

Chapter 6 Indefinite Integrals: tasks for self-controlling

Task 1. Find the following indefinite integrals.

$$6.1.1. a) \int \sin^3 x \cos x dx; b) \int \frac{x}{\sin^2 x} dx; c) \int \frac{dx}{\sin x + \cos x};$$

$$d) \int \frac{(x+2)dx}{\sqrt{x^2+3x+1}}; e) \int \frac{3x^2-6x+1}{x^3-3x^2+2x} dx; f) \int \frac{(1+\sqrt{x})^2}{x^4\sqrt{x^3}} dx.$$

$$6.1.2. a) \int \frac{\arcsin \ln x}{x} dx; b) \int x^2 e^{-x} dx; c) \int \frac{dx}{2 \sin x + 3 \cos x + 1};$$

$$d) \int \frac{(4x+1)dx}{\sqrt{-x^2-2x+1}}; e) \int \frac{2x^2-6x+1}{(x-1)^2(x-2)} dx; f) \int \frac{(1+\sqrt[3]{x})^3}{x\sqrt{x}} dx.$$

$$6.1.3. a) \int \sin 4x e^{\sin 2x} dx; b) \int \frac{x}{\cos^2 x} dx; c) \int \frac{\cos^2 x dx}{\sin^4 x};$$

$$d) \int \frac{(7x-1)dx}{\sqrt{3x^2-6x+4}}; e) \int \frac{x^2-9x+14}{(x^2-4x+3)(x-4)} dx; f) \int \frac{e^{\sqrt{x}} \arctg e^{\sqrt{x}}}{2\sqrt{x}} dx.$$

$$6.1.4. a) \int \sin^3 x \cos^{15} x dx; b) \int x^2 \sin 2x dx; c) \int \frac{dx}{7 \cos x - 6 \sin x + 9};$$

$$d) \int \frac{(4x+3)dx}{\sqrt{-x^2+2x+4}}; e) \int \frac{3x^2-12x+10}{(x^2-5x+6)(x-2)} dx; f) \int \frac{(1+\sqrt[3]{x^2})^2}{x^9\sqrt{x^8}} dx.$$

$$6.1.5. a) \int \frac{dx}{2 \sin x + 3 \cos x + 2}; b) \int x^2 \cos 5x dx; c) \int \frac{\arcsin \frac{1}{x+1}}{(x+1)^2} dx;$$

$$d) \int \frac{(3x+4)dx}{\sqrt{9x^2+6x-5}}; e) \int \frac{2x^2+21x+50}{(x-6)^2(x-2)} dx; f) \int \frac{(1+\sqrt[3]{x^2})^2}{x^2\sqrt[9]{x}} dx.$$

$$6.1.6. a) \int e^{\frac{x^2}{2}} dx; b) \int x \ln(x^2-6x-27) dx; c) \int \frac{\sin^3 x}{\cos^{17} x} dx;$$

$$d) \int \frac{(1-4x)dx}{\sqrt{-x^2-x+2}}; e) \int \frac{5x^2-18x+8}{(x^2-3x+2)(x-6)} dx; f) \int \frac{(1+\sqrt[3]{x^2})^3}{x^2} dx.$$

$$6.1.7. a) \int \sin^3 x \cos^9 x dx; b) \int \ln(x-1) dx; c) \int \frac{dx}{18 \cos x - \sin x + 17};$$

