HIGHER MATHEMATICS

SYLLABUS

Code and name of the specialty	**171 Electronics**	**Institute / faculty**	**Institute of Education and Science in Power Engineering, Electronics and Electromechanics**
Name of the program	**Electronics; Electric cars and automotive electronics**	**Department**	**Applied Mathematics**
Program type	**Educational and professional**	**Language of study**	**English**

Teacher

Larysa Dzyubak, Larysa.Dzyubak@khpi.edu.ua

Scientific degree: Ph.D. in Physics and Mathematics
Academic title: Associate Professor
Position: Associate Professor of the Department of Applied Mathematics
Number of publications: 70 of Scientific and Educational Publications
Basic courses: Higher Mathematics, Mathematical Analysis, Linear Algebra, Analytic Geometry

General information about the course

Summary	The academic course is aimed at mastering the theoretical and practical skills of applying the main sections of the higher mathematics course. The academic course is taught during 4 semesters and includes: 144 hours of lectures, 160 hours of practical classes, 266 hours of independent work. The final control is an exam at the end of each semester.
Course goals	familiarization and mastering of modern mathematical methods, necessary for solving theoretical and practical problems (by applying methods of studying the function of one and several variables, using limits, integrals and series, etc.); development of skills in mathematical research of applied problems, construction of mathematical models and formation of mathematical knowledge for mastering other disciplines of the mathematical cycle; development of students' ability to independently deepen and expand mathematical knowledge and implement it in the analysis of applied problems.
Format	Lectures, practical classes, independent work, consultations. The final control is an exam.
Scope	The general scope of the academic course is 570 hours: 144 hours of lectures, 160 hours of practical classes, 266 hours of independent work.

Learning achievements	Acquisition of mathematical knowledge by students, necessary for studying a number of general scientific disciplines and disciplines of the professional cycle.
Topics covered	
Semester 1	
Content module 1. Linear Algebra	
Theme 1	Matrices and Determinants
Theme 2	Systems of linear algebraic equations (SLAE)
Content module 2. Analytic Geometry	
Theme 1	Vectors and operations on vectors
Theme 2	Plane and straight line in space R^3
Theme 3	Straight line and the second order curves in plane R^2
Content module 3. The theory of limits and function continuity	
Theme 1	Limits of sequences
Theme 2	Limits and continuity of functions
Content module 4. Differential calculus of function of one variable	
Theme 1	Derivative and differential of function of one variable
Semester 2	
Content module 4. Differential calculus of function of one variable	
Theme 2	Application of the derivative
Content module 5. Integral calculus of a function of one variable	
Theme 1	Indefinite integral

Theme 2	Definite integral
Content module 6. Differential calculus of the multivariable functions	
Theme 1	The concept of the multivariable function
Theme 2	Derivative and differential of the function of many variables
Semester 3	
Content module 7. Differential equations	
Theme 1	Differential equations of the first order
Theme 2	Differential equations of the second and higher orders. Systems of differential equations
Content module 8. Integral calculus for functions with several variables	
Theme 1	Multiple integrals
Theme 2	Curvilinear integrals and surface integrals
Semester 4	
Content module 9. Field Theory	
Theme 1	Scalar and vector fields
Content module 10. Series	
Theme 1	Numerical series
Theme 2	Functional series

