

# I. Individual Compulsory Homework: Limits and Continuity

**Problem 1.** Find the limits:

## Variant 1

1.  $\lim_{x \rightarrow \infty} \frac{4x^3 - 2x + 4}{2x^3 - 4x^2 + 3}$

4.  $\lim_{x \rightarrow 0} \frac{\sqrt[3]{2+x} - \sqrt[3]{2-x}}{2x^3 - 7x}$

7.  $\lim_{x \rightarrow 0} \frac{\cos 7x - 1}{x \sin 3x}$

10.  $\lim_{x \rightarrow \infty} \left( \frac{5-x}{11-2x} \right)^{\frac{1+3x^2}{x}}$

2.  $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^4 + 5x - 3} + \sqrt[3]{2x^3 + 5x^2 + 4}}{\sqrt[4]{7x^3 + 4x^2 + 3} - \sqrt{2x^5 + 7x^4 - 5}}$

5.  $\lim_{x \rightarrow 8} \frac{\sqrt{x^2 - 15} - 7}{8 + 7x - x^2}$

8.  $\lim_{x \rightarrow 3} \frac{\ln(2x-5)}{\tan 3 - \tan x}$

11.  $\lim_{x \rightarrow \infty} \left( \frac{5-2x}{11-2x} \right)^{\frac{1+3x^2}{x}}$

3.  $\lim_{x \rightarrow 2} \frac{x^3 - 3x - 2}{x^3 + 2x - 12}$

6.  $\lim_{x \rightarrow 0} \frac{2^x - 3^x}{\tan 2\pi(x + \frac{1}{2}))}$

9.  $\lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2 \cos x}{\sin(\pi - 3x)}$

12.  $\lim_{x \rightarrow \infty} \left( \frac{5-2x}{11-x} \right)^{\frac{1+3x^2}{x}}$

## Variant 2

1.  $\lim_{x \rightarrow \infty} \frac{2x^3 - 3x^2 + 5}{4x^3 + 5x - 2}$

4.  $\lim_{x \rightarrow -3} \frac{\sqrt[3]{x-5} + 2}{-2x^2 - 5x + 3}$

7.  $\lim_{x \rightarrow 0} \frac{e^{2x^2+3x} - 1}{2 \sin \frac{x}{2}}$

10.  $\lim_{x \rightarrow \infty} \left( \frac{3x-7}{3x+1} \right)^{\frac{x+1}{4}}$

2.  $\lim_{x \rightarrow \infty} \left( \sqrt{3x^2 + 4x - 3} - \sqrt{3x^2 - 2x + 7} \right)$

5.  $\lim_{x \rightarrow 1} \frac{\sqrt[4]{x-1}}{1 - \sqrt{x}}$

8.  $\lim_{x \rightarrow 2} \frac{\ln(7-3x)}{\arctan(x^2 - 2x)}$

3.  $\lim_{x \rightarrow \frac{1}{2}} \frac{2x^2 - 5x + 2}{6x^3 + 7x^2 - x - 2}$

6.  $\lim_{x \rightarrow 0} \frac{\cos^2 \pi x - 1}{\log_2(1 + 3x^2)}$

9.  $\lim_{x \rightarrow 1} \frac{\sin \pi(x+1)}{2 - 2^{4x-3}}$

12.  $\lim_{x \rightarrow \infty} \left( \frac{3x-7}{x+1} \right)^{\frac{x+1}{4}}$

## Variant 3

1.  $\lim_{x \rightarrow \infty} \frac{7x^3 + 5x - 3}{2x^3 + 4x^2 + 5}$

4.  $\lim_{x \rightarrow 8} \frac{\sqrt[3]{x^2} - 4}{8 + 7x - x^2}$

7.  $\lim_{x \rightarrow 0} \frac{\sin 2x - \sin 6x}{\tan \left( \frac{3\pi}{2} (x^2 - 3x) \right)}$

10.  $\lim_{x \rightarrow \infty} \left( \frac{4x+2}{x-3} \right)^{\frac{1-x^2}{x+2}}$

2.  $\lim_{x \rightarrow +\infty} \left( \sqrt{x^3 + 2x + 5} - \sqrt{x^3 - x + 4} \right) \sqrt{x}$

5.  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x^2 - 9}$

8.  $\lim_{x \rightarrow 1} \frac{1 + \cos 3\pi x}{\tan^2 \pi x}$

3.  $\lim_{x \rightarrow -3} \frac{x^3 + 4x^2 + 3x}{x^2 + 2x - 3}$

6.  $\lim_{x \rightarrow 0} \frac{\log_2(2x+1)}{1 - e^{2\pi x}}$

9.  $\lim_{x \rightarrow -1} \frac{3^{3+2x} - 3}{\sin \pi(x+1)}$

12.  $\lim_{x \rightarrow \infty} \left( \frac{x+2}{4x-3} \right)^{\frac{1-x^2}{x+2}}$

## Variant 4.

1.  $\lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 14}{3x^4 + 2x^2 - 3}$

4.  $\lim_{x \rightarrow 2} \frac{\sqrt[3]{4x-2}}{2x-x^2}$

7.  $\lim_{x \rightarrow 0} \frac{2^{-5x} - 1}{\ln(1-\pi x)}$

2.  $\lim_{x \rightarrow \infty} \frac{\sqrt[5]{6x^3 + 5x + 3} + \sqrt[3]{x^4 + 6x^2 - 3}}{\sqrt{x^3 - 5x + 3} + \sqrt[6]{x^2 - 4x + 5}}$

5.  $\lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x^2 - 8x + 15}$

8.  $\lim_{x \rightarrow 4} \frac{3^x - 81}{\sin \pi(x+3)}$

3.  $\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^2 + 2x - 8}$

6.  $\lim_{x \rightarrow 0} \frac{\cos \pi x - \cos 3\pi x}{\arctan 5x^2}$

9.  $\lim_{x \rightarrow \pi} \frac{e^{2\pi} - e^{\pi+x}}{\tan 2x}$

$$10. \lim_{x \rightarrow \infty} \left( \frac{1+4x}{2+4x} \right)^{\frac{2+4x^2}{3x}}$$

### Variant 5.

$$1. \lim_{x \rightarrow \infty} \frac{6x^3 + 18x - 3}{3x^3 + 5x + 10}$$

$$4. \lim_{x \rightarrow 0} \frac{\sqrt{9+2x} - 3}{x + 2\sqrt[5]{x^7}}$$

$$7. \lim_{x \rightarrow 0} \frac{1 - \cos^3 \sqrt{x}}{\ln(1+5x)}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2-3x+x^2}{1+x+2x^2} \right)^{\frac{3x^3}{x^2-4}}$$

### Variant 6.

$$1. \lim_{x \rightarrow \infty} \frac{7x^2 - 14x + 12}{13x^4 + 7x - 3}$$

$$4. \lim_{x \rightarrow 0} \frac{\sqrt[4]{x+1} - 1}{x^3 - 4x}$$

$$7. \lim_{x \rightarrow 0} \frac{4 - 4^{2x+1}}{\sin 3x}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{x^3 - 2}{4x^3 + 2} \right)^{\frac{x^2+2}{3}}$$

### Variant 7.

$$1. \lim_{x \rightarrow \infty} \frac{12x^3 + 7x^2 - 2}{6x^3 + 5x + 100}$$

$$4. \lim_{x \rightarrow 1} \frac{\sqrt[4]{15+x} - 2}{x^3 - 2x^2 + 1}$$

$$7. \lim_{x \rightarrow 0} \frac{\cos^3 x - \cos^2 x}{\cos 3x - \cos x}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{7x^2 - 5x + 4}{3 + 2x + 7x^2} \right)^{\frac{x+2}{3}}$$

