

The control tasks to Definite Integrals

Task 1. Calculate the definite integrals:

Tasks 1.1

$$7.1.1. \int_{-2}^0 (x+2)^2 \cos 3x dx; 7.1.2. \int_0^1 \frac{x+1}{x^2+6x+9} dx; 7.1.3. \int_0^4 x^2 \sqrt{16-x^2} dx;$$

$$7.1.4. \int_0^2 \frac{x}{1+x+x^2} dx; 7.1.5. \int_0^{2\pi} (3-7x^2) \cos 2x dx; 7.1.6. \int_0^{\frac{\pi}{2}} x^2 \sin x dx;$$

$$7.1.7. \int_0^3 (x^2-3x) \sin 2x dx; 7.1.8. \int_0^{\frac{\pi}{4}} \frac{dx}{1+2\sin^2 x}; 7.1.9. \int_1^4 x \ln x dx;$$

$$7.1.10. \int_0^1 x \sqrt{1-x^2} dx; 7.1.11. \int_1^3 \frac{\ln^2 x}{\sqrt[3]{x^2}} dx; 7.1.12. \int_1^2 x \ln(1+x) dx;$$

$$7.1.13. \int_1^{e^2} \frac{\ln^2 x}{\sqrt{x}} dx; 7.1.14. \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1+\cos^2 x} dx; 7.1.15. \int_1^e \sqrt{x} \ln^2 x dx;$$

$$7.1.16. \int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x dx; 7.1.17. \int_2^3 (x-1)^2 \ln(x-1) dx; 7.1.18. \int_0^{\pi} \cos^2 x dx;$$

$$7.1.19. \int_{-1}^0 (x+2) \ln^2(x+2) dx; 7.1.20. \int_0^4 \frac{\sqrt{x}+1}{\sqrt{x}+2} dx; 7.1.21. \int_{-1}^1 x^2 e^{\frac{-x}{2}} dx;$$

$$7.1.22. \int_{-2}^0 x^3 \sqrt{x^2+9} dx; 7.1.23. \int_0^1 x^2 e^{3x} dx; 7.1.24. \int_{\frac{\sqrt{2}}{2}}^1 \frac{\sqrt{1-x^2}}{x^2} dx;$$

$$7.1.25. \int_0^1 \frac{4\operatorname{arctg} x - x}{1+x^2} dx; 7.1.26. \int_0^1 \frac{x}{x^2+3x+2} dx; 7.1.27. \int_0^{\frac{\pi}{4}} \frac{4-7\operatorname{tg} x}{2+3\operatorname{tg} x} dx;$$

$$7.1.28. \int_0^1 x e^{-2x} dx; 7.1.29. \int_1^e \frac{x^2 + \ln x^2}{x} dx; 7.1.30. \int_0^{\pi} x \sin x dx;$$

