Syllabus
Course Program

HIGHER MATHEMATICS

Specialty
121 – Software Engineering

Institute
Institute of Computer Science and Information Technology

Educational program
Software Engineering

Department
Computer mathematics and data analysis (324)

Level of education
Bachelor's level

Course type
Special (professional), Mandatory

Semester
1 – 2

Language of instruction
Ukrainian, English

Lecturers and course developers

Oksana Dubinina

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Doctor of Pedagogical Sciences, Candidate of Technical Sciences, Professor, Professor of the Department of Computer Mathematics and Data Analysis of National Technical University "KhPI".

Work experience – more than 30 years. Author and coauthor of more than 100 scientific and educational publications. Lecturer in the disciplines: "Higher Mathematics", "Blockchain Technology".
More about the lecturer on the department's website

General information

Summary

Discipline "Higher Mathematics" is a discipline in the cycle of general training for the specialty 121 "Software Engineering". The course covers the main sections of higher mathematics. The course provides three modified modules and taking into account the formation of modern mathematical thinking, learning the basic mathematical tools needed for analysis and modeling of processes and the emergence in finding optimal solutions and choosing the best means of implementing these solutions, research methods and solving mathematically formalized problems, use analysis and synthesis of the obtained results and input facts.

Course objectives and goals

Mastering by students the mathematical apparatus necessary for further study and work, development of logical and algorithmic thinking of students; mastering by students of methods of research and the decision of mathematical problems; developing students' ability to independently expand their mathematical knowledge and conduct mathematical analysis of applied and engineering problems.

Format of classes

Lectures, practical classes, independent work. Final control – exam.

Competencies

К01. Ability to think abstractly, analyze and synthesize.
К05. Ability to learn and master modern knowledge.
К06. Ability to search, process and analyze information from various sources.
К20. Ability to apply fundamental and interdisciplinary knowledge to successfully solve software engineering problems.

Learning outcomes

PLO01. Analyze, purposefully search and select information and reference resources and knowledge necessary for solving professional problems, taking into account modern achievements of science and technology.

Student workload

The total volume of the course is 330 hours (11 ECTS credits): lectures – 90 hours, workshops – 74 hours, self–study – 166 hours.

Course prerequisites

School mathematics course

Features of the course, teaching and learning methods, and technologies

Interactive lectures with presentations, "lecture bug-tracking", practical classes using group dynamics, project–based learning

Program of the course

Topics of the lectures

Part 1
Topic 1. Basics of linear algebra and analytic geometry.
1. Determinants, their calculations and properties.
2–3. Matrices, basic concepts, types of matrices, operations on matrices. Inverse matrix. Matrix equations.
4. The rank of the matrix.
5–6. Systems of linear algebraic equations. Kramer's rule. The inverse matrix method. The Kronecker–Capelli theorem. Gauss method. Systems of linear algebraic homogeneous equations. Fundamental system of solutions.
7. Basic concepts of vector algebra and analytic geometry.
Topic 2. Theory of borders.
8 – 9. Basic concepts and symbols of set theory. Numerical sets. Variable values. Function, methods of its assignment. The limit of a numerical sequence and its simplest properties.
10. Infinitely small and infinitely large sequences. The limit of a monotonic sequence.
11 – 12. Boundary of a function at a point and infinity (according to Heine). Properties of boundaries. Determining the boundary of a function using inequalities (according to Cauchy). The first and second significant boundaries and their consequences.
13. Continuity of a function at a point and on an interval. Basic theorems about continuous functions. Classification of function breakpoints. Continuity of basic elementary functions.
Topic 3. Differential calculus of a function of one variable.
14 – 15. Concept of derivative, its geometric and physical meaning. Connection between continuity and differentiability of a function. Differentiation rules. The derivative of a composite function. Differentiation of an implicit function. Logarithmic differentiation. Derivatives of basic elementary functions.
16. Functions and lines defined parametrically. Differentiation of functions given parametrically.
17 – 18. Derivatives of higher orders, Leibniz's formula. Differential function. Invariance of the form of the first differential of a function. Differentials of higher orders.
19. Cases of non–differentiability of functions continuous at a given point. Fermat's, Rolle's, Lagrange's and Cauchy's theorems.
20 – 21. Lopital's rule. Disclosure of exponent uncertainties. Formulas of Taylor and McLaren.
22. Study of functions for monotonicity and extremum. Finding the maximum and minimum value of a function on a segment.

23 – 24. Inflection points of the function, intervals of convexity and concavity. The second rule of investigation of the function at the extremum. Asymptotes of the function graph. Scheme of a complete study of the function and construction of its graph.

Part 2

Topic 4. Indefinite integral.

25. Primitive and indefinite integral.
26. Integration by parts and replacement of a variable in an indefinite integral.
27. Factoring a polynomial. Rational fractions and their decomposition into the simplest.
28. Integration of rational fractions and functions that rationally depend on trigonometric ones.