### Variant 8.

$$1. \lim_{x \rightarrow \infty} \frac{6x^2 + 7x + 2}{8x^2 - 2x + 5}$$

$$4. \lim_{x \rightarrow 2} \frac{1 - \sqrt{7-3x}}{\sqrt[3]{x^2 + 4} + 2\sqrt[3]{7-2x^2}}$$

$$7. \lim_{x \rightarrow 0} \frac{\log_{10}(x+10) - 1}{\sin 3x - \sin 5x}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{1+x}{2+4x} \right)^{\frac{2+4x^2}{3x}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[4]{12x^5 + 5x - 3} + \sqrt[3]{x^2 - 3x + 2}}{\sqrt[5]{5x^3 - 4x + 3} + \sqrt[5]{x^6 + 5x - 3}}$$

$$5. \lim_{x \rightarrow -8} \frac{2 + \sqrt[3]{x}}{3 - \sqrt{1-x}}$$

$$8. \lim_{x \rightarrow 2} \frac{\tan 2\pi x}{4 - x^2}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{2-3x+2x^2}{1+x+2x^2} \right)^{\frac{3x^3}{x^2-4}}$$

$$2. \lim_{x \rightarrow \infty} \left( \sqrt{2x^2 + 5x - 3} - \sqrt{2x^2 - 3x} \right)$$

$$5. \lim_{x \rightarrow 7} \frac{4 - \sqrt[3]{x^2 + 15}}{\sqrt{x+2} - 3}$$

$$8. \lim_{x \rightarrow 2} \frac{\tan \frac{x}{2} - \tan 1}{2 \arctan(3x - 6)}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{4x^3 - 2}{x^3 + 2} \right)^{\frac{x^2+2}{3}}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{1+4x}{2+x} \right)^{\frac{2+4x^2}{3x}}$$

$$3. \lim_{x \rightarrow 2} \frac{x^3 - 3x - 2}{x^3 - 6x^2 + 12x - 8}$$

$$6. \lim_{x \rightarrow 0} \frac{5^{-2x} - 1}{\arctan(4x - x^2)}$$

$$9. \lim_{x \rightarrow 0} \frac{\sin(x-2) - \sin(x+2)}{e^{2x} - e^{3x}}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{2-3x+x^2}{1+x+x^2} \right)^{\frac{3x^3}{x^2-4}}$$

$$3. \lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^2 - 4x + 4}$$

$$6. \lim_{x \rightarrow 0} \frac{\log_2(1-2x)}{\cos \sqrt{x} - 1}$$

$$9. \lim_{x \rightarrow 1} \frac{e - e^x}{(x^2 + 1)\sin(x^2 - 1)}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{4x^3 - 2}{4x^3 + 2} \right)^{\frac{x^2+2}{3}}$$

$$3. \lim_{x \rightarrow 2} \frac{x^3 - 6x^2 + 12x - 8}{x^3 - 3x^2 + 4}$$

$$6. \lim_{x \rightarrow 0} \frac{2^{\frac{x^2-x+1}{3}} - 2}{\log_3(1-2x)}$$

$$9. \lim_{x \rightarrow 4} \frac{\tan \pi x}{4x - x^2}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 5x + 4}{3 + 2x + 7x^2} \right)^{\frac{x+2}{3}}$$

$$3. \lim_{x \rightarrow -1} \frac{x^3 + 4x^2 + 5x + 2}{x^3 - 7x^2 - 5x + 3}$$

$$6. \lim_{x \rightarrow 0} \frac{x^2 + 8x}{\sqrt{25 - x^3} - 5}$$

$$9. \lim_{x \rightarrow 2} \frac{(4x^2 - 3)\sin(4 - x^2)}{e^{\sin \pi x} - e^{\tan \pi x}}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2+3x^2-x^3}{3-x^2-x^3} \right)^{\frac{x-1}{3}}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{2+3x^2-4x^3}{3-x^2-x^3} \right)^{\frac{x-1}{3}}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{2+3x^2-4x^3}{3-x^2-6x^3} \right)^{\frac{x-1}{3}}$$

### Variant 9.

$$1. \lim_{x \rightarrow \infty} \frac{7x^2-5x+4}{3x^4+2x-2}$$

$$4. \lim_{x \rightarrow 1} \frac{\sqrt{5-x}-\sqrt[3]{7x+1}}{\sqrt{x+1}-\sqrt{3x^2-1}}$$

$$7. \lim_{x \rightarrow 0} \frac{\cos(x+x^2)-1}{\sin 3x^2};$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3-2x^3}{5-2x^3} \right)^{\frac{2x^3+1}{4}};$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+4x-2}+\sqrt[5]{x^3-4x+3}}{\sqrt[3]{7x^3+5x+4}+\sqrt{x+5}}$$

$$5. \lim_{x \rightarrow 2} \frac{(x\sqrt{x+2}-x^2)}{\sin(2-x)};$$

$$8. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan 8x}{\cos(x+\frac{\pi}{4})}.$$

$$3. \lim_{x \rightarrow 3} \frac{x^3-10x+3}{2x^3-7x^2+9}.$$

$$6. \lim_{x \rightarrow 0} \frac{2^x+5^x-2}{\log_3(1+10x)}.$$

$$9. \lim_{x \rightarrow \frac{\pi}{3}} \frac{8\cos^3 x-1}{\frac{x}{2}-\frac{\pi}{6}}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3-x^3}{5-2x^3} \right)^{\frac{2x^3+1}{4}}.$$

### Variant 10.

$$1. \lim_{x \rightarrow \infty} \frac{12x^3+4x-7}{5x^2+6x+3}$$

$$4. \lim_{x \rightarrow -2} \frac{\sqrt[3]{4x^2-15}-1}{\sqrt{x+3}-1}.$$

$$7. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot x-1}{\sqrt[3]{\cos 2x}}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{x^2+2x-4}{2x^2-3x+5} \right)^{\frac{2x+7}{4}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{5x^2+4x-2}-\sqrt{x^5+3x^2+2}}{\sqrt[3]{x^7-14x^5+10}+\sqrt[7]{x^2-x-2}}$$

$$5. \lim_{x \rightarrow 0} \frac{\sqrt{3x+2}-\sqrt{x+2}}{4^x+5^x-2};$$

$$8. \lim_{x \rightarrow 2} \frac{\sin \sqrt{x}-\sin \sqrt{2}}{2x-4}.$$

$$3. \lim_{x \rightarrow -2} \frac{x^3+5x^2+8x+4}{2x^3+3x^2-7x-10}.$$

$$6. \lim_{x \rightarrow 0} \frac{\log_2(16-x^2)-2\log_3 9}{3\arctan(x^2-3x)};$$

$$9. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 5x}{\cos 3x}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3x^2+2x-4}{x^2-3x+5} \right)^{\frac{2x+7}{4}}$$

### Variant 11.

$$1. \lim_{x \rightarrow \infty} \frac{3x^4+5x+10}{2x^3+15x+21}$$

$$4. \lim_{x \rightarrow -5} \frac{3-\sqrt[3]{x^2+2}}{\sqrt[3]{6+x}-1}.$$

$$7. \lim_{x \rightarrow 0} \frac{\log_2 \cos 3x}{2 \sin \frac{2x}{3} \tan \frac{12x}{5}};$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3x^3-3x+2}{x^3+2x-4} \right)^{\frac{x^2-1}{4x}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{5x^3-4x+3}+\sqrt[5]{x^6+x-3}}{\sqrt[5]{x^4+3x^3-5}+\sqrt[4]{2x^6+x^5+7}}$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{x^2+5}-3}{2-\sqrt{2x^2-4}};$$

$$8. \lim_{x \rightarrow 3} \frac{\log_4(4-x)}{\tan \pi x}.$$

$$3. \lim_{x \rightarrow 3} \frac{6x^3-23x^2+15x}{x^3-6x^2+27}.$$

$$6. \lim_{x \rightarrow 0} \frac{(3^x-7^x)^2}{\sqrt[6]{1+3x^2}-1}.$$

$$9. \lim_{x \rightarrow \frac{\pi}{6}} \frac{1+2\sin x}{2\sin(6x+\pi)}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3x^3-3x+2}{3x^3+2x-4} \right)^{\frac{x^2-1}{4x}}$$

