

Task 1. In the items "1", "2" "3", "4", find the derivatives of the given functions;
in the item "5", find the derivative of the implicit function;
in the item "6", calculate approximately the value of function at a given x by means
of the differential;
in the item "7", solve a problem using the geometrical meaning of the derivative.

1) $y = e^{2x} \arctg 2x + \frac{\ln x}{x}$; 2) $y = \ln^5(\operatorname{tg} \sqrt[3]{x})$; 3) $y = (\ln x)^{\sin 3x}$;

1.1. 4) $\begin{cases} x = \frac{t+1}{t}, \\ y = \frac{t-1}{t}; \end{cases}$ 5) $\sqrt{xy} + \sin x + \sin a = 0$; 6) $y = x^3 + 3x^2 - 7, x = 2,03$.

7) Find the equation of line tangent to the curve $y = x \cdot \ln x$, which is parallel to the straight line $y - x - 5 = 0$.

1) $y = (x^2 - 2x + 3)e^{3x} - \frac{x}{\ln x}$; 2) $y = 5^{\arcsin^2(x^3 - x + 1)}$; 3) $y = (\cos 3x)^x$;

1.2. 4) $\begin{cases} x = a(t - \sin t), \\ y = a(1 - \cos t); \end{cases}$ 5) $y^3 \sin x + a^2 \cos 2x = 5a$; 6) $y = e^{x^2 - x}, x = 1,12$.

7) Find the equation of line normal to the curve $y = \frac{x-1}{x}$, which is parallel to the straight line $2y + x + 3 = 0$.

1) $y = x \operatorname{tg} 3x - \frac{5^x}{\sqrt{7x}}$; 2) $y = \arccos^2\left(\ln \frac{x}{1+x^2}\right)$; 3) $y = (\sin 3x)^{\ln x}$;

1.3. 4) $\begin{cases} x = \ln(1+t^2), \\ y = t - \arctg t; \end{cases}$ 5) $y^2 - 2xy + \sin(x+y) = \cos a$; 6) $y = e^{2-x}, x = 1,97$.

7) Find the equation of line tangent to the curve $y = 2 - \sqrt{x}$, which is perpendicular to the straight line $y + 4x - 4 = 0$.

1) $y = \frac{e^{2x}(\cos 3x + \sin 3x)}{\ln x}$; 2) $y = \sin^3(\cos 3x)$; 3) $y = \left(\sin \frac{3}{x}\right)^{x^3}$;

1.4. 4) $\begin{cases} x = t \ln t, \\ y = \frac{t-1}{\sin t}; \end{cases}$ 5) $x^4 + y^4 = x^2 y^2$; 6) $y = (x^2 - 3)^2(x + 2), x = 3,011$.

7) Find the equations of lines tangent and normal to the curve $y = \frac{x^2 + 1}{x^3 - 1}$ at the point with abscissa $x_0 = -1$.

1) $y = x^{\frac{2}{3}} \arccos 2x - \frac{2x + 3}{\operatorname{tg} 3x}$; 2) $y = \ln^3 x + 3^{\operatorname{tg} 3x}$;

1.5. 3) $y = (\sin x)^{\sqrt{x}}$; 4) $\begin{cases} x = a \cos^3 t, \\ y = b \sin^3 t; \end{cases}$ 5) $\sin(x + y) + \cos(x + y) = \sin a$;

6) $y = \arcsin 3x, x = 0,05$.

7) Find the equations of lines tangent and normal to the curve $y = \frac{x^2 - 3x + 6}{x^2}$ at the point with abscissa $x_0 = 3$.

1.6. 1) $y = \sqrt{x} \cos x + \frac{3x^2 + 7}{\arcsin 2x}$; 2) $y = e^{\cos \left(\ln \frac{1-x}{x^2} \right)}$; 3) $y = x^{\sin x}$;

4) $\begin{cases} x = a \cos t, \\ y = b \sin t; \end{cases}$ 5) $x^y = y^x$; 6) $y = \operatorname{arctg} x, x = 0,98$.

7) Find the equation of line tangent to the curve $y = \frac{1}{1 + x^2}$, which is parallel to the straight line $y = 2x$.

1) $y = 3^{\sin^2 \ln x}$; 2) $y = \frac{x^2 e^{2x}}{\operatorname{arctg} 2x}$; 3) $y = x^{\cos 2x}$;

1.7. 4) $\begin{cases} x = 4 - t^2, \\ y = t - t^3; \end{cases}$ 5) $\cos(xy) = \sin(xy)$; 6) $y = 2^{x-3}, x = 2,08$.

7) 7) Find the equations of lines tangent and normal to the curve $y = \frac{1 + 3x^2}{3 + x^2}$ at the point with abscissa $x_0 = 1$.

1) $y = \frac{x \cos 4x}{1 + \operatorname{tg} 4x}$; 2) $y = \sin^5 \left(4^{\operatorname{arctg} 2x} \right)$; 3) $y = (\cos 2x)^{\operatorname{tg} 2x}$;

1.8. 4) $\begin{cases} x = t(1 - \sin t), \\ y = t \cos t; \end{cases}$ 5) $ye^x - \operatorname{tg} xy = e^a$; 6) $y = \ln x, x = 1,13$.

