

Obligatory homework on Ordinary Differential Equations

Task 1. Solve the separable differential equation:

$$1.1 \quad 4x dx - 3y dy = 3x^2 y dy - 2xy^2 dx$$

$$1.2 \quad x\sqrt{1+y^2} + yy'\sqrt{1+x^2} = 0$$

$$1.3 \quad \sqrt{4+y^2} dx - y dy = x^2 y dy$$

$$1.4 \quad \sqrt{3+y^2} dx - y dy = x^2 y dy$$

$$1.5 \quad 6x dx - 6y dy = 2x^2 y dy - 3xy^2 dx$$

$$1.6 \quad x\sqrt{3+y^2} dx + y\sqrt{2+x^2} dy = 0$$

$$1.7 \quad (e^{2x} + 5) dy + ye^{2x} dx = 0$$

$$1.8 \quad y'y \sqrt{\frac{1-x^2}{1-y^2}} + 1 = 0$$

$$1.9 \quad 6x dx - 6y dy = 3x^2 y dy - 2xy^2 dx$$

$$1.10 \quad x\sqrt{5+y^2} dx + y\sqrt{4+x^2} dy = 0$$

Task 2. Solve the homogeneous differential equation:

$$2.1 \quad y' = \frac{y^2}{x^2} + 4\frac{y}{x} + 2$$

$$2.2 \quad xy' = \frac{3y^3 + 2yx^2}{2y^2 + x^2}$$

$$2.3 \quad y' = \frac{x+y}{x-y}$$

$$2.4 \quad xy' = \sqrt{x^2 + y^2} + y$$

$$2.5 \quad 2y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 3$$

$$2.6 \quad xy' = \frac{3y^3 + 4yx^2}{2y^2 + 2x^2}$$

$$2.7 \quad y' = \frac{x+2y}{2x-y}$$

$$2.8 \quad xy' = 2\sqrt{x^2 + y^2} + y$$

$$2.9 \quad 3y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4$$

$$2.10 \quad xy' = \frac{3y^3 + 6yx^2}{2y^2 + 3x^2}$$

Task 3. Solve the homogeneous differential equation, where the numerator and denominator of the right part are the equations of intersecting straight lines.

$$3.1 \quad y' = \frac{x+2y-3}{2x-2}$$

$$3.2 \quad y' = \frac{x+y-2}{2x-2}$$

$$3.3 \quad y' = \frac{3y-x-4}{3x+3}$$

$$3.4 \quad y' = \frac{2y-2}{x+y-2}$$

$$3.5 \quad y' = \frac{x+y-2}{3x-y-2}$$

$$3.6 \quad y' = \frac{2x+y-3}{x-1}$$

$$3.7 \quad y' = \frac{x+7y-8}{9x-y-8}$$

$$3.8 \quad y' = \frac{x+3y+4}{3x-6}$$

$$3.9 \quad y' = \frac{3y+3}{2x+y-1}$$

$$3.10 \quad y' = \frac{x+2y-3}{4x-y-3}$$

Task 4. Solve the initial value problem for the linear differential equation of first order:

$$4.1 \quad y' - \frac{y}{x} = x^2, \quad y(1) = 0.$$

$$4.2 \quad y' - y \operatorname{ctgx} = 2x \sin x, \quad y(\pi/2) = 0$$

$$4.3 \quad y' + y \cos x = \frac{1}{2} \sin 2x, \quad y(0) = 0$$

$$4.4 \quad y' + y \operatorname{tgx} = \cos^2 x, \quad y(\pi/4) = 1/2$$

$$4.5 \quad y' - \frac{y}{x+2} = x^2 + 2x, \quad y(-1) = 3/2.$$

$$4.6 \quad y' - \frac{1}{x+1} y = e^x (x+1), \quad y(0) = 1$$

$$4.7 \quad y' - \frac{y}{x} = x \sin x, \quad y(\pi/2) = 1$$

$$4.8 \quad y' + \frac{y}{x} = \sin x, \quad y(\pi) = 1/\pi$$

$$4.9 \quad y' + \frac{y}{2x} = x^2, \quad y(1) = 1.$$

$$4.10 \quad y' + \frac{2x}{1+x^2} y = \frac{2x^2}{1+x^2}, \quad y(0) = \frac{2}{3}$$

Task 5. Solve the initial value problem for the Bernoulli equation:

$$5.1 \quad y' + xy = (1+x)e^{-x} y^2, \quad y(0) = 1.$$

$$5.2 \quad xy' + y = 2y^2 \ln x, \quad y(1) = 1/2$$

$$5.3 \quad 2(xy' + y) = xy^2, \quad y(1) = 2$$

$$5.4 \quad y' + 4x^3 y = 4(x^3 + 1)e^{-4x} y^2, \quad y(0) = 1$$

$$5.5 \quad xy' - y = -y^2(\ln x + 2) \ln x, \quad y(1) = 1$$

$$5.6 \quad 2(y' + xy) = (1+x)e^{-x} y^2, \quad y(0) = 2$$

$$5.7 \quad 3(xy' + y) = y^2 \ln x, \quad y(1) = 3$$

$$5.8 \quad 2y' + y \cos x = y^{-1} \cos x(1 + \sin x)$$

$$y(0) = 1$$

$$5.9 \quad y' + 4x^3 y = 4y^2 e^{4x} (1 - x^3), \quad y(0) = -1.$$

$$5.10 \quad 3y' + 2xy = 2xy^{-2} e^{-2x^2}, \quad y(0) = -1$$