Higher mathematics

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Specialty	**Institute**
122 – Computer Science	Educational and Scientific Institute of Computer Sciences and Information Technologies
Educational program	**Department**
Computer Science and Intelligent Systems	Computer mathematics and data analysis (324)
Level of education	**Course type**
Bachelor's level	General, Mandatory
Semester	**Language of instruction**
1-2	English, Ukrainian

Lecturers and course developers
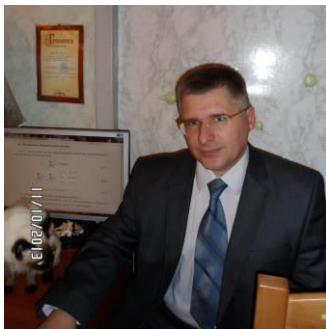
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Candidate of physical-mathematical sciences, associate professor, associate professor of Computer mathematics and data analysis Department (NTU "KhPI").

23 years of work experience. Authored and co-authored over 60 scientific publications. Leading lecturer of the courses: "Higher Mathematics", "Computer's discrete mathematics".
More about the lecturer on the department's website
http://web.kpi.kharkov.ua/kmmm/uk/o_kafedre_ua/profesorsko-vikladatskij-sklad/tonitsa-oleg-volodimirovich/

General information

Summary

The discipline "Higher Mathematics (Part 1)" is a discipline in the cycle of general training in the specialty 122 "Computer Science". The course covers the main sections of higher mathematics. The course provides three semantic modules and involves the formation of modern mathematical thinking, learning the basic mathematical tools needed to analyze and model processes and phenomena in finding optimal solutions and choosing the best means of implementing these solutions, research methods and solving mathematically formalized problems, the ability to analysis and synthesis of the obtained results and input facts.

The discipline "Higher Mathematics (Part 2)" is a discipline in the cycle of general training in the specialty 122 "Computer Science". The course covers the main sections of higher mathematics. The course provides five content modules and provides for the formation of modern mathematical thinking, learning the basic mathematical tools needed to analyze and model processes and phenomena in finding optimal solutions and choosing the best means of implementing these solutions, research methods and solving mathematically formalized problems, ability to analyze and synthesize the obtained results and input facts.

Course objectives and goals

Mastery of the mathematical apparatus necessary for further study and work, development of students' logical and algorithmic thinking; mastery of methods of research and solving mathematical problems; development of students' ability to independently expand their mathematical knowledge and conduct mathematical analysis of applied and engineering problems.

Format of classes

Lectures, practical classes, self-study, calculation task. Final control - exam.

Competencies

GC1. Ability to think abstractly, analyze and synthesize.
GC2. Ability to apply knowledge in practical situations.
GC6. Ability to learn and master modern knowledge.
PC1. Ability to mathematically formulate and study continuous and discrete mathematical models, justify the choice of methods and approaches for solving theoretical and applied problems in the field of computer science, analysis and interpretation.

Learning outcomes

PLO2. To use the modern mathematical apparatus of continuous and discrete analysis, linear algebra, analytical geometry in professional activities to solve theoretical and applied problems in the design and implementation of information objects.

Student workload

The total scope of the discipline is 330 hours. (11 ECTS credits): lectures – 90 hours, practical classes 74 hours, independent work – 166 hours.

Course prerequisites

Algebra in the school curriculum. Geometry in the scope of the school curriculum.

Features of the course, teaching and learning methods, and technologies

Interactive lectures with presentations, "bugtracking of lectures", practical classes with the use of group dynamics, project training.

Program of the course

Topics of the lectures

Topic 1. Determinants, their calculations and properties.
Topic 2. Matrices, basic concepts, types of matrices, actions on matrices. Inverse matrix.
Topic 3. Matrix equations.
Topic 4. The rank of the matrix.
Topic 5. Systems of linear algebraic equations. Kramer's rule. The inverse matrix method.
Topic 6. The Kronecker-Cappelli theorem. Gauss method.
Topic 7. Systems of linear algebraic homogeneous equations. Fundamental system of solutions.
Topic 8. Basic concepts and symbols of set theory. Numerical sets. Variable values. Function, methods of its assignment.
Topic 9. The limit of a numerical sequence and its simplest properties.
Topic 10. Infinitely small and infinitely large sequences. The limit of a monotonic sequence.
Topic 11. The limit of a function at a point and infinity (according to Heine). Properties of boundaries. Determining the boundary of a function using inequalities (according to Cauchy).
Topic 12. The first and second significant boundaries and their consequences.
Topic 13. Continuity of a function at a point and on an interval. Basic theorems about continuous functions. Classification of function breakpoints. Continuity of basic elementary functions.