$$\begin{aligned}
& d) \int \frac{(2-5x)dx}{\sqrt{4x^2-8x+1}}; e) \int \frac{3x^2+2x-1}{(x+3)(x-1)(x-5)} dx; f) \int \frac{\arccos \frac{1}{\ln x}}{x \ln^2 x} dx. \\
6.1.8. & a) \int \frac{\arctg \frac{1}{x}}{x^2} dx; b) \int e^{\frac{x}{2}} x^2 dx; c) \int \frac{dx}{\sin^2 x \cos x}; \\
& d) \int \frac{(4x-1)dx}{\sqrt{-x^2-2x+3}}; e) \int \frac{2x^2+3x-2}{(x^2-4x-12)(x+2)} dx; f) \int \frac{(1+\sqrt{x})^3}{x^8 \sqrt[7]{x}} dx. \\
6.1.9. & a) \int \frac{\arccos^2 x}{\sqrt{1-x^2}} dx; b) \int \ln^2 x dx; c) \int \frac{dx}{3 \sin x + 11 \cos x + 12}; \\
& d) \int \frac{(5x-3)dx}{\sqrt{2x^2-4x+5}}; e) \int \frac{6x^2+25x+16}{(x+4)(x+1)(x+2)} dx; f) \int \frac{(1+\sqrt[3]{x^2})^3}{x^2 \sqrt[6]{x}} dx. \\
6.1.10. & a) \int \frac{\arctg^3 x}{1+x^2} dx; b) \int x^2 e^{5x} dx; c) \int \frac{dx}{\cos^3 x \sin x}; \\
& d) \int \frac{(3x+2)dx}{\sqrt{-x^2+x+4}}; e) \int \frac{-4x^2+14x+11}{(x+1)^2(x-6)} dx; f) \int \frac{(1+\sqrt[4]{x^3})^2}{\sqrt[3]{x^2}} dx. \\
6.1.11. & a) \int \sin^3 x \cos^{12} x dx; b) \int x^2 e^x dx; c) \int \frac{dx}{8 \cos x + \sin x + 9}; \\
& d) \int \frac{(2x+4)dx}{\sqrt{4x^2-x-11}}; e) \int \frac{2x^2-3x-7}{(x-1)(x+2)(x-5)} dx; f) \int \frac{\sqrt{x+1}}{1+\sqrt[3]{x+1}} dx. \\
6.1.12. & a) \int \frac{e^{\operatorname{tg} x}}{\cos^2 x} dx; b) \int \frac{x}{\sin^2 2x} dx; c) \int \frac{dx}{\cos^2 x \sin x}; \\
& d) \int \frac{(-3x+1)dx}{\sqrt{-x^2+3x-1}}; e) \int \frac{5x^2-31x+31}{(x-1)^2(x-6)} dx; f) \int \frac{(1+\sqrt{x})^2}{x^{10} \sqrt[9]{x}} dx. \\
6.1.13. & a) \int \sin^{13} x \cos^3 x dx; b) \int x^2 3^x dx; c) \int \frac{dx}{5 \cos x - 5 \sin x + 7}; \\
& d) \int \frac{(-3x+6)dx}{\sqrt{6x^2-2x-7}}; e) \int \frac{7x^2-20x-3}{(x^2-1)(x-5)} dx; f) \int \frac{dx}{(1+\sqrt[3]{x})\sqrt{x}}. \\
6.1.14. & a) \int \frac{e^{\operatorname{ctg} x}}{\sin^2 x} dx; b) \int x^3 e^{3x} dx; c) \int \frac{dx}{\cos x \sin^3 x};
\end{aligned}$$

$$d) \int \frac{(3x-2)dx}{\sqrt{-x^2-3x-2}}; e) \int \frac{3x^2+29x+69}{(x+5)^2(x+4)} dx; f) \int \frac{(1+\sqrt[3]{x^2})^2}{x^2 \sqrt[5]{x}} dx.$$

$$6.1.15. a) \int \frac{\cos x}{\sin^{11} x} dx; b) \int \frac{x}{\sin^2 x} dx; c) \int \frac{dx}{25 \cos x + 2 \sin x + 23};$$

$$d) \int \frac{(4-x)dx}{\sqrt{x^2+8x+3}}; e) \int \frac{3x^2-20x+22}{(x^2-x+1)(x-4)} dx; f) \int \frac{e^{\sqrt{x^3+1}} x^2}{\sqrt{x^3+1}} dx.$$