### Variant 12.

$$1. \lim_{x \rightarrow \infty} \frac{2x^5-4x+7}{6x^7+2x-10}$$

$$4. \lim_{x \rightarrow -2} \frac{\sqrt[3]{x^2-3}-1}{\sqrt[3]{3+x}-1}.$$

$$7. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\ln \tan x}{4 \cos 6x}.$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{5x^3-4x+3}-\sqrt[5]{x^6+x-3}}{\sqrt{2x^3+3x-1}-\sqrt[4]{3x^5+4x-2}}$$

$$5. \lim_{x \rightarrow 1} \frac{5-\sqrt{3x^2+22}}{\sqrt{3+x}-\sqrt{5-x}};$$

$$8. \lim_{x \rightarrow 1} \frac{3 \cos \frac{5\pi x}{2}}{\tan 5\pi x}.$$

$$3. \lim_{x \rightarrow 2} \frac{-2x^3+9x^2-12x+4}{3x^3-8x^2-4x+16};$$

$$6. \lim_{x \rightarrow 0} \frac{1-\sqrt[8]{1+5x^3}}{(e^2-e^{3x+2})\tan x^2}.$$

$$9. \lim_{x \rightarrow \pi} \frac{\arctan(1-\cos 2x)}{\frac{\pi}{2^x}-2}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 2x + 7}{3x^2 + 7x - 1} \right)^{\frac{x+2}{3}}$$

### Variant 13.

$$1. \lim_{x \rightarrow \infty} \frac{10x^2 - 7x + 8}{5x^3 - x^2 - 10}$$

$$4. \lim_{x \rightarrow 5} \frac{\sqrt[3]{x^2 + 2} - 3}{2 + \sqrt[3]{x - 13}}.$$

$$7. \lim_{x \rightarrow \pi} \frac{2^{\frac{\cos^2 x}{2}} - 1}{\ln(2 + \cos x)}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{x^2 - x + 4}{2x^2 + x + 5} \right)^{\frac{x^3 - 5}{4x+3}}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 2x + 7}{x^2 + 7x - 1} \right)^{\frac{x+2}{3}}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3x^2 - 2x + 7}{x^2 + 7x - 1} \right)^{\frac{x+2}{3}}$$

$$2. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x - 5} - \sqrt{x^2 - x + 10})$$

$$5. \lim_{x \rightarrow 0} \frac{\sqrt{5-2x} - \sqrt{5+2x}}{\sin\left(\frac{\pi}{6} - 3x\right) \sin 4x};$$

$$8. \lim_{x \rightarrow 3} \frac{5 \arcsin \frac{x-3}{4}}{3^{x^2-8} - 3}.$$

$$3. \lim_{x \rightarrow -1} \frac{x^3 - x^2 - 5x - 3}{3x^3 - 4x^2 + x + 2}.$$

$$6. \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{\sin^2 3x}.$$

$$9. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 3x - \cos x}{5(2x - \pi)^2}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{x^2 - x + 4}{x^2 + x + 5} \right)^{\frac{x^3 - 5}{4x+3}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[3]{6x^5 + 5x^4 - 3} + \sqrt[5]{x^6 + 5x + 3}}{\sqrt[6]{4x^{10} - 7x^5 + 10} + x + \sqrt{x - 3}}$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - 3}{\sqrt{x+2} - 2};$$

$$8. \lim_{x \rightarrow 4} \frac{1 - 2^{\sin \pi x}}{1 - \cos \frac{\pi x}{2}}.$$

$$3. \lim_{x \rightarrow -2} \frac{x^3 + 5x^2 + 8x + 4}{x^3 + 5x^2 + 5x - 2}.$$

$$6. \lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + \tan^2 \frac{x}{2}} - 1}{(e^{3x} - e^{2x})^2}.$$

$$9. \lim_{x \rightarrow -1} \frac{4(x^3 + 1)}{\sin(x + 1)}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{-3x^2 + x + 5}{-3x^2 - x + 7} \right)^{\frac{2x^3 + 5}{2-3x}}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{-x^2 + x + 5}{-3x^2 - x + 7} \right)^{\frac{2x^3 + 5}{2-3x}}$$

$$3. \lim_{x \rightarrow -3} \frac{2x^3 + 15x^2 + 36x + 27}{2x^3 + 7x^2 + 6x + 9}.$$

$$6. \lim_{x \rightarrow 0} \frac{e^2 - e^{2-x^3}}{(x^2 - 3x) \cdot \sin(2x^2)};$$

$$9. \lim_{x \rightarrow \pi} \left( \sin \frac{\pi - x}{2} \cdot \tan \frac{\pi^2}{2x} \right).$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{x^3 + 4}{5 + x^2 + 2x^3} \right)^{\frac{(x^2+4)x}{7x+2}}$$

$$2. \lim_{x \rightarrow \infty} (\sqrt{x^4 + 2x^2 + 8} - \sqrt{x^4 + x + 3})$$

$$5. \lim_{x \rightarrow 1} \frac{\sqrt{2x^2 + 3x + 4} - 3}{2 - \sqrt{3x^2 + 1}};$$

$$3. \lim_{x \rightarrow -2} \frac{3x^3 + 17x^2 + 32x + 20}{5x^3 - 18x^2 - 12x + 8};$$

$$6. \lim_{x \rightarrow 0} \frac{\arctan(1 - \cos x)}{(2^{x^2} - 3^{x^2})(4x + 1)};$$

### Variant 15.

$$1. \lim_{x \rightarrow \infty} \frac{7x^4 - 3x^2 + 5}{6x^3 + 3x - 4}$$

$$4. \lim_{x \rightarrow 0} \frac{\sqrt[3]{2x^2 + 8} - 2}{3 + \sqrt[3]{5x^2 - 27}}.$$

$$7. \lim_{x \rightarrow 0} \frac{\ln(1 + \sin^2 2x)}{(\cos 3x - 1) \sin\left(3x + \frac{\pi}{6}\right)}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2x^3 + 4}{5 + x^2 + x^3} \right)^{\frac{(x^2+4)x}{7x+2}}$$

### Variant 16.

$$1. \lim_{x \rightarrow \infty} \frac{8x^4 + 17x^2 + 9}{5x^4 + 6x - 3}$$

$$4. \lim_{x \rightarrow -2} \frac{x^2 + 2x}{3\sqrt[3]{5x + 2} + 2}.$$

$$7. \lim_{x \rightarrow 0} \frac{\sin(x^3 + 4x) \cdot \tan 3x}{\log_2(1 + 2x^2)}.$$

$$8. \lim_{x \rightarrow -1} \frac{\cos \frac{\pi x}{2}}{\cos \frac{3\pi x}{2}}.$$

$$9. \lim_{x \rightarrow \frac{\pi}{2}} (\pi^2 - 4x^2) \cdot \tan 3x.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 3x + 2}{3x^2 + x - 2} \right)^{\frac{4-x^2}{4-3x}}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{3x^2 - 3x + 2}{3x^2 + x - 2} \right)^{\frac{4-x^2}{4-3x}}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3x^2 - 3x + 2}{x^2 + x - 2} \right)^{\frac{4-x^2}{4-3x}}$$

