

The control tasks to Definite Integrals

Task 1. Find the area of the region bounded by the graphs of the functions:

Tasks 1.1

Variant	Tasks	Variant	Tasks
1	a) $y = x^2 + 3x - 7, y = -x - 7$; b) $y = 4 - x^2, y = 0$; c) $y = (x - 2)^2, y = 4x - 8$	2	a) $y = x^2 + 3x - 5, y = x + 3$; b) $xy = 4, x = 1, x = 4, y = 0$; c) $y = x\sqrt{9 - x^2}, y = 0, (0 \leq x \leq 3)$
3	a) $y = -x^2 + x, y = x - 9$; b) $y^2 = 2x + 4, x = 0$; c) $y = 4 - x^2, y = x^2 - x$	4	a) $y = -x^2 + 2x - 4, y = -3x$; b) $y = x^2, y = 2 - x^2$; c) $y = \sin x \cos^2 x, y = 0, \left(0 \leq x \leq \frac{\pi}{2}\right)$
5	a) $y = x^2 + 5x - 1, y = -2x^2 - x - 1$; b) $y = \ln x, x = e, y = 0$; c) $y = \sqrt{4 - x^2}, y = 0, x = 0, x = 1$	6	a) $y = x^2 - 2x - 2, y = -x$; b) $y = x^3, x = 0, y = 8$; c) $y = x^2\sqrt{4 - x^2}, y = 0, (0 \leq x \leq 2)$
7	a) $y = 2x^2 + 2x, y = -x^2 + 1$; b) $y = \sin x, x \in [0, \pi], y = 0$; c) $y = \sin^2 x \cos x, y = 0, \left(0 \leq x \leq \frac{\pi}{2}\right)$	8	a) $y = x^2 + 5x, y = 4x + 6$; b) $xy = 6, x + y = 7$; c) $y = \sqrt{e^x - 1}, y = 0, x = \ln 2$
9	a) $y = -x^2 + 8x + 3, y = 5x + 3$; b) $y = x, y = 2x, x = 3$; c) $y = 1/(x\sqrt{1 + \ln x}), y = 0, x = 1, x = e^3$	10	a) $y = -x^2 - 3x + 2, y = -6 - x$; b) $y = x, y = 3x, x = 2$; c) $y = \arccos x, y = 0, x = 0$
11	a) $y = x^2 + 4x - 3, y = -x^2 + 4x + 5$; b) $y = 1 - x^2, y = 0$; c) $y = (x + 1)^2, y^2 = x + 1$	12	a) $y = x^2 + 4x + 11, y = -x^2 - x + 9$; b) $y = 3 - x^2, x = 0, y = 0$; c) $y = 2x - x^2 + 3, y = x^2 - 4x + 3$
13	a) $y = x^2 + 7x + 2, y = x - 6$; b) $y = x, x = 2, y = 0$; c) $y = x\sqrt{36 - x^2}, y = 0, (0 \leq x \leq 6)$	14	a) $y = x^2 - 13x, y = -10x$; b) $y = e^x, y = e^{2x}, x = 1$; c) $x = \arccos y, y = 0, x = 0$
15	a) $y = x^2 + 2x - 3, y = 1 - x$; b) $y = 2^x, y = 2^{2x}, x = 1$; c) $y = x \arctg x, y = 0, x = \sqrt{3}$	16	a) $y = -x^2 + 6x - 6, y = -x + 4$; b) $y = \cos x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y = 0$; c) $y = x^2\sqrt{8 - x^2}, y = 0, (0 \leq x \leq 2\sqrt{2})$

