MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
National Technical University
«Kharkiv Polytechnic Institute»

FUNDAMENTALS OF KINEMATIC AND FORCE ANALYSIS OF MATERIAL BODIES

Methodological instructions for the implementation of practical tasks, independent work and individual tasks in the disciplines «Applied Mechanics» for foreign applicants of the specialty 185 – Oil and Gas Engineering and Technology of the first (bachelor's) level of education of full-time education

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Fundamentals of kinematic and force analysis of material bodies. Methodological instructions for the implementation of practical tasks, independent work and individual tasks in the disciplines «Applied Mechanics» for foreign applicants of the specialty 185 – Oil and Gas Engineering and Technology of the first (bachelor's) level of education of full-time education / Compiled by V. V. Klitnoi. Kharkiv: NTU «KhPI», 2024. 39 p.

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CONTENTS

Introduction

The course "Applied Mechanics" is a synthetic course, which includes the main sections of the following disciplines:

1. Theoretical Mechanics.
2. Theory of mechanisms and machines.
3. Strength of materials.
4. Machine parts.

The development of the course is based on the knowledge of the basic laws of physics, higher mathematics, descriptive geometry, engineering, and computer graphics. It is assumed that the trainees must have logical thinking, as well as knowledge of dimensional physical characteristics.

The course "Applied Mechanics" is one of the oldest general engineering training courses in technical colleges. It completes the cycle of engineering disciplines and is a link between the general technical and specialized disciplines.

Practical training 1. Structural Analysis of the Mechanism. Basic Concepts and Notation.

1.1. Links. Kinematic pairs and their classification.

Each machine consists of mobile elements. The simplest mobile elements are called links. The link may include one or more items.

The junction of mobile units is called the kinematic pair.

The immediate area of contact - the element of the kinematic pair, which can be presented by a surface, line or point.

Kinematic pairs, mating on the surface, are called lower kinematic pairs (Fig. 1, a, b), and mating on the line (Fig. 1, c) or point (Fig. 1, d) - the highest.

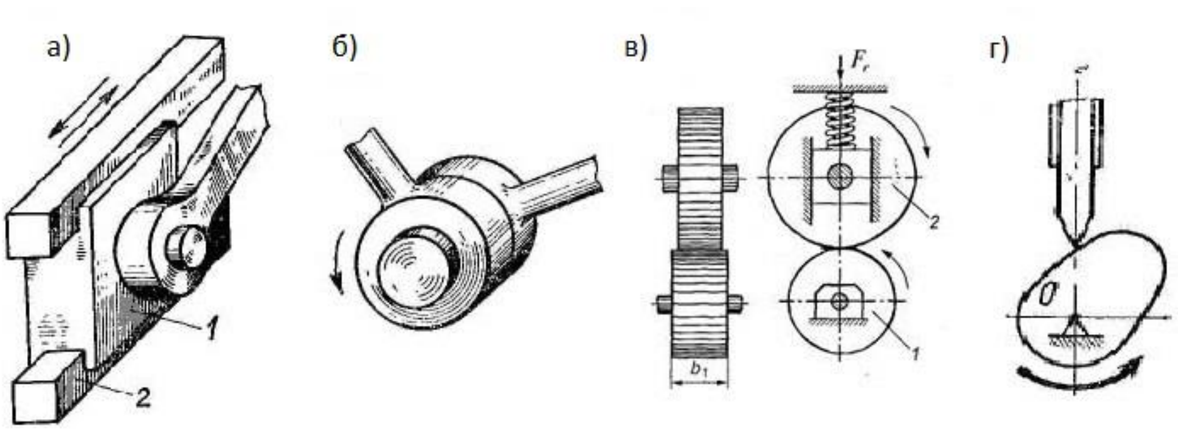


Fig. 1.1. - Examples of lower and higher kinematic pairs.

Lower kinematic pairs produce heavy loads. Higher kinematic pairs have a high mechanical efficiency. Locking in kinematic pairs can be locus (Fig. 1, a, b) and load-bearing (Fig. 1c, d).

1.2. Kinematic chains and their structures. The degree of link mobility. Mechanism.

A set of links and kinematic pairs form kinematic chains, which, depending on the structure, can be closed or opened, simple or complex.

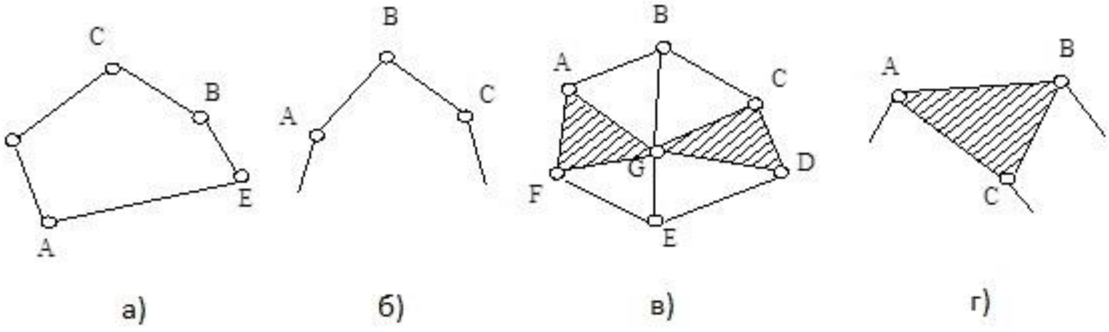


Fig. 1.2. - Schematic representation of kinematic chains.

In a closed kinematic chain (Fig. 2a, b) each unit includes at least two kinematic pairs, in the open circuit (Fig. 1, b, d) there are links that are only included into one kinematic pair.

The simple chain (Fig. 1, b) is called a kinematic chain in which each link is a connection to another link with one or two kinematic pairs. Otherwise, the circuit is called a complex one (Fig. 1c, d).

Kinematic chains are flat or spatial. In planar kinematic chains all the links lie and move in the same or in parallel planes. In spatial connections - in intersecting planes.

Each link in the space has six degrees of freedom (three linear movements and three rotational movements) (Fig. 3).

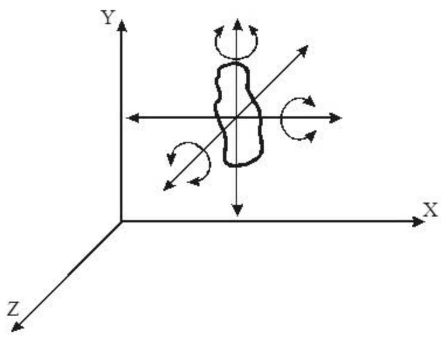


Fig. 1.3.