### Variant 17.

$$1. \lim_{x \rightarrow \infty} \frac{11x^5 + x^4 - 19}{12x^3 + 10x + 2}$$

$$4. \lim_{x \rightarrow -3} \frac{\sqrt[3]{2x-2} + \sqrt[3]{x^2-1}}{3 + \sqrt[3]{5x-12}}.$$

$$7. \lim_{x \rightarrow 0} \frac{4^{x+1} - 4^{1-x}}{\cos \frac{3x}{2} \sin(5x^2 + x)};$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{5x - x^2 + x^3}{7 - 2x + 3x^3} \right)^{\frac{2+3x^4}{x^3+2}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{4x^4 + 8x^3 - 5} + \sqrt[6]{x^5 + 4x + 5}}{(x^2 + 5)^3 \sqrt{x^6 + 3x + 2}}$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{2x^3 - 15} - 1}{\sqrt{7 - x^2} - \sqrt{x^2 - 1}};$$

$$8. \lim_{x \rightarrow \pi} \frac{e^{2 \sin x} - e^{\sin 2x}}{4(\pi^2 - x^2)}.$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{5x - x^2 + x^3}{7 - 2x + x^3} \right)^{\frac{2+3x^4}{x^3+2}}$$

$$3. \lim_{x \rightarrow 4} \frac{x^3 - 6x^2 + 7x + 4}{x^3 - 2x^2 - 4x - 16}.$$

$$6. \lim_{x \rightarrow 0} \frac{\log_4(1 + \tan 2x)}{4 \sin\left(\frac{\pi}{3} + 2x\right) \arcsin 3x}.$$

$$9. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt[5]{1 + \tan 2x} - 1}{3 \cos 3x (1 + \sin 3x)}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{5x - x^2 + 3x^3}{7 - 2x + x^3} \right)^{\frac{2+3x^4}{x^3+2}}$$

### Variant 18.

$$1. \lim_{x \rightarrow \infty} \frac{15x^3 + 7x^2 + 5}{23x^4 - 17x + 8}$$

$$4. \lim_{x \rightarrow 1} \frac{\sqrt[3]{5x^2 + 3} + \sqrt[3]{x^3 - 7}}{1 - \sqrt[3]{3x^2 - 2}}.$$

$$7. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin^3 x - \sin \frac{\pi}{6}}{\tan x - \cot x}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3 + 2x - x^2}{4 - x - x^2} \right)^{\frac{3x^4+1}{x+2}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[10]{x^{11} + 8x^{10} + 5} + \sqrt[7]{x^{12} + 7x^2 + 5}}{\sqrt[8]{x^9 + x^5 + 1} + \sqrt{x + 8}}$$

$$5. \lim_{x \rightarrow 4} \frac{\sqrt{3x-8} - \sqrt{x}}{\sqrt{2x-4} - \sqrt{8-x}};$$

$$8. \lim_{x \rightarrow 3} \frac{\tan \frac{\pi}{2x} \cdot \cot \frac{3\pi}{2x}}{2 \arcsin\left(\frac{x}{3} - 1\right)}.$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{3 + 2x - 2x^2}{4 - x - x^2} \right)^{\frac{3x^4+1}{x+2}}$$

$$3. \lim_{x \rightarrow 3} \frac{-x^3 + 8x^2 - 21x - 18}{x^3 - 3x^2 - 9x + 27};$$

$$6. \lim_{x \rightarrow 0} \frac{4 \cos 5x (1 - \cos 3x)}{7^x + 7^{-x} - 2};$$

$$9. \lim_{x \rightarrow \pi} \frac{\ln(1 + 2 \sin^2 x)}{3(\cos 2x - 1)}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3 + 2x - x^2}{4 - x - 2x^2} \right)^{\frac{3x^4+1}{x+2}}$$

$$3. \lim_{x \rightarrow -3} \frac{x^3 - 4x + 15}{x^3 + 5x^2 + 3x - 9}.$$

$$6. \lim_{x \rightarrow 0} \frac{2(e^{x+1} - e^{1-x}) \cdot \cos \frac{3x}{2}}{\ln(1 - \tan 2x)}.$$

$$9. \lim_{x \rightarrow 1} \left( \sin \frac{1-x}{2} \cdot \tan \frac{\pi x}{2} \right)^2.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{x^4 - 3}{x^4 + 2x - 1} \right)^{\frac{3x^2-1}{5}}.$$

### Variant 19.

$$1. \lim_{x \rightarrow \infty} \frac{11x^5 + 7x^4 - 12}{3x^5 + 6x^3 - 13x}$$

$$4. \lim_{x \rightarrow -2} \frac{\sqrt[3]{x^2 + 4} - \sqrt[3]{3x^2 - 4}}{\sqrt[3]{6 - x} - 2}.$$

$$7. \lim_{x \rightarrow -\frac{\pi}{4}} \frac{\cos^3 x + \sin^3 x}{3 \sin 4x}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2x^4 - 3}{x^4 + 2x - 1} \right)^{\frac{3x^2-1}{5}}$$

$$2. \lim_{x \rightarrow \infty} \left( \sqrt{x^3 + 4x + 5} - \sqrt{x^3 - 1} \right)$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{6x-3} - \sqrt{4x+1}}{\sqrt{1+x^3} - 3};$$

$$8. \lim_{x \rightarrow \frac{\pi}{6}} \frac{(2 \cos x - \sqrt{3}) \sin 3x}{(2 \cos 2x - 1) \cos 2x}.$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{x^4 - 3}{2x^4 + 2x - 1} \right)^{\frac{3x^2-1}{5}}$$

### Variant 20.

$$1. \lim_{x \rightarrow \infty} \frac{8x^3 + 5x^2 - 4}{3x^4 + 6x + 11}$$

$$4. \lim_{x \rightarrow 1} \frac{\sqrt[3]{5+3x} + \sqrt[3]{4x-12}}{\sqrt[3]{x^3+7}-2}.$$

$$7. \lim_{x \rightarrow \pi} \frac{3\sin^2 x}{1 + \cos^3 x}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2+3x-x^3}{1-x-2x^3} \right)^{\frac{x^2+5}{3}};$$

### Variant 21.

$$1. \lim_{x \rightarrow \infty} \frac{4x^3 + 5x^2 + 11}{13x^2 - 5x - 7}$$

$$4. \lim_{x \rightarrow 2} \frac{\sqrt[3]{4+2x} - \sqrt[3]{5x-2}}{5 - \sqrt{3x^3+1}}$$

$$7. \lim_{x \rightarrow \frac{\pi}{6}} \frac{3\sin x \cdot \lg(\sin 3x)}{2 \tan\left(2x + \frac{2\pi}{3}\right) \tan\left(x - \frac{\pi}{3}\right)}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2-3x+3x^2}{1+x+x^2} \right)^{\frac{2x^2+5}{4-x}}$$

### Variant 22.

$$1. \lim_{x \rightarrow \infty} \frac{25x^4 + 7x^3 - 3}{8x^6 + 5x + 4}$$

$$4. \lim_{x \rightarrow -1} \frac{\sqrt[3]{2x+10} - \sqrt[3]{x^2+7}}{\sqrt[3]{2+x}-1}$$

$$7. \lim_{x \rightarrow \frac{\pi}{3}} \frac{1-4\cos^2 x}{\sin(3x-\pi)}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{x^3 + 2x - 1}{4x^3 - x^2 + 5} \right)^{\frac{3-x^4}{x^3+1}}$$

