Syllabus
Course Program

Higher Mathematics

Specialty
141 – Electric power engineering
171 – Electronics

Institute
IES in Power Engineering, Electronics and Electromechanics

Educational program
Electronics; Electric cars and automotive electronics

Department
Applied Mathematics (170)

Level of education
Bachelor

Course type
General Education Course, mandatory

Semesters
1,2,3 and 4

Language of instruction
English

Lecturers and course developers

Larysa Dzyubak
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Ph.D. in Physics and Mathematics,
Associate Professor of the Department of Applied Mathematics NTU "KhPI"

Work experience: over 25 years
Number of scientific and educational-methodological works: over 70
Number of papers indexed in Scopus and Web of Science: 19
Chapters in books: 3
Number of presentations at international scientific conferences: 30
Main courses: Higher Mathematics, Mathematical Analysis, Linear Algebra, Analytic Geometry

More details on the web-page:
https://web.kpi.kharkov.ua/apm/personal/larisa-p-dzyubak/

General information

Summary

The present Higher Mathematics course is organized into four parts (a four-semester course). It covers the following main topics: Elements of linear algebra, Analytical geometry, Theory of limits and continuity of functions, Differential and integral calculus of functions of one variable, Differential and integral calculus of functions of several variables, Differential equations, Field theory and Series.

Course objectives and goals

The course objectives and goals are familiarization and mastering of modern mathematical methods, necessary for solving theoretical and practical problems (by applying methods of studying the function of one and several variables, using limits, integrals and series, etc.); development of skills in mathematical research of applied problems, construction of mathematical models and formation of mathematical knowledge for mastering other disciplines of the mathematical cycle; development of students' ability to independently deepen and expand mathematical knowledge and implement it in the analysis of applied problems.

Format of classes

Lectures, practical classes, tests, self-study, consultations. The final control is an exam.

Competencies

GC01. Ability to abstract thinking, analysis and synthesis.
GC02. Ability to apply knowledge in practical situations.
GC05. Ability to search, process and analyze information from various sources.
GC06 Ability to learn and master modern knowledge.
GC12. The ability to solve practical problems involving the methods of mathematics, physics and engineering.

Learning outcomes

LR01. Students are expected to acquire proficiency in linear and vector algebra, analytic geometry, and the theory of limits. Furthermore, they should be adept at applying fundamental concepts and methods from the course to solve specialized problems within the domains of industrial and electric power engineering.

Student workload

510 hours (17 ECTS credits): lectures – 112 hours, practical classes – 160 hours, individual independent work – 238 hours

Course prerequisites

To successfully pass the course, students must have the knowledge and skills from the elementary mathematics course of high school.

Features of the course, teaching and learning methods, and technologies

The 'Higher Mathematics' course is delivered through lectures, practical classes, tests, consultations that incorporate multimedia technologies, in particualar, Microsoft Office 365 Teams. Additionally, students engage in individual independent study to master the educational material and complete individual educational tasks.

Program of the course

Topics of the lectures

Semester 1

Topic 1. Linear Algebra

Matrices. Types of matrices. Operations on matrices. The determinants of the 2nd and the 3rd orders. Determinants of the n^{th} order, their calculation. Properties of the determinants. Algebraic cofactors and minors of the matrix element. Rule by Cramer. Inverse matrix. Solving the matrix equations by means of inverse matrix. Rank of matrix. Calculation of the rank. The theorem about basic minor. Theorem by Kronecker-Capelli. General solution of SLAE. Methods by Gauss and by Jordan-Gauss. Finding the fundamental system of solutions.

Topic 2. Analytic Geometry

Concept of vector. Linear operations on vectors. Decomposition of the vector in basis. Projection of vector on axis. Coordinates in Cartesian coordinate system. Scalar product of two vectors. Definition and properties. Condition of perpendicularity. Vector product of two vectors. Definition and properties. Condition of collinearity. Mixed product of three vectors. Definition and properties. Condition of complanarity.