17	<p>a) $y = -x^2 - x + 17, y = x + 17;$ b) $y = x^2, y^2 = 4 - x^2;$ c) $x = \sqrt{e^y - 1}, x = 0, y = \ln 2$</p>	18	<p>a) $y = -x^2 + 2x, y = -3;$ b) $xy = 3, x = 1, x = 3, y = 0;$ c) $y = x\sqrt{4 - x^2},$ $y = 0, (0 \leq x \leq 2)$</p>
19	<p>a) $y = x^2 - 1, y = -x^2 + 17;$ b) $y^2 = x + 1, x = 0;$ c) $y = x/(1 + \sqrt{x}), y = 0, x = 1$</p>	20	<p>a) $y = x^2 + x - 12, y = -12;$ b) $y^2 = x, y^2 = 2x, x = 1;$ c) $y = \frac{1}{1 + \cos x}, y = 0, x = \frac{\pi}{2}, x = -\frac{\pi}{2}$</p>
21	<p>a) $y = x^2 - 4x + 8, y = -2x^2 - 3x + 10;$ b) $y = x^2, y = 2x^2, x = 1;$ c) $x = (y - 2)^2, x = 4y - 8$</p>	22	<p>a) $y = x^2 - 3x - 1, y = 2 - x;$ b) $y = 5 - x^2, x = 0, y = 0;$ c) $y = \sin 2x \cos^5 x,$ $y = 0, \left(0 \leq x \leq \frac{\pi}{2}\right)$</p>
23	<p>a) $y = -x^2 + 2x + 3, y = 7x - 3;$ b) $xy = 1, x = 1, x = 2, y = 0;$ c) $y = \frac{x}{(x^2 + 1)^2}, y = 0, x = 1$</p>	24	<p>a) $y = x^2 + 7x + 1, y = 5x + 4;$ b) $y = x^{-1}, x = 2, x = 4, y = 0;$ c) $x = 4 - y^2, x = y^2 - y$</p>
25	<p>a) $y = 4 - x^2, y = x + 2;$ b) $y = x, y = 5x^{-1}, x = 3;$ c) $x = \frac{1}{y\sqrt{1 + \ln y}},$ $x = 0, y = 1, y = e^3$</p>	26	<p>a) $y = -x^2 + 8x + 1, y = 8x;$ b) $y = \lg x, x = 10, y = 0;$ c) $y = \frac{e^{1/x}}{x^2}, y = 0, x = 1, x = 2$</p>
27	<p>a) $y = x^2 + 4x, y = 3x + 6;$ b) $xy = 6, x + y = 5;$ c) $y = x^2\sqrt{16 - x^2},$ $y = 0, (0 \leq x \leq 4)$</p>	28	<p>a) $y = 3x^2 - x, y = -2x^2 + 4x;$ b) $y = x^2, y = 9 - x^2;$ c) $x = \sqrt{4 - y^2}, x = 0, y = 0, y = 1$</p>
29	<p>a) $y = -x^2 - 7x, y = -11x + 3;$ b) $y = \sin 2x, x \in \left[0; \frac{\pi}{2}\right], y = 0;$ c) $y = (x - 1)^2, y^2 = x - 1$</p>	30	<p>a) $y = 2x^2, y = 3 - x^2;$ b) $y = 3^x, y = 3^{2x}, x = 1;$ c) $x = 4 - (y - 1)^2,$ $x = y^2 - 4x + 3$</p>

Tasks 1.2 Find the area of the region bounded by curves defined with respect to parameter

1.	$\begin{cases} x = 4\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \end{cases}$ $x = 2 \quad (x \geq 2)$	2.	$\begin{cases} x = 4(t - \sin t) \\ y = 4(1 - \cos t) \end{cases}$ $y = 6, (0 < x < 8\pi, y \geq 6)$
3.	$\begin{cases} x = 3 \cos t \\ y = 8 \sin t \end{cases}$ $y = 4\sqrt{3} \quad (y \geq 4\sqrt{3})$	4.	$\begin{cases} x = 32 \cos^3 t \\ y = 3 \sin^3 t \end{cases}$ $x = 12\sqrt{3} \quad (x \geq 12\sqrt{3})$
5.	$\begin{cases} x = 8(t - \sin t) \\ y = 8(1 - \cos t) \end{cases}$ $y = 12, (0 < x < 16\pi, y \geq 4\sqrt{3})$	6.	$\begin{cases} x = 2\sqrt{2} \cos t \\ y = 5\sqrt{2} \sin t \end{cases}$ $y = 5 \quad (y \geq 5)$
7.	$\begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \end{cases}$ $y = 2, (0 < x < 4\pi, y \geq 2)$	8.	$\begin{cases} x = 9 \cos t \\ y = 4 \sin t \end{cases}$ $y = 2 \quad (y \geq 2)$
9.	$\begin{cases} x = 24 \cos^3 t \\ y = 2 \sin^3 t \end{cases}$ $x = 9\sqrt{3} \quad (x \geq 9\sqrt{3})$	10.	$\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$ $y = 1, (0 < x < 2\pi, y \geq 1)$
11.	$\begin{cases} x = 8 \cos^3 t \\ y = 8 \sin^3 t \end{cases}$ $x = 1 \quad (x \geq 1)$	12.	$\begin{cases} x = \sqrt{2} \cos t \\ y = 4\sqrt{2} \sin t \end{cases}$ $y = 4 \quad (y \geq 4)$
13.	$\begin{cases} x = 2\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \end{cases}$ $x = 1 \quad (x \geq 1)$	14.	$\begin{cases} x = 6 \cos t \\ y = 4 \sin t \end{cases}$ $y = 2\sqrt{3} \quad (y \geq 2\sqrt{3})$
15.	$\begin{cases} x = 6(t - \sin t) \\ y = 6(1 - \cos t) \end{cases}$ $y = 6, (0 < x < 12\pi, y \geq 6)$	16.	$\begin{cases} x = 32 \cos^3 t \\ y = \sin^3 t \end{cases}$ $x = 4 \quad (x \geq 4)$
17.	$\begin{cases} x = 2\sqrt{2} \cos t \\ y = 3\sqrt{2} \sin t \end{cases}$ $y = 3 \quad (y \geq 3)$	18.	$\begin{cases} x = 3(t - \sin t) \\ y = 3(1 - \cos t) \end{cases}$ $y = 3, (0 < x < 6\pi, y \geq 3)$
19.	$\begin{cases} x = 16 \cos^3 t \\ y = \sin^3 t \end{cases}$ $x = 6\sqrt{3} \quad (x \geq 6\sqrt{3})$	20.	$\begin{cases} x = 2 \cos t \\ y = 6 \sin t \end{cases}$ $y = 3 \quad (y \geq 3)$