### Variant 23.

$$1. \lim_{x \rightarrow \infty} \frac{7x^3 + 8x^2 + 1}{12x^3 - 9x + 5}$$

$$4. \lim_{x \rightarrow 2} \frac{\sqrt{3x^2-8} - \sqrt{12-2x^2}}{1-\sqrt{3-x}}$$

$$7. \lim_{x \rightarrow \frac{1}{4}} \frac{\cos \pi x - \sin \pi x}{\tan 4\pi x \cdot \cot \pi x}.$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[3]{6x^2+3} - \sqrt[5]{x^3-4x+5}}{\sqrt[4]{3x^2+5x-4} - \sqrt[4]{3x^2-x-1}}$$

$$5. \lim_{x \rightarrow -3} \frac{\sqrt{3x^2-2} - \sqrt{2x^2+7}}{\sqrt{4+x}-1};$$

$$8. \lim_{x \rightarrow -\frac{\pi}{6}} \frac{2(1+\sin 3x)\cos x}{\cot 3x}.$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{2+3x-x^3}{1-x-x^3} \right)^{\frac{x^2+5}{3}};$$

$$3. \lim_{x \rightarrow -1} \frac{-x^3 - 3x^2 + x + 3}{3x^3 + 5x^2 + x - 1}.$$

$$6. \lim_{x \rightarrow 0} \frac{\log_5(2 \cos x - 1)}{\tan\left(\frac{\pi}{4} - 2x\right)(2^{x^2} - 3^{x^2})}.$$

$$9. \lim_{x \rightarrow -2} \frac{\cos \frac{\pi}{2x} + \sin \frac{\pi}{2x}}{\sqrt[5]{1 + \tan \pi x} - 1}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{2+3x-3x^3}{1-x-x^3} \right)^{\frac{x^2+5}{3}}.$$

$$3. \lim_{x \rightarrow 3} \frac{x^3 + 5x^2 - 8x - 48}{x^3 - 5x^2 + 5x + 3};$$

$$6. \lim_{x \rightarrow 0} \frac{(3x-5)(\cos 3x - \cos 5x)}{(3x+5)(\sin 3x - \sin 5x) \tan 2x};$$

$$9. \lim_{x \rightarrow -1} \frac{2 \tan(x^2 - 1)}{e^{x^2+3x+2} - 1}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{2-3x+x^2}{1+x+x^2} \right)^{\frac{2x^2+5}{4-x}}$$

$$2. \lim_{x \rightarrow +\infty} \left( \sqrt{x^2 + 4x + 5} - \sqrt{x^2 - 2x - 1} \right)$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{3x^2-4} - 2\sqrt{6-x^2}}{3 - \sqrt{x^2+5}};$$

$$8. \lim_{x \rightarrow b} \frac{1 - 3^{b^2-x^2}}{\tan\left(\ln \frac{x}{b}\right) \sin(x+b)}.$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{x^3 + 2x - 1}{x^3 - x^2 + 5} \right)^{\frac{3-x^4}{x^3+1}}$$

$$3. \lim_{x \rightarrow 1} \frac{x^3 + 3x^2 - 4}{3x^3 - 10x^2 + 11x - 4};$$

$$6. \lim_{x \rightarrow 0} \frac{2(e^{5x} - e^{7x})}{\cos 4x (\sin 5x - \sin 7x)};$$

$$9. \lim_{x \rightarrow \frac{1}{2}} \frac{3 \sin \pi x \cos 3\pi x}{\log_3(4x-1)}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{4x^3 + 2x - 1}{x^3 - x^2 + 5} \right)^{\frac{3-x^4}{x^3+1}}.$$

$$3. \lim_{x \rightarrow 1} \frac{x^3 - 5x^2 + 7x - 3}{2x^3 + 5x^2 - 4x + 1}.$$

$$6. \lim_{x \rightarrow 0} \frac{(7^x - 9^x)3^{x+1}}{4 \cos^2 \frac{x}{2} (1 - \cos 5\sqrt{x})};$$

$$9. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot^3 x - 1}{2 - \cot x - \cot^3 x}.$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[4]{3x^2+2x-1} + \sqrt[5]{x^2+4x-3}}{\sqrt[6]{5x^7+4x+5} + \sqrt{x+3}}$$

$$5. \lim_{x \rightarrow 3} \frac{\sqrt[3]{x+5} - \sqrt[3]{2x^2-10}}{\sqrt[3]{x^2-1}-2};$$

$$8. \lim_{x \rightarrow -\frac{\pi}{6}} \frac{2^{\cos^2 3x} - 1}{1 - \sqrt[3]{\sin 3x}}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{-x^2 + 5x + 4}{4 - 2x - 3x^2} \right)^{\frac{5x^3}{4-x^2}}$$

### Variant 24.

$$1. \lim_{x \rightarrow \infty} \frac{12x^5 + 6x - 1}{5x^4 - 4x^3 + 3}$$

$$4. \lim_{x \rightarrow -2} \frac{\sqrt[3]{3x-2} - \sqrt[3]{x+10}}{\sqrt[3]{x^2-3}-1}$$

$$7. \lim_{x \rightarrow \frac{\pi}{3}} \frac{(1 + \cos 3x)(1 - \sin 3x)}{\tan^2 6x}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{4 - x + x^3}{7 + 3x + x^3} \right)^{\frac{3x^4 - 1}{4+x^3}}$$

### Variant 25.

$$1. \lim_{x \rightarrow \infty} \frac{6x^5 + 4x + 3}{10x^3 + 5x^2 - 1}$$

$$4. \lim_{x \rightarrow -1} \frac{\sqrt[3]{3x^3 + 11} - \sqrt[3]{5x^2 + 3}}{\sqrt[3]{2 + x} - 1}$$

$$7. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos\left(3x - \frac{\pi}{4}\right) \sin 2x}{2(\cos x - \sin x)}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2x^2 - 4x + 1}{x^2 + 2x - 3} \right)^{\frac{5-2x^4}{x^3+1}}$$

### Variant 26.

$$1. \lim_{x \rightarrow \infty} \frac{4x^3 + 18x - 3}{3x^3 + 5x + 10}$$

$$4. \lim_{x \rightarrow -2} \frac{\sqrt[3]{9 + x^3} - \sqrt[3]{10x^2 - 39}}{\sqrt[3]{3 + x} - 1}$$

$$7. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{\cos 2x}}{\tan^2 x - 1}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{4 - 2x^2 + x^3}{8 + x + 2x^3} \right)^{\frac{4x^3}{2x^2-3}}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{-3x^2 + 5x + 4}{4 - 2x - 3x^2} \right)^{\frac{5x^3}{4-x^2}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[3]{8x^3 + 4x - 5} + \sqrt{x-3}}{\sqrt{x^2 - 2x - 15} - \sqrt[4]{x+5}}$$

$$5. \lim_{x \rightarrow 1} \frac{\sqrt{5x^2 - 4} - \sqrt{3 - 2x^2}}{\sqrt{1 + 3x} - 2}$$

$$8. \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - 3^{\cos 2x} - 3^{\sin 4x}}{5 \sin 4x \cos 3x}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{4 - x + 2x^3}{7 + 3x + x^3} \right)^{\frac{3x^4 - 1}{4+x^3}}$$

$$2. \lim_{x \rightarrow +\infty} \left( \sqrt{x^2 + 7x - 4} - \sqrt{x^2 + 3} \right)$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{3x^2 + 4} - 2\sqrt{8x - 3x^2}}{x^2 - 2x}$$

$$8. \lim_{x \rightarrow \pi} \frac{\sin 3x}{\pi^2 - x^2}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 4x + 1}{2x^2 + 2x - 3} \right)^{\frac{5-2x^4}{x^3+1}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[4]{12x^5 + 5x - 3} + \sqrt[3]{x^2 - 3x + 2}}{\sqrt[5]{5x^3 - 4x + 3} + \sqrt[5]{x^6 + 5x - 3}}$$

$$5. \lim_{x \rightarrow 3} \frac{\sqrt{3x^2 - 2} - \sqrt{2x^2 + 7}}{4(x^2 - 3x)}$$

