The control tasks to Functions of Several Variables

Task 1. Find the domain of the functions and plot their graphs.

1.1 $z=\dfrac{y}{\sqrt{4-x^2-2y^2}}$;

1.2 $z=\dfrac{1}{9-x^2-y^2}$;

1.3 $z=\ln\left(1-x^2-y^2\right)$

1.4 $z=\arcsin(x^2+y^2-3)$;

1.5 $z=\dfrac{1}{ln(x^2-y^2)}$;

1.6 $z=\sqrt{x^2+y^2-1}$

1.7 $z=\ln(y^2-x^2)$;

1.8 $z=arccos\dfrac{x}{x+y}$;

1.9 $z=\dfrac{1}{\sqrt{1-x^2-y^2}}$;

Task 2. Calculate

2.1 $u=x+ye^{\frac{x}{y}}, \quad du=?$

2.2 $u=(\frac{x}{y})^{\frac{1}{z}}, \quad du(1;1;1)=?$

2.3 $z=\dfrac{y}{x^2-y^2}$. Prove $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=-z.$

2.4 $u=e^{\frac{x}{y}}+e^{\frac{z}{y}}, \quad du=?$

2.5 $z=\dfrac{x^3}{x-y}$ Prove $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=2z.$

2.6 $u=\dfrac{z}{\sqrt{x^2+y^2}}, \quad du(3;4;5)=?$

2.7 $z=e^{\frac{x}{y}}\ln y$ Prove $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=\dfrac{z}{\ln y}.$

2.8 $u = arctg\,\frac{x+y}{1-xy}, \quad du = ?$

2.9 $u = x + \frac{x-y}{y-z}.$ Prove $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 1.$

Task 3. Calculate using the Chain Rule

3.1 $u = \frac{yz}{x}$, where $x = e^t, y = ln\,t, z = t^2 - 1$. Find $\frac{dz}{dt}$.

3.2 $u = arcsin\frac{x}{z}$, where $z = \sqrt{x^2+1}$. Find $\frac{\partial u}{\partial x}$ and $\frac{du}{dx}$.

3.3 $z = arctg\,\frac{y}{x}$, where $x = e^{2t+1}, y = e^{2t-1}$. Find $\frac{dz}{dt}$.

3.4 $u = xyz$, where $x = t^2 + 1,\ y = \ln t,\ z = \tan t$. Find $\frac{du}{dt}$.

3.5 $z = ln(x^2 + y^2)$, where $y = \frac{1}{3}x^3 + x$. Find $\frac{dz}{dx}$.

3.6 $z = tan(3t + 2x^2 - y)$, where $x = \frac{1}{t}, y = \sqrt{t}$. Find $\frac{dz}{dt}$.

3.7 $z = arctan(xy)$, where $y = e^x$. Find $\frac{dz}{dx}$.

3.8 $z = x^2y - y^2x$, where $x = u\cos v,\ y = u\sin v$. Find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$.

3.9 $z = x^2\,ln\,y$, where $x = \frac{u}{v}, y = 3u - 2v$. Find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$.

Task 4. Find $\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial x \partial y}$ for the following functions:

4.1 $u = \left(\frac{x}{y}\right)^z$

4.2 $u = x^{\frac{y}{z}}$

4.3 $u = e^{xyz}$

4.4 $u = sin(x + y + z)$

4.5 $u = ze^x\cos(y)$

4.6 $u = \arcsin(xyz)$

4.7 $u = xz\cos(y) + y\sin(xz)$

4.8 $u = \left(\frac{y}{x}\right)^z$

4.9 $u = \cos(xyz)$

Task 5. Find partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of an implicitly given function z:

5.1 $ye^{x-yz} - \cos(xz) = 0$

5.2 $\frac{\tan(z+1)}{y-x} = e^{x+y}$

5.3 $3^{xy} - \ln(z^2 - xy) = 5$

5.4 $\sin x = z \ln\frac{x}{y}$

5.5 $\arcsin\frac{x}{y} + 2^{x+y+z} = 0$

5.6 $x - z + \arctan\frac{y}{z-x} = 0$

5.7 $x\sin y + y\cos z + z\tan x = 1$

5.8 $\cot(x+z) = xy2^z$

5.9 $2xe^{yz} - y\sin xz = 0$

Task 6.