7) Find the equation of line tangent to the curve $y = x \cdot \cos x$, which is perpendicular to the straight line $y + x + 3 = 0$.

$$1) y = \frac{\sqrt[3]{x} 2^x}{1 + \cos 5x}; \quad 2) y = e^{\sqrt{\cos^3 x} \operatorname{tg} 3x}; \quad 3) y = (\operatorname{tg} 3x)^x;$$

$$1.9. \quad 4) \begin{cases} x = \frac{1+t^3}{t^2-1}, \\ y = \frac{t}{t^2-1}; \end{cases} \quad 5) y - x = \operatorname{arctg} \frac{x}{y}; \quad 6) y = \sqrt{1+x}, x = 3,01$$

7) Find the equation of line normal to the curve $y = e^{1-x^2}$, which is perpendicular to the straight line $y + 2x - 4 = 0$.

$$1) y = \left(\sqrt[3]{x + \sqrt{x+x}} \right) \cos 4x; \quad 2) y = \ln^3 \operatorname{tg} \frac{x-1}{1-2x}; \quad 3) y = (\sin 5x)^{x^5};$$

$$1.10. \quad 4) \begin{cases} x = e^t \sin t, \\ y = e^t \cos t; \end{cases} \quad 5) y \sin x - \cos(x-y) = 0; \quad 6) y = \sqrt[3]{\frac{1-x}{1+x}}, x = 0,02.$$

7) Find the equation of line normal to the curve $y = \sqrt[3]{1-x}$ at the point with abscissa $x_0 = 1$.

$$1) y = \left(\sqrt{x+x^3} \right) \operatorname{tg} 3x; \quad 2) y = \cos(5^{\ln x}); \quad 3) y = \frac{x^3 e^{3x} \sin 2x}{\cos 3x};$$

$$1.11. \quad 4) \begin{cases} x = 5 \cos^3 t, \\ y = 5 \sin^3 t; \end{cases} \quad 5) x \sin y - \cos y + \cos 2x = 0; \quad 6) y = \arccos x, x = 0,01.$$

7) Find the equation of line tangent to the curve $y = \arcsin \frac{x-1}{2}$, which is parallel to the straight line $2y - x + 5 = 0$.

$$1.12. \quad 1) y = \frac{\arccos 3x}{\arcsin 6x}; \quad 2) y = \operatorname{tg}^3 \sin(\ln 2x); \quad 3) y = (1+x^2)^{x^3}; \quad 4) \begin{cases} x = \frac{3t}{1+t}, \\ y = \frac{3t^2}{1+t^3}; \end{cases}$$

$$5) x + y = \arcsin x + \arcsin y; \quad 6) y = x^3 - 4x^2 + 6x + 3, x = 1,04.$$

7) Find the equation of line tangent to the curve $y = \operatorname{arctg} x$, which is perpendicular to the straight line $y + 2x + 3 = 0$.

$$1.13. \quad 1) y = \frac{3x^3 - 5x^2 + 7}{\operatorname{arctg} 3x}; \quad 2) y = \sqrt[3]{\ln^2 \frac{\sin x}{5 - x^2}}; \quad 3) y = x^2 \sin 2x (x + 3)^5;$$

$$4) \begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t); \end{cases} \quad 5) 3^{x+y} = 3^x - 3^y; \quad 6) y = \sqrt[3]{1+x}, x = 6,93.$$

7) Find the equation of line normal to the curve $y = \arccos 3x$, which is perpendicular to the straight line $y + 3x + 3 = 0$.

$$1) y = \cos 4x \cdot 2^x - 5 \operatorname{tg} 4x; \quad 2) y = 3^{\arccos^2(\operatorname{tg} 4x)}; \quad 3) y = (\operatorname{tg} 4x)^{\ln x};$$

$$1.14. \quad 4) \begin{cases} x = 5^t (t^2 + 1), \\ y = 5^t (1 - t); \end{cases} \quad 5) y \ln(x + y) = \ln a; \quad 6) y = e^{2-x-x^2}, x = -1,94.$$

7) Find the equation of line tangent to the curve $y = \frac{x}{1+x^2}$ at the point with abscissa $x_0 = 2$.

$$1) y = e^{3x} \ln x - 3 \arccos 3x; \quad 2) y = \sqrt[3]{\operatorname{tg}^2 \ln \sqrt{x}}; \quad 3) y = (\sin 2x)^{\operatorname{ctg} 2x};$$

$$1.15. \quad 4) \begin{cases} x = \frac{1-t^2}{t}, \\ y = \frac{t^3}{1+t}; \end{cases} \quad 5) x^{2/3} + y^{2/3} = a^{2/3}; \quad 6) y = \sqrt{9+x^2}, x = 4,01.$$

7) Find the equation of line normal to the curve $y = \sqrt{1+x^2}$ at the point with abscissa $x_0 = 0$.

$$1) y = \frac{\cos 7x + 3}{\ln x + 4}; \quad 2) y = \sin^2 \frac{x^5 - x}{2 - x}; \quad 3) y = (\sin 5x)^{\cos 5x};$$

$$1.15. \quad 4) \begin{cases} x = 3^t \cos t, \\ y = 3^t \sin t; \end{cases} \quad 5) \operatorname{tgy} = \operatorname{atgx}; \quad 6) y = \sqrt[3]{3x + \cos x}, x = 0,01.$$

7) Find the equation of line tangent to the curve $y = x \cdot \sin 2x$, which is parallel to the straight line $y + \pi x + 9 = 0$.

$$1) y = 4x^{-3/2} + x^4 \sqrt{x} - 1; \quad 2) y = \sin \sqrt[3]{1+x^3}; \quad 3) y = (x^3 + 5)^{\operatorname{ctg} 5x};$$

$$1.16. \quad 4) \begin{cases} x = t^3 \ln t, \\ y = (1-t)\sqrt{t}; \end{cases} \quad 5) y = \operatorname{tg}^2(y - x); \quad 6) y = \operatorname{arctg} x^2, x = 0,97.$$

7) Find the equations of lines tangent and normal to the curve $y = \frac{x^3 + 1}{x^2 + 4}$ at the point with abscissa $x_0 = -1$.