$$8. \lim_{x \rightarrow 5} \frac{\log_5 x - 1}{2 \sin(5 - x)}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{4 - 2x^2 + 2x^3}{8 + x + 2x^3} \right)^{\frac{4x^3}{2x^2-3}}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{-3x^2 + 5x + 4}{4 - 2x - x^2} \right)^{\frac{5x^3}{4-x^2}}$$

$$3. \lim_{x \rightarrow -4} \frac{x^3 + 8x^2 + 16x}{x^3 + x^2 - 11x + 4};$$

$$6. \lim_{x \rightarrow 0} \frac{\ln \cos 2x}{\log_2 \cos 3x};$$

$$9. \lim_{x \rightarrow \frac{\pi}{6}} \frac{(2 \sin x - 1) \sin 3x}{2 \cos x - \sqrt{3}}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{4 - x + x^3}{7 + 3x + 2x^3} \right)^{\frac{3x^4 - 1}{4+x^3}}$$

$$3. \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{3x^3 - 10x^2 + 11x - 4}$$

$$6. \lim_{x \rightarrow 0} \frac{e^{2+x^2} - e^{2-x^2}}{\arcsin \frac{3x}{2} \tan(5x^2 - x)}$$

$$9. \lim_{x \rightarrow e} \frac{(\ln x)^2 - 1}{x - e}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 4x + 1}{x^2 + 2x - 3} \right)^{\frac{5-2x^4}{x^3+1}}$$

$$3. \lim_{x \rightarrow 1} \frac{3x^3 - 10x^2 + 11x - 4}{x^3 + 3x^2 - 4}$$

$$6. \lim_{x \rightarrow 0} \frac{\cos 3x^2 \arctan \frac{2x}{3}}{4 \sin(3x - x^2)}$$

$$9. \lim_{x \rightarrow 1} \frac{\sin 2x - \sin 2}{6x - 6x^2}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{4 - 2x^2 + x^3}{8 + x + x^3} \right)^{\frac{4x^3}{2x^2-3}}$$

$$3. \lim_{x \rightarrow -1} \frac{3x^3 + 5x^2 + x - 1}{3x^3 + 3x^2 + 4x + 4}$$

### Variant 27.

$$1. \lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 14}{7x^3 + 2x^2 - 3}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[5]{6x^3 + 5x + 3} + \sqrt[3]{x^4 + 6x^2 - 3}}{\sqrt[3]{x^3 - 5x + 3} + \sqrt[6]{x^2 - 4x + 5}}$$

$$4. \lim_{x \rightarrow 1} \frac{\sqrt[3]{5x^2 - 4} + \sqrt[3]{3 - 4x^2}}{\sqrt[3]{x + 7} - 2}$$

$$7. \lim_{x \rightarrow \sqrt{3}} \frac{3 \arctan x - \pi}{(3 - x^2) \arcsin \frac{x}{2}}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3x^2 - 2x + 5}{x^2 + 3x + 1} \right)^{\frac{5x^4+3}{4-x^3}}$$

### Variant 28.

$$1. \lim_{x \rightarrow \infty} \frac{7x^3 + 5x - 3}{2x^3 + 4x^2 + 5}$$

$$4. \lim_{x \rightarrow 2} \frac{\sqrt[3]{9 - 2x^2} - \sqrt[3]{3x^2 - 11}}{\sqrt[3]{1 - x} + 1}$$

7.

$$\lim_{x \rightarrow \sqrt{2}} \frac{\arccos \frac{x}{2} - \arcsin \frac{x}{2}}{4 \arctan \frac{x}{\sqrt{2}} (x^2 - 2)}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3 - 4x^2 + x^3}{2 + 2x + 4x^3} \right)^{\frac{5-2x^4}{4+x^2}}$$

$$5. \lim_{x \rightarrow -2} \frac{\sqrt{x^2 + 5} - \sqrt{2x + 13}}{2x^2 + 3x - 2}$$

$$8. \lim_{x \rightarrow 3} \frac{(8 - 2^x)(3 + 2^x)}{\sin \pi x}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 2x + 5}{x^2 + 3x + 1} \right)^{\frac{5x^4+3}{4-x^3}}$$

$$6. \lim_{x \rightarrow 0} \frac{3 \operatorname{tg} \left( \frac{\pi}{4} - 3x \right) (1 - \cos 2\sqrt{x})}{(5^x + 3^x - 2) \cos \left( 2x + \frac{\pi}{3} \right)}$$

$$9. \lim_{x \rightarrow 1} \frac{2^{\sin \pi x} - 1}{\tan(\ln(3x - 2))}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{x^2 - 2x + 5}{3x^2 + 3x + 1} \right)^{\frac{5x^4+3}{4-x^3}}$$

### Variant 29.

$$1. \lim_{x \rightarrow \infty} \frac{4x^2 + 3x - 2}{5x^2 + 3x - 1}$$

$$4. \lim_{x \rightarrow 1} \frac{\sqrt{3x + 1} - \sqrt{x^2 + 3}}{x^2 - x}$$

$$7. \lim_{x \rightarrow 1} \frac{e^x - e}{\sin 2(x^2 - 1)}$$

$$2. \lim_{x \rightarrow +\infty} \left( \sqrt{x^6 + 4x^3 - 3} - \sqrt{x^6 - x + 4} \right)$$

$$5. \lim_{x \rightarrow 1} \frac{\sqrt{3x + 6} - \sqrt{4x^2 + 5}}{2x - 2x^2}$$

$$8. \lim_{x \rightarrow 1} \frac{2 \sin(x^2 - x) \cos(3x^2 + x - 4)}{\ln(4x^2 + 2x - 5)}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{3 - 4x^2 + 4x^3}{2 + 2x + x^3} \right)^{\frac{5-2x^4}{4+x^2}}$$

$$3. \lim_{x \rightarrow 3} \frac{x^3 - 4x^2 - 3x + 18}{x^3 - 3x^2 - 4x + 12}$$

$$6. \lim_{x \rightarrow 0} \frac{e^{\sin 3x} - e^{\sin x}}{2 \cos 5x \cdot \arctan 4x}$$

$$9. \lim_{x \rightarrow \pi} \frac{1 - 3^{\sin 2x}}{\arcsin(\tan 2x)}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3 - 4x^2 + x^3}{2 + 2x + x^3} \right)^{\frac{5-2x^4}{4+x^2}}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{2 - 3x + x^3}{3 + 2x + 5x^3} \right)^{\frac{4x^4+3}{x^3-4}}$$

$$2. \lim_{x \rightarrow +\infty} \left( \sqrt{x^3 + 2x + 5} - \sqrt{x^3 - x + 4} \right) \sqrt{x}$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt[3]{3x^2 - 4} - 2}{\sqrt[3]{x + 6} + \sqrt[3]{4 - 3x^2}}$$

$$8. \lim_{x \rightarrow \frac{\pi}{3}} \frac{\ln \cos 6x}{\arcsin(3x - \pi)^2}$$

$$3. \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^3 + 5x^2 + 3x - 9}$$

$$6. \lim_{x \rightarrow 0} \frac{\log_2(1 + 2x^2)}{3^{x^2+1} - 3}$$

$$9. \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - \sqrt{\sin 3x}}{\sqrt{\tan \frac{3x}{2}} - 1}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{2 - 3x + x^3}{3 + 2x + x^3} \right)^{\frac{4x^4+3}{x^3-4}}$$