In the plane the element has three degrees of freedom (two linear movements and one rotational movement).

Kinematic pairs impose restrictions on the mobility of units. These restrictions are called <u>constraints</u>. In the space the kinematic pair cannot impose restrictions over 5; otherwise the links will be fixed and degenerate into a single unit. In the plane the number of connections cannot exceed 2.

The number of links imposed by the kinematic pair defines the class of kinematic pair:

• Grade 5 – one-degree-of-freedom kinematic pairs (reciprocating (Fig. 1 a), rotational (Fig. 1b), and screw pairs);

• Class 4 – two-degree-of-freedom kinematic pairs (cylinder in cylinder);

• Grade 3 -three - degree-of-freedom kinematic pairs (plane lying on the plane);

• Class 2 - four-degree-of-freedom kinematic pairs (cylinder lying in the plane);

• Class 1 – five-degree-of-freedom kinematic pairs (ball lying on the plane).

The kinematic chain one link of which is fixed is called a mechanism. The fixed link is called - front.

A mechanism designed for performing of useful work is called - machine.

1.3. Schemes of planar mechanisms.

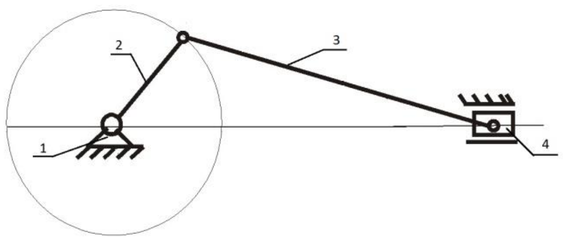
<table>
<tr><td colspan="2">a) slider-crank mechanism</td></tr>
<tr><td>

Fig. 1.4.</td><td>1– Front;
2– crank;
3–rod;

4–slider;

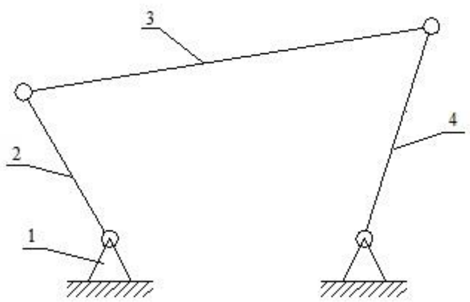
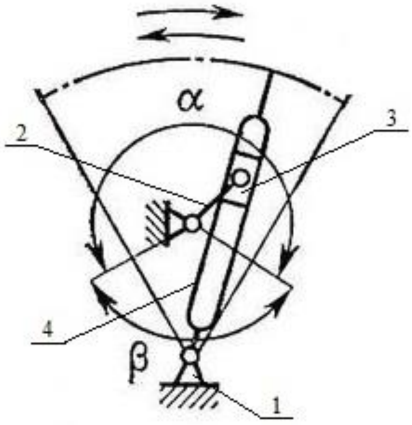
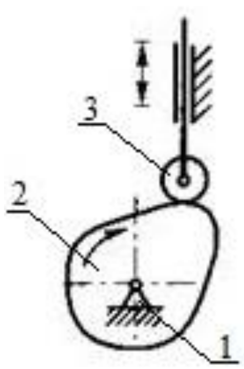
The mechanism is designed to convert the rotational motion of the crank (2) into the reciprocating motion of the slider (4).</td></tr>
<tr><td colspan="2">b) The crank-balancing mechanism</td></tr>
<tr><td>

Fig. 1.5.</td><td>1– Front;
2– crank;
3–rod;

4-balancer.

The mechanism is designed to convert the rotational motion of the crank (2) during rotational movement of the balance bar (4).</td></tr>
<tr><td colspan="2">c) The crank-rocker mechanism</td></tr>
</table>

 Figure 1.6.	1– Front; 2– crank; 3-slide block; 4-shifter. The mechanism is designed to convert the oscillating motion of the crank (2) into a swinging, motion of scenes (4).
g) The cam mechanism	
 Fig. 1.7.	1– Front; 2 - cam; 3-pusher. The mechanism is designed to convert the rotational motion of the cam (2) into the reciprocating movement of the pusher (3).

1.4. Structural analysis of planar mechanisms.

The purpose of the structural analysis is:

1. Determine whether the set of links and kinematic pairs form a mechanism.
2. What degree of mobility a mechanism features.

<u>The degree of mobility of a mechanism</u> is the number of independent coordinates that must be set so that the position of a mechanism could be identified unambiguously.

Suppose a mechanism has n-links, then the number of mobile units will be equivalent to $n-1$. Each unit on the plane has three degrees of freedom and all the mobile units will have $3(n-1)$ - degrees of freedom. Each kinematic pair imposes a constraint (limits the degree of freedom of the mechanism.) For the plane problem of the kinematic pair of class 5 $P_5 = P_H$ has two constraints, class 4 has a single constraint. Then the degree of mobility of the mechanism will be:

.
$$W = 3(n-1) - 2P_H - P_B \tag{1.1}$$

Expression (1.1) – Chebyshev's formula for a flat mechanism. By analogy, one can determine the degree of mobility for the spatial mechanism:

$$W = 6(n-1) - 5P_5 - 4P_4 - 3P_3 - 2P_2 - 1P_1 \tag{1.2}$$

The presented mechanisms (Fig. 1.4 - 1.7.) have one degree of mobility, i.e. it's enough to know the position of link 2 to determine the position of all the other units.

The link, which can be released from the mechanism without interrupting its operation is known as an unnecessary link.

Practical training 2. Kinematics.

2.1. General notation and performance.

Kinematics is a branch of mechanics which studies the motion of material bodies regardless of the reasons causing the movement and the weight of the bodies proper.

Kinematics deals with the following problems:
- the laws of motion of the body;
- velocity of the body;
- acceleration of the body.

As a model of the bodies under study they accept: the material point - a geometric point having a mass but not having a size; a perfectly rigid body is a body, the distance between the two points of which remain constant (rigid body).

2.2. Kinematics of a point.

The motion of a material point can be presented in three ways:
- vector;
- coordinate;
- naturally.

Under the vector method of defining the movement the position of a material point is defined by its radius vector $\vec{r}(t)$, drawn from the initial point O (Fig. 2.1.). Their ends define the curve, which is called the locus of the radius vector and corresponds to the trajectory of the point.