Tasks 1.2

$$7.2.1. \int_0^1 \frac{(x^2 + 1)dx}{(x^3 + 3x + 1)^3}$$

$$7.2.3. \int_0^{\pi/4} \frac{2 \cos x + 3 \sin x}{(3 \cos x - 2 \sin x)^3} dx$$

$$7.2.5. \int_0^{\sqrt{3}} \frac{2 \operatorname{arctg} x + x}{1 + x^2} dx$$

$$7.2.7. \int_1^3 \frac{1 - \sqrt{x}}{\sqrt{x}(x+1)} dx$$

$$7.2.9. \int_1^e \frac{x^2 + \ln x^2}{x} dx$$

$$7.2.11. \int_{e+1}^{e^2+1} \frac{4 + \ln(x-1)}{x-1} dx$$

$$7.2.13. \int_{\pi}^{2\pi} \frac{x + \cos x}{x^2 + 2 \sin x} dx$$

$$7.2.15. \int_0^1 \frac{x dx}{x^4 + 1}$$

$$7.2.17. \int_0^{\sin 1} \frac{(\arcsin x)^2 - 2}{\sqrt{1-x^2}} dx$$

$$7.2.19. \int_0^1 \frac{x dx}{\sqrt{1+x^2} + x^4}$$

$$7.2.21. \int_1^4 \frac{1/(2\sqrt{x}) + 1}{(\sqrt{x} + x)^2} dx$$

$$7.2.23. \int_{\sqrt{2}}^{\sqrt{3}} \frac{x dx}{\sqrt{x^4 - x^2 - 1}}$$

$$7.2.25. \int_{\pi}^{2\pi} \frac{1 - \cos x}{(x - \sin x)^2} dx$$

$$7.2.2. \int_0^2 \frac{x^3}{x^2 + 4} dx$$

$$7.2.4. \int_{\sqrt{3}}^{\sqrt{8}} \frac{x - 2/x}{\sqrt{1+x^2}} dx$$

$$7.2.6. \int_0^1 \frac{x^3}{x^2 + 1} dx$$

$$7.2.8. \int_1^e \frac{1 + \ln x}{x} dx$$

$$7.2.10. \int_0^1 \frac{x^3}{(x^2 + 1)^2} dx$$

$$7.2.12. \int_0^1 \frac{3 \operatorname{arctg} x + x}{x^2 + 1} dx$$

$$7.2.14. \int_0^{1/2} \frac{8x + \operatorname{arctg} 2x}{4x^2 + 1} dx$$

$$7.2.16. \int_{\sqrt{3}}^{\sqrt{8}} \frac{x - 1/x}{\sqrt{1+x^2}} dx$$

$$7.2.18. \int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x\sqrt{1+x^2}}$$

$$7.2.20. \int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2 - 1}}$$

$$7.2.22. \int_0^{\sqrt{3}} \frac{2x + \operatorname{arctg}^3 x}{1 + x^2} dx$$

$$7.2.24. \int_{\pi/4}^{\pi/2} \frac{\sin x + x \cos x}{(x \sin x)^2} dx$$

$$7.2.26. \int_{-1}^0 \frac{\operatorname{tg}^2(x+1)}{\cos^2(x+1)} dx$$

$$7.2.27. \int_2^9 \frac{xdx}{\sqrt[3]{x-1}}$$

$$7.2.29. \int_0^{\pi/4} \frac{\sin x - \cos x}{(\cos x + \sin x)^4} dx$$

$$7.2.28. \int_0^{\pi/4} \operatorname{tg} x \cdot \ln \cos x dx$$

$$7.2.30. \int_0^1 \frac{x^3 + x}{x^4 + 1} dx$$

Tasks 1.3

$$7.3.1. \int_0^{\pi/4} \frac{dx}{\cos x(1 + \cos x)}$$

$$7.3.3. \int_0^{\pi/2} \frac{\sin x dx}{2 + \cos x}$$

$$7.3.5. \int_{\pi/2}^{2\operatorname{arctg} 2} \frac{dx}{\sin x(1 + \sin x)}$$

$$7.3.7. \int_0^{\pi/2} \frac{\sin^2 x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.9. \int_{-2\pi/3}^0 \frac{\cos^2 x dx}{(1 + \cos x - \sin x)^2}$$

$$7.3.11. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.13. \int_0^{2\operatorname{arctg}(1/2)} \frac{(1 - \sin x) dx}{\cos x(1 + \cos x)}$$

$$7.3.15. \int_{-\pi/2}^0 \frac{\cos x dx}{(1 + \cos x - \sin x)^2}$$

$$7.3.17. \int_0^2 \frac{\cos x dx}{(1 + \cos x + \sin x)}$$

$$7.3.19. \int_{\pi/3}^{\pi/2} \frac{\cos x dx}{(1 - \cos x + \sin x)}$$

$$7.3.2. \int_0^{\pi/2} \frac{\sin x dx}{3 + 2 \sin x}$$

$$7.3.4. \int_0^{\pi/4} \frac{dx}{(1 + \sin x + \cos x)^2}$$

$$7.3.6. \int_0^{2\pi/3} \frac{\cos^2 x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.8. \int_{-\pi/2}^0 \frac{\sin x dx}{(1 + \cos x - \sin x)^2}$$

$$7.3.10. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \sin x)^2}$$

$$7.3.12. \int_0^{\pi/2} \frac{\cos x dx}{(1 + \cos x + \sin x)^2}$$

$$7.3.14. \int_{-2\pi/3}^0 \frac{\cos x dx}{(1 + \cos x - \sin x)}$$

$$7.3.16. \int_0^{2\operatorname{arctg}(1/3)} \frac{\cos x dx}{(1 + \cos x)(1 - \sin x)}$$

$$7.3.18. \int_0^{\pi/2} \frac{\sin x dx}{(1 + \cos x + \sin x)}$$

$$7.3.20. \int_0^{\pi/2} \frac{\sin x dx}{5 + 4 \sin x}$$

$$7.3.21. \int_{2\operatorname{arctg}(1/3)}^{2\operatorname{arctg}(1/2)} \frac{dx}{\sin x(1-\sin x)}$$

$$7.3.23. \int_{\pi/2}^{2\operatorname{arctg} 2} \frac{dx}{\sin^2 x(1+\cos x)}$$

$$7.3.25. \int_{2\operatorname{arctg}(1/2)}^{\pi/2} \frac{\cos x dx}{(1-\cos x)^3}$$

$$7.3.27. \int_{2\operatorname{arctg}(1/2)}^{\pi/2} \frac{dx}{(1+\sin x-\cos x)^2}$$