1) $y = \cos 2x \cdot 2^x - 5 \operatorname{tg} 3x$; 2) $y = \frac{\ln x}{\cos^3 3x}$; 3) $y = (\cos 3x)^{\sin 5x}$;

1.18. 4) $\begin{cases} x = t^3 \cos t, \\ y = (t^2 - 1) \sin t; \end{cases}$ 5) $e^{xy} = \arcsin x$; 6) $y = \sqrt[3]{2 + x^2}, x = 4,97$.

7) Find the equations of lines tangent and normal to the curve $y = e^{\sqrt{x}-1}$ at the point with abscissa $y_0 = e$.

1) $y = \frac{\sqrt[3]{x} \cdot \operatorname{tg} 3x}{1 + \cos 4x}$; 2) $y = \sqrt{\frac{2 - \sin x}{2 + \sin x}}$; 3) $y = (\operatorname{ctg} 5x)^{x/3}$;

1.19. 4) $\begin{cases} x = \frac{t^3 - 3}{2t}, \\ y = \frac{t^2 + 1}{\ln t}; \end{cases}$ 5) $x^3 + ax^2y + y^3 = a$; 6) $y = \frac{x^3 - 2}{x^2 + 1}, x = 1,07$.

7) Find the equation of line tangent to the curve $y = e^{\cos x}$, which is parallel to the straight line $y + x + 3 = 0$.

1) $y = \frac{x + \arcsin 2x}{3x + \operatorname{arctg} 3x}$; 2) $y = \arcsin(\cos x^3)$; 3) $y = (\arcsin 3x)^x$;

1.20. 4) $\begin{cases} x = \cos^3 t, \\ y = \sin^3 t; \end{cases}$ 5) $\cos(xy) = e^{x+y}$; 6) $y = \sqrt[4]{2x - \sin \frac{\pi x}{2}}, x = 1,03$.

7) Find the equation of line tangent to the curve $y = x^3 - 3x^2 - 5$, which is perpendicular to the straight line $2x - 6y + 1 = 0$.

1) $y = \sqrt[3]{x}(x^2 - 3\sqrt{x} + 6)$; 2) $y = \arccos(\sin x^2)$; 3) $y = (\operatorname{arctg} 3x)^{x+3}$;

1.21. 4) $\begin{cases} x = t - \sin t, \\ y = 1 - \cos t; \end{cases}$ 5) $2y \ln y = e^x$; 6) $y = (x^2 - 1)^3(x + 2), x = 2,03$.

7) Find the equations of lines tangent and normal to the curve $y = x \cdot \ln(1 + x^2)$ at the point with abscissa $x_0 = 1$.

$$1) y = ((x + \sqrt{x} - 1) \operatorname{tg} 5x) / x^3; \quad 2) y = \sqrt[4]{\ln \operatorname{tg} \frac{x}{3}}; \quad 3) y = (\operatorname{arctg} 5x)^{7+x};$$

$$1.22. \quad 4) \begin{cases} x = t / (1 - t^2), \\ y = t^2 / (1 - t^2); \end{cases} \quad 5) 2^{x+y} = \ln(x + y); \quad 6) y = \frac{x^2 + 1}{1 - x}, x = -0,93.$$

7) Find the equations of lines tangent and normal to the curve $y = \sin x + \cos x$ at the point with abscissa $x_0 = \pi/4$.

$$1) y = \frac{e^{2x} \operatorname{arctg} 3x}{\cos 4x}; \quad 2) y = \ln^3 x + \ln(\operatorname{ctg} 3x); \quad 3) y = (\sin 3x) \frac{5}{x};$$

$$1.23. \quad 4) \begin{cases} x = \sqrt{1 - t^3}, \\ y = t \sqrt{1 + t}; \end{cases} \quad 5) \operatorname{arctg} \frac{x}{y} = \ln y; \quad 6) y = \arcsin 2x, x = 0,249.$$

7) Find the equations of lines tangent and normal to the curve $y = 4^{x-x^2}$ at the point with abscissa $x_0 = 1$.

$$1) y = \sqrt[4]{x^3} + 3^{-2x}; \quad 2) y = \sqrt[3]{\ln \sin \frac{x}{2}}; \quad 3) y = (\arccos 5x)^{x+7};$$

$$1.24. \quad 4) \begin{cases} x = \frac{t-2}{t}, \\ y = \frac{1-t}{\sqrt{t}}; \end{cases} \quad 5) y^3 - 3y + 2a \ln x = 0; \quad 6) y = \frac{1-x^2}{1+x^2}, x = 1,02.$$

7) Find the equation of line tangent to the curve $y = 1 - \frac{1}{x^2}$, which is parallel to the straight line $2y + 32x + 7 = 0$.

$$1) y = \frac{x e^{2x}}{\sin 2x}; \quad 2) y = 4^{\frac{x}{\cos 2x}}; \quad 3) y = \frac{(x-1)^2 \sqrt[3]{(x+1)^5} (x-3)^3}{(x^2+1) \sqrt[3]{(1-x)^5}};$$

$$1.25. \quad 4) \begin{cases} x = t^5, \\ y = 1 - \cos t; \end{cases} \quad 5) \operatorname{arctg} \sqrt{x/y} + \sin y = \ln a; \quad 6) y = \sqrt{5 - x^2}, x = 0,98.$$

7) Find the equations of lines tangent and normal to the curve $y = x - \operatorname{arctg} x$ at the point with abscissa $x_0 = 1$.

$$1) y = \frac{\sqrt[3]{x} \cdot 2^x}{\sin 3x}; \quad 2) y = 5^{\operatorname{ctg}^3(\ln \sqrt{x})}; \quad 3) y = (\operatorname{ctg} 5x) \frac{7}{x};$$

$$1.26. \quad 4) \begin{cases} x = \sin t + \ln t, \\ y = \cos t + \ln t; \end{cases} \quad 5) \sin \sqrt{x+y} = \ln \operatorname{tg} y; \quad 6) y = e^{2x-x^2}, x = 2,014.$$

