Vector Algebra. Practice 2.

Problem 1. Check the collinearity of vectors $\bar{c}_1$ and $\bar{c}_2$.

where $\vec{a}(2,0,-5)$

$\vec{b}(1,-3,4)$

$$\bar{c}_1 = 2\bar{a} - 5\bar{b}, \quad \bar{c}_2 = 5\bar{a} - 2\bar{b}$$

Solution.

First of all, let's define coordinates of vectors $\bar{c}_1$ and $\bar{c}_2$.

$$\bar{c}_1 = 2(2;0;-5) - 5(1;-3;4) =$$

$$= (-1; 3; -14)$$

$$\bar{c}_2 = 5\bar{a} - 2\bar{b} = (8; 6; -33).$$

If vectors are collinear then

$$\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = \lambda$$

Check

$$\frac{-1}{8} \neq \frac{6}{8} \neq \frac{-14}{-33}$$

Vectors are not collinear.

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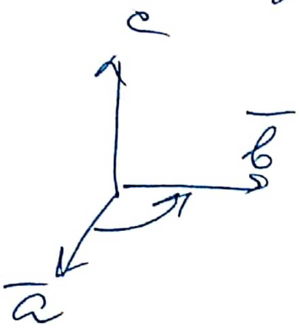
Vector algebra. ~~Practice 3.~~

Vector product.



$\bar{c} = \bar{a} \times \bar{b}$ – vector!!!
if

1) $\bar{a} \perp \bar{c}$, $\bar{b} \perp \bar{c}$
2) $|\bar{c}| = |\bar{a}| \cdot |\bar{b}| \sin\alpha$,
   where α is angle between $\bar{a}$ and $\bar{b}$
3) $\bar{a}, b, \bar{c}$ form right-hand triple.

if vectors $\bar{a}$ and $\bar{b}$ are given by their coordinates then

$$\vec{a} = (a_x, a_y, a_z) \quad \bar{b} = (b_x, b_y, b_z)$$

$$\bar{c} = \bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

<u>Task 1</u>. Find area of triangle constructed on vectors $\vec{a}$ and $\vec{b}$ and the value of altitude dropped on side of vector $\vec{a}$.

$$\vec{a} = 2\vec{i} - 5\vec{k}, \ \vec{b} = \vec{i} - 3\vec{j} + 4\vec{k}$$

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Solution. Area of triangle based on vectors $\bar{a}$ and $\vec{b}$ coinsides with $\frac{1}{2}$ of module of their vector product.

$$S = \frac{1}{2} |\bar{a} \times \bar{b}|$$

Let's find vector product

$$\bar{a} \times \bar{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -5 \\ 1 & -3 & 4 \end{vmatrix} = \vec{i} \begin{vmatrix} 0 & -5 \\ -3 & 4 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -5 \\ 1 & 4 \end{vmatrix} +$$

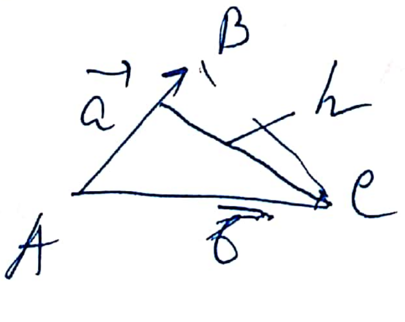
$$+ \vec{k} \begin{vmatrix} 2 & 0 \\ 1 & -3 \end{vmatrix} = -15\vec{i} - 13\vec{j} - 6\vec{k}$$

$$|\bar{a} \times \bar{b}| = \sqrt{(-15)^2 + (-13)^2 + (-6)^2} = \sqrt{225 + 169 + 36} =$$

$$= \sqrt{430}$$

$$S = \frac{1}{2}\sqrt{430}$$

Value of altitude dropped on side of vector $\vec{a}$



$$S = \frac{1}{2} AB \cdot h$$

h - is altitude

$$AB = |\bar{a}| = \sqrt{4 + 25} = \sqrt{29}$$

$$h = \frac{2S}{AB} = \frac{2 \cdot \frac{1}{2}\sqrt{430}}{\sqrt{29}} = \frac{\sqrt{430}}{\sqrt{29}}$$

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Task 2. Find area of parallelogram constructed on vectors $\vec{a}$ and $\vec{b}$.

$\vec{a} = 2\vec{p} - 3\vec{q}$, $\vec{b} = 5\vec{p} + \vec{q}$, $|\vec{p}| = 2$, $|\vec{q}| = 3$

$(\widehat{\vec{p}, \vec{q}}) = \frac{\pi}{2}$.

Solution. Area of parallelogram is equal to the modulus of vector product of vectors $\vec{a}$ and $\vec{b}$.

Let's find vector product

$$\bar{a} \times \bar{b} = (2\bar{p} - 3\bar{q}) \times (5\bar{p} + \bar{q}) =$$

$$= \underbrace{10\,\bar{p} \times \bar{p}}_{=0} + 2\bar{p} \times \bar{q} - 15\underbrace{\bar{q} \times \bar{p}}_{-\bar{p} \times \bar{q}} - \underbrace{3\bar{q} \times \bar{q}}_{=0}$$

Pay attention that

$$\bar{p} \times \bar{p} = |\bar{p}| \cdot |\bar{p}| \cdot \sin \overbrace{0^\circ}^{0} = 0.$$

$$\bar{q} \times \bar{q} = 0.$$

$\bar{p} \times \bar{q} = -\bar{q} \times \bar{p}$ according to vector product property.

So.

$\bar{a} \times \bar{b} = 17 \cdot \bar{p} \times \bar{q}$ then

$$|\bar{a} \times \bar{b}| = 17 \cdot |\bar{p} \times \bar{q}| = 17 \cdot |\bar{p}| \cdot |\bar{q}| \cdot \overbrace{\sin \frac{\pi}{2}}^{1} =$$

$$= 17 \cdot 2 \cdot 3 = 102.$$

$\underline{S = 102}$.

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