Topic 5. The definite integral and its application.

29. Definite integral; definition and geometric meaning. The simplest properties of the definite integral. Theorems about the mean integral.
30. Sumy Darbu. Necessary and sufficient conditions for the existence of a definite integral. Integral with variable upper bound, its properties. Newton-Leibnitz formula, connection between definite and indefinite integrals.
31. Calculation of the area of a figure using a definite integral.
32. Calculation of arc length, body volume, surface area of rotation.
33. Improper integrals of the first kind, their calculation. Signs of convergence. Improper integrals of the second kind, their convergence.

Topic 6. The theory of the function of several variables.

34. Functions of several variables, their scope. Limit of a function, continuity and discontinuities. Basic properties of continuous functions. Partial derivatives of functions of several variables. Differential function of several variables and its application to approximate calculations.
35. Partial derivatives of composite functions. Invariance of the form of the first differential of a function.
36. Extremum of a function of several variables. An extremum condition is necessary. The concept of a quadratic form and its significance. Sufficient conditions of extremum. Conditional extremum of functions.
37. Tangent line and normal plane to a line in space; tangent plane and normal to the surface. The geometric content of the complete differential of a function of two variables.
38. Definition of the double integral, its properties and geometric meaning. Calculation of the double integral over a rectangular area.
39. Calculation of the double integral over an arbitrary domain. Double integral in the polar coordinate system.
40. Curvilinear integral of the second kind (by coordinates). Definition, properties and physical content. Vector and scalar form.
41. Integral over a closed loop. Independence of the curve integral from the line of integration. The Green–Riemann formula.

Topic 7. Differential equations.

42. Differential equations, basic concepts. Ordinary differential equations. Differential equations with separated variables. Homogeneous functions of two variables and homogeneous differential equations.
43. Differential equations "in complete differentials". Integrating multiplier. Cauchy's problem, Cauchy's theorem.
44. Linear differential equations of the first order and the Bernoulli equation.
45. Types of singular points of differential equations: node, center, focus. Differential equations of the second order, basic concepts.
46. Linear homogeneous differential equations of the second and higher orders, their general properties. Linear inhomogeneous differential equations and their properties.
47. The principle of superposition of decisions. Linear dependence and independence of functions. Fundamental system of solutions of differential equations. Theorem on the structure of the general solution of a linear homogeneous differential equation of the second and higher orders.
48. Systems of linear differential equations with constant coefficients.

Topic 8. Rows.

49. Numerical constant sign series, basic concepts. A convergence sign is necessary. Properties of convergent series. Sufficient features based on series comparison. Dalembert's sign, Cauchy's radical and integral signs.
50. Interspersed series, Leibniz's sign. Interchangeable rows. Absolute and conditional convergence.

51. Functional series, basic concepts. Power series, Abel's theorem. Interval and radius of convergence of power series. Properties of power series.
52. Development of functions in power series by Taylor and McLaren. Approximate calculations of functions and integrals using power series.
Topic 9. Additional sections of higher mathematics.
53. Elements of the theory of functions of a complex variable.
54. Operational calculation. Laplace transform, basic concepts and properties (linearity theorem, similarity theorem, shift theorem and delay theorem).
55. Theorems of differentiation and integration of the original and the image. Convolution of two functions. Image multiplication theorem.
56. Solving differential equations and their systems by the operational method.