6.1 Find the equations of the tangent plane and the normal to the surface defined by the function $x + 2y - \ln z + 4 = 0$ at a point (2,-3,1)

6.2 Find the equations of the tangent plane and the normal to the surface defined by the function $z = 2x^2y^2 + 2xy + 1$ at a point (-1,1,1)

6.3 Find the equations of the tangent plane and the normal to the surface defined by the function $x^2y^3z^2 + 1 = 0$ at a point (-1,-1,-1)

6.4 Find the equations of the tangent plane and the normal to the surface defined by the function $x^3 + y^3 + z^3 + xyz = 6$ at a point (1,2,-1)

6.5 Find the equations of the tangent plane and the normal to the surface defined by the function $z = ye^{-2x} + x - 2xy$ at a point (0,2,2)

6.6 Find the equations of the tangent plane and the normal to the surface defined by the function $z = x^3y^2 - 2x + 1$ at a point (1,1,0)

6.7	Find the equations of the tangent plane and the normal to the surface defined by the function $x^2z + zy^2 - 4 = 0$ at a point (-2,0,1)
6.8	Find the equations of the tangent plane and the normal to the surface defined by the function $z = y(x+1)^2 + 1$ at a point (0,1,2)
6.9	Find the equations of the tangent plane and the normal to the surface defined by the function $z = x^2y^2 - 1$ at a point (-1,1,0)

Task 7. Find a directional derivative of function $w = w(x, y, z)$ in the direction of vector $\vec{u}$ at a point M_0

7.1	$w(x, yz) = xyz$, $\vec{u} = 2\vec{j} - \vec{k}$, $M_0(1,1,1)$
7.2	$w(x, y, z) = x^2 + y^2 + z^2$, $\vec{u} = \vec{i} + 2\vec{j}$, $M_0(2,3,1)$
7.3	$w(x, y, z) = x^2y^2z + \ln(z+1)$, $\vec{u} = 2\vec{i} + \vec{k}$, $M_0(4,-3,0)$
7.4	$w(x, y, z) = \ln(xy + yz + xz)$, $\vec{u} = \vec{i} - \vec{j} - \vec{k}$, $M_0(0,1,1)$
7.5	$w(x, y, z) = 3yz - \frac{z}{x}$, $\vec{u} = \vec{i} - \vec{j} + \sqrt{2}\vec{k}$, $M_0(-1,2,3)$
7.6	$w(x, y, z) = \sqrt{xy} + \sqrt{4 - z^2}$, $\vec{u} = 2\vec{i} - 2\vec{j} + \vec{k}$, $M_0(-1,-1,0)$
7.7	$w(x, y, z) = x^2 + y^2 + z^2$, $\vec{u} = \vec{i} + 2\vec{j}$, $M_0(2,3,1)$
7.8	$w(x, y, z) = x^2 + y^2 - 3x + 2z$, $\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$, $M_0(3,4,0)$
7.9	$w(x, y, z) = x^2z - 2xyz + z^2$, $\vec{u} = \vec{i} + \sqrt{2}\vec{j} + \vec{k}$, $M_0(3,1,1)$

Task 8. Find Local Extrema of the Function:

8.1	$z = x^2 + xy + y^2 + x + y - 1$
8.2	$z = 3x^2 + 3y^2 - 2x - 2y + 2$
8.3	$z = x^2 + 2xy - 3y^2 + 1$
8.4	$z = -3x^2 + 2xy - 2y^2 + 8$
8.5	$z = -x^2 - xy - y^2 + 3x + 6y$
8.6	$z = x^2 + xy + y^2 - 6x - 9y$
8.7	$z = x^3 + y^3 - 9xy$
8.8	$z = x^3 - 6xy + 8y^3 + 1$
8.9	$z = x^2y - xy + xy^2$

Task 9. Finding Absolute Maximum and Minimum Values of z on D

9.1	$z = 3x^2 + 3y^2 - 2x - 2y + 2$, D: $x = 0$, $y = 0$, $y = 1 - x$
9.2	$z = x^2 + y^2 - 6x + 4y + 3$, D: a rectangle given by $A(1, -3)$, $B(1,2)$, $C(4,2)$, $D(4, -3)$
9.3	$z = -9x^2 + 6xy - 9y^2 + 4x + 4y$, D: $x = 0$, $x = 1$, $y = 0$, $y = 2$
9.4	$z = x^2 + y^2 - 2x - 2y + 3$, D: $x \geq 0$, $y \geq 0$, $4y + 3x \leq 12$
9.5	$z = 3x^2 + 3y^2 - x - y + 1$, D: $x = 5$, $y = 0$, $y = x - 1$
9.6	$z = 5x^2 - xy + y^2 - 4x$, D: $x = -1$, $y = -1$, $y = 1 - x$
9.7	$z = x^2 + 3y^2 + x - y$, D: $x = 0$, $y = 0$, $y = 1 - x$
9.8	$z = x^2 - xy + 2y^2 + 3x + 2y + 1$ D: $x = 0$, $y = 0$, $y + x + 5 = 0$
9.9	$z = x^3 + 3xy + y^3 - 1$, D: $0 \leq x \leq 2$, $-1 \leq y \leq 2$