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Topic 14. The concept of derivative, its geometric and physical meaning. Connection between continuity and differentiability of a function. Differentiation rules.

Topic 15. Derivative of a composite function. Differentiation of an implicit function. Logarithmic differentiation. Derivatives of basic elementary functions.

Topic 16. Functions and lines defined parametrically. Differentiation of functions given parametrically.

Topic 17. Derivatives of higher orders, Leibniz's formula. Differential function. Invariance of the form of the first differential of a function.

Topic 18. Differentials of higher orders.

Topic 19. Cases of non-differentiability of functions continuous at a given point. Fermat's, Rolle's, Lagrange's and Cauchy's theorems.

Topic 20. Lopital's rule. Disclosure of exponent uncertainties.

Topic 21. Formulas of Taylor and McLaren.

Topic 22. Study of functions for monotonicity and extremum. Finding the maximum and minimum value of a function on a segment.

Topic 23. Inflection points of a function, intervals of convexity and concavity. The second rule of investigation of the function at the extremum.

Topic 24. Asymptotes of the function graph. Scheme of a complete study of the function and construction of its graph.

Topic 25. Primitive and indefinite integral.

Topic 26. Integration by parts and replacement of a variable in an indefinite integral.

Topic 27. Factoring a polynomial. Rational fractions and their decomposition into the simplest.

Topic 28. Integration of rational fractions and functions that rationally depend on trigonometric ones.

Topic 29. Definite integral; definition and geometric meaning. The simplest properties of the definite integral. Theorems about the mean integral.

Topic 30. Darbu's songs. Necessary and sufficient conditions for the existence of a definite integral. Integral with variable upper bound, its properties. Newton-Leibnitz formula, connection between definite and indefinite integrals.

Topic 31. Calculation of the area of a figure using a definite integral.

Topic 32. Calculation of arc length, body volume, surface area of rotation.

Topic 33. Improper integrals of the first kind, their calculation. Signs of convergence. Improper integrals of the second kind, their convergence.

Topic 34. Functions of several variables, their domain of definition. Limit of a function, continuity and discontinuities. Basic properties of continuous functions. Partial derivatives of functions of several variables. Differential function of several variables and its application to approximate calculations.

Topic 35. Partial derivatives of composite functions. Invariance of the form of the first differential of a function.

Topic 36. Extremum of a function of several variables. An extremum condition is necessary. The concept of a quadratic form and its significance. Sufficient conditions of extremum. Conditional extremum of functions.

Topic 37. Tangent line and normal plane to a line in space; tangent plane and normal to the surface. The geometric content of the complete differential of a function of two variables.

Topic 38. Definition of the double integral, its properties and geometric meaning. Calculation of the double integral over a rectangular area.

Topic 39. Calculation of the double integral over an arbitrary domain. Double integral in the polar coordinate system.

Topic 40. Curvilinear integral of the second kind (by coordinates). Definition, properties and physical content. Vector and scalar form.

Topic 41. Integral over a closed loop. Independence of the curve integral from the line of integration. The Green-Riemann formula.

Topic 42. Differential equations, basic concepts. Ordinary differential equations. Differential equations with separated variables. Homogeneous functions of two variables and homogeneous differential equations.

Topic 43. Differential equations "in complete differentials". Integrating factor. Cauchy's problem, Cauchy's theorem.

Topic 44. Linear differential equations of the first order and Bernoulli's equation.

Topic 45. Types of singular points of differential equations: node, center, focus. Differential equations of the second order, basic concepts.

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Topic 46. Linear homogeneous differential equations of the second and higher orders, their general properties. Linear inhomogeneous differential equations and their properties.

Topic 47. The principle of superposition of solutions. Linear dependence and independence of functions. Fundamental system of solutions of differential equations. Theorem on the structure of the general solution of a linear homogeneous differential equation of the second and higher orders.

Topic 48. Systems of linear differential equations with constant coefficients.