7) Find the equation of line normal to the curve $y = x - \sqrt{x}$, which is perpendicular to the straight line $4y - 3x + 5 = 0$.

$$1) y = \frac{x \cdot \operatorname{arctg} 4x}{\ln x}; \quad 2) y = \ln^2 x + \ln(\ln x); \quad 3) y = (\cos 2x) \frac{3}{x};$$

1.27.

$$4) \begin{cases} x = \sqrt{t}, \\ y = \sqrt[3]{t}; \end{cases} \quad 5) \operatorname{arcsin} xy = 2^x; \quad 6) y = (3x - 1)^2 (x + 1), x = 1, 01.$$

7) Find the equations of lines tangent and normal to the curve $y = \frac{x^3 - 2}{x^2 - 4}$ at the point with abscissa $x_0 = 3$.

$$1) y = \frac{e^{2x} \cdot \sin 3x}{\sqrt[4]{x} + x^2}; \quad 2) y = 3^{\operatorname{tg}^2(\ln \sqrt{x})}; \quad 3) y = (\operatorname{tg} 4x) \frac{6}{x};$$

1.28.

$$4) \begin{cases} x = \cos t + t \sin t, \\ y = \sin t - t \cos t; \end{cases} \quad 5) x \ln(x + y) = a; \quad 6) y = \operatorname{arctg} \frac{x}{2}, x = 2, 031.$$

7) Find the equations of lines tangent and normal to the curve $y = \sqrt[3]{x^2 - 1}$ at the point with abscissa $x_0 = 1$.

$$1) y = \frac{\sqrt{x} \cdot \cos 5x}{e^{2x}}; \quad 2) y = 5 \frac{\sin 3x}{x}; \quad 3) y = 3 \sqrt{\frac{x^2(x^3 + 1) \sin 2x}{x^2 - 2}};$$

1.29.

$$4) \begin{cases} x = \sin t, \\ y = 1 + t^3; \end{cases} \quad 5) \operatorname{arccos} \frac{x}{y} = 2^a; \quad 6) y = x^4 - 2x^2 + 5, x = 2, 03.$$

7) Find the equations of lines tangent and normal to the curve $y = xe^{-x^2}$ at the point with abscissa $x_0 = 1$.

$$1) y = \frac{1 - x^2}{1 + x^2} \operatorname{arcsin} 2x; \quad 2) y = \cos^3(\sin 2x); \quad 3) y = \left(\cos \frac{5}{x} \right)^{x^2};$$

1.30

$$4) \begin{cases} x = e^t (t^3 + 1); \\ y = e^t (1 - t^3); \end{cases} \quad 5) \ln \operatorname{tg} \frac{y}{x} = a; \quad 6) y = \sqrt{1 + x + \sin x}; x = 0, 02.$$

7) Find the equations of lines tangent and normal to the curve $y = x^3 + \sqrt[3]{x}$ at the point with abscissa $x_0 = -1$.

1) $y = e^{2x} \operatorname{arctg} 2x + \frac{\ln x}{x}$; 2) $y = \ln^5(\operatorname{tg} \sqrt[3]{x})$; 3) $y = (\ln x)^{\sin 3x}$;

1.31. 4) $\begin{cases} x = \frac{t+1}{t}, \\ y = \frac{t-1}{t}; \end{cases}$ 5) $\sqrt{xy} + \sin x + \sin a = 0$; 6) $y = x^3 + 3x^2 - 7, x = 2, 0, 3$.

7) Find the equation of line tangent to the curve $y = x \cdot \ln x$, which is parallel to the straight line $y - x - 5 = 0$.

1) $y = (x^2 - 2x + 3)e^{3x} - \frac{x}{\ln x}$; 2) $y = 5^{\arcsin^2(x^3 - x + 1)}$; 3) $y = (\cos 3x)^x$;

1.32. 4) $\begin{cases} x = a(t - \sin t), \\ y = a(1 - \cos t); \end{cases}$ 5) $y^3 \sin x + a^2 \cos 2x = 5a$; 6) $y = e^{x^2 - x}, x = 1, 1, 2$.

7) Find the equation of line normal to the curve $y = \frac{x-1}{x}$, which is parallel to the straight line $2y + x + 3 = 0$.

1) $y = x \operatorname{ctg} 5x - \frac{3^x}{\sqrt{x^3}}$; 2) $y = \arcsin^4\left(\ln \frac{1+x^2}{x}\right)$; 3) $y = (\cos 3x)^{\ln x^2}$;

1.33. 4) $\begin{cases} x = \ln(1-t^2), \\ y = t^3 - \operatorname{arcctg} 2t; \end{cases}$ 5) $y^2 + 5x^2 y + \operatorname{tg}(x+y) = \cos a, a = \text{const}$; 6) $y = e^{1-x}, x = 0, 9, 7$.

7) Find the equation of line tangent to the curve $y = 2 - \sqrt{x}$, which is perpendicular to the straight line $y + 4x - 4 = 0$.

Task 2. Find the differential of the function: dy .

2.1. $y = x \arcsin(1/x) + \ln|x + \sqrt{x^2 - 1}|, x > 0$.

2.2. $y = \operatorname{tg}\left(2 \arccos \sqrt{1 - 2x^2}\right), x > 0$.

2.3. $y = \sqrt{1 + 2x} - \ln|x + \sqrt{1 + 2x}|$. 2.4. $y = x^2 \operatorname{arctg} \sqrt{x^2 - 1} - \sqrt{x^2 - 1}$.