General equation of plane in Cartesian coordinate system. Mutual relationship of two planes. Different types of equations of straight line in space. Mutual relationship of two straight lines. Mutual relationship of a plane and a straight line. Mixed problems. Different types of equations of a straight line in plane.

Circle, parabola, ellipse and hyperbola. Canonical equations. Properties of graphs. Reducing of the general equation of the second order curves to canonical equation.

Topic 3. The theory of limits and function continuity

Sets and operations on sets. Classification of functions and their characteristics. Numerical sequences. Limit of numerical sequence. Properties of convergent sequences. Infinitesimals and infinitely large values. Properties. Monotonic sequence and its limit. Limit of function at point and at infinity. One-side-limits of functions. Properties of limits. Solving problems with undefined expressions. The first and the second remarkable limits. Their consequences. Table of equivalences and their application to limit calculation. Continuous functions at the point. Points of discontinuity and their classification. Theorems about functions continuous in the segment.

Topic 4. Differential calculus of function of one variable

Derivative of function. Tangent and normal lines to the graph of function. Derivative of sum, product, quotient. Table of derivatives. Differential of function. Approximate calculations. Derivatives and differentials of higher orders. Formula by Leibnitz.

Semester 2

Topic 1. Differential calculus of function of one variable

Basic theorems of differential calculus (theorems by Fermat, Rolle, Lagrange, Cauchy). L'Hospital's Rule. Taylor's formula for a polynomial. Taylor's formula for functions. McLaren's formula.

Monotonicity and extrema of the function. Necessary and sufficient conditions of the extremum. The largest and smallest values of the function on the interval. Convexity conditions and inflection points of the graph of the function. Asymptotes. General scheme of graphing the function.

Topic 2. Integral calculus of a function of one variable

Indefinite integral. Properties indeterminate integrals. Table of basic integrals. Basic methods of integration. Complex numbers and operations with complex numbers. Trigonometric Form of a Complex Number. Formula by Moivre. Expanding a rational fraction into the Partial Rational Fractions. Integration of Partial Rational Fractions. Integration of rational fraction. Integration functions that are rationally dependent on trigonometric functions. Integration of Some Irrational Functions.

Definite integral. Problems that lead to the concept of integral, the conditions of its existence. Properties of a definite integral. Fundamental theorem of calculus. The Formula by Newton-Leibniz. Replacement of a variable, integration by parts in definite integral. Application of a definite integral for calculation Area of Plane Figures. Application of a definite integral for calculation of the surface area and volume of a body of rotation. Application of a definite integral for calculation of Arc Length of a Curve.

Integrals with Infinite Limits (I-kind). Improper Integrals of the 2nd Kind (Integrals of Unbounded Functions, Integrals of Discontinuous Functions)

Topic 3. Differential calculus of the multivariable functions

The function of two or more variables. Geometric content of the function of two variables. Domain of the function of two and three variables. Limit of the function of two variables. Continuity of the Functions of two variables. Partial derivatives. Geometric content of the partial derivative of the function of two variables. Necessary and sufficient conditions for the differentiation of the function of two variables. Differential of the function of many variables. Derivative of a composite function. The Total Derivative. The derivative of the implicit function. Invariance of the form of the first differential. Derivatives and differentials of higher orders. Mixed derivatives. Partial Derivatives of Higher Orders. Taylor's Formula for a Function of Two Variables. Quadratic forms. Sylvester's criterion. Extrema of Two Variables Function. The Equation of a Tangent Line to a Curve in Space. Equation of a Tangent Plane and Normal to a Surface.

Semester 3

Topic 1. Differential equations

Problems leading to necessity of solving differential equations. The general and partial solution. Problem by Cauchy. Equations with separable variables. Homogeneous differential equations in Euler's sense. Linear differential equations and solving methods for linear equations of the first order. Bernoulli's equation. Exact differential equations (equations in total differentials). Equations not solved for derivative. Equations by Lagrange and Clairaut. Differential equations of higher orders. General conceptions. Some types of the second order differential equations reducible to the first order equations.

