Obligatory homework on Higher Order Ordinary Differential Equations

Task 1. Find the general solution and a solution of the IVP of the following higher order differential equations which allow the order reduction:

Variant	(a)	(b)	
1.1	$y''' \cdot x \cdot \ln x = y''$.	$4y^3 y'' = y^4 - 1$,	$y(0) = \sqrt{2}$, $y'(0) = 1/(2\sqrt{2})$
1.2	$x \cdot y''' + y'' = 1$.	$y'' = 128 y^3$,	$y(0) = 1$, $y'(0) = 8$.
1.3	$2xy''' = y''$.	$y'' y^3 + 64 = 0$,	$y(0) = 4$, $y'(0) = 2$.
1.4	$xy''' + y'' = x + 1$.	$y'' + 2\sin y \cdot \cos^3 y = 0$,	$y(0) = 0$, $y'(0) = 1$.
1.5	$tgx \cdot y'' - y' + \dfrac{1}{\sin x} = 0$.	$y'' = 32 \sin^3 y \cdot \cos y$,	$y(1) = \pi/2$, $y'(1) = 4$.
1.6	$x^2 y'' + xy' = 1$.	$y'' = 98 y^3$,	$y(1) = 1$, $y'(1) = 7$.
1.7	$y''' ctg 2x + 2y'' = 0$.	$y'' y^3 + 49 = 0$,	$y(3) = -7$, $y'(3) = -1$.
1.8	$x^3 y''' + x^2 y'' = 1$.	$4y^3 y'' = 16 y^4 - 1$,	$y(0) = \sqrt{2}/2$, $y'(0) = 1/\sqrt{2}$.
1.9	$tgx \cdot y''' = 2y''$.	$y'' + 8 \cdot \sin y \cdot \cos^3 y = 0$	$y(0) = 0$, $y'(0) = 2$.
1.10	$x^4 y'' + x^3 y' = 1$.	$y'' y^3 + 36 = 0$,	$y(0) = 3$, $y'(0) = 2$.

Task 2. Find the general solution of the following *Linear Homogeneous Differential Equations* with constant coefficients.

Variant	(a)	(b)
2.1	$y''' + 4y' = 0$	$y''' + y'' + y' + y = 0$
2.2	$y^{IV} + 16y'' = 0$	$y''' - 13y'' + 12y' = 0$
2.3	$y^{IV} + 4y''' + 4y'' = 0$	$y''' + 2y'' + 2y' + y = 0$
2.4	$y''' + 8y = 0$	$y^{IV} - 5y'' + 4y = 0$
2.5	$y^{IV} - 3y''' + 3y'' = 0$	$y''' + 2y'' - y' - 2y = 0$
2.6	$y''' + y' = 0$	$3y^{IV} + y''' = 0$
2.7	$y''' + 4y' = 0$	$y''' - 5y'' + 8y' - 4y = 0$

2.8	$y^{IV}+y''=0$	$y'''-3y''+3y'-y=0$
2.9	$y^{IV}+3y'''+5y''=0$	$y'''-2y''+y'=0$
2.10	$y^{IV}+16y''=0$	$y'''-y''+2y'+4y=0$

Task 3. Solve the Linear Nonhomogeneous Differential Equations with constant coefficients.