Topics of the workshops

Part 1
Topic 1. Implementation of the mathematical apparatus of linear and vector algebra and analytical geometry in engineering.
1. Methods of calculating determinants of the second, third and higher orders. Reduction of determinants of higher orders to upper–triangular and lower–triangular forms.
2. Performing operations with matrices: addition of matrices, subtraction, multiplication of matrices by a number, product of two matrices, transposition. Finding the inverse matrix. Solving matrix equations.
3. Calculation of matrix ranks by the method of separating minors and reducing the matrix to a trapezoidal form. The Vandermonde determinant, the determinant of the product of two matrices. Determinant of the inverse matrix.
4. Solving systems by the method of Gauss, Kramer, inverse matrix. Basic, partial and general solutions. Finding a fundamental system of solutions.
5. Elements of vector algebra and analytic geometry.
Topic 2. Application of the theory of determinants.
6. Calculation of the limit of a numerical sequence by definition. Classification of functions. Basic elementary functions, their properties and graphs. Concept of elementary function. The domain of value and the domain of definition of elementary functions.
7. Analytical calculation of limits of sequences. Arithmetic properties of boundaries. Supremum and infimum of the numerical set. The number "e" as the limit of a monotonic sequence.
8. Calculation of the limits of the functions of such types of uncertainties, which are revealed with the help of the first and second significant limits and their consequences. Table of infinitesimal functions. Comparison of infinitesimal quantities. Properties of equivalent infinitesimals.
9. Study of functions for continuity. Finding break points and determining the type of breaks. Graphical schematic representation of the graph of the function around the breakpoints.
Topic 3. Applied questions of the theory of differential calculus of functions of one variable.
10. Finding derivatives of composite functions, functions given implicitly. Consideration of cases for the application of logarithmic differentiation.
11. Finding derivatives of functions given parametrically. Improvement of the differentiation technique.
12. Calculation of derivatives of higher orders. Application of the method of mathematical induction.
13. Application of the differential to approximate calculations.
14. Application of Lopital's rule to reveal various types of uncertainties that occur when calculating limits of functions of one variable. Decomposition of functions according to Taylor's and McLaren's formulas. Use of Taylor's formula when calculating the limits of functions.
15. Definition of monotonicity of a function. Finding extremes. Compiling and solving problems for finding the largest and smallest value of a function on a line segment.
16. Complete function study and graphing.
Part 2
Topic 4. Indefinite integral.
17. The history of the emergence and development of the theory of integration in mathematics. Properties, compilation of a table of indefinite integrals. The simplest methods of integration.
18. Finding integrals using the theorem on the invariance of integration formulas. Integration by parts and change of variable in the indefinite integral. Complex numbers in algebraic form and operations on

them. Geometric interpretation of a complex number. Parametric and trigonometric forms of complex numbers. Moivre's formula.
19. Finding integrals that contain a quadratic trinomial. Binomial integral. Chebyshev's theorem.
20. Technique of integration of rational fractions. Methods of integration of trigonometric functions. Integration of some irrational and hyperbolic functions.

Topic 5. The definite integral and its application.

21. Problems that lead to the concepts of the definite integral.
22. Features of methods for calculating the definite integral. Integration by parts and change of variable for the definite integral. The average value of the function on the interval; evaluation of the definite integral; an integral with variable upper and lower limits of integration.
23. Finding the area of a flat figure bounded by lines given analytically in the Cartesian coordinate system explicitly, in parametric form, and implicitly. Application of the definite integral to the solution of physical problems.
24. Problems for finding the length of an arc, the volume of a body, the area of the surface of rotation with different methods of assigning lines. Application of the definite integral for calculating limits of infinite sums.
25. Formulas for integration by parts and replacement of a variable in improper integrals. Determination of convergence by signs.

Topic 6. Function of several variables.

26. Finding the area of definition, the limits of a function of several variables. Study of the continuity of a function of several variables.
27. Calculation of partial derivatives and differentials. Taylor's formula for a function of several variables and its application to approximate calculations.
28. Finding the extremum of a function of several variables. Problems on the largest and smallest value of a function in the domain. Scalar field. Derivative in direction, its properties and physical meaning. Gradient and its properties.
29. Geometric applications of functions of several variables.
30. Calculation of the double integral in the Cartesian coordinate system. Problems leading to the concept of a double integral.
31. Calculation of the double integral in the polar coordinate system. Finding volumes of bodies using the double integral. Some geometric and physical applications of double integrals.
32. Calculation of the curvilinear integral. Application to the calculation of the work of a variable force along a curved path. The first and second forms of the condition of independence of the integral from the path of integration. Finding a function of two or three variables by its complete differential.
33. Application of the Green–Riemann formula.

Topic 7. Differential equations.

34. Problems that lead to the solution of differential equations. Methods of solving homogeneous differential equations and some types of equations that reduce to homogeneous ones.
35. Solving differential equations "in complete differentials". Differential equations that reduce to isolated variables by substituting an unknown function.
36. Solving linear differential equations of the first order.
37. Differential equations of the second and higher orders, which assume a decrease in order.
38. Linear homogeneous differential equations with constant coefficients. Method of variation of arbitrary constants. Differential equations of Euler and Bessel.
39. Linear inhomogeneous differential equations with right–hand sides of a special form. Vronsky's determinant. Theorem on the existence of a fundamental system of decisions. Theorem on the non–singular linear transformation of the fundamental system of solutions.
40. Improvement of the technique of solving differential equations and systems of differential equations of various types.

Topic 8. Rows.

41. Number series. Research on the convergence of sign-constant series.
42. Research on absolute and conditional convergence of sign-changing series.
43. Functional series. Finding the area of convergence. Power series. Finding the interval of convergence. Uniform convergence, Weierstrass sign. Basic properties of uniformly convergent functional series.
44. Development of functions in power series. Fourier series.

Topic 9. Additional sections of higher mathematics.