Linear differential equations of the second order. The general theorems. Wronskian's determinant. Ostrogradskiy-Liuvill's formula. Homogeneous linear equations of the second order with constant coefficients. The general theorems for inhomogeneous linear equations of the second order (INHLDE). The linear differential equation of the second order with constant coefficients and special right part. The method of variation of an arbitrary constants. Homogeneous linear equations of the n-th order with constant coefficients. Higher order inhomogeneous linear equations. Systems of Differential Equations. General Concept and Definition. Systems of Linear Differential Equations. Method of Elimination.

Topic 2. Integral calculus for functions with several variables

Double integral. Some problems leading to concept of the double integrals. Calculation of the double integrals in the Cartesian coordinate system. Curvilinear coordinates on the plane. Change of the variables in the double integrals. Physical applications of the double integral. Triple integral calculation and applications. Calculation of the triple integral in the Cartesian coordinates system, in cylindrical and spherical coordinate systems.

Curvilinear integral of the first kind. Applications of the curvilinear integrals of the first kind. Curvilinear integrals of the II-nd kind (line integrals). Calculation, physical and geometrical properties. Connection between line integrals and curvilinear integrals of the first kind. Green's formula. Condition for line integral to be independent of the integration path. Finding function by its total differential. Surface integrals of the I-st and of the II-nd kind. Physical and geometric properties of these integrals. Surface integrals over a closed surface. Ostrogradsky-Gauss formula. Stokes' formula.

Semester 4

Topic 1. Field Theory

Definitions of scalar and vector fields. Level surfaces and lines, directional derivatives, gradient of scalar fields. Vector lines of vector fields. Concept of the flux of the vector field. The divergence of the field. The theorem by Ostragradsky-Gauss. Linear integral of the vector field. Circulation. Concept of the curl. The theorem by Stokes in vector form. Types of the vector fields. Finding the potential of the potential fields.

Topic 2. Series

Concepts of the numerical series and the convergence of it. Properties of the convergent series. Necessary condition for convergence. Comparison of the series with positive terms. Two comparison tests for convergence. D'Alembert's ratio test for convergence. Integral and radical tests by Cauchy for convergence. Plus-and-minus series. Absolute and conditional Convergence. Alternating series. Theorem by Leibniz.

Functional series. Uniform convergence. Test by Weierstrass. Integration and differentiation of the functional series. Power series. Abel's theorem. Taylor's series and Maclaurin's series. Expansion the functions in Taylor's series and Maclaurin's series. Fourier's series. Problem formulation. Orthogonality of the trigonometric system of functions. The Fourier's series for a function with period 2l. Fourier's series for even and odd functions. The expansion of a nonperiodic function in a Fourier's series.

Topics of the workshops

Semester 1

Topic 1. Linear Algebra

Linear operations on matrices and multiplication of matrices. Calculation of the determinants of the 2nd and the 3rd orders. Calculation of the determinants. Calculation of algebraic cofactors and minors of the matrix elements. Rule by Cramer. Inverse matrix. Solving the matrix equations by means of inverse matrix. Calculation of the rank by means of elementary transformations. Theorem by Kronecker-Capelli. Solving of SLAE by the method by Gauss. Solving SLAE by the method by Gauss and Jordan-Gauss. Homogeneous systems.

Topic 2. Analytic Geometry

Linear operations on vectors. Decomposition of vector. Coordinates in new basis. Problems on the concept of scalar product of vectors. Problems on the concept of vector product of vectors. Problems on the concept of mixed product of vectors.

Solving the problems on finding the equation of plane. Straight line in space. Mixed problems on a plane and a straight line. Problems on finding the equation of straight line in plane.

Finding the canonical equations of the second order curves by allocating of the full square. Plotting the graphs. Reducing the general equation of the second order curves to canonical equation.