$$6.1.16. a) \int \frac{\arcsin^3 x}{\sqrt{1-x^2}} dx; b) \int x^3 e^{-2x} dx; c) \int \frac{dx}{\cos x \sin^2 x};$$

$$d) \int \frac{(3x-4)dx}{\sqrt{-x^2-x+5}}; e) \int \frac{x^2-12x+14}{(x^2-8x+15)(x-3)} dx; f) \int \frac{(1+\sqrt[5]{x^4})^2}{x^2 \sqrt[25]{x^{11}}} dx.$$

$$6.1.17. a) \int \frac{\sin x}{\cos^{12} x} dx; b) \int x \arctg x dx; c) \int \frac{dx}{5 \cos x + 3 \sin x + 3};$$

$$d) \int \frac{(1-x)dx}{\sqrt{3x^2-2x-8}}; e) \int \frac{3x^2+x+3}{(x^2+3x-4)(x-3)} dx; f) \int \frac{dx}{\sqrt{x+2} + \sqrt[3]{(x+2)^2}}.$$

$$6.1.18. a) \int \frac{\arctg \sqrt{x}}{\sqrt{x}} dx; b) \int x^3 e^{-x} dx; c) \int \frac{\sin^2 x}{\cos^4 x} dx;$$

$$d) \int \frac{(2x+5)dx}{\sqrt{-x^2+2x+1}}; e) \int \frac{x^2+7x-37}{(x-4)^2(x+3)} dx; f) \int \frac{(1+\sqrt{x})^2}{x^3 \sqrt{x^2}} dx.$$

$$6.1.19. a) \int \cos^{15} x \sin x dx; b) \int e^{\frac{3x}{4}} \cos 3x dx; c) \int \frac{dx}{2 \cos x - 3 \sin x + 1};$$

$$d) \int \frac{(x+3)dx}{\sqrt{4x^2+6x+1}}; e) \int \frac{x^2-6x+6}{(x^2-5x+4)(x-2)} dx; f) \int \frac{x+\sqrt{x}+\sqrt[3]{x^2}}{x(1+\sqrt[3]{x})} dx.$$

$$6.1.20. a) \int \frac{x}{1+x^4} dx; b) \int x^3 e^x dx; c) \int \frac{\sin^4 x}{\cos^6 x} dx;$$

$$d) \int \frac{(2x+3)dx}{\sqrt{-x^2-3x+4}}; e) \int \frac{x^2-9x+22}{(x^2-6x+8)(x-4)} dx; f) \int \frac{(1+\sqrt[3]{x})^3}{x^9 \sqrt{x^4}} dx.$$

$$6.1.21. a) \int \sin^{11} x \cos x dx; b) \int \frac{\ln x}{x^2} dx; c) \int \frac{dx}{7 \cos x - \sin x + 5};$$

$$d) \int \frac{(1-5x)dx}{\sqrt{x^2+x+9}}; e) \int \frac{5x^2+16x+2}{(x^2-5x+4)(x+2)} dx; f) \int \frac{\arccos \frac{x-1}{x}}{x^2} dx.$$

6.1.22. a) $\int \frac{\operatorname{arctg} \sqrt{1-2x}}{\sqrt{1-2x}} dx; b) \int x^3 e^{-\pi^2 x} dx; c) \int \frac{\cos^4 x}{\sin^6 x} dx;$

$$d) \int \frac{(1-2x)dx}{\sqrt{-x^2-3x+4}}; e) \int \frac{x^2-3x-13}{(x^2+4x+4)(x+5)} dx; f) \int \frac{(1+\sqrt[3]{x})^2}{x^9 \sqrt{x^5}} dx.$$

6.1.23. a) $\int \frac{\sin^5 x}{\cos^9 x} dx; b) \int \operatorname{arccotg} 4x dx; c) \int \frac{dx}{11 \cos x + 8 \sin x + 13};$

$$d) \int \frac{(3x+2)dx}{\sqrt{6x^2+3x+5}}; e) \int \frac{4x^2-13x+3}{(x^2-1)(x-4)} dx; f) \int \frac{(1+\sqrt{x})^2}{x^6 \sqrt{x^5}} dx.$$