Topic 49. Numerical series with constant signs, basic concepts. A convergence sign is necessary. Properties of convergent series. Sufficient features based on series comparison. Dalembert's sign, Cauchy's radical and integral signs.

Topic 50. Interspersed series, Leibniz's sign. Interchangeable rows. Absolute and conditional convergence.

Topic 51. Functional series, basic concepts. Power series, Abel's theorem. Interval and radius of convergence of power series. Properties of power series.

Topic 52. Development of functions in power series of Taylor and McLaren. Approximate calculations of functions and integrals using power series.

Topic 53. Elements of the theory of functions of a complex variable.

Topic 54. Operational arithmetic. Laplace transform, basic concepts and properties (linearity theorem, similarity theorem, shift theorem and delay theorem).

Topic 55. Theorems of differentiation and integration of the original and the image. Convolution of two functions. Image multiplication theorem.

Topic 56. Solving differential equations and their systems by the operational method.

Topics of the workshops

Topic 1. Methods of calculating determinants of the second, third and higher orders.

Topic 2. Performing operations with matrices: addition of matrices, subtraction, multiplication of matrices by a number, product of two matrices, transposition.

Topic 3. Calculation of the ranks of matrices by the method of extracting minors and reducing the matrix to a trapezoidal form.

Topic 4. Solving systems by the method of Gauss, Kramer, inverse matrix. Basic, partial and general solutions.

Topic 5. Finding a fundamental system of solutions

Topic 6. Calculation of the limit of a numerical sequence by definition.

Topic 7. Analytical calculation of limits of sequences.

Topic 8. Calculation of the limits of the functions of such types of uncertainties, which are revealed with the help of the first and second significant limits and their consequences. Table of infinitesimal functions.

Topic 9. Study of functions for continuity. Finding break points and determining the type of breaks. Graphical schematic representation of the graph of the function around the breakpoints.

Topic 10. Finding the derivatives of composite functions, functions given implicitly. Consideration of cases for the application of logarithmic differentiation.

Topic 11. Finding derivatives of functions given parametrically.

Topic 12. Calculation of derivatives of higher orders. Application of the method of mathematical induction.

Topic 13. Application of the differential to approximate calculations.

Topic 14. Application of Lopital's rule to reveal various types of uncertainties that occur when calculating the limits of functions of one variable. Decomposition of functions according to Taylor's and McLaren's formulas.

Topic 15. Determination of monotonicity of a function. Finding extremes.

Topic 16. Complete function study and graphing.

Topic 17. Properties, compiling a table of indefinite integrals. The simplest methods of integration.

Topic 18. Finding integrals using the theorem on the invariance of integration formulas. Integration by parts and change of variable in the indefinite integral.

Topic 19. Finding integrals that contain a quadratic trinomial.

Topic 20. Technique of integration of rational fractions. Methods of integration of trigonometric functions. Integration of some irrational and hyperbolic functions.

Topic 21. Problems that lead to the concepts of the definite integral.

Topic 22. Peculiarities of methods for calculating the definite integral. Integration by parts and change of variable for the definite integral.

Topic 23. Finding the area of a flat figure bounded by lines given analytically in the Cartesian coordinate system explicitly, parametrically and implicitly.

Topic 24. Problems for finding the length of an arc, the volume of a body, the area of the surface of rotation with different methods of assigning lines.

Topic 25. Formulas for integration by parts and replacement of a variable in improper integrals. Determination of convergence by signs.

Topic 26. Finding the area of definition, limits of a function of several variables.

Topic 27. Calculation of partial derivatives and differentials.

Topic 28. Finding the extremum of a function of several variables. Problems on the largest and smallest value of a function in the domain.

Topic 29. Geometric applications of functions of several variables.

Topic 30. Calculation of the double integral in the Cartesian coordinate system.

Topic 31. Calculation of the double integral in the polar coordinate system. Finding volumes of bodies using the double integral.

Topic 32. Calculation of the curvilinear integral. Application to the calculation of the work of a variable force along a curved path.

Topic 33. Application of the Green-Riemann formula.

Topic 34. Methods of solving homogeneous differential equations and some types of equations that reduce to homogeneous ones.

Topic 35. Solving differential equations "in complete differentials".

Topic 36. Solving linear differential equations of the first order.

Topic 37. Differential equations of the second and higher orders, which assume a decrease in order.