45. Differentiation and integration of functions of a complex variable. Calculation of integrals by Cauchy's integral formulas. Laurent series of functions of a complex variable. Finding singular points of functions and excesses. Application of the theory of remainders to the calculation of integrals.
46. Finding images of functions. Duhamel's integral. Finding the original by its image. Table of images of the main functions.
47. Application of the Duhamel integral to the solution of differential equations. Development formula.
48. Solving linear differential equations with constant coefficients using operational calculus.

Topics of the laboratory classes

The plan does not include laboratory work.

Self-study

Written individual tasks for calculation and graphic works, preparation for practical classes, express survey, online testing, final/semester control in the form of a semester exam, according to the schedule of the educational process.

Course materials and recommended reading

Basic literature

1. Dorogovtsev A. Ya. Mathematical analysis: Textbook: In two parts. Part 1. – K.: Lybid, 1993. – 321 p.
2. Dubinina O. M. Definite integral and computer mathematics system MathCad: a study guide. – Kharkiv: NTU "KhPI", 2017. – 225 p.
3. Zavalo S. T. Algebra course. – Kyiv: Higher School, 1985. – 278 p.
4. Linear algebra. A collection of tasks and a method of solving them: educational and methodological manual / L. P. Dzyubak, S. P. Iglin, G. B. Linnik, I. O. Morachkovska. – Kh.: NTU "KhPI", 2013. – 240 p.
5. Mathematical analysis. Modular training: Practical course for students of technical specialties: education. manual: in 3 parts – Part 1 / N. M. Yasnytska, O. B. Ahiezer, A. A. Boyeva, etc. – Kh.: "Textbook of NTU "KhPI", 2014. – 384 p.
6. Mathematical analysis. Modular training: Practical course for students of technical specialties: education. manual: in 3 parts – Part 2 / N. M. Yasnytska, O. B. Ahiezer, A. A. Boyeva, etc. – Kh.: "Textbook of NTU "KhPI", 2014. – 244 p.
7. Osadcha L. K. Linear algebra and analytic geometry: teaching. manual. – Rivne: NUVHP, 2020. – 205 p.

Additional literature

1. V.V. Veretelynyk Theory of functions of a complex variable / V.V. Veretelynyk, H.M. Timchenko. – Kh.: NTU "KhPI", 2012. – 208 p.
2. Dubinina O. M., Lyemesheva L. P., Mezerna M. V. Operational computing as a method of building automated control systems: methodological instructions for practical classes. – Kharkiv: NTU "KhPI", 2012. – 52 p.
3. Kostrobiy P.P. Elements of the theory of a complex variable. Integral Fourier and Laplace transforms. Collection of problems and exercises / P. P. Kostrobiy, D. V. Uhanska, T. M. Salo and others. – Lviv: Lviv Polytechnic, 2011. – 200 p.
4. Mathematical analysis. Modular training: Practical course for students of technical specialties: education. manual: in 3 parts – Part 3 / N. M. Yasnytska, O. B. Ahiezer, A. A. Boyeva, etc. – Kh.: "Textbook of NTU "KhPI", 2014. – 384 p.
5. H. G. Shvachych and others. Introduction to the theory of functions of a complex variable: a study guide / H. G. Shvachych, V. S. Konovalenkov, etc. – Dnipropetrovsk: NMetAU, 2016. - 33 p.
6. Shvets V. T. Higher mathematics: operational calculus: teaching. manual / V. T. Shvets. – Odesa: Grin D. S., 2015. – 228 p.
7. Shkil M. I., Sotnichenko M. A. Ordinary differential equations: Education. manual. – K.: Higher School, 1992. – 303 p.

Assessment and grading

Criteria for assessment of student performance, and the final score structure

100% final assessment in the form of an exam (40%) and ongoing assessment (60%).
40% exam: semester exam, according to the schedule of the educational process.
60% current assessment:
• 20% assessment of tasks in practical classes;
• 30% written individual calculation and graphic tasks;
• 10% intermediate control (3 online tests)

Grading scale

Total points	National	ECTS
90–100	Excellent	A
82–89	Good	B
75–81	Good	C
64–74	Satisfactory	D
60–63	Satisfactory	E
35–59	Unsatisfactory (requires additional learning)	FX
1–34	Unsatisfactory (requires repetition of the course)	F

Norms of academic integrity and course policy

The student must adhere to the Code of Ethics of Academic Relations and Integrity of NTU "KhPI": to demonstrate discipline, good manners, kindness, honesty, and responsibility. Conflict situations should be openly discussed in academic groups with a lecturer, and if it is impossible to resolve the conflict, they should be brought to the attention of the Institute's management.
Regulatory and legal documents related to the implementation of the principles of academic integrity at NTU "KhPI" are available on the website: http://blogs.kpi.kharkov.ua/v2/nv/akademichna–dobrochesnist/

Approval

Approved by 08.06.2023 Head of the department
Olena AKHIEZER

08.06.2023 Guarantor of the educational program
Uliya LITVINOVA