### Variant 30.

$$1. \lim_{x \rightarrow \infty} \frac{3x^3 - 2x + 5}{2x^2 + 4x - 3}$$

$$4. \lim_{x \rightarrow 3} \frac{3 - \sqrt[3]{4x^2 - 9}}{1 + \sqrt[3]{x - 4}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[3]{5x^2 - 4x + 3} + \sqrt[3]{x^3 - 4x^2 + 5}}{x^2 + \sqrt{3x^2 + 5x - 3} + \sqrt[4]{x - 3}}$$

$$5. \lim_{x \rightarrow 0} \frac{\cos 2x - \cos 4x}{x \cdot \sin 3x}$$

$$3. \lim_{x \rightarrow -2} \frac{\sqrt{x + 6} - \sqrt{2x^2 - 4}}{x^2 + 5x + 6}$$

$$6. \lim_{x \rightarrow 0} \frac{2^x - 5^x}{\ln(1 + \tan 2x)}$$

$$7. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot x - 1}{\sqrt[3]{\cos 2x}}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3+2x^2-3x^3}{5-x^2-x^3} \right)^{\frac{5-x^4}{4-x^3}}$$

$$8. \lim_{x \rightarrow \frac{\pi}{3}} \frac{8 \sin^3 \frac{x}{2} - 1}{\sin(\pi - 3x)}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{3+2x^2-3x^3}{5-x^2-3x^3} \right)^{\frac{5-x^4}{4-x^3}}$$

$$9. \lim_{x \rightarrow 10} \frac{\log_{10} x - 1}{x^2 - 11x + 10}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3+2x^2-x^3}{5-x^2-3x^3} \right)^{\frac{5-x^4}{4-x^3}}$$

### Variant 31.

$$1. \lim_{x \rightarrow \infty} \frac{3x^2 - x + 4}{2x^3 - 4x^2 + 3}$$

$$4. \lim_{x \rightarrow 2} \frac{3x^3 - 7x^2 - 8x + 20}{5x^3 + 22x^2 - 28x + 8};$$

$$7. \lim_{x \rightarrow \pi} \frac{\tan^2 5x}{(e^\pi - e^x)^2}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{1-x}{10-2x} \right)^{\frac{1+3x^2}{x}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{3x^4 + 5x^3 - 3} + \sqrt[3]{2x^3 + 5x^2 + 4}}{\sqrt[4]{7x^3 + 4x^2 + 3} - \sqrt{2x^5 + 7x^4 - 5}}$$

$$5. \lim_{x \rightarrow 1} \frac{\sqrt{3x+1} - \sqrt{2x+2}}{\sqrt{8x+1} - 3}$$

$$8. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 5x - \cos 3x}{1 - \cos 4x}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{3-2x}{7-2x} \right)^{\frac{1+3x^2}{x}}$$

$$3. \lim_{x \rightarrow 3} \frac{3x^2 - 7x - 6}{2x^2 + x - 21}$$

$$6. \lim_{x \rightarrow 0} \frac{\sin 3x}{\tan 2x}$$

$$9. \lim_{x \rightarrow \frac{1}{3}} \frac{(1-3x)^2}{\sin 3\pi x \tan 6\pi x}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{5-2x}{11-x} \right)^{\frac{1+3x^2}{x}}$$

### Variant 32.

$$1. \lim_{x \rightarrow \infty} \frac{3x^5 + 4x^4 - 3x - 2}{7x^3 - 2x + 5}$$

$$4. \lim_{x \rightarrow 3} \frac{x^3 - 3x^2 - 9x + 27}{x^3 + 11x^2 - 39x + 45}$$

$$7. \lim_{x \rightarrow 3} \frac{\ln(2x-5)}{\tan 3 - \tan x}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3x-7}{3x+1} \right)^{\frac{x+1}{4}}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt[3]{5x^4 - 3x^3 + 7+x} + \sqrt[3]{x^2 - 4x + 3}}{\sqrt[5]{x^2 + 3x - 2} + \sqrt[6]{x+5}}$$

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{4} - \sqrt[3]{x+6}}{\sqrt{x+2} - 2}$$

$$8. \lim_{x \rightarrow 1} \frac{1 + \cos 3\pi x}{\tan^2 \pi x}$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{x-7}{3x+1} \right)^{\frac{x+1}{4}}$$

$$3. \lim_{x \rightarrow 3} \left( \frac{x+2}{x-3} - \frac{x^2 - 7x}{x^2 - 2x - 3} \right)$$

$$6. \lim_{x \rightarrow 0} \frac{\arcsin^2 x}{x \sin x}$$

$$9. \lim_{x \rightarrow 1} \frac{\sin \pi(x+1)}{2 - 2^{4x-3}}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{3x-7}{x+1} \right)^{\frac{x+1}{4}}$$

### Variant 33.

$$1. \lim_{x \rightarrow \infty} \frac{2x^3 - 4x^2 + 3}{4x^2 + 7x - 5}$$

$$4. \lim_{x \rightarrow 5} \frac{\sqrt[3]{x^2 + 2} - 3}{2 + \sqrt[3]{x-13}}.$$

$$7. \lim_{x \rightarrow \pi} \frac{2^{\frac{\cos^2 x}{2}} - 1}{\ln(2 + \cos x)}.$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{x^2 - x + 4}{2x^2 + x + 5} \right)^{\frac{x^3 - 5}{4x+3}}$$

$$2. \lim_{x \rightarrow \infty} \left( \sqrt{3x^2 + 4x - 3} - \sqrt{3x^2 - 2x + 7} \right)$$

$$5. \lim_{x \rightarrow 0} \frac{\sqrt{5-2x} - \sqrt{5+2x}}{\sin \left( \frac{\pi}{6} - 3x \right) \sin 4x};$$

$$8. \lim_{x \rightarrow 3} \frac{5 \arcsin \frac{x-3}{4}}{3^{x^2-8} - 3}.$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{2x^2 - x + 4}{x^2 + x + 5} \right)^{\frac{x^3 - 5}{4x+3}}$$

$$3. \lim_{x \rightarrow -1} \frac{x^3 - x^2 - 5x - 3}{3x^3 - 4x^2 + x + 2}.$$

$$6. \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{\sin^2 3x}.$$

$$9. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 3x - \cos x}{5(2x - \pi)^2}.$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{x^2 - x + 4}{x^2 + x + 5} \right)^{\frac{x^3 - 5}{4x+3}}$$

**Problem 2.** a) Find points of discontinuity, determine their type and draw a sketch of the function behavior near the discontinuity points; b) Find the parameter at which a give function to be continuous (if it is possible).