Topic 38. Linear homogeneous differential equations with constant coefficients. Method of variation of arbitrary constants.

Topic 39. Linear inhomogeneous differential equations with right-hand sides of a special form.

Topic 40. Improvement of the technique of solving differential equations and systems of differential equations of various types.

Topic 41. Number series. Research on the convergence of sign-constant series.

Topic 42. Research on absolute and conditional convergence of sign-changing series.

Topic 43. Functional series. Finding the area of convergence. Power series. Finding the interval of convergence.

Topic 44. Development of functions in power series.

Topic 45. Differentiation and integration of functions of a complex variable.

Topic 46. Finding images of functions. Duhamel's integral. Finding the original by its image.

Topic 47. Application of the Duhamel integral to the solution of differential equations.

Topic 48. Solving linear differential equations with constant coefficients using operational calculus.

Topics of the laboratory classes

Laboratory classes are not provided within the discipline.

Self-study

Independent work: study of lecture material, preparation for practical classes, independent study of topics and issues that are not taught in lecture classes, performance of individual tasks. Individual tasks contain tasks from all studied topics.

Course materials and recommended reading

Key literature

1. Kurpa L. V. Higher mathematics: Problems solving and variants of typical calculation. Volume 1,2. Kharkiv: NTU KhPI, 2004. - 316 p.
2. Kurpa L.V., Shmatko T.V. Differential and Integral Calculus for One Variable Functions – Kharkiv: NTU KhPI: 2017. – 324 p.

3. Mulyk O. Calculus: part I. Differential calculus of function of one variable - Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2022. – 117 p.

4. Zhuravska G.V. Higher Mathematics. Differential Calculus of a Function of One Variable. Elements of Theory – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2019. – 81 p.

5. Pershina Yu. I. [etc.] Indefinite and definite integrals: teaching method. manual / - Kharkiv: NTU KhPI, Madrid Printing House, 2022. - 188 p.

Additional literature and Internet resources

1. Lyubchyk L. M. [etc.] Higher mathematics. Theory of the function of a complex variable. Operational calculus (Practical course for students of technical specialties of correspondence and distance learning forms): teacher. manual / Kharkiv: NTU "KhPI", 2016. – 148 p.

2. Lyubchyk L. M. [etc.] Higher mathematics. Improper integrals. Integral calculus of functions of many variables. Ryady (Practical course for students of technical specialties of correspondence and distance learning forms): training. Manual – Kharkiv: NTU "KhPI", 2016. – 150 p.

3. Ahiezer O. B. Methodological instructions for individual tasks on the topic "Curvilinear integrals": from higher mathematics for students of correspondence and distance learning forms / Kharkiv: NTU "KhPI", 2016. – 55 p.

4. Ahiezer O. B. Methodological instructions for individual tasks on the topic "Rows": from higher mathematics for students of correspondence and distance forms of education / Kharkiv: NTU "KhPI", 2016. – 88 p.

5. Ahiezer O. B. Methodological instructions for individual tasks on the topic "Double integrals": from higher mathematics for students of correspondence and distance forms of education / Kharkiv: NTU "KhPI", 2016. – 51 p.

Assessment and grading

Criteria for assessment of student performance, and the final score structure

100% of the sub-bag assessment is added up from the results of the stream assessment (60%) and the exam (40%). More precisely, the assessment is made up of the assessment of IDZ (40%), work in the classroom and control work (20%).

Grading scale

Total points	National	ECTS
90–100	Excellent	A
82–89	Good	B
75–81	Good	C
64–74	Satisfactory	D
60–63	Satisfactory	E
35–59	Unsatisfactory (requires additional learning)	FX
1–34	Unsatisfactory (requires repetition of the course)	F

Norms of academic integrity and course policy

The student must adhere to the Code of Ethics of Academic Relations and Integrity of NTU "KhPI": to demonstrate discipline, good manners, kindness, honesty, and responsibility. Conflict situations should be openly discussed in academic groups with a lecturer, and if it is impossible to resolve the conflict, they should be brought to the attention of the Institute's management.

Regulatory and legal documents related to the implementation of the principles of academic integrity at NTU "KhPI" are available on the website: http://blogs.kpi.kharkov.ua/v2/nv/akademichna-dobrochesnist/

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Approval

Higher mathematics

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