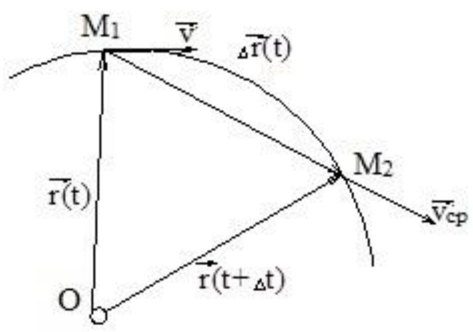


Fig. 2.1.

The expression $\vec{r} = \vec{r}(t)$ is the equation of motion of a material point in a vector form. The function $\vec{r}(t)$ is considered unique, continuous and twice differentiable with respect to time.

Suppose that the point was in the position M_1 at the time t_1 and M_2 for the time t_2 (Fig. 2.1), let's denote $\Delta t = t_2 - t_1$, then $\Delta\vec{r}(t) = \vec{r}(t+\Delta t) - \vec{r}(t)$. The ratio of increasing the radius vector to the time interval Δt is called the mean speed $\overrightarrow{V_{cp}}$.

$$\overrightarrow{V_{cp}} = \frac{\Delta\vec{r}(t)}{\Delta t}. \tag{2.1}$$

The extent of this relationship, when $\Delta t \to 0$, is called the instantaneous velocity of a material point:

$$\vec{V} = \lim_{\Delta t\to 0}\overrightarrow{V_{cp}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt} \tag{2.2}$$

Thus, the velocity of a material point can be determined as the first derivative of the radius vector with respect to time. The velocity vector is tangent to the trajectory of the given material point movement. The unit of speed measurement is m / s.

The ratio of the rate increase $\Delta\vec{V}(t) = \vec{V}(t+\Delta t) - \vec{V}(t)$ to the increment of time Δt is called the average acceleration $\overrightarrow{a_{cp}}$.

$$\overrightarrow{a_{cp}} = \frac{\Delta\vec{V}(t)}{\Delta t} \tag{2.3}$$

Letting the period of time to zero, by which acceleration is calculated, we'll obtain the value of the instantaneous acceleration:

$$\vec{a} = \lim_{\Delta t\to 0}\overrightarrow{a_{cp}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{V}(t)}{\Delta t} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{r}}{dt^2} \tag{2.4}$$

Then, the acceleration of a point - vector physical quantity is equal to the second derivative of the radius vector with respect to time and, accordingly, of the first derivative depending on the instantaneous velocity with respect to time. It characterizes the rate of velocity change. The unit of acceleration measurement is m/s2.

Under <u>the coordinate method</u> the position of a material point is described by its coordinates. As an example, let's consider the fixed Cartesian coordinate system (figure 2.2), then the equations of motion of a material point can be represented as follows:

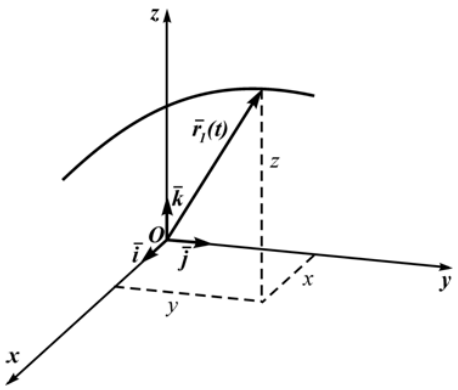


Fig. 2.2.

$$\begin{cases} x = x(t); \\ y = y(t); \\ z = z(t). \end{cases} \tag{2.5}$$

Both the function $\vec{r}(t)$ and functions $x(t)$, $y(t)$, $z(t)$ are considered to be unambiguous, continuous and having two continuous derivatives with respect to time.

The relationship between the radius vector $\vec{r}(t)$ and the Cartesian coordinates of the point is expressed by the equation

$$\vec{r}(t) = x(t)\cdot\vec{i} + y(t)\cdot\vec{j} + z(t)\cdot\vec{k} \tag{2.6}$$

here $\vec{i}$, $\vec{j}$, -$\vec{k}$ are the unit vectors (unitary vectors) of coordinate axes.

For the rate we have the following expression

,
$$\vec{V} = \frac{d\vec{r}}{dt} = \dot{x}\cdot\vec{i} + \dot{y}\cdot\vec{j} + \dot{z}\cdot\vec{k} = v_x\cdot\vec{i} + v_y\cdot\vec{j} + v_z\cdot\vec{k} \tag{2.7}$$

where v_x, v_y, v_z - the projection of the velocity V on the axis OX, OY, OZ. The rate of speed is given by

$$\left|\vec{V}\right| = \sqrt{v_x^{\ 2} + v_y^{\ 2} + v_z^{\ 2}}\ , \qquad (2.8)$$

and its direction is given by the direction of cosines

$$\cos(x\hat{\ }\vec{V}) = \frac{v_x}{V}, \qquad \cos(y\hat{\ }\vec{V}) = \frac{v_y}{V}, \qquad \cos(z\hat{\ }\vec{V}) = \frac{v_z}{V} \qquad (2.9)$$

Similarly, for acceleration we obtain:

$$\vec{a} = \frac{d\vec{V}}{dt} = \ddot{x}\cdot\vec{i} + \ddot{y}\cdot\vec{j} + \ddot{z}\cdot\vec{k} = a_x\cdot\vec{i} + a_y\cdot\vec{j} + a_z\cdot\vec{k} \qquad (2.10)$$

where a_x, a_y, - a_z - the projection of acceleration on the axis OX, OY, OZ. And then the magnitude and direction of the acceleration are given by:

$$\left|\vec{a}\right| = \sqrt{a_x^{\ 2} + a_y^{\ 2} + a_z^{\ 2}}\ ; \qquad (2.11)$$

$$\cos(x\hat{\ }\vec{a}) = \frac{a_x}{a}; \qquad \cos(y\hat{\ }\vec{a}) = \frac{a_y}{a}; \qquad \cos(z\hat{\ }\vec{a}) = \frac{a_z}{a}. \qquad (2.12)$$

Under a natural method of defining the motion of a material point the following items must be known:

- The trajectory of movement.
- The initial position on the path.
- The direction of motion in a path.
- The law of motion along the path $S = S(t)$ is the law of motion of a material point in a natural form. At the same time $S(t)$ is a twice continuously differentiable function.