Topic 3. The theory of limits and function continuity

Operations on sets. Graphs of main elementary functions. Domain of definition of functions. Calculation of limits of sequences. Limit of rational fractions at a point and at infinity. Calculation of limits by means of equivalences. Investigation of points of discontinuity and their classification.

Topic 4. Differential calculus of function of one variable

Calculation of derivatives by means of table of derivatives and rules of differentiating. Logarithmic differentiation. Approximate calculations. Equations of tangent and normal lines to graph.

Semester 2

Topic 1. Differential calculus of function of one variable

Calculation of derivatives of higher orders. Formula by Leibnitz. Calculation of limits using L'Hospital's Rule. Expanding the polynomials by means of Taylor's formula and functions by means of McLaren's formulas. Finding the extremum points and intervals of monotonicity of the function graph. Finding the largest and smallest values of a function. Convexity and concavity of the function graph. Finding inflection points and asymptotes of the function graph. General scheme of graphing the function.

Topic 2. Integral calculus of a function of one variable

Calculation of the indefinite integral according to the table. Changing the variable in the indefinite integral. Calculation of the indefinite integral – integration by parts. Expanding a rational fraction into the partial rational fractions. Integration of partial rational fractions of the 1-4 types. Integration of fractional rational functions. Integration of trigonometric expressions. Integration of irrational expressions. Integration of irrational expressions using trigonometric expressions.

Calculation of the definite integral according to the Newton-Leibniz formula. Calculation of the definite integral by substitution of variable and integration by parts. Calculation of the area of a plane figure in the Cartesian coordinate system using the definite integral. Calculation of the area of a plane figure in the polar coordinate system and given by parametric equations. Calculation of surface area and volume of a body of rotation, volumes of bodies with a given law of change of cross-sectional area. Calculation of the arc length in Cartesian and polar coordinate systems, as well as given by parametric equations, using the definite integral. Application of the definite integral for solving physics and mechanics problems.

Calculation and investigation of the convergence of improper integrals of the first and second kind.

Topic 3. Differential calculus of the multivariable functions

Finding the domain of a function of two variables. The method of intersections for constructing surfaces of the 2nd order. Finding the limits of functions of two variables. Application of the differential and Taylor's formula for approximate calculations. Solving problems for finding the largest and smallest values of a function in a given domain. Finding unconditional and conditional extrema of functions. Application of the differential and Taylor's formula for approximate calculations. Solving problems for finding the largest and smallest values of a function in a given domain. Finding unconditional and conditional extrema of functions.

Semester 3

Topic 1. Differential equations

Integration of the equations with separable variables. Integration of the homogeneous differential equations in Euler's sense. Integration of the linear equations. Integration of the Bernoulli's Equations.

Integration of the exact differential equations (equations in total differentials). Different types of the differential equations. Integration of the equations by Lagrange and Clairaut. Integration of the second order differential equations reducible to the first order equations. Integration of the homogeneous linear equations of the second order with constant coefficients. Integration of the inhomogeneous linear equations of the second order with constant coefficients. Integration of the linear equations of the n-th order with constant coefficients. Integration of the Systems of Differential Equations.

Topic 2. Integral calculus for functions with several variables

Calculation of the double integrals in Cartesian system coordinates. Calculation of the solid volumes in Cartesian system coordinates. Calculation of double integrals in polar system of coordinates. Physical applications of the double integral. Calculation of the triple integrals in Cartesian system of coordinates, in cylindrical and spherical systems of coordinates.

Curvilinear integral of the first kind: calculation and applications. Curvilinear integral of the II-nd kind: calculation and applications. Application of the Green's formula. Independence of the line integrals on form of the integration path. Finding antiderivative. Evaluation of the surface integrals of the I-st and of

the II-nd kind. Evaluation of the surface integrals over a closed surface. Ostrogradsky-Gauss formula. Stokes' formula.

