{3} = \frac{z+3}{2}, \quad 3x+4y+7z-16=0.$$

$$3.19. \frac{x-5}{-2} = \frac{y-2}{0} = \frac{z+4}{-1}, \quad 2x-5y+4z+24=0.$$

$$3.20. \frac{x-1}{8} = \frac{y-8}{-5} = \frac{z+5}{12}, \quad x-2y-3z+18=0.$$

$$3.21. \frac{x-3}{1} = \frac{y-1}{-1} = \frac{z+5}{0}, \quad x+7y+3z+11=0.$$

$$3.22. \frac{x-5}{-1} = \frac{y+3}{5} = \frac{z-1}{2}, \quad 3x+7y-5z-11=0.$$

$$3.23. \frac{x-1}{7} = \frac{y-2}{1} = \frac{z-6}{-1}, \quad 4x + y - 6z - 5 = 0.$$

$$3.24. \frac{x-3}{1} = \frac{y+2}{-1} = \frac{z-8}{0}, \quad 5x + 9y + 4z - 25 = 0.$$

$$3.25. \frac{x+1}{-2} = \frac{y}{0} = \frac{z+1}{3}, \quad x + 4y + 13z - 23 = 0.$$

$$3.26. \frac{x-1}{6} = \frac{y-3}{1} = \frac{z+5}{3}, \quad 3x - 2y + 5z - 3 = 0.$$

$$3.27. \frac{x-2}{4} = \frac{y-1}{-3} = \frac{z+3}{-2}, \quad 3x - y + 4z = 0.$$

$$3.28. \frac{x-1}{2} = \frac{y+2}{-5} = \frac{z-3}{-2}, \quad x + 2y - 5z + 16 = 0.$$

$$3.29. \frac{x-1}{1} = \frac{y-3}{0} = \frac{z+2}{-2}, \quad 3x - 7y - 2z + 7 = 0.$$

$$3.30. \frac{x+3}{0} = \frac{y-2}{-3} = \frac{z+5}{11}, \quad 5x + 7y + 9z - 32 = 0.$$

Task 4. Find projection of point M on the given plane.