- Variant 1.** a)  $f(x) = \frac{\arctan \frac{1}{x-2}}{x+1}$ , b)  $f(x) = \begin{cases} x^2, & -\infty < x < 1, \\ Ax - 1, & 1 \leq x < \infty. \end{cases}$
- Variant 2** a)  $f(x) = \frac{\sin(x-3)}{x^2-9}$ , b)  $f(x) = \begin{cases} \frac{1}{x^2+1}, & -\infty < x < 2, \\ -\frac{A}{10}x, & 2 \leq x < \infty. \end{cases}$
- Variant 3.** a)  $f(x) = 2^{\frac{x-1}{x^2-1}}$ , b)  $f(x) = \begin{cases} x^2 + 3, & -4 < x \leq 3, \\ \frac{A}{x-3}, & 3 < x < \infty. \end{cases}$
- Variant 4.** a)  $f(x) = \frac{x^2 - 16}{x-4} e^{\frac{1}{x}}$ , b)  $f(x) = \begin{cases} \sin x, & x \leq 0, \\ Ax + 2, & x > 0. \end{cases}$
- Variant 5.** a)  $f(x) = \frac{x^2 - 1}{\sqrt{x-1}} 3^{\frac{1}{x-2}}$ , b)  $f(x) = \begin{cases} \cos \frac{\pi}{2}x, & -\infty < x < 1, \\ Ax - 1, & 1 \leq x < \infty. \end{cases}$
- Variant 6.** a)  $f(x) = \frac{1}{\ln(x-3)}$ , b)  $f(x) = \begin{cases} x + 1, & -\infty < x \leq -1, \\ \frac{x^2 - 1}{x + 1}, & -1 < x < \infty. \end{cases}$
- Variant 7.** a)  $f(x) = (1+x) \arctan \frac{1}{1-x^2}$ , b)  $f(x) = \begin{cases} -2 \cos x, & x \leq \pi, \\ \sin x + A, & \pi < x < 2\pi. \end{cases}$
- Variant 8.** a)  $f(x) = \frac{x-4}{\sqrt{x-2}} 2^{\frac{1}{x-5}}$ , b)  $f(x) = \begin{cases} \ln x, & 1 < x \leq e, \\ Ax - e, & x > e. \end{cases}$
- Variant 9.** a)  $f(x) = \frac{x^2 - 4}{x+2} 3^{\frac{1}{x}}$ , b)  $f(x) = \begin{cases} x, & 0 < x \leq 1, \\ -x^2 + 4x - A, & 1 < x < 5x. \end{cases}$
- Variant 10.** a)  $f(x) = \frac{\sin x}{x(x+2)}$ , b)  $f(x) = \begin{cases} x+2, & x \leq 1, \\ -Ax^2, & -1 < x < 4. \end{cases}$
- Variant 11.** a)  $f(x) = \frac{x}{\ln(1+x)}$ , b)  $f(x) = \begin{cases} 2^x, & -\infty < x < 0, \\ A-x, & 0 \leq x < \infty. \end{cases}$
- Variant 12.** a)  $f(x) = 2^{\frac{1}{x-1}} \frac{\sin(x-4)}{x-4}$ , b)  $f(x) = \begin{cases} x+2, & -\infty < x \leq 0, \\ Ae^{\frac{x}{x+1}}, & 0 < x < \infty. \end{cases}$
- Variant 13.** a)  $f(x) = \frac{\arctan \frac{1}{1-x}}{x+2}$ , b)  $f(x) = \begin{cases} x^3, & -\infty \leq x \leq 1, \\ Ax + 1, & 1 < x < \infty. \end{cases}$
- Variant 14.** a)  $f(x) = \frac{\sin(x-1)}{x^2-1}$ , b)  $f(x) = \begin{cases} \frac{1}{x^2+1}, & -\infty < x < 2, \\ -\frac{A}{4}x, & 2 \leq x < \infty. \end{cases}$
- Variant 15.** a)  $f(x) = 2^{\frac{x-4}{x^2-16}}$ , b)  $f(x) = \begin{cases} x^2 - 1, & -4 < x \leq 1, \\ \frac{A}{x+2}, & 1 < x < \infty. \end{cases}$

**Variant 16.** a)  $f(x) = \frac{x^2 - 9}{x - 3} e^{\frac{1}{x}},$

b)  $f(x) = \begin{cases} \sin x, & x \leq -\frac{\pi}{2}, \\ Ax + 1, & -\frac{\pi}{2} < x < \infty. \end{cases}$

**Variant 17.** a)  $f(x) = \frac{x^2 - 4}{x - 2} 3^{\frac{1}{x-1}},$

b)  $f(x) = \begin{cases} \cos \frac{\pi}{2} x, & -\infty < x \leq 2, \\ Ax + 2, & 2 < x < \infty. \end{cases}$

**Variant 18.** a)  $f(x) = \frac{x - 9}{\sqrt{x - 3}} 2^{\frac{1}{x}},$

b)  $f(x) = \begin{cases} 2x, & 0 < x \leq 1, \\ -x^2 + Ax, & 1 < x < \infty. \end{cases}$

**Variant 19.** a)  $f(x) = \frac{\sin 2x}{x(x+2)},$

b)  $f(x) = \begin{cases} x + 3, & x \leq 1, \\ -Ax^3, & 1 < x < 4. \end{cases}$

**Variant 20.** a)  $f(x) = \frac{x}{\ln(1+4x)},$

b)  $f(x) = \begin{cases} 3^x, & -\infty < x < 0, \\ A+x, & 0 \leq x < \infty. \end{cases}$

**Variant 21.** a)  $f(x) = 2^{\frac{1}{x}} \frac{\sin(x+1)}{x+1},$

b)  $f(x) = \begin{cases} x-2, & -\infty < x \leq 0, \\ Ae^{\frac{x}{x+2}}, & 0 < x < \infty. \end{cases}$

**Variant 22**  
a)  $f(x) = \frac{\arctan \frac{1}{x-1}}{x},$

b)  $f(x) = \begin{cases} x^2 + 4, & -\infty < x \leq 1, \\ Ax, & 1 < x < \infty. \end{cases}$

**Variant 23.** a)  $f(x) = \frac{x^2 - 4}{x - 2} e^{\frac{1}{x}},$

b)  $f(x) = \begin{cases} \frac{1}{x^2}, & -\infty < x \leq -3, \\ \frac{A}{5}x, & -3 < x. \end{cases}$

**Variant 24.** a)  $f(x) = \frac{\sin(x-2)}{x^2 - 4},$

b)  $f(x) = \begin{cases} \ln x, & 1 < x \leq e, \\ Ax^2 - 5, & e < x < \infty. \end{cases}$

**Variant 25.** a)  $f(x) = 2^{\frac{x+1}{x^2-1}},$

b)  $f(x) = \begin{cases} \arctan \frac{1}{x}, & -\infty < x < 0, \\ x^2 + A, & 0 \leq x < \infty. \end{cases}$

**Variant 26.** a)  $f(x) = \frac{x^3 - 1}{x - 1} 2^{\frac{1}{x-2}},$

b)  $f(x) = \begin{cases} \sin x, & -\infty < x \leq 2\pi, \\ Ax^2 + 3, & 2\pi < x < \infty. \end{cases}$

**Variant 27.** a)  $f(x) = \frac{\ln(1+x)}{x} e^{\frac{1}{x-1}},$

b)  $f(x) = \begin{cases} \cos \frac{\pi}{2} x, & -\infty < x \leq -2, \\ Ax + 4, & -2 < x < \infty. \end{cases}$

**Variant 28.** a)  $f(x) = \frac{x^2 - 4}{x - 2} 2^{\frac{1}{x}},$

b)  $f(x) = \begin{cases} \arctan x, & 0 < x \leq \frac{\pi}{4}, \\ Ax + 4, & \frac{\pi}{4} < x < \infty. \end{cases}$

**Variant 29.** a)  $f(x) = \frac{2^x - 1}{x(x+2)},$

b)  $f(x) = \begin{cases} 3x, & 0 < x \leq 1, \\ Ax^2 - 1, & 1 < x < \infty. \end{cases}$

**Variant 30.** a)  $f(x) = \frac{\arctan \frac{1}{x}}{x+3},$

b)  $f(x) = \begin{cases} \ln x, & 0 < x \leq e^2, \\ \frac{A}{x} + 1, & e^2 < x < \infty. \end{cases}$

**Variant 31.** a)  $f(x) = 2^x \frac{\sin(x+1)}{x+1},$

b)  $f(x) = \begin{cases} x - 2, & -\infty < x \leq 0, \\ Ae^{\frac{x}{x+2}}, & 0 < x < \infty. \end{cases}$

**Variant 32.** a)  $f(x) = \frac{\ln(1+x)}{x} e^{\frac{1}{x-1}},$

b)  $f(x) = \begin{cases} \cos \frac{\pi}{2} x, & -\infty < x \leq -2, \\ Ax + 4, & -2 < x < \infty. \end{cases}$

**Variant 33.** a)  $f(x) = 2^{\frac{x-1}{x^2-1}},$

b)  $f(x) = \begin{cases} x^2 + 3, & -4 < x \leq 3, \\ \frac{A}{x-3}, & 3 < x < \infty. \end{cases}$