21.	$\begin{cases} x = 4(t - \sin t) \\ y = 4(1 - \cos t) \end{cases}$ $y = 4, (0 < x < 8\pi, y \geq 4)$	22.	$\begin{cases} x = 4\sqrt{2} \cos^3 t \\ y = 2\sqrt{2} \sin^3 t \end{cases}$ $x = 2 \quad (x \geq 2)$
23.	$\begin{cases} x = 10(t - \sin t) \\ y = 10(1 - \cos t) \end{cases}$ $y = 15, (0 < x < 20\pi, y \geq 15)$	24.	$\begin{cases} x = 8 \cos^3 t \\ y = 4 \sin^3 t \end{cases}$ $x = 3\sqrt{3} \quad (x \geq 3\sqrt{3})$
25.	$\begin{cases} x = 3 \cos t \\ y = 8 \sin t \end{cases}$ $y = 4 \quad (y \geq 4)$	26.	$\begin{cases} x = 6(t - \sin t) \\ y = 6(1 - \cos t) \end{cases}$ $y = 9, (0 < x < 12\pi, y \geq 9)$
27.	$\begin{cases} x = 8\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \end{cases}$ $x = 4 \quad (x \geq 4)$	28.	$\begin{cases} x = 6 \cos t \\ y = 2 \sin t \end{cases}$ $y = \sqrt{3} \quad (y \geq \sqrt{3})$
29.	$\begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \end{cases}$ $y = 3, (0 < x < 4\pi, y \geq 3)$	30.	$\begin{cases} x = 16 \cos^3 t \\ y = 2 \sin^3 t \end{cases}$ $x = 2 \quad (x \geq 2)$

Tasks 1.3 Find the area of the region bounded by curves defined in polar coordinate system.

1	$\rho = \frac{1}{2} - \sin \varphi$	2	$\rho = 3 \cos 2\varphi$
3	$\rho = 2(1 - \cos \varphi)$	4	$\rho^2 = \frac{3}{2} \cos 3\varphi$
5	$\rho = 2 \sin 3\varphi$	6	$\rho = 4 + \cos \varphi$
7	$\rho^2 = 3 \cos 2\varphi$	8	$\rho = \frac{1}{2}(1 + \sin \varphi)$
9	$\rho = 1 + \cos 2\varphi$	10	$\rho = \frac{2}{5}(1 + \cos \varphi)$
11	$\rho = 5 \sin 4\varphi$	12	$\rho = 2 + \sin 4\varphi$
13	$\rho^2 = 1 + \cos 2\varphi$	14	$\rho = 1 + \cos \varphi$
15	$\rho = 3 \cos^2 \varphi$	16	$\rho^2 = \frac{3}{2} \sin 4\varphi$
17	$\rho = 1 + \sin \varphi$	18	$\rho = 5 + \frac{2}{3} \cos 4\varphi$
19	$\rho = \sin 2\varphi$	20	$\rho = \frac{1}{2} \cos 3\varphi$
21	$\rho^2 = 1 + \cos 2\varphi$	22	$\rho = 1 + \sin \varphi$
23	$\rho = 1 + \sqrt{3} \cos \varphi$	24	$\rho = \cos \varphi, \rho = 3 \cos \varphi$
25	$\rho = 4 \sin \varphi, \rho = 2 \sin \varphi$	26	$\rho = 4 \sin 4\varphi$
27	$\rho = 2 \cos 4\varphi$	28	$\rho = \frac{1}{2} + \cos \varphi$

Task 2. Calculate the arc length of the graph of $y = f(x)$ over the given interval:

Tasks 2.1.