2.5. $y = \arccos\left(1/\sqrt{1 + 2x^2}\right), x > 0$. 2.6. $y = x \ln|x + \sqrt{x^2 + 3}| - \sqrt{x^2 + 3}$.

2.7. $y = \operatorname{arctg}(\operatorname{sh} x) + (\operatorname{sh} x) \operatorname{lnch} x$. 2.8. $y = \arccos\left(\frac{(x^2 - 1)}{(x^2 \sqrt{2})}\right)$.

$$2.9. y = \ln(\cos^2 x + \sqrt{1 + \cos^4 x}).$$

$$2.10. y = \ln(x + \sqrt{1 + x^2}) - \sqrt{1 + x^2} \operatorname{arctg} x.$$

$$2.11. y = \frac{\ln|x|}{1+x^2} - \frac{1}{2} \ln \frac{x^2}{1+x^2}$$

$$2.12. y = \ln(e^x + \sqrt{e^{2x} - 1}) + \operatorname{arcsine}^x.$$

$$2.13. y = x\sqrt{4 - x^2} + a \operatorname{arcsin}(x/2).$$

$$2.14. y = \operatorname{Intg}(x/2) - x/\sin x.$$

$$2.15. y = 2x + \ln|\sin x + 2\cos x|.$$

$$2.16. y = \sqrt{\operatorname{ctg} x} - \sqrt{\operatorname{tg}^3 x}/3.$$

$$2.17. y = \ln \left| \frac{x + \sqrt{x^2 + 1}}{2x} \right|.$$

$$2.18. y = \sqrt[3]{\frac{x+2}{x-2}}.$$

$$2.19. y = \operatorname{arctg} \frac{x^2 - 1}{x}.$$

$$2.20. y = \ln|x^2 - 1| - \frac{1}{x^2 - 1}.$$

$$2.21. y = \operatorname{arctg} \left(\operatorname{tg} \frac{x}{2} + 1 \right).$$

$$2.22. y = \ln|2x + 2\sqrt{x^2 + x + 1}|.$$

$$2.23. y = \ln|\cos \sqrt{x}| + \sqrt{x} \operatorname{tg} \sqrt{x}.$$

$$2.24. y = e^x (\cos 2x + 2\sin 2x).$$

$$2.25. y = x(\sin \ln x - \cos \ln x).$$

$$2.26. y = \left(\sqrt{x-1} - \frac{1}{2} \right) e^{2\sqrt{x-1}}.$$

$$2.27. y = \cos x \cdot \operatorname{Intg} x - \operatorname{Intg} \frac{x}{2}.$$

$$2.28. y = \sqrt{3 + x^2} - x \ln|x + \sqrt{3 + x^2}|.$$

$$2.29. y = \sqrt{x} - (1 + x) \operatorname{arctg} \sqrt{x}.$$

$$2.30. y = x \operatorname{arctg} x - \ln \sqrt{1 + x^2}.$$

$$2.31. y = x\sqrt{x^2 - 1} + \ln|x + \sqrt{x^2 - 1}|.$$

$$2.32. y = \operatorname{arctg} x + \frac{5}{6} \ln \frac{x^2 + 1}{x^2 + 4}.$$

$$2.33. y = \operatorname{arctg} \frac{\sqrt{1-x}}{1-\sqrt{x}}.$$

Task 3. Find the derivative of the high order.

3.1. $y = (2x^2 - 7) \ln(x - 1), y''' = ?$

3.2. $y = (3 - x^2) \ln^2 x, y''' = ?$

3.3. $y = x \cos x^2, y''' = ?$

3.4. $y = \frac{\ln(x-1)}{\sqrt{x-1}}, y''' = ?$

3.5. $y = \frac{\log_2 x}{x^3}, y''' = ?$

3.6. $y = (4x^3 + 5)e^{2x+1}, y''' = ?$

3.7. $y = x^2 \sin(5x - 3), y''' = ?$

3.8. $y = \frac{\ln x}{x^2}, y''' = ?$

3.9. $y = (2x + 3) \ln^2 x, y''' = ?$

3.10. $y = (1 + x^2) \operatorname{arctg} x, y''' = ?$

3.11. $y = \frac{2^x}{x^3}, y''' = ?$

3.12. $y = (4x + 3)2^{-x}, y''' = ?$

3.13. $y = e^{1-2x} \sin(2x + 3), y''' = ?$

3.14. $y = \frac{\ln(3+x)}{3+x}, y''' = ?$

3.15. $y = (2x^3 + 1) \cos x, y''' = ?$

3.16. $y = (x^2 + 3) \ln(x - 3), y''' = ?$

3.17. $y = (1 - x - x^2)e^{x+1}, y''' = ?$

3.18. $y = \frac{1}{x} \sin 2x, y''' = ?$

3.19. $y = (x + 7) \ln(x + 4), y''' = ?$

3.20. $y = (3x - 7)3^{-x}, y''' = ?$

3.21. $y = \frac{\ln(2x+5)}{2x+5}, y''' = ?$

3.22. $y = e^{\frac{x}{2}} \sin(2x), y''' = ?$

3.23. $y = \frac{\ln x}{x^5}, y''' = ?$

3.24. $y = x \ln(1 - 3x), y''' = ?$

3.25. $y = (x^2 + 3x + 1)e^{3x+1}, y''' = ?$

3.26. $y = (5x - 8)2^{-x}, y''' = ?$

3.27. $y = \frac{\ln(x-2)}{x+2}, y''' = ?$

3.28. $y = e^{-x}(\sin 2x - \cos 3x), y''' = ?$

3.29. $y = (5x - 1) \ln^2 x, y''' = ?$

3.30. $y = \frac{\log_3 x}{x^2}, y''' = ?$

3.31. $y = (x^3 + 1)e^{4x}, y''' = ?$

3.32. $y = \frac{\ln(2x)}{x^2}, y''' = ?$

3.33. $y = x \cos x^2, y''' = ?$

Task 4. Find the second order derivative of the function given parametrical equations:

$$4.1. \begin{cases} x = \cos 2t, \\ y = 2\sec^2 t. \end{cases}$$

$$4.2. \begin{cases} x = \sqrt{1-t^2}, \\ y = 1/t. \end{cases}$$

$$4.3. \begin{cases} x = e^t \cos t, \\ y = e^t \sin t. \end{cases}$$

$$4.4. \begin{cases} x = \operatorname{sh}^2 t, \\ y = 1/\operatorname{ch}^2 t. \end{cases}$$

$$4.5. \begin{cases} x = t + \sin t, \\ y = 2 - \cos t. \end{cases}$$

$$4.6. \begin{cases} x = 1/t, \\ y = 1/(1+t^2). \end{cases}$$

$$4.7. \begin{cases} x = \sqrt{t}, \\ y = 1/\sqrt{1-t}. \end{cases}$$

$$4.8. \begin{cases} x = \sin t, \\ y = \sec t. \end{cases}$$

$$4.9. \begin{cases} x = \operatorname{tg} t, \\ y = 1/\sin 2t. \end{cases}$$

$$4.10. \begin{cases} x = \sqrt{t-1}, \\ y = t/\sqrt{1-t}. \end{cases}$$

$$4.11. \begin{cases} x = \sqrt{t}, \\ y = \sqrt[3]{t-1}. \end{cases}$$

$$4.12. \begin{cases} x = \cos t/(1+2\cos t), \\ y = \sin t/(1+2\cos t). \end{cases}$$

$$4.13. \begin{cases} x = \sqrt{t^3-1}, \\ y = \ln t. \end{cases}$$

$$4.14. \begin{cases} x = \operatorname{sh} t, \\ y = \operatorname{th}^2 t. \end{cases}$$

$$4.15. \begin{cases} x = \sqrt{t-1}, \\ y = 1/\sqrt{t}. \end{cases}$$

$$4.16. \begin{cases} x = \cos^2 t, \\ y = \operatorname{tg}^2 t. \end{cases}$$

$$4.17. \begin{cases} x = \sqrt{t-3}, \\ y = \ln(t-2). \end{cases}$$

$$4.18. \begin{cases} x = \sin t, \\ y = \ln \cos t. \end{cases}$$

$$4.19. \begin{cases} x = t + \sin t, \\ y = 2 + \cos t. \end{cases}$$

$$4.20. \begin{cases} x = t - \sin t, \\ y = 2 - \cos t. \end{cases}$$

$$4.21. \begin{cases} x = \cos t, \\ y = \ln \sin t. \end{cases}$$

$$4.22. \begin{cases} x = \cos t + t \sin t, \\ y = \sin t - t \cos t. \end{cases}$$

$$4.23. \begin{cases} x = e^t, \\ y = \arcsin t. \end{cases}$$

$$4.24. \begin{cases} x = \cos t, \\ y = \sin^4(t/2). \end{cases}$$

$$4.25. \begin{cases} x = \operatorname{ch} t, \\ y = \sqrt[3]{\operatorname{sh}^2 t}. \end{cases}$$

$$4.26. \begin{cases} x = \operatorname{arctg} t, \\ y = t^2/2. \end{cases}$$

$$4.27. \begin{cases} x = 2(t - \sin t), \\ y = 4(2 + \cos t). \end{cases}$$

$$4.28. \begin{cases} x = \sin t - t \cos t, \\ y = \cos t + t \sin t. \end{cases}$$

$$4.29. \begin{cases} x = 1/t^2, \\ y = 1/(t^2 + 1). \end{cases}$$

$$4.30. \begin{cases} x = \cos t + \sin t, \\ y = \sin 2t. \end{cases}$$

$$4.31. \begin{cases} x = \ln t, \\ y = \operatorname{arctg} t. \end{cases}$$

$$4.32. \begin{cases} x = t + \sin t, \\ y = 2 - \cos t. \end{cases}$$

$$4.33. \begin{cases} x = \sqrt{t^3 - 1}, \\ y = \ln t. \end{cases}$$

Task 5. Find the limits using H'Lopital rule:

$$5.1 \quad \text{a) } \lim_{x \rightarrow 0} \frac{\ln \sin 3x}{\ln \sin 7x},$$

$$\text{b) } \lim_{x \rightarrow 0} \left(\ln \frac{1}{x} \right)^{\sin x}.$$

$$5.2. \quad \text{a) } \lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right),$$

$$\text{b) } \lim_{x \rightarrow 0} x^{\frac{3}{4 + \ln x}}.$$

$$5.3. \quad \text{a) } \lim_{x \rightarrow 0} \frac{\sin^3 x}{\sin 2x - \operatorname{tg} 2x},$$

$$\text{b) } \lim_{x \rightarrow \frac{\pi}{2} - 0} (\operatorname{tg} x)^{2x - \pi}.$$

$$5.4. \text{ a) } \lim_{x \rightarrow 2} \frac{2^x - x^2}{x - 2},$$

$$\text{b) } \lim_{x \rightarrow \infty} \left(\frac{2}{\pi} \operatorname{arctg} x \right)^x.$$

$$5.5. \text{ a) } \lim_{x \rightarrow 3} \frac{x^x - 3^3}{x - 3},$$

$$\text{b) } \lim_{x \rightarrow 0} (\operatorname{arcsin} x + x)^x.$$

$$5.6. \text{ a) } \lim_{x \rightarrow 0} \frac{\ln x}{1 + 2 \ln \sin x},$$

$$\text{b) } \lim_{x \rightarrow +0} \ln(\operatorname{ctg} x)^{\operatorname{tg} x}.$$