Academic course structure

Semester 1

Lecture 1	Matrices. Types of matrices. Operations on matrices. The determinants of the 2nd and the 3rd orders.	**Practical class 1**	Linear operations on matrices and multiplication of matrices. Calculation of the determinants of the 2nd and the 3rd orders.
Lecture 2	Determinants of the n^{th} order, their calculation. Properties of the determinants. Algebraic cofactors and minors of the matrix element.	**Practical class 2**	Calculation of the determinants. Calculation of algebraic cofactors and minors of the matrix elements.
Lecture 3	Rule by Cramer. Inverse matrix. Solving the matrix equations by means of inverse matrix.	**Practical class 3**	Rule by Cramer. Inverse matrix. Solving the matrix equations by means of inverse matrix.
Lecture 4	Rank of matrix. Calculation of the rank. The theorem about basic minor.	**Practical class 4**	Calculation of the rank by means of elementary transformations.
Lecture 5	Theorem by Kronecker-Capelli. General solution of SLAE.	**Practical class 5**	Theorem by Kronecker-Capelli. Solving of SLAE by the method by Gauss.
Lecture 6	Methods by Gauss and by Jordan-Gauss. Finding the fundamental system of solutions.	**Practical class 6**	Solving SLAE by the method by Gauss and Jordan-Gauss. Homogeneous systems.
Lecture 7	Concept of vector. Linear operations on vectors. Decomposition of the vector in basis. Projection of vector on axis. Coordinates in Cartesian coordinate system.	**Practical class 7**	Linear operations on vectors. Decomposition of vector. Coordinates in new basis.
Lecture 8	Scalar product of two vectors. Definition and properties. Condition of perpendicularity.	**Practical class 8**	Problems on the concept of scalar product of vectors.
Lecture 9	Vector product of two vectors. Definition and properties. Condition of collinearity.	**Practical class 9**	Problems on the concept of vector product of vectors.
Lecture 10	Mixed product of three vectors. Definition and properties. Condition of complanarity.	**Practical class 10**	Problems on the concept of mixed product of vectors.
Lecture 11	General equation of plane in Cartesian coordinate system. Mutual relationship of two planes.	**Practical class 11**	Solving the problems on finding the equation of plane.
Lecture 12	Different types of equations of straight line in space. Mutual relationship of two straight lines.	**Practical class 12**	Straight line in space.
Lecture 13	Mutual relationship of a plane and a straight line. Mixed problems.	**Practical class 13**	Mixed problems on a plane and a straight line.
Lecture 14	Different types of equations of a straight line in plane.	**Practical class 14**	Problems on finding the equation of straight line in plane.

Lecture 15	Circle, parabola, ellipse and hyperbola. Canonical equations. Properties of graphs.	**Practical class 15**	Finding the canonical equations of the second order curves by allocating of the full square. Plotting the graphs.
Lecture 16	Reducing of the general equation of the second order curves to canonical equation.	**Practical class 16**	Reducing the general equation of the second order curves to canonical equation.
Lecture 17	Sets and operations on sets. Classification of functions and their characteristics.	**Practical class 17**	Operations on sets. Graphs of main elementary functions. Domain of definition of functions.
Lecture 18	Numerical sequences. Limit of numerical sequence. Properties of convergent sequences.	**Practical class 18**	Calculation of limits of sequences.
Lecture 19	Infinitesimals and infinitely large values. Properties. Monotonic sequence and its limit.	**Practical class 19**	Calculation of limits of sequences.
Lecture 20	Limit of function at point and at infinity. One-side-limits of functions. Properties of limits. Solving problems with undefined expressions.	**Practical class 20**	Limit of rational fractions at a point and at infinity.
Lecture 21	The first and the second remarkable limits. Their consequences. Table of equivalences and their application to limit calculation.	**Practical class 21**	Calculation of limits by means of equivalences.
Lecture 22	Continuous functions at the point. Points of discontinuity and their classification. Theorems about functions continuous in the segment.	**Practical class 22**	Investigation of points of discontinuity and their classification.
Lecture 23	Derivative of function. Tangent and normal lines to the graph of function. Derivative of sum, product, quotient. Table of derivatives.	**Practical class 23**	Calculation of derivatives by means of table of derivatives and rules of differentiating. Logarithmic differentiation.
Lecture 24	Differential of function. Approximate calculations. Derivatives and differentials of higher orders. Formula by Leibnitz.	**Practical class 24**	Approximate calculations. Equations of tangent and normal lines to graph.

Semester 2

Lecture 1	Basic theorems of differential calculus (theorems by Fermat, Rolle, Lagrange, Cauchy). L'Hospital's Rule.	**Practical class 1**	Calculation of derivatives of higher orders. Formula by Leibnitz.
Lecture 2	Taylor's formula for a polynomial. Taylor's formula for functions. McLaren's formula.	**Practical class 2**	Calculation of limits using L'Hospital's Rule.
Lecture 3	Monotonicity and extrema of the function. Necessary and sufficient conditions of the extremum. The largest and smallest values of the function on the interval.	**Practical class 3**	Expanding the polynomials by means of Taylor's formula and functions by means of McLaren's formulas.
Lecture 4	Convexity conditions and inflection points of the graph of the function. Asymptotes. General scheme of graphing the function.	**Practical class 4**	Finding the extremum points and intervals of monotonicity of the function graph.
Lecture 5	Indefinite integral. Properties indeterminate integrals. Table of basic integrals. Basic methods of integration.	**Practical class 5**	Finding the largest and smallest values of a function.