$$7.3.29. \int_0^{2\operatorname{arctg}(1/2)} \frac{1+\sin x}{(1-\sin x)^2} dx$$

$$7.3.22. \int_0^{\pi/2} \frac{\cos x - \sin x}{(1+\sin x)^2} dx$$

$$7.3.24. \int_{\pi/2}^{2\operatorname{arctg} 2} \frac{dx}{\sin^2 x(1-\cos x)}$$

$$7.3.26. \int_0^{\pi/2} \frac{\cos x - \sin x}{(1+\cos x)^2} dx$$

$$7.3.28. \int_0^{2\pi/3} \frac{1+\sin x}{(1+\cos x+\sin x)} dx$$

$$7.3.30. \int_0^{\pi/2} \frac{1+\cos x}{(1+\sin x+\cos x)} dx$$

Tasks 1.4

$$7.4.1. \int_0^{\pi/4} \frac{2\operatorname{tg}^2 x - \operatorname{tg} x + 2}{3 - \operatorname{tg} x} dx$$

$$7.4.2. \int_0^{\arccos \sqrt{2/3}} \frac{(2 + \operatorname{tg} x) dx}{\sin^2 x + 2\cos^2 x - 3}$$

$$7.4.3. \int_{\pi/4}^{\operatorname{arctg} 3} \frac{4\operatorname{tg} x - 3}{4\cos^2 x + 1 - \sin 2x} dx$$

$$7.4.4. \int_{\arccos 4/\sqrt{17}}^{\pi/4} \frac{(1 + 2\operatorname{ctg} x) dx}{(2\sin x + \cos x)^2}$$

$$7.4.5. \int_{-\operatorname{arctg} 1/3}^0 \frac{(3\operatorname{tg} x + 1) dx}{1 + 2\sin 2x - 5\cos 2x}$$

$$7.4.6. \int_{\operatorname{arcsin} 1/\sqrt{37}}^{\pi/4} \frac{\operatorname{tg} x dx}{3\sin 2x + 5\cos^2 x}$$

$$7.4.7. \int_0^{\operatorname{arctg} 1/3} \frac{(8 + \operatorname{tg} x) dx}{9\sin^2 x + \cos^2 x}$$

$$7.4.8. \int_{\pi/4}^{\operatorname{arctg} 3} \frac{dx}{(3\operatorname{tg} x + 5)\sin 2x}$$

$$7.4.9. \int_0^{\arccos 1/\sqrt{7}} \frac{(3 + 2\operatorname{tg} x) dx}{\sin^2 x + 3\cos^2 x - 1}$$

$$7.4.10. \int_0^{\arccos 1/\sqrt{6}} \frac{(3\operatorname{tg}^2 x - 1) dx}{\operatorname{tg}^2 x + 5}$$

$$7.4.11. \int_{\arccos 1/\sqrt{10}}^{\arccos 1/\sqrt{26}} \frac{dx}{(6 + 5\operatorname{tg} x)\sin 2x}$$

$$7.4.12. \int_{-\operatorname{arcsin} 2/\sqrt{5}}^{\pi/4} \frac{(2 - \operatorname{tg} x) dx}{(\sin x + 3\cos x)^2}$$

$$7.4.13. \int_{\pi/4}^{\arccos 1/\sqrt{26}} \frac{dx}{(6 - \operatorname{tg} x) \sin 2x}$$

$$7.4.14. \int_{-\arccos 1/\sqrt{5}}^0 \frac{(7 - 3\operatorname{tg} x) dx}{\operatorname{tg} x + 3}$$

$$7.4.15. \int_{\pi/4}^{\arcsin 2/\sqrt{5}} \frac{(4\operatorname{tg} x - 5) dx}{4 \cos^2 x - \sin 2x + 1}$$

$$7.4.16. \int_{-\arccos 1/\sqrt{10}}^0 \frac{(3\operatorname{tg}^2 x - 50) dx}{2\operatorname{tg} x + 7}$$

$$7.4.17. \int_0^{\pi/4} \frac{(3 + \operatorname{tg} x) dx}{(\sin x + 2 \cos x)^2}$$

$$7.4.18. \int_0^{\operatorname{arctg} 2/3} \frac{(4 + \operatorname{tg} x) dx}{5 \sin^2 x + 3 \cos^2 x}$$

$$7.4.19. \int_0^{\operatorname{arctg} 3} \frac{(2 + \operatorname{tg} x) dx}{\sin^2 x + 9 \cos^2 x}$$

$$7.4.20. \int_{\pi/4}^{\arccos 1/\sqrt{3}} \frac{\operatorname{tg} x dx}{\sin^2 x - 5 \cos^2 x + 4}$$

$$7.4.21. \int_{\pi/4}^{\arcsin \sqrt{2/3}} \frac{\operatorname{tg} x dx}{7 \sin^2 x + \cos^2 x}$$