Semester 4

Topic 1. Field Theory

Calculation of surface integrals of the first and the second kind. Geometric and physical applications of these integrals. Theorem by Ostrogradsky-Gauss, theorem by Stockes. Finding the level surfaces and lines, the directional derivatives and the gradient of scalar fields. Finding the vector lines of the vector field. Calculation of the flux and the divergence of the vector field. Application of the theorem by Ostrogradsky-Gauss to find the flux. Finding the circulation and the curl of the vector field. Application of the Stokes's theorem. Analyzing the type of the vector field.

Topic 2. Series

Finding the sums of convergent series. Application of the convergence tests to the series with positive terms. Plus-and-minus series. Investigation of the absolute and conditional convergence. Alternating series. Sign by Leibniz.

Finding the domains of convergence of the functional and power series. Integration and differentiation of the functional series. Power functions. Intervals of the convergence. Expansion the functions in Taylor's series and Maclaurin's series. Application of the Series for approximate calculations. The expansion of a function in a Fourier's series.

Topics of the laboratory classes

Not provided

Self-study

The course involves students studying lecture material, preparing for practical classes, completing individual assignments, preparing for tests, and independently processing material on course topics.

The topics for individual assignments and tests throughout the course are as follows:

1.1. Determinants, matrices and systems of linear algebraic equations.
1.2. Vector algebra. Analytical geometry.
1.3. The theory of limits.
2.1. Differential calculus of a function of one variable.
2.2. Integral calculus of a function of one variable.
2.3. Differential calculus of functions of several variables.
3.1. Differential equations and systems of differential equations.
3.2 Integral calculus of a function of several variables.
4.1. Field theory.
4.2. Series.

Course materials and recommended reading

1. Rudnyeva G.V. Elements of linear algebra and analytic geometry. Second revised and expanded edition: textbook / G.V.Rudnyeva – Kharkiv: Panov A.M., 2020. – 236 p.
2. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU "KhPI", 2004. – Volume 1.
3. Kurpa L.V., Shmatko T.V. Differential and Integral Calculus for One Variable Functions: Textbook. – Kharkiv: NTU "KhPI": 2017.
4. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU "KhPI", 2004. – Volume 2.
5. L.V. Kurpa, T.V.Shmatko. Differential and integral calculus for functions with several variables: Textbook. – Kharkiv: NTU "KhPI": 2012.
6. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU ""KhPI"", 2004. – Volume 3.
7. L.V. Kurpa, O.S. Mazur, T.V.Shmatko. Differential Equations and Series: Textbook. – Kharkiv: NTU "KhPI": 2013.

8. Higher mathematics. Problem solving and variants of typical calculations. Edited by Dr.Sci.Tech. Kurpa L.V. – Kharkiv: NTU ""KhPI"", 2004. – Volume 4.

Assessment and grading

Criteria for assessment of student performance, and the final score structure

Description of the final score structure, course requirements, and necessary steps to earn points, especially paying attention to self-study and individual assignments.

Grading scale

Total points	National	ECTS
90–100	Excellent	A
82–89	Good	B
75–81	Good	C
64–74	Satisfactory	D
60–63	Satisfactory	E
35–59	Unsatisfactory (requires additional learning)	FX
1–34	Unsatisfactory (requires repetition of the course)	F

Norms of academic integrity and course policy

The student must adhere to the Code of Ethics of Academic Relations and Integrity of NTU "KhPI": to demonstrate discipline, good manners, kindness, honesty, and responsibility. Conflict situations should be openly discussed in academic groups with a lecturer, and if it is impossible to resolve the conflict, they should be brought to the attention of the Institute's management.
Regulatory and legal documents related to the implementation of the principles of academic integrity at NTU "KhPI" are available on the website: http://blogs.kpi.kharkov.ua/v2/nv/akademichna-dobrochesnist/

Approval

Approved by	Date, signature	Head of the AM Department Vyacheslav BURLAYENKO
	Date, signature	Guarantor of the educational program Hanna OMELIANENKO