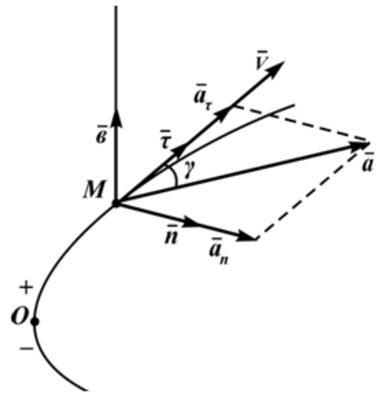


Fig. 2.3.

Let's obtain the expressions for velocity and acceleration of the point in the natural way of defining the movement. We'll introduce a natural trihedral formed by unit vectors $\vec{\tau}$, $\vec{n}$, $\vec{b}$ (Fig. 2.3). The radius vector $\vec{r}(t)$ relative to a fixed point will be a complex function of time. Then:

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}(S(t))}{dt} = \frac{d\vec{r}}{dS}\cdot\frac{dS}{dt} = \vec{\tau}\cdot V_\tau \ , \tag{2.13}$$

where V_τ is the projection of the velocity vector on the tangent axis. The value V_τ is positive when the point moves upward the arc coordinate and is negative otherwise. According to (2.13) the speed is always tangential to the trajectory.

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\vec{\tau}\cdot V_\tau) = \frac{d\vec{\tau}}{dS}\cdot\frac{dS}{dt}\cdot V_\tau + \frac{dV_\tau}{dt}\cdot\vec{\tau} = \frac{V_\tau^2}{\rho}\cdot\vec{n} + \frac{dV_\tau}{dt}\cdot\vec{\tau} = \vec{a_n} + \vec{a_\tau}$$

(2.14)

Here ρ is the radius of curvature of the trajectory at this point. $\vec{a_n}$ is the normal acceleration of the point, $\vec{a_\tau}$ is shear (tangential) acceleration. From (2.14) it follows that the total acceleration in the natural form can be represented as a vector sum of the normal and tangential acceleration, and always lies in the osculating plane (see Fig. 2.3).

If $\vec{a_n} = 0$- rectilinear motion of the point ($\rho \to \infty$); $\vec{a_\tau} = 0$ - the path velocity is constant; $\vec{a_\tau} \triangleright 0$ - the velocity and acceleration have the same direction; $\vec{a_\tau} \triangleleft 0$ – the speed and acceleration are opposite.

2.3. Kinematics of rigid bodies.

The most common types of motion of a rigid body are as follows:

• The progressive movement;
• The rotational movement;
• A complex motion.

<u>A translational motion</u> is called a motion in which every line drawn on the body during the movement of the given body remains parallel to itself (Figure 2.4).

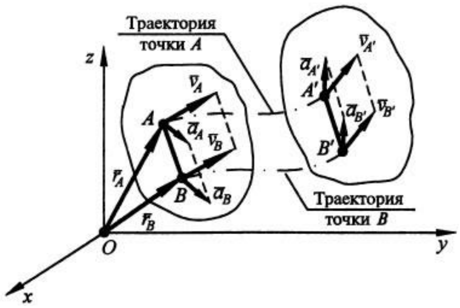


Fig. 2.4.

Segment $AB = A'B'$; $AB \parallel A'B'$. The motion path of point A is equal and parallel to the motion path of point B. Based on the definition, the law of motion of a rigid body can be defined in a vector form:

$$\overrightarrow{r_B} = \overrightarrow{r_A} + \overrightarrow{r_{AB}} \tag{2.15}$$

For the velocity and acceleration of an arbitrary point we obtain:

$$\begin{aligned} \frac{d\overrightarrow{r_B}}{dt} = \frac{d\overrightarrow{r_A}}{dt} + \frac{d\overrightarrow{r_{AB}}}{dt} = \frac{d\overrightarrow{r_A}}{dt} + 0 \qquad &\Rightarrow \overrightarrow{V_B} = \overrightarrow{V_A}; \\ \frac{d\overrightarrow{V_B}}{dt} = \frac{d\overrightarrow{V_A}}{dt} \qquad &\Rightarrow \overrightarrow{a_B} = \overrightarrow{a_A}. \end{aligned} \tag{2.16}$$

From these expressions, it follows that during the forward motion of a rigid body the kinematics of its motion is equal to the kinematics of movement of any point of the given body.

<u>The rotational movement</u> is a movement in which one line of the body remains stationary (called the axis of rotation), and all other points of the body move in circular orbits in the planes perpendicular to the axis of rotation (Fig. 2.5).

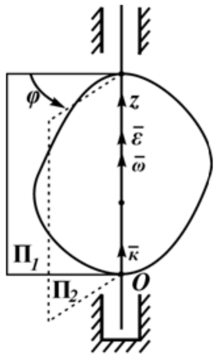


Fig. 2.5.

The dihedral angle φ completely determines the position of the body rotation (set out from some initial position). The change in angle with time is the law of rotary motion proper:

$$\varphi = \varphi(t) \tag{2.17}$$

The positive is the angle-delayed anti-clockwise when viewed towards the chosen direction of the axis of rotation. The angle is measured in radians.

The rate of change of the angle φ - is the angular velocity. By taking $\vec{k}$ as the unit vector of the positive direction of the axis, we get:

$$\vec{\omega} = \vec{k} \cdot \frac{d\varphi}{dt}$$

(2.18)

The angular velocity vector is always directed along the axis, with a positive angular velocity directions $\vec{\omega}$ and $\vec{k}$ are the same, with a negative - the opposite.

The angular speed change is characterized by angular acceleration:

$$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt} \tag{2.19}$$

Angular acceleration vector $\vec{\varepsilon}$ is directed along the same axis of rotation. With accelerated rotation directions $\vec{\varepsilon}$ and $\vec{\omega}$ are the same, with slow one - opposite.

As a rigid body rotates around a fixed axis, each point of the body moves along a circle. The radius of the circle R is the distance from a point to the rotation axis (Figure 2.6).

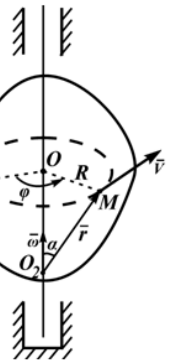


Fig. 2.6.

Since the trajectory of the point is known - a circle, the reference point (the point O1) and the positive direction of movement are chosen, the length of the arc (arc coordinate) is given by:

$$S(t) = \cup O_1M = \varphi \cdot R \tag{2.20}$$

then the law of motion of a point can be defined in a natural way.