1. $y = \ln \frac{5}{2x}, \quad \sqrt{3} \leq x \leq \sqrt{8}$
2. $y = e^x + 6, \quad \ln \sqrt{8} \leq x \leq \ln \sqrt{15}$
3. $y = 1 - \ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$
4. $y = \sqrt{1-x^2} + \arcsin x, \quad 0 \leq x \leq \frac{7}{9}$
5. $y = \ln(x^2 - 1), \quad 2 \leq x \leq 3$
6. $y = -\ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$
7. $y = \arcsin x - \sqrt{1-x^2},$
 $0 \leq x \leq \frac{15}{16}$
8. $y = 2 + \operatorname{ch} x, \quad 0 \leq x \leq 1$
9. $y = \ln(1-x^2), \quad 0 \leq x \leq \frac{1}{4}$
10. $y = 2 + \arcsin \sqrt{x} + \sqrt{x-x^2},$
 $\frac{1}{4} \leq x \leq 1$
11. $y = 1 - \ln(x^2 - 1), \quad 3 \leq x \leq 4$
12. $y = (1 - e^x - e^{-x})/2, \quad 0 \leq x \leq 3$
13. $y = 4 + \arccos \sqrt{x} - \sqrt{x-x^2},$
 $0 \leq x \leq \frac{1}{2}$
14. $y = e^x + e, \quad \ln \sqrt{3} \leq x \leq \ln \sqrt{15}$
15. $y = \ln \frac{7}{4} - \ln x, \quad \sqrt{3} \leq x \leq \sqrt{8}$
16. $y = 2 + \ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$
17. $y = (e^{2x} + e^{-2x} + 3)/4,$
 $0 \leq x \leq 2$
18. $y = 3 + \operatorname{ch} x, \quad 0 \leq x \leq 1$
19. $y = 1 + \arcsin x - \sqrt{1-x^2},$
 $0 \leq x \leq \frac{3}{4}$
20. $y = e^x + 7, \quad \ln \sqrt{8} \leq x \leq \ln \sqrt{24}$
21. $y = \ln \sin x, \quad \frac{\pi}{3} \leq x \leq \frac{\pi}{2}$
22. $y = -\arccos x + \sqrt{1-x^2} + 2,$
 $0 \leq x \leq \frac{9}{16}$
23. $y = -\arccos \sqrt{x} - \sqrt{x-x^2} + 5,$
 $\frac{1}{9} \leq x \leq 1$
24. $y = 1 - \ln \sin x, \quad \frac{\pi}{3} \leq x \leq \frac{\pi}{2}$
25. $y = e^x - 2, \quad \ln \sqrt{3} \leq x \leq \ln \sqrt{8}$
26. $y = \ln x, \quad \sqrt{3} \leq x \leq \sqrt{15}$
27. $y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2$
28. $y = 6 - e^x, \quad \ln \sqrt{8} \leq x \leq \ln \sqrt{15}$
29. $y = 3 + (e^x + e^{-x})/2, \quad 0 \leq x \leq 2$
30. $y = 1 - \ln \cos x, \quad 0 \leq x \leq \frac{\pi}{6}$

Tasks 2.2 Calculate the arc length of the graph of $y = f(x)$ in polar coordinates over the given interval:

1. $\rho = 2(1 - \sin \varphi), \quad -\frac{\pi}{4} \leq \varphi \leq 0$

2. $\rho = 3(1 + \sin \varphi), \quad 0 \leq \varphi \leq \frac{\pi}{2}$

3. $\rho = 4(1 - \cos \varphi), \quad -\frac{\pi}{2} \leq \varphi \leq -\frac{\pi}{4}$

4. $\rho = 12e^{12\varphi/5}, \quad 0 \leq \varphi \leq \frac{\pi}{3}$

5. $\rho = 4e^{4\varphi/3}, \quad \frac{\pi}{4} \leq \varphi \leq \frac{\pi}{2}$

6. $\rho = 3(1 - \cos \varphi), \quad -\frac{\pi}{4} \leq \varphi \leq 0$

7. $\rho = 21(1 + \sin \varphi), \quad -\frac{\pi}{3} \leq \varphi \leq \frac{\pi}{3}$

8. $\rho = 1 - \sin \varphi, \quad 0 \leq \varphi \leq \frac{\pi}{2}$

9. $\rho = 7e^{7\varphi/12}, \quad 0 \leq \varphi \leq \frac{\pi}{4}$

10. $\rho = \sqrt{3}e^{2\varphi}, \quad -\frac{\pi}{2} \leq \varphi \leq -\frac{\pi}{4}$

11. $\rho = \sqrt{2}e^\varphi, \quad 0 \leq \varphi \leq \frac{\pi}{2}$

12. $\rho = 6e^{12\varphi/5}, \quad 0 \leq \varphi \leq \frac{\pi}{4}$

13. $\rho = 8(1 - \cos \varphi), \quad -\frac{2\pi}{3} \leq \varphi \leq 0$

14. $\rho = 3 \cos \varphi, \quad 0 \leq \varphi \leq \frac{\pi}{6}$

15. $\rho = 6 \sin \varphi, \quad 0 \leq \varphi \leq \frac{\pi}{2}$

16. $\rho = 11\varphi, \quad 0 \leq \varphi \leq \frac{8}{7}$

17. $\rho = 3e^{3\varphi}, \quad -\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{4}$

18. $\rho = 7e^{7\varphi/3}, \quad 0 \leq \varphi \leq \frac{\pi}{3}$

19. $\rho = 2 \sin \varphi, \quad 0 \leq \varphi \leq \frac{2\pi}{3}$

20. $\rho = 3\varphi, \quad 0 \leq \varphi \leq \frac{4}{7}$

21. $\rho = 5(1 - \cos \varphi), \quad -\frac{\pi}{3} \leq \varphi \leq 0$

22. $\rho = a \sin^3 \frac{\varphi}{3}, \quad 0 \leq \varphi \leq \frac{\pi}{2}$

23. $\rho = a\varphi, \quad 0 \leq \varphi \leq 2\pi$

24. $\rho = a(1 + \cos \varphi), \quad 0 \leq \varphi \leq 2\pi$

25. $\rho = \frac{1}{\varphi}, \quad \frac{1}{2} \leq \varphi \leq 2$

26. $\rho = a \sec^2 \frac{\varphi}{2}, \quad 0 \leq \varphi \leq \frac{\pi}{2}$

27. $\rho = 4\varphi, \quad 0 \leq \varphi \leq \frac{3}{4}$

28. $\rho = 6 \cos \varphi, \quad 0 \leq \varphi \leq \frac{\pi}{4}$

29. $\rho = \sqrt{3}e^{2\varphi}, \quad -\frac{\pi}{3} \leq \varphi \leq \frac{\pi}{3}$

30. $\rho = 2e^{4\varphi/3}, \quad -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$

Tasks 3. Find the volumes of the solids of revolution generated by revolving the region between the graphs of the functions around (a) the x -axis; and (b) the y -axis.

1	$y = 2x^2; y = -2x + 4$	2	$y = \frac{1}{4}x^2; y = -\frac{1}{2}x + 2$
3	$y = \frac{1}{4}x^2; y = -2x + 6$	4	$y = x^2; y = -x + 2$
5	$y = 4x^2; y = -2x + 6$	6	$y = 2x^2; y = -x + 10$
7	$y = 3x^2; y = -x + 4$	8	$y = x^2; y = -x + 3$
9	$y = 3x^2; y = -3x + 6$	10	$y = 2x^2; y = -3x + 14$
11	$y = \frac{1}{4}x^2; y = -x + 3$	12	$y = 2x^2; y = -2x + 5$
13	$y = \frac{1}{2}x^2; y = -3x + 8$	14	$y = \frac{1}{3}x^2; y = -x + 6$
15	$y = \frac{1}{2}x^2; y = -x + 3$	16	$y = \frac{1}{3}x^2; y = -3x + 12$
17	$y = 3x^2; y = -2x + 5$	18	$y = 3x^2; y = -5x + 8$
19	$y = 4x^2; y = -2x + 2$	20	$y = \frac{1}{3}x^2; y = -x + 9$
21	$y^2 = 4 - x; x = 0$	22	$y^2 = 9x; y = -x$
23	$y = 2x - x^2; y = 2 - x$	24	$y = 4x - x^2; y = x$
25	$y = x^2; y^2 = 8x$	26	$y^2 = 4x; x^2 = 4y$
27	$xy = 4; y = -2x + 6$	28	$y^2 = x; y = \frac{x}{2}$