$$7.4.22. \int_0^{\pi/4} \frac{(2 - 3\operatorname{tg} x) dx}{2 + 3\operatorname{tg} x}$$

$$7.4.23. \int_0^{\arcsin 3/\sqrt{10}} \frac{(2\operatorname{tg} x - 5) dx}{(4 \cos x - \sin x)^2}$$

$$7.4.24. \int_0^{\arcsin \sqrt{7/8}} \frac{\sin^2 x dx}{2 + 3 \cos 2x}$$

$$7.4.25. \int_0^{\pi/4} \frac{(3\operatorname{tg} x + 2) dx}{\sin 2x + 5}$$

$$7.4.26. \int_{\arcsin 2/\sqrt{5}}^{\arcsin 3/\sqrt{10}} \frac{(2 + \operatorname{tg} x) dx}{(5 - \operatorname{tg} x) \sin 2x}$$

$$7.4.27. \int_0^{\arcsin \sqrt{3/7}} \frac{\operatorname{tg} x^2 dx}{3 \sin^2 x + 4 \cos^2 x - 7}$$

$$7.4.28. \int_0^{\operatorname{arctg} 2} \frac{(11 - \operatorname{tg} x) dx}{3 \sin^2 x + \cos^2 x}$$

$$7.4.29. \int_0^{\pi/4} \frac{\sin^2 x dx}{3 \cos 2x - 4}$$

$$7.4.30. \int_{\pi/4}^{\operatorname{arctg} 3} \frac{(1 + \operatorname{ctg} x) dx}{(\sin x + 2 \cos x)^2}$$

Tasks 1.5

$$7.5.1. \int_2^4 \frac{\sqrt{x^2 - 4}}{x^4} dx$$

$$7.5.2. \int_3^6 \frac{\sqrt{x^2 - 9}}{x^4} dx$$

$$7.5.3. \int_0^2 \frac{dx}{(16 - x^2)^{3/2}}$$

$$7.5.4. \int_{-3}^3 x^2 \sqrt{9 - x^2} dx$$

$$7.5.5. \int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2 - 2}}{x^4} dx$$

$$7.5.7. \int_0^{9/2} \frac{x^2}{\sqrt{81 - x^2}} dx$$

$$7.5.9. \int_0^2 \frac{x^2}{\sqrt{16 - x^2}} dx$$

$$7.5.11. \int_1^2 \frac{\sqrt{x^2 - 1}}{x^4} dx$$

$$7.5.13. \int_0^2 x^2 \sqrt{4 - x^2} dx$$

$$7.5.15. \int_0^3 \frac{dx}{(9 + x^2)^{3/2}}$$

$$7.5.17. \int_0^{1/\sqrt{2}} \frac{x^4}{\sqrt{(1 - x^2)^3}} dx$$

$$7.5.19. \int_0^{\sqrt{3}} \sqrt{3 - x^2} dx$$

$$7.5.21. \int_0^7 x^2 \sqrt{49 - x^2} dx$$

$$7.5.23. \int_0^{2/\sqrt{2}} \frac{x^4 dx}{(16 - x^2)\sqrt{16 - x^2}}$$

$$7.5.25. \int_0^2 \frac{x^4}{\sqrt{(8 - x^2)^3}} dx$$

$$7.5.27. \int_0^{3/2} \frac{x^2}{\sqrt{9 - x^2}} dx$$

$$7.5.29. \int_0^1 \frac{x^2}{\sqrt{4 - x^2}} dx$$

$$7.5.6. \int_0^3 \sqrt{9 - x^2} dx$$

$$7.5.8. \int_0^2 \frac{dx}{(4 + x^2)^{3/2}}$$

$$7.5.10. \int_0^{\sqrt{3}} \frac{dx}{(4 - x^2)^{3/2}}$$

$$7.5.12. \int_0^3 \frac{dx}{(9 + x^2)^{3/2}}$$

$$7.5.14. \int_0^4 \sqrt{16 - x^2} dx$$

$$7.5.16. \int_0^{\sqrt{5}/2} \frac{dx}{(5 - x^2)^{3/2}}$$

$$7.5.18. \int_0^1 \frac{x^4 dx}{(2 - x^2)^{3/2}}$$

$$7.5.20. \int_0^{\sqrt{2}} x^2 \sqrt{2 - x^2} dx$$

$$7.5.22. \int_0^{4\sqrt{3}} \frac{dx}{(64 - x^2)^{3/2}}$$

$$7.5.24. \int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1 + x^2)^3}}$$

$$7.5.26. \int_0^1 \sqrt{4 - x^2} dx$$

$$7.5.28. \int_0^{\sqrt{2}/2} \frac{dx}{(1 - x^2)\sqrt{1 - x^2}}$$

$$7.5.30. \int_0^{\sqrt{2}} \frac{x^4 dx}{(4 - x^2)^{3/2}}$$