The linear velocity of the point:

$$\vec{V} = \vec{\tau} \cdot \frac{dS}{dt} = \vec{\tau} \cdot R \cdot \frac{d\varphi}{dt} = \vec{\tau} \cdot R \cdot \omega \tag{2.21}$$

The acceleration in the natural way of motion specification is defined as the sum of the tangent and normal acceleration.

$$\vec{a} = \overrightarrow{a_n} + \overrightarrow{a_\tau};$$
$$\overrightarrow{a_n} = \frac{V_\tau^2}{\rho} \cdot \vec{n} = \omega^2 \cdot R \cdot \vec{n}; \qquad \overrightarrow{a_\tau} = \frac{dV_\tau}{dt} \cdot \vec{\tau} = R \cdot \frac{d\omega}{dt} \cdot \vec{\tau} = R \cdot \varepsilon \cdot \vec{\tau}. \tag{2.22}$$

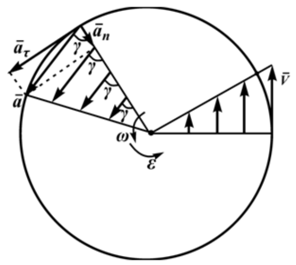


Fig. 2.7.

Figure 2.7 indicates the direction of the velocity and acceleration vectors. The angle of total acceleration with a radius can be determined from the formula:

$$tg\gamma = \frac{a_\tau}{a_n} = \frac{R \cdot \varepsilon}{\omega^2 \cdot R} = \frac{\varepsilon}{\omega^2} \tag{2.23}$$

In this same figure there are depicted the laws of distribution of velocities and accelerations of points in a rotating body, depending on their distance from the axis of rotation. These distributions correspond to the formulas:

$$V = R \cdot \omega; \qquad a = R \cdot \sqrt{\omega^4 + \varepsilon^2}. \qquad 2.24)$$

Complex motion is called a motion in which the movement is seen in two coordinate systems (stationary and mobile systems).

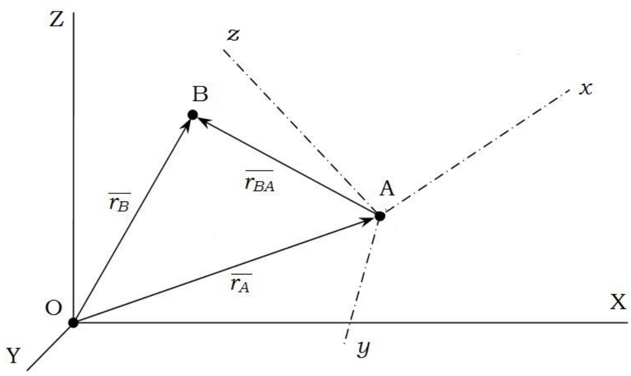


Fig. 2.8.

Let's consider the point B, moving with respect to the moving frame of reference Axyz, which in turn somehow moves relative to another frame of reference OXYZ, which we call the primary or conditionally fixed one (Fig. 2.8). We shall introduce the following definitions.

The movement executed by point *B* with respect to the moving frame of reference Axyz, is called a relative movement. The speed of point *B* with respect to the axes Axyz is called the relative velocity V_{BA}, while the acceleration - the relative acceleration of a_{BA}. The definition implies that in calculating V_{BA} and a_{BA} the movement of axes Axyz can be neglected (considered as fixed ones).

The movement, executed by the moving frame of reference Axyz (and all invariably associated with it points in space) with respect to the fixed system OXYZ, is transport motion for point *B*. The transport velocity V_A and transport acceleration a_A are called the speed and acceleration relative to the fixed frame of reference of the point, invariably associated with a moving frame of reference Axyz, which the moving point B coincides with at the given moment.

The movement, executed by point *B* with respect to the fixed frame of reference OXYZ, is called an absolute or a *complex* one. The speed of this point is called the absolute velocity V_B and acceleration - the absolute acceleration a_B.

The main objective of the study of the complex motion of the point is to establish the relationship between relative velocities and accelerations, transport and absolute motion of a point.

The law of motion of point *B* can be written as follows:

$$. \quad \overrightarrow{r_B} = \overrightarrow{r_A} + \overrightarrow{r_{AB}} \tag{2.25}$$

For the velocity and acceleration of an arbitrary point we get:

$$\begin{aligned} \frac{d\overrightarrow{r_B}}{dt} &= \frac{d\overrightarrow{r_A}}{dt} + \frac{d\overrightarrow{r_{BA}}}{dt} \quad \Rightarrow \overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{BA}}; \\ \frac{d\overrightarrow{V_B}}{dt} &= \frac{d\overrightarrow{V_A}}{dt} + \frac{d\overrightarrow{V_{BA}}}{dt} \quad \Rightarrow \overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{BA}}. \end{aligned} \tag{2.26}$$

If the figurative movement is progressive, then the following equation is valid:

$$, \overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{BA}^{\,n}} + \overrightarrow{a_{BA}^{\,\tau}}$$

At rotational transport motion there develops Coriolis acceleration (characterizes the change of the relative velocity in a transport movement and velocity relative), then

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{BA}}^{n} + \overrightarrow{a_{BA}}^{\tau} + \overrightarrow{a_K} \qquad (2.27)$$

where

$$\overrightarrow{a_K} = 2 \cdot \left(\overrightarrow{V_{BA}} \times \vec{\omega}\right)$$

$\vec{\omega}$ - angular-velocity vector of transport rotation.

2.4. Kinematic study of planar mechanisms.

Kinematic analysis is the study of movement of units of the mechanism, excluding the forces causing this movement. The objectives of the kinematic study are:

• determination of positions of all the units that they hold at operation of the mechanism as well as draw the trajectories of individual points of the mechanism;

• determination of linear velocities of characteristic points of the mechanism and determination of angular velocity of its units;

• identification of linear accelerations of individual points of the mechanism and angular accelerations of its units.

At kinematic analysis there should be given a block diagram of the mechanism, chosen the initial unit, which is taken as an input element, and set its law of motion.

The kinematic study of planar mechanisms is carried out in two ways.

<u>The analytical method.</u> With this method the mechanism links, its specific dimensions and displacement of links are represented as vectors. The result is vector polygons on the basis of which they generate the vector equation, the solution of which allows deriving equations for determination the motion (linear or angular) of investigated links and differentiating them they determine the velocity and acceleration.

<u>Graphical-analytical method.</u> This method is implemented in two ways – construction of diagrams of displacements, velocities and accelerations using graphical differentiation and integration; building plans of positions, velocities and accelerations.

The velocity diagram (acceleration) of the mechanism is called the drawing in which the velocity (acceleration) of various points are shown in the form of vectors indicating the direction and magnitude (scale) of these velocities (accelerations) at any given time.