$$5.7. \text{ a) } \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3},$$

$$\text{b) } \lim_{x \rightarrow +0} x^{\frac{1}{\ln(e^x - 1)}}.$$

$$5.8. \text{ a) } \lim_{x \rightarrow 0} \frac{e^x \sin x - x(1 + x)}{x^3},$$

$$\text{b) } \lim_{x \rightarrow +0} \left(\ln \frac{1}{\operatorname{arcsin} x} \right)^{\operatorname{tg} x}.$$

$$5.9. \text{ a) } \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right),$$

$$\text{b) } \lim_{x \rightarrow 0} (\operatorname{ctg} x)^{1/\ln x}$$

$$5.10. \text{ a) } \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x} - \operatorname{ctg} x \right),$$

$$\text{b) } \lim_{x \rightarrow \infty} x^{\frac{3}{1 + \ln x}}.$$

$$5.11. \text{ a) } \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right),$$

$$\text{b) } \lim_{x \rightarrow +\infty} \left(\ln x \right)^{1/x}.$$

$$5.12. \text{ a) } \lim_{x \rightarrow 0} \frac{\ln \operatorname{arcsin} x - \ln x}{\operatorname{tg}^2 2x},$$

$$\text{b) } \lim_{x \rightarrow 0} (\operatorname{tg} x)^x.$$

$$5.13. \text{ a) } \lim_{x \rightarrow 0} \frac{\operatorname{arcsin} 2x - 2 \operatorname{arcsin} x}{x^3},$$

$$\text{b) } \lim_{x \rightarrow \infty} \left(\operatorname{tg} \frac{\pi x}{2x + 1} \right)^{1/x}.$$

$$5.14. \text{ a) } \lim_{x \rightarrow 0} \frac{\ln(e^x - 1) - \ln x}{x},$$

$$\text{b) } \lim_{x \rightarrow 0} \left(\frac{2^x - x \ln 2}{3^x - x \ln 3} \right)^{1/x}.$$

$$5.15. \text{ a) } \lim_{x \rightarrow 0} \frac{\ln(\ln(1+x)) - \ln x}{x}, \quad \text{b) } \lim_{x \rightarrow 0} \left(\frac{\arcsin x}{x} \right)^{\operatorname{ctg}^2 x}.$$

$$5.16. \text{ a) } \lim_{x \rightarrow 0} \frac{x \operatorname{ctg} x - 1}{\operatorname{tg}^2 x}, \quad \text{b) } \lim_{x \rightarrow 0} \left(\frac{1}{x} \right)^{\sin x}.$$

$$5.17. \text{ a) } \lim_{x \rightarrow 0} \left(\frac{1}{\ln(x + \sqrt{x^2 + 1})} - \frac{1}{\ln(1+x)} \right), \quad \text{b) } \lim_{x \rightarrow 0} x^x.$$

$$5.18. \text{ a) } \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - x}{x - \sin x}, \quad \text{b) } \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x^2}.$$

$$5.19. \text{ a) } \lim_{x \rightarrow 0} x \ln x, \quad \text{b) } \lim_{x \rightarrow 0} \left(\frac{(1+x)^{1/x}}{e} \right)^{1/x}.$$

$$5.20. \text{ a) } \lim_{x \rightarrow 0} x^3 e^{-x}, \quad \text{b) } \lim_{x \rightarrow 0} (\sin x)^{\operatorname{tg} x}.$$

$$5.21. \text{ a) } \lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x - \ln(1+2x)}{x^2}, \quad \text{b) } \lim_{x \rightarrow +\infty} \left(\frac{\pi}{2} - \operatorname{arctg} x \right)^{1/\ln x}.$$

$$5.22. \text{ a) } \lim_{x \rightarrow 1} \operatorname{arcsin} \left(\frac{x-1}{1} \right) \operatorname{ctg}(x-1), \quad \text{b) } \lim_{x \rightarrow +\infty} \left(\operatorname{arcctg} x \right)^{1/\ln x}.$$

$$5.23. \text{ a) } \lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \operatorname{ctg}^2 x \right), \quad \text{b) } \lim_{x \rightarrow 0} \left(x \operatorname{ctg} x \right)^{1/\ln(e^x + x)}.$$

$$5.24. \text{ a) } \lim_{x \rightarrow 1} \frac{x^x - 1}{\ln x - x + 1}, \quad \text{b) } \lim_{x \rightarrow 0} (1 - \ln \cos x)^{\operatorname{ctg}^2 x}.$$

$$5.25. \text{ a) } \lim_{x \rightarrow 0} \frac{(3+x)^3 - 3^3}{x^2},$$

$$\text{b) } \lim_{x \rightarrow 1} \left(\frac{1 - \ln x}{x} \right)^{\operatorname{ctg}(x-1)}.$$

$$5.26. \text{ a) } \lim_{x \rightarrow 0} \frac{\sin x - x \cos x}{x - \sin x},$$

$$\text{b) } \lim_{x \rightarrow 0} \left(\frac{2}{\pi} \arccos x \right)^{\frac{1}{x}}.$$

$$5.27. \text{ a) } \lim_{x \rightarrow 0} \left(\frac{1}{x} - \operatorname{ctgx} \right),$$

$$\text{b) } \lim_{x \rightarrow 0} (\ln \cos x)^{\operatorname{tg} x}.$$

$$5.28. \text{ a) } \lim_{x \rightarrow \infty} \left(x - x^2 \ln \left(1 + \frac{1}{x} \right) \right),$$

$$\text{b) } \lim_{x \rightarrow 0} \left(e^x + \operatorname{arctg} x \right)^{1/x}.$$