$$4.1. M(0, -3, -2), \quad 3x + y - z - 1 = 0.$$

$$4.2. M(2, -1, 1), \quad x + 3y - z + 1 = 0.$$

$$4.3. M(1, 1, 1), \quad x + y - z = 0.$$

$$4.4. M(1, 2, 3), \quad 9x + y - z - 1 = 0.$$

$$4.5. M(1, 0, -1), \quad 2x + y + 4 = 0.$$

$$4.6. M(2, 1, 0), \quad x - 2y + z + 4 = 0.$$

$$4.7. M(-2, -3, 0), \quad 3x + 4z - 1 = 0.$$

$$4.8. M(-1, 0, -1), \quad x + 2y - z - 1 = 0.$$

$$4.9. M(0, 2, 1), \quad x + y + z = 0.$$

$$4.10. M(3, -3, -1), \quad 5x + 4y + 1 = 0.$$

$$4.11. M(3, 3, 3), \quad x - y - z - 1 = 0.$$

- 4.12. $M(-1, 2, 0)$, $2x + 2y + z - 3 = 0$.
- 4.13. $M(2, -2, -3)$, $x - y - z + 2 = 0$.
- 4.14. $M(-1, 0, 1)$, $2x + y + 3z + 1 = 0$.
- 4.15. $M(0, -3, -2)$, $-x + y + 5z = 0$.
- 4.16. $M(1, 0, 1)$, $4x + 6y + 4z - 25 = 0$.
- 4.17. $M(-1, 0, -1)$, $2x + 6y - 2z + 11 = 0$.
- 4.18. $M(0, 2, 1)$, $2x + 4y - 3 = 0$.
- 4.19. $M(2, 1, 0)$, $y + z + 2 = 0$.
- 4.20. $M(-1, 2, 0)$, $4x - 5y - z - 7 = 0$.
- 4.21. $M(2, -1, 1)$, $x - y + 2z - 2 = 0$.
- 4.22. $M(1, 1, 1)$, $x + 4y + 3z + 5 = 0$.
- 4.23. $M(1, 2, 3)$, $2x + 10y + 10z - 1 = 0$.
- 4.24. $M(0, -3, -2)$, $2x + 10y + 10z - 1 = 0$.
- 4.25. $M(1, 0, -1)$, $2y + 4z - 1 = 0$.
- 4.26. $M(3, -3, -1)$, $2x - 4y - 4z - 13 = 0$.
- 4.27. $M(-2, -3, 0)$, $x + 5y + 4 = 0$.
- 4.28. $M(2, -2, -3)$, $y + z + 2 = 0$.
- 4.29. $M(-1, 0, 1)$, $2x + 4y - 3 = 0$.
- 4.30. $M(3, 3, 3)$, $8x + 6y + 8z - 25 = 0$.

Task 5. Coordinates of the triangle vertices are given. Find:

- 1) the equations of the triangle sides;
- 2) the equation of the altitude dropped from the vertex A ;
- 3) the equation of the middle line of the triangle parallel to the side CB ;

5.1. $A(-6;-3), B(-4;3), C(9;2)$

5.2. $A(2;-1), B(8;7), C(-10;4)$

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| 5.3. | $A(-3;1), B(-1;7), C(12;6)$ | 5.4. | $A(5;-3), B(1;0), C(7;2)$ |
| 5.5. | $A(-1;3), B(1;9), C(4;7)$ | 5.6. | $A(4;-6), B(2;2), C(-2;-1)$ |
| 5.7. | $A(0;0), B(2;6), C(7;2)$ | 5.8. | $A(3;4), B(-1;7), C(-4;0)$ |
| 5.9. | $A(-2;-6), B(0;0), C(3;-2)$ | 5.10. | $A(1;-2), B(7;6), C(0;2)$ |
| 5.11. | $A(-2;-5), B(6;2), C(0;0)$ | 5.12. | $A(2;-1), B(-2;-3), C(-6;4)$ |
| 5.13. | $A(-2;0), B(-4;-7), C(5;5)$ | 5.14. | $A(5;-8), B(3;-2), C(-3;-6)$ |
| 5.15. | $A(1;2), B(3;8), C(-4;-1)$ | 5.16. | $A(8;-2), B(-6;-5), C(0;4)$ |
| 5.17. | $A(4;4), B(1;-3), C(9;0)$ | 5.18. | $A(7;5), B(3;2), C(4;0)$ |
| 5.19. | $A(5;6), B(7;2), C(-6;0)$ | 5.20. | $A(3;-7), B(6;0), C(1;1)$ |
| 5.21. | $A(-6;-4), B(-1;2), C(6;1)$ | 5.22. | $A(5;3), B(-1;-2), C(-3;7)$ |
| 5.23. | $A(2;0), B(7;2), C(0;5)$ | 5.24. | $A(3;1), B(-2;8), C(-5;3)$ |
| 5.25. | $A(-2;-6), B(-6;-3), C(10;-1)$ | 5.26. | $A(9;2), B(-5;7), C(0;-3)$ |
| 5.27. | $A(8;2), B(-2;1), C(-4;7)$ | 5.28. | $A(-3;3), B(3;1), C(-1;4)$ |
| 5.29. | $A(2;-4), B(-2;-1), C(4;1)$ | 5.30. | $A(7;9), B(-2;0), C(-3;2)$ |

Task 6. Reduce the equations of the second order curves to the canonical form and plot graphs of these curves.