The analytical method is precise, but time-consuming and not an illustrative one. The graphical-analytical method has a calculation error of 5 ... 7% due to the need of graphic constructions, but it is quite easy to implement and intuitive.

Practical training 3. Kinetics.

<u>Kinetics</u> is a branch of mechanics which studies the motion of material bodies under the action of forces applied to these bodies.

The result of interaction between the two bodies is power.

Power is a vector which is characterized by the magnitude, direction and point of application. Force is measured in Newtons [N]. Usually, the power is given by its projections on coordinate axes.

3.1. Basic concepts and laws (axioms) of dynamics

The basic laws of dynamics were formulated by Newton with respect to an absolute (fixed) space. The coordinate systems, fixed with respect to this space or moving relative to it progressively, uniformly and rectilinearly, are called inertial reference systems.

1. *Newton's first law* (axiom of inertia). In an inertial reference frame the isolated point saves the state of rest or of uniform rectilinear motion. The isolated point is the point which no force on the part of other material objects is acting on.

2. *The second law (the basic axiom of dynamics)*. If in the inertial reference frame the force $\vec{F}$ is acting on the body, then it accumulates acceleration $\vec{a}$ in the direction of the force action.

$$\vec{F} = m \cdot \vec{a} \tag{3.1}$$

m - the body weight, which is a measure of its inertia.

3. Newton's third law (axiom of action and reaction). Forces, which two material points interact with or two material bodies are equal in magnitude, opposite in direction and have a common line of action.

4. The law of superposition (axiom of forces composition). The material point under the influence of several forces is given acceleration equal to the geometric sum of those accelerations, which it receives from each of the force, acting separately, independently of the other.

Let's assume that a system of forces ($\vec{F}_1, \vec{F}_2, \ldots \vec{F}_n$) is acting on the body, then, according to the fourth axiom: $\vec{a} = \vec{a_1} + \vec{a_2} + \ldots + \vec{a_n}$,if m - is the mass of the body, then we can write:

$$m \cdot \vec{a} = m \cdot \vec{a_1} + m \cdot \vec{a_2} + \ldots + m \cdot \vec{a_n} \,,$$

whence

$$\vec{F} = \vec{F_1} + \vec{F_2} + \ldots + \vec{F_n} = \sum_{i=1}^{n} \vec{F}_i \tag{3.2}$$

This is the axiom of forces composition proper. It is similar to the wording of the law of superposition.

If a system of forces can be replaced by another, and thus the state of the body does not change (motion or standstill), such systems are called equivalent.

If a system of forces can be replaced by a force (equivalent), this force is called – resultant.

$$\vec{R} = \sum_{i=1}^{n} \vec{F}_i$$

3.2. The differential equation of motion of a material point. Problems of dynamics.

Let's assume that *Oxyz* is an inertial coordinate system, M – is a moving point of the mass m, $\vec{R}$ - the resultant of all the forces applied to the point, $\vec{a}$ - acceleration of the point (Fig. 3.1).

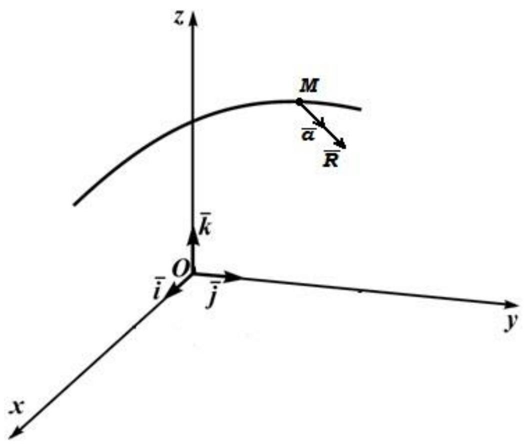


Figure 3.1.

At any specific time for point *M* the fundamental law of dynamics can be written in vector form $\vec{R} = m \cdot \vec{a}$. Let's project the components of the given vector equation on the coordinate axes: $R_x = m \cdot a_x$; $R_y = m \cdot a_y$; $R_z = m \cdot a_z$. From kinematics it is known that $a_x = \ddot{x}$, $a_y = \ddot{y}$, $a_z = \ddot{z}$, whereas these equations can be written as follows:

$$\begin{cases} R_x = m \cdot \ddot{x}; \\ R_y = m \cdot \ddot{y}; \\ R_z = m \cdot \ddot{z}. \end{cases} \tag{3.3}$$

The resulting equations (3.3) are called the differential equations of motion of a material point in the Cartesian coordinate system.

Let's write down the differential equations of motion of a point in projections on the natural coordinate axes. Recalling the formulae of kinematics for projections of acceleration on the natural axes, we can write down:

$$\begin{cases} R_n = m \cdot a_n = m \cdot \dfrac{V_\tau^{\ 2}}{\rho}; \\ R_\tau = m \cdot a_\tau = m \cdot \dfrac{dV_\tau}{dt}; \\ R_b = m \cdot a_b = 0. \end{cases} \tag{3.4}$$

The system of equations (3.4) represents differential equations of motion of a material point in a natural form. Many problems concerning the dynamics of a point can be solved much easier if we use the differential equations of motion in a natural form.

When considering the dynamics of motion we can solve two problems:

1. <u>The direct problem,</u> when given the law of motion and the weight of the mass they determine the forces acting on the body.
2. <u>The inverse problem</u>, when for a given body weight and known forces acting on the body, they determine the law of motion of the body proper.

In a particular case under the action of forces the body is in equilibrium, that is, the state of rest or uniform, rectilinear motion is preserved. This state is studied by another branch of mechanics - statics.

3.3. The axioms of statics and their consequences.

Axiom 1. Two forces applied to a completely solid body will be balanced if and only if they are equal in magnitude, acting on the same line and vectored in opposite directions (Fig. 3.2).

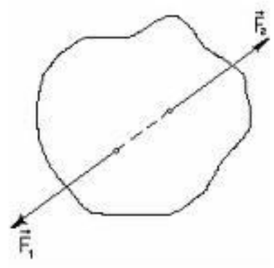


Figure 3.2.

This means that if the fulcrum is at rest under the action of two forces, these forces are equal in magnitude $|\vec{F_1}| = |\vec{F_2}|$, act on the same line and vectored in opposite directions, whence $\vec{F_1} + \vec{F_2} = 0$. Such a system of forces is equivalent to zero $(\vec{F_1}, \vec{F_2}) \sim 0$.