Variant	(a)	(b)
3.1	$y'''-4y''+5y'-2y=(16-12x)e^{-x}$	$y''+2y'=4e^{x}(\sin x+\cos x)$
3.2	$y'''-3y''+2y'=(1-2x)e^{x}$.	$y''-4y'+4y=-e^{2x}\sin 6x$.
3.3	$y'''-y''-y'+y=(3x+7)e^{2x}$.	$y''+2y'=-2e^{x}(\sin x+\cos x)$.
3.4	$y'''-2y''+y'=(2x+5)e^{2x}$.	$y''+y=2\cos 7x+3\sin 7x$.
3.5	$y'''-3y''+4y=(18x-21)e^{-x}$.	$y''+2y'+5y=-\sin 2x$.
3.6	$y'''-5y''+8y'-4y=(2x-5)e^{x}$.	$y''-4y'+8y=e^{x}(5\sin x-3\cos x)$
3.7	$y'''-4y''+4y'=(x-1)e^{x}$.	$y''-4y'+4y=-e^{2x}\sin 2x$.
3.8	$y'''+2y''+y'=(18x+21)e^{2x}$.	$y''-4y'+4y=e^{2x}\sin 3x$.
3.9	$y'''+y''-y'-y=(8x+4)e^{x}$.	$y''+6y'+13y=e^{-3x}\cos 4x$.
3.10	$y'''-3y'-2y=-4xe^{x}$.	$y''+y=2\cos 3x-3\sin 3x$.

Task 4. Solve the initial value problem for the linear nonhomogeneous differential equation by the Lagrange method

Variant		
4.1	$y''+\pi^2 y=\pi^2/\cos\pi x$	$y(0)=3\ y'(0)=0$
4.2	$y''+3y'=9e^{3x}/(1+e^{3x})$,	$y(0)=\ln 4\ y'(0)=3(1-\ln 2)$
4.3	$y''+4y=8ctg 2x$	$y(\pi/4)=5\ y'(\pi/4)=4$
4.4	$y''-6y'+8y=4/(1+e^{-2x})$	$y(0)=1+2\ln 2\ y'(0)=6\ln 2$
4.5	$y''-9y'+18y=9e^{3x}/(1+e^{-3x})$	$y(0)=0\ y'(0)=0$
4.6	$y''+\pi^2 y=\pi^2/\sin\pi x$	$y\left(\frac{1}{2}\right)=1\ y'\left(\frac{1}{2}\right)=\pi^2/2$
4.7	$y''+16y=16/\sin 4x$	$y(\pi/8)=3\ y'(\pi/8)=2\pi$
4.8	$y''+16y=16/\cos 4x$	$y(0)=3\ y'(0)=0$

4.9	$y'' - 3y' + 2y = e^x/(1+e^{-x})$	$y(0) = 0$ $y'(0) = 0$
4.10	$y'' + y = 1/\sin x$	$y(\pi/2) = 1$ $y'(\pi/2) = \pi/2$

Task 5. Solve the systems of linear nonhomogeneous differential equations:

Variant	
5.1	$\begin{cases} y_1' = y_2 - \cos x, \\ y_2' = -y_1 + \sin x. \end{cases}$
5.2	$\begin{cases} y_1' = y_1 - y_2 + 8x, \\ y_2' = 5y_1 - y_2. \end{cases}$
5.3	$\begin{cases} y_1' = 2y_1 - 4y_2, \\ y_2' = y_1 - 2y_2 + 2\sin 2x. \end{cases}$
5.4	$\begin{cases} y_1' = -y_2 + x^2, \\ y_2' = y_1 + x. \end{cases}$
5.5	$\begin{cases} y_1' = y_1 - y_2 + \dfrac{1}{\cos x}, \\ y_2' = 2y_1 - y_2. \end{cases}$
5.6	$\begin{cases} y_1' = y_1 + 2y_2 + e^x \sin 2x, \\ y_2' = -2y_1 + y_2. \end{cases}$
5.7	$\begin{cases} y_1' = y_2, \\ y_2' = -y_1 + \dfrac{1}{\cos x}. \end{cases}$
5.8	$\begin{cases} y_1' = -2y_2 + 3, \\ y_2' = 2y_1 - 2x. \end{cases}$
5.9	$\begin{cases} y_1' = -y_2 + \sin x \\ y_2' = y_1 + \cos x. \end{cases}$
5.10	$\begin{cases} y_1' = y_2 + \operatorname{tg}^2 x - 1, \\ y_2' = -y_1 + \operatorname{tg} x. \end{cases}$