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| 6.1. | $x^2 - 6x - 4y + 29 = 0;$
$9x^2 + 4y^2 - 18x - 8y - 23 = 0.$ | 6.2. | $x^2 + y^2 + 2x - 6y + 1 = 0;$
$3y^2 + 5x + 6y + 13 = 0.$ |
| 6.3. | $x^2 - 4y^2 + 6x + 16y - 11 = 0;$
$x^2 + 4x + 3y = 0.$ | 6.4. | $x^2 + 4y^2 + 18x + 16y - 11 = 0;$
$2y^2 - 4y + 5x = 0.$ |
| 6.5. | $y^2 - 2x + 8y + 10 = 0;$
$4x^2 - y^2 - 2y - 5 = 0.$ | 6.6. | $7x^2 - 2y^2 + 28x + 14 = 0;$
$2x^2 - 8x - y = 0.$ |
| 6.7. | $x^2 + 2y^2 + 4y - 6 = 0;$ | 6.8. | $5x^2 + 9y^2 - 30x + 18y + 9 = 0;$ |

$$4y^2 - 8y + x = 0.$$

6.9. $4x^2 - 9y^2 - 8x - 36y - 68 = 0;$
 $y + x^2 + 2x = 0.$

6.11. $-x^2 + 3y^2 + 6x - 12y = 0;$
 $4y^2 + 2x + 8y + 1 = 0.$

6.13. $2y^2 + x - 8y + 3 = 0;$
 $x^2 - y^2 + x - y + 1 = 0.$

6.15. $x^2 + y^2 - 14x - 8y + 40 = 0;$
 $10x - 5x^2 - 2y + 3 = 0.$

6.17. $-3x^2 + y^2 - 6x - 4y - 11 = 0;$
 $x^2 + x + 2y - 1 = 0.$

6.19. $x^2 - 4y^2 - 8x + 12 = 0;$
 $3x^2 - 2y + 6x + 1 = 0.$

6.21. $x^2 - y^2 - 4x + 2y + 2 = 0;$
 $3y^2 + 3y + 2x = 0.$

6.23. $2x^2 - 8x + y + 5 = 0;$
 $4x^2 + y^2 + 8x - 2y - 11 = 0.$

6.25. $x^2 + y^2 - 2x + 6y - 6 = 0;$
 $2y - 5x^2 + 10x = 0.$

6.27. $4x^2 + 3y^2 - 8x + 12y - 32 = 0;$
 $5y^2 + 10y + x = 0.$

6.29. $4x^2 - y^2 - 8x - 6y - 25 = 0;$
 $y = 8x - 2x^2 - 5.$

$$y^2 - 2x + 4y + 2 = 0.$$

15.10. $x^2 - 10x - 4y - 3 = 0;$
 $4x^2 + 16y^2 + 24x - 28 = 0.$

6.12. $x^2 + y^2 - 6x - 8y + 9 = 0;$
 $3x^2 + 12x + 16y - 12 = 0.$

6.14. $x^2 - 4y^2 - 2x - 8y + 13 = 0;$
 $3y^2 - 12x - 6y + 11 = 0.$

6.16. $x^2 - y^2 - 6x + 4y + 6 = 0;$
 $2y^2 + x - 4y - 8 = 0.$

6.18. $9x^2 + 25y^2 - 18x + 100y - 116 = 0;$
 $x^2 - 2y + 6x + 1 = 0.$

6.20. $5x^2 + 9y^2 - 30x + 18y + 9 = 0;$
 $3x^2 + y + 6x = 0.$

6.22. $16x^2 + 25y^2 + 64x - 50y - 311 = 0;$
 $3y^2 + 3y + 2x + 2 = 0.$

6.24. $9x^2 + 4y^2 - 18x - 8y - 23 = 0;$
 $x^2 + 6x - 2y + 1 = 0.$

6.26. $9x^2 - 16y^2 + 90x + 32y - 367 = 0;$
 $x^2 + 6x + 2y = 0.$

6.28. $x^2 + y^2 + 4x - 5 = 0;$
 $4x^2 + 8x - y = 0.$

6.30. $4x^2 - 8x + y + 7 = 0;$
 $x^2 - 4y^2 + 6x + 16y - 11 = 0.$