Axiom 2. The action of the given system of forces on the body will not change if we add to it or take away from it a balanced system of forces.

The second axiom is a logical consequence of the first. If two mutually balanced forces have no effect on the body, we may say that any balanced system of forces does not affect the solid body, regardless of whether the body is at rest or in motion, before we discarded or applied to it a balanced system of forces.

From the corollary of this axiom: without disturbing the state of the body, the point of force application can be moved along the line of its action.

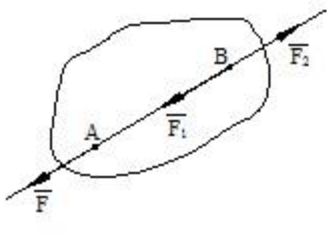


Figure 3.3.

Indeed, suppose that at point *A* the force $\vec{F}$ is applied to the body (Fig. 3.3). This force must be moved to the point B through the line of its action. We shall apply in the point B on the line of action of the force $\vec{F}$ two balanced forces $\vec{F_1}$ and $\vec{F_2}$, assuming that $|\vec{F}| = |\vec{F_1}| = |\vec{F_2}|$. As a consequence, the effect of the force on the body will not change. But forces $\vec{F}$ and $\vec{F_2}$, according to the first axiom form a balanced system of forces that can be discarded. As a result, the body will only be affected by one force $\vec{F}_1$ equal to $\vec{F}$, but applied at point B. On this basis, we conclude that the force applied to a completely solid body can be moved to any point on the line of its action, i. e. it’s a sliding vector.

Axiom 3. The resultant of the two forces applied to the rigid body at one point and directed at an angle to each other, attached to the same point and is represented in magnitude and direction by the diagonal of the parallelogram formed by these forces as on the two sides (Fig. 3.4) $\vec{R} = \vec{F}_1 + \vec{F}_2$.

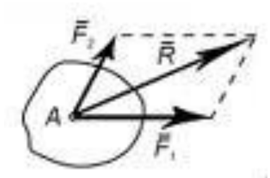


Figure 3.4.

The consequence of the first three axioms is the theorem of three non-parallel forces. If three non-parallel forces attached to a solid body that lie in the same plane, are in equilibrium, then the line of action of these forces intersect at one point.

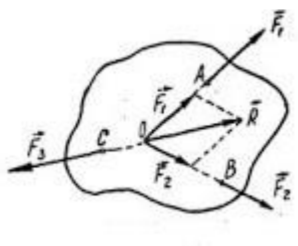


Figure 3.5.

Let’s assume that the body is in equilibrium under the action of the three forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ (Fig. 3.5). According to the third axiom of statics the resultant of the first two forces can be determined according to the rule of the parallelogram, constructed **on forces** $\vec{F}_1$ and $\vec{F}_2$, carried forward along the lines of their actions to the point of intersection, here $\vec{R} = \vec{F}_1 + \vec{F}_2$. According to the first axiom of statics for the equilibrium of the body it is necessary and sufficient for the force $\vec{F}_3$ to be the counterbalance of the first two forces. This is only possible when the forces $\vec{R}$ and $\vec{F}_3$ lie on the same straight line and have opposing directions. But then the lines of action of forces $\vec{F}_1$, $\vec{F}_2$ will intersect at one point *O*.

Axiom 4. The forces of interaction between two bodies are equal in magnitude and directed along a straight line in opposite directions (Newton's third law or the principle of action and reaction).

Note that the forces of interaction between two bodies do not compose a system of balanced forces, as they are applied to different objects.

Axiom 5. The equilibrium of a deformable body is not violated if its points are rigidly bound and the body is considered to be absolutely solid (non-deformable).

The given axiom allows studying the balance of deformable physical bodies. It establishes the connection between the conditions of equilibrium of a rigid and deformable bodies. Conditions of equilibrium of a rigid body are necessary but not sufficient for the balance of the deformable body. If the equations of rigid body equilibrium are not sufficient, they derive additional equations, where they take into account the deformation of the body, as it is practiced in the field of the strength of materials and elasticity theory.

3.4. Constraints. Constraint reactions.

Restrictions that prevent the free movement of the body are called *constraints*. The action of constraints on the body is called the reaction of constraints.

Let’s consider the basic types of constraints.

1. The smooth surface.

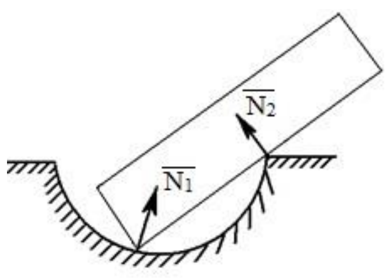


Fig. 3.6.

The reaction of constraints is aimed at the common normal to the surfaces of bodies in contact at the point of tangency and is affixed to this point (see Figure 3.6).

2. Torsion fiber.

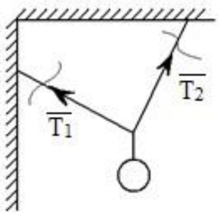


Fig. 3.7.

The reaction of constraint is directed along the filament to the point of suspension (Fig. 3.7).

3. Hinge bearing.

The joint can be rotated around the point of attachment. There are two types of joints.

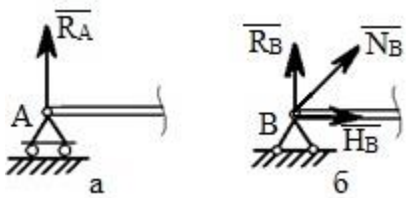


Fig. 3.8.

The movable joint (cylindrical). A rod attached to the joint can rotate about the joint, and the fixing point can be moved along the guide (Fig. 3.8 a). The reaction proceeds via the joint axis and perpendicular to the supporting surface.

The fixed joint (cylindrical). The attachment point cannot move. The rod can be freely rotate around the joint axis (Fig. 3.8 b). The reaction of such a support $\overrightarrow{N_B}$ passes through the axis of the joint, but its direction is unknown. It is usually decomposed into two components $\overrightarrow{R_B}$ and $\overrightarrow{H_B}$ in two mutually perpendicular directions $\left|\overrightarrow{N_B}\right| = \sqrt{{R_B}^2 + {H_B}^2}$.

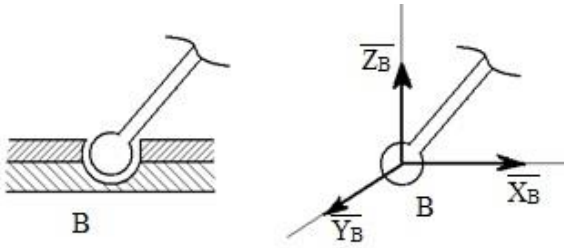


Fig. 3. 9

If a fixed joint is spherical (Figure 3.9), its back action is decomposed into three components.