6.1.24. a) $\int \frac{\sqrt{\ln x}}{x} dx; b) \int x^3 e^x dx; c) \int \sin^2 x \cos^4 x dx;$

$$d) \int \frac{(3-2x)dx}{\sqrt{-x^2+5x-6}}; e) \int \frac{x^2-9x-21}{(x+2)^2(x-5)} dx; f) \int \frac{(1+\sqrt[3]{x})^3}{x^{12} \sqrt{x^7}} dx.$$

6.1.25. a) $\int \frac{\cos^3 x}{\sin^{18} x} dx; b) \int x \operatorname{arctg} 2x dx; c) \int \frac{dx}{\cos x + 2 \sin x + 2};$

$$d) \int \frac{(4x+6)dx}{\sqrt{4x^2+5x+3}}; e) \int \frac{-x^2+x+6}{(x^2-3x+2)(x-4)} dx; f) \int \frac{\sqrt{x+1}}{1+\sqrt[3]{x+1}} dx.$$

6.1.26. a) $\int \frac{\ln^3(x-1)}{x-1} dx; b) \int x e^{-2x} dx; c) \int \cos^2 x \sin^4 x dx;$

$$d) \int \frac{(5-2x)dx}{\sqrt{-x^2+6x-5}}; e) \int \frac{x^2-9x+32}{(x-5)^2(x-2)} dx; f) \int \frac{(1+\sqrt[4]{x^3})^2}{x^2 \sqrt[3]{x}} dx.$$

6.1.27. a) $\int \frac{\cos^3 x}{\sin^{12} x} dx; b) \int x^2 e^{\frac{x}{4}} dx; c) \int \frac{dx}{7 \sin x - 19 \cos x - 17};$

$$d) \int \frac{(3x+2)dx}{\sqrt{3x^2+x+8}}; e) \int \frac{2x^2-3x-6}{x(x^2+5x+6)} dx; f) \int \frac{(1+\sqrt[4]{x^3})^2}{x^2 \sqrt[4]{x}} dx.$$

6.1.28. a) $\int \frac{e^{2x} \arccos e^{2x}}{\sqrt{1-e^{4x}}} dx; b) \int x e^{4x} dx; c) \int \frac{\sin^2 x}{\cos^6 x} dx;$

$$d) \int \frac{(3+x)dx}{\sqrt{-x^2+6x-1}}; e) \int \frac{4x^2-18x+43}{(x^2-8x+4)(x-4)} dx; f) \int \frac{(1+\sqrt[3]{x})^2}{x^5\sqrt{x^3}} dx.$$

$$6.1.29. a) \int \frac{\sin^5 x}{\cos^3 x} dx; b) \int x^2 e^{\frac{x}{2}} dx; c) \int \frac{dx}{-3\cos x + \sin x - 1};$$

$$d) \int \frac{(2x-3)dx}{\sqrt{3x^2+x+1}}; e) \int \frac{3x^2-7x+1}{(x^2-3x+2)(x+3)} dx; f) \int \frac{(1+\sqrt[4]{x^3})^2}{x^2 \sqrt[20]{x^7}} dx.$$

$$6.1.30. a) \int \frac{\operatorname{arctg}(x+1)}{2+2x+x^2} dx; b) \int x \cos^2 x dx; c) \int \frac{\cos^2 x}{\sin^8 x} dx;$$

$$d) \int \frac{(x-5)dx}{\sqrt{-3x^2+x+1}}; e) \int \frac{5x^2-6x-7}{(x^2-x-6)(x-3)} dx; f) \int \frac{(1+\sqrt[5]{x^4})^2}{x^2 \sqrt[5]{x}} dx.$$