$$5.29. \text{ a) } \lim_{x \rightarrow 0} \frac{x - \operatorname{arctg} x}{x^3},$$

$$\text{b) } \lim_{x \rightarrow 0} (5^x + x \ln 5)^{\operatorname{ctg} x}.$$

$$5.30. \text{ a) } \lim_{x \rightarrow 0} \left(\frac{1}{x \sin x} - \frac{1}{x^2} \right),$$

$$\text{b) } \lim_{x \rightarrow 1} \left(\frac{x}{\ln x} \right)^{\operatorname{tg}(x-1)}.$$

$$5.31. \text{ a) } \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right),$$

$$\text{b) } \lim_{x \rightarrow +\infty} \left(\ln x \right)^{1/x}.$$

$$5.32. \text{ a) } \lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right),$$

$$\text{b) } \lim_{x \rightarrow 0} x^{\frac{3}{4 + \ln x}}.$$

$$5.33. \text{ a) } \lim_{x \rightarrow 0} \frac{\arcsin 2x - 2 \arcsin x}{x^3},$$

$$\text{b) } \lim_{x \rightarrow \infty} \left(\operatorname{tg} \frac{\pi x}{2x+1} \right)^{1/x}.$$

Task 6. Using the methods of differential calculus to investigate the following functions and plot their graphs

$$6.1. \text{ a) } y = \frac{x}{2(x-1)^2};$$

$$\text{b) } y = \frac{x}{e^x}.$$

$$6.16 \text{ a) } y = \frac{x^4 - 81}{3x^2};$$

$$\text{b) } y = \frac{\sqrt{x}}{\ln x}.$$

6.2. a) $y = \frac{8x}{4 + x^2}$;

b) $y = \frac{e^x}{x}$.

6.3. a) $y = \frac{3x - 2}{x^2}$;

b) $y = \frac{e^{\frac{1}{2}(x+1)}}{x+1}$.

6.4. a) $y = \frac{x^2}{x-1}$;

b) $y = \ln(1 + x^2)$

6.5. a) $y = \frac{x^3 + 125}{12x}$;

b) $y = \ln(x+1) - x$.

6.6. a) $y = \frac{x^2}{x^2 - 1}$;

b) $y = 2e^{-x^2} x$.

6.7. a) $y = \frac{4x - 12}{(x-2)^2}$;

b) $y = \frac{\ln x}{x}$.

6.8. a) $y = \frac{2x - 1}{(x-1)^2}$;

b) $y = 4xe^{-x}$.

6.17. a) $y = \frac{16}{x^2(x-4)}$;

b) $y = 2 \ln \frac{x}{x+1} - 1$.

6.18. a) $y = \frac{x^2 + 1}{x^2 - 1}$;

b) $y = 4e^{-x^2/2}$.

6.19. a) $y = \frac{3x^4 + 1}{x^3}$;

b) $y = (3-x)e^{x-2}$.

6.20. a) $y = \frac{2x}{1-x^2}$;

b) $y = x \ln x$.

6.21. a) $y = x^2 - \frac{2}{x}$;

b) $y = 2xe^{-1/2x}$.

6.22. a) $y = \frac{1-x^3}{x^2}$;

b) $y = x + \frac{\ln x}{x}$.

6.23. a) $y = \frac{x^2 - 2x + 2}{x-1}$;

b) $y = \frac{e^{2(x-1)}}{2(x-1)}$.

6.9. a) $y = \frac{x^4 - 3}{x}$;

b) $y = (2x + 3)e^{-2(x+1)}$.

6.10. a) $y = \frac{2x}{x^2 + 1}$;

b) $y = \frac{4 \ln x}{x}$.

6.11. a) $y = \frac{x^4 + 4}{x^2}$;

b) $y = 3 - 3 \ln \frac{x}{x+4}$.

6.12. a) $y = x - 1 + \frac{1}{x+1}$;

b) $y = 2xe^{-1/2^x}$.

6.13. a) $y = \frac{x}{\sqrt[3]{x^2 - 1}}$;

b) $y = \frac{e^{2(x+2)}}{2(x+2)}$.

6.14. a) $y = \frac{2x^3}{x^2 + 1}$;

b) $y = \ln(x^2 - 2x + 2)$.

6.15. a) $y = \frac{x^2 - 1}{x^2 + 1}$;

b) $y = 8xe^{-x/2}$.

6.24. a) $y = \frac{(x^2 + 1)^2}{x^2 + 2}$;

b) $y = \frac{x}{\ln x}$.

6.25. a) $y = \frac{4x}{(x+1)^2}$;

b) $y = \ln \frac{x}{x-2} - 2$.

6.26. a) $y = \frac{x}{x^2 - 4}$;

b) $y = 3x \ln x$.

6.27. a) $y = \frac{1 - 2x}{x^2}$;

b) $y = \frac{e^{3-x}}{3-x}$.

6.28. a) $y = \frac{x}{3 - x^2}$;

b) $y = \frac{e^{2x}}{x}$.

6.29. a) $y = \frac{x^3 - 4}{2x^2}$;

b) $y = 2 \ln \frac{x+3}{x} - 3$.

6.30. a) $y = \frac{x}{(1-x)^2}$;

b) $y = \frac{\ln x}{2x}$.

6.31. a) $y = \frac{x}{2(x-1)^2};$

b) $y = \frac{x}{e^x}.$

6.32. a) $y = x - 1 + \frac{1}{x+1};$

b) $y = 2xe^{-1/2^x}.$

6.33 a) $y = \frac{x^4 - 81}{3x^2};$

b) $y = \frac{\sqrt{x}}{\ln x}.$

6.34. a) $y = \frac{1-2x}{x^2};$

b) $y = \frac{e^{3-x}}{3-x}.$