4. Anchorage (jamming).

Anchorage does not allow any movement of parts (Fig. 3.10).

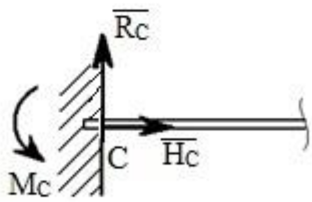


Figure 3.10.

Finding the reaction of anchorage is reduced to the determination of the components $\overrightarrow{R_C}$ and $\overrightarrow{H_C}$, preventing the line movement of the beam in the plane of forces action, and the reaction moment M_C, which prevents the rotation of the beam under the influence of forces applied to it.

Practical training 4. Kinetics (contd). Power analysis of the planar mechanism.

4.1. The transformation of forces and moments.

1. *Addition* of converging forces

The system of forces, the line of action of which intersects at one point, is called a system of converging forces.

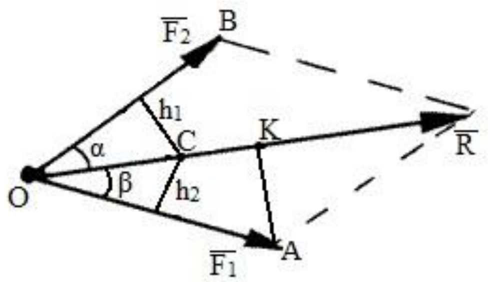


Figure 4.1.

Let's determine the magnitude of the resultant force (Fig. 4.1). From the third axiom of statics for two converging forces in magnitude and direction is the diagonal of the parallelogram formed **by these forces as two sides**. Its value can be determined by the law of cosines:

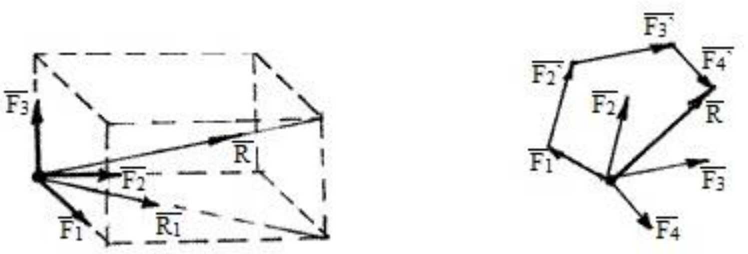
$$\left|\vec{R}\right| = \sqrt{F_1^2 + F_1^2 + 2 \cdot F_1 \cdot F_2 \cdot \cos(\alpha + \beta)} \tag{4.1}$$

We shall drop from an arbitrary point C perpendiculars to the directrix line of forces $\vec{F_1}$ and $\vec{F_2}$. As a result, we'll get the segments h_1, h_2- shoulders forces with respect to point C (Fig. 4.1). Thus, $\frac{h_1}{h_2} = \frac{\sin(\alpha)}{\sin(\beta)}$. On the other hand, analyzing the length of the line *AK*, we can write:

$$\cdot \frac{F_2}{F_1} = \frac{\sin(\alpha)}{\sin(\beta)} = \frac{h_1}{h_2} \quad \Rightarrow \quad F_1 \cdot h_1 = F_2 \cdot h_2$$

(4.2)

The product of the force on the shoulder is called the moment of the force with respect to a point. Then, according to (4.2) the moments of converging forces with respect to any point lying on the line of action of the resultant are equal.

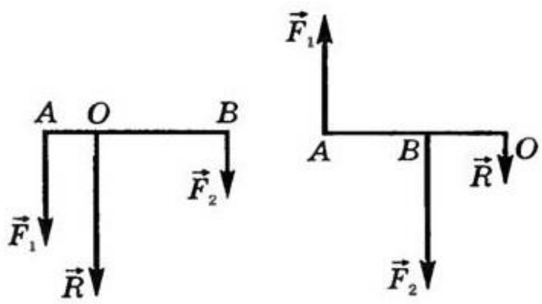


a b

Fig. 4.2.

When considering a system of converging forces, their resultant can be obtained either by successive addition of forces (Fig. 4.2a), or by building a power polygon (Fig. 4.2b).

2. Addition of parallel forces



a b

Fig. 4.3.

The resultant of two parallel forces vectored in the same direction (Fig. 4.3 a) is equal in magnitude to their algebraic sum $|\vec{R}| = |\vec{F}_1| + |\vec{F}_2|$, parallel to them, vectored in the same direction and attached to the point *O*, which divides the segment *AB* in the ratio inversely proportional to the magnitude of forces $\vec{F}_1$, $\vec{F}_2$

$$\frac{|\vec{F}_1|}{|\vec{F}_2|} = \frac{OB}{OA}.$$

.

The resultant of two parallel forces vectored in opposite directions (Fig. 4.3 b) is equal in magnitude to their algebraic difference $|\vec{R}| = |\vec{F}_2| - |\vec{F}_1|$, parallel to them, directed towards the action of the greater force and applied to the point *O*, **with respect** to which the sum of the initial moments of forces is equal to zero.

$$\frac{|\vec{F}_1|}{|\vec{F}_2|} = \frac{OA}{OB}$$

3. *Force couple.*

A force couple is called a system of two equal in size, parallel and opposing forces ($\vec{F}_1$, $\vec{F}_2$) (see Fig. 4.4).

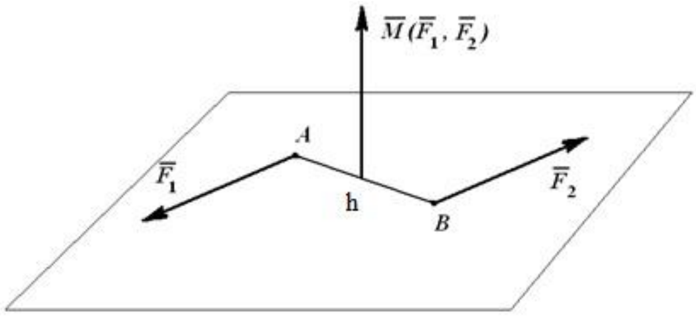


Fig. 4.4

The couple has no resultant force. The sum of projections of forces of the couple on any axis is equal to zero, since their projections are always equal and opposite in sign. The plane, passing through the lines of forces action, is called the plane of couple action.

A couple of forces have a