Lecture 6	Complex numbers and operations with complex numbers. Trigonometric Form of a Complex Number. Formula by Moivre.	**Practical class 6**	Convexity and concavity of the function graph. Finding inflection points and asymptotes of the function graph. General scheme of graphing the function.
Lecture 7	Expanding a rational fraction into the Partial Rational Fractions. Integration of Partial Rational Fractions. Integration of rational fraction.	**Practical class 7**	Calculation of the indefinite integral according to the table. Changing the variable in the indefinite integral.
Lecture 8	Integration functions that are rationally dependent on trigonometric functions.	**Practical class 8**	Calculation of the indefinite integral – integration by parts.
Lecture 9	Integration of Some Irrational Functions.	**Practical class 9**	Expanding a rational fraction into the partial rational fractions. Integration of partial rational fractions of the 1-4 types.
Lecture 10	Definite integral. Problems that lead to the concept of integral, the conditions of its existence. Properties of a definite integral.	**Practical class 10**	Integration of fractional rational functions.
Lecture 11	Fundamental theorem of calculus. The Formula by Newton-Leibniz. Replacement of a variable, integration by parts in definite integral.	**Practical class 11**	Integration of trigonometric expressions.
Lecture 12	Application of a definite integral for calculation Area of Plane Figures.	**Practical class 12**	Integration of irrational expressions. Integration of irrational expressions using trigonometric expressions.
Lecture 13	Application of a definite integral for calculation of the surface area and volume of a body of rotation.	**Practical class 13**	Calculation of the definite integral according to the Newton-Leibniz formula. Calculation of the definite integral by substitution of variable and integration by parts.
Lecture 14	Application of a definite integral for calculation of Arc Length of a Curve.	**Practical class 14**	Calculation of the area of a plane figure in the Cartesian coordinate system using the definite integral. Calculation of the area of a plane figure in the polar coordinate system and given by parametric equations.
Lecture 15	Integrals with Infinite Limits (I-kind)	**Practical class 15**	Calculation of surface area and volume of a body of rotation, volumes of bodies with a given law of change of cross-sectional area. Calculation of the arc length in Cartesian and polar coordinate systems, as well as given by parametric equations, using the definite integral.
Lecture 16	Improper Integrals of the 2nd Kind (Integrals of Unbounded Functions, Integrals of Discontinuous Functions)	**Practical class 16**	Application of the definite integral for solving physics and mechanics problems.
Lecture 17	The function of two or more variables. Geometric content of the function of two variables. Domain of the function of two and three variables.	**Practical class 17**	Calculation and investigation of the convergence of improper integrals of the first and second kind.
Lecture 18	Limit of the function of two variables. Continuity of the Functions of two variables.	**Practical class 18**	Finding the domain of a function of two variables.
Lecture 19	Partial derivatives. Geometric content of the partial derivative of the function of two variables. Necessary and sufficient conditions for the differentiation of the function of two variables. Differential of the function of many variables.	**Practical class 19**	The method of intersections for constructing surfaces of the 2nd order.

Lecture 20	Derivative of a composite function. The Total Derivative. The derivative of the implicit function. Invariance of the form of the first differential.	**Practical class 20**	Finding the limits of functions of two variables.
Lecture 21	Derivatives and differentials of higher orders. Mixed derivatives. Partial Derivatives of Higher Orders.	**Practical class 21**	Application of the differential and Taylor's formula for approximate calculations.
Lecture 22	Taylor's Formula for a Function of Two Variables. Quadratic forms. Sylvester's criterion.	**Practical class 22**	Solving problems for finding the largest and smallest values of a function in a given domain. Finding unconditional and conditional extrema of functions.
Lecture 23	Extrema of Two Variables Function.	**Practical class 23**	Application of the differential and Taylor's formula for approximate calculations.
Lecture 24	The Equation of a Tangent Line to a Curve in Space. Equation of a Tangent Plane and Normal to a Surface.	**Practical class 24**	Solving problems for finding the largest and smallest values of a function in a given domain. Finding unconditional and conditional extrema of functions.

Semester 3

Lecture 1	Problems leading to necessity of solving differential equations. The general and partial solution. Problem by Cauchy. Equations with separable variables. Homogeneous differential equations in Euler's sense.	**Practical class 1**	Integration of the equations with separable variables. Integration of the homogeneous differential equations in Euler's sense.
Lecture 2	Linear differential equations and solving methods for linear equations of the first order. Bernoulli's equation.	**Practical class 2**	Integration of the linear equations. Integration of the Bernoulli's Equations.
Lecture 3	Exact differential equations (equations in total differentials). Equations not solved for derivative. Equations by Lagrange and Clairaut.	**Practical class 3**	Integration of the exact differential equations (equations in total differentials). Different types of the differential equations. Integration of the equations by Lagrange and Clairaut.
Lecture 4	Differential equations of higher orders. General conceptions. Some types of the second order differential equations reducible to the first order equations.	**Practical class 4**	Integration of the second order differential equations reducible to the first order equations.
Lecture 5	Linear differential equations of the second order. The general theorems. Wronskian's determinant. Ostrogradskiy-Liuvill's formula. Homogeneous linear equations of the second order with constant coefficients.	**Practical class 5**	Integration of the homogeneous linear equations of the second order with constant coefficients.
Lecture 6	The general theorems for inhomogeneous linear equations of the second order (INHLDE). The linear differential equation of the second order with constant coefficients and special right part. The method of variation of an arbitrary constants.	**Practical class 6**	Integration of the inhomogeneous linear equations of the second order with constant coefficients.
Lecture 7	Homogeneous linear equations of the n-th order with constant coefficients. Higher order inhomogeneous linear equations.	**Practical class 7**	Integration of the linear equations of the n-th order with constant coefficients.
Lecture 8	Systems of Differential Equations. General Concept and Definition. Systems of Linear Differential Equations. Method of Elimination.	**Practical class 8**	Integration of the Systems of Differential Equations.

Lecture 9	Double integral. Some problems leading to concept of the double integrals. Calculation of the double integrals in the Cartesian coordinate system.	**Practical class 9**	Calculation of the double integrals in Cartesian system coordinates. Calculation of the solid volumes in Cartesian system coordinates.
Lecture 10	Curvilinear coordinates on the plane. Change of the variables in the double integrals. Physical applications of the double integral.	**Practical class 10**	Calculation of double integrals in polar system of coordinates. Physical applications of the double integral.
Lecture 11	Triple integral calculation and applications. Calculation of the triple integral in the Cartesian coordinates system, in cylindrical and spherical coordinate systems.	**Practical class 11**	Calculation of the triple integrals in Cartesian system of coordinates, in cylindrical and spherical systems of coordinates.
Lecture 12	Curvilinear integral of the first kind. Applications of the curvilinear integrals of the first kind.	**Practical class 12**	Curvilinear integral of the first kind: calculation and applications.
Lecture 13	Curvilinear integrals of the II-nd kind (line integrals). Calculation, physical and geometrical properties. Connection between line integrals and curvilinear integrals of the first kind. Green's formula.	**Practical class 13**	Curvilinear integral of the II-nd kind: calculation and applications. Application of the Green's formula.
Lecture 14	Condition for line integral to be independent of the integration path. Finding function by its total differential.	**Practical class 14**	Independence of the line integrals on form of the integration path. Finding antiderivative.
Lecture 15	Surface integrals of the I-st and of the II-nd kind. Physical and geometric properties of these integrals.	**Practical class 15**	Evaluation of the surface integrals of the I-st and of the II-nd kind.
Lecture 16	Surface integrals over a closed surface. Ostrogradsky-Gauss formula. Stokes' formula.	**Practical class 16**	Evaluation of the surface integrals over a closed surface. Ostrogradsky-Gauss formula. Stokes' formula.

Semester 4

Lecture 1	Definitions of scalar and vector fields. Level surfaces and lines, directional derivatives, gradient of scalar fields. Vector lines of vector fields.	**Practical class 1**	Calculation of surface integrals of the first and the second kind. Geometric and physical applications of these integrals.
		Practical class 2	Theorem by Ostrogradsky-Gauss, theorem by Stockes.
Lecture 2	Concept of the flux of the vector field. The divergence of the field. The theorem by Ostragradsky-Gauss.	**Practical class 3**	Finding the level surfaces and lines, the directional derivatives and the gradient of scalar fields.
		Practical class 4	Finding the vector lines of the vector field. Calculation of the flux and the divergence of the vector field.
Lecture 3	Linear integral of the vector field. Circulation. Concept of the curl. The theorem by Stokes in vector form. Types of the vector fields. Finding the potential of the potential fields.	**Practical class 5**	Application of the theorem by Ostrogradsky-Gauss to find the flux.
		Practical class 6	Finding the circulation and the curl of the vector field. Application of the Stockes's theorem.

Lecture 4	Concepts of the numerical series and the convergence of it. Properties of the convergent series. Necessary condition for convergence. Comparison of the series with positive terms. Two comparison tests for convergence. D'Alembert's ratio test for convergence.	**Practical class 7**	Analyzing the type of the vector field.
		Practical class 8	Test on content module 9.
Lecture 5	Integral and radical tests by Cauchy for convergence. Plus-and-minus series. Absolute and conditional Convergence. Alternating series. Theorem by Leibniz.	**Practical class 9**	Finding the sums of convergent series. Comparison tests for convergence.
		Practical class 10	Application of the convergence tests to the series with positive terms.
Lecture 6	Functional series. Uniform convergence. Test by Weierstrass. Integration and differentiation of the functional series.	**Practical class 11**	Plus-and-minus series. Investigation of the absolute and conditional convergence. Alternating series. Sign by Leibniz.
		Practical class 12	Finding the domains of convergence of the functional and power series. Integration and differentiation of the functional series.
Lecture 7	Power series. Abel's theorem. Taylor's series and Maclaurin's series. Expansion the functions in Taylor's series and Maclaurin's series.	**Practical class 13**	Integration and differentiation of the functional series.
		Practical class 14	Power functions. Intervals of the convergence. Expansion the functions in Taylor's series and Maclaurin's series.
Lecture 8	Fourier's series. Problem formulation. Orthogonality of the trigonometric system of functions. The Fourier's series for a function with period 2l. Fourier's series for even and odd functions. The expansion of a nonperiodic function in a Fourier's series.	**Practical class 15**	Application of the Series for approximate calculations.
		Practical class 16	The expansion of a function in a Fourier's series.

Independent work

Order No	Name of types of independent work	Hours
1	Lecture material processing	36
2	Preparation for practical (laboratory) classes	80
3	Individual assignments execution	54
4	Preparation for tests	38
5	Independent processing of the material according to the course topic	58
	Total (hours)	266

LITERATURE AND EDUCATIONAL MATERIALS

Basic

1. Rudnyeva G.V. Elements of linear algebra and analytic geometry. Second revised and expanded edition: textbook / G.V.Rudnyeva – Kharkiv: Panov A.M., 2020. – 236 p.
2. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU “KhPI”, 2004. – Volume 1.
3. Kurpa L.V., Shmatko T.V. Differential and Integral Calculus for One Variable Functions: Textbook. – Kharkiv: NTU "KhPI": 2017.
4. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU “KhPI”, 2004. – Volume 2.
5. L.V. Kurpa, T.V.Shmatko. Differential and integral calculus for functions with several variables: Textbook. – Kharkiv: NTU "KhPI": 2012.
6. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU “"KhPI"”, 2004. – Volume 3.
7. L.V. Kurpa, O.S. Mazur, T.V.Shmatko. Differential Equations and Series: Textbook. – Kharkiv: NTU "KhPI": 2013.
8. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU “"KhPI"”, 2004. – Volume 4.

LIST OF EQUIPMENT

No special equipment is required

Knowledge and skills assessment scale: national and ECTS rating

Distribution of points for evaluating the success of a graduate student	The sum of points for all types of educational activity	ECTS assessment	National assessment	Scoring	
	90 – 100	A	Excellent		
	82 – 89	B	Good		
	75 – 81	C			
	64 – 74	D	Satisfactory		
	60 – 63	E			
	35 – 59	FX	Unsatisfactory with the possibility of retaking the exam		
	0 – 34	F	Unsatisfactory with mandatory repeated study of the academic course		

Points are awarded according to the following ratio:

- control works and test tasks: 45% of the semester assessments;
- independent work: 35% of the semester assessments;
- exam: 20% of the semester assessments.

NORMS OF ACADEMIC ETHICS

The student must adhere to the "Code of Ethics of Academic Relations and Integrity of NTU "KhPI"": show discipline, education, benevolence, honesty, responsibility. Conflict situations should be openly discussed in study groups with the teacher, and if the conflict cannot be resolved, it should be brought to the directorate's staff.

The content of the syllabus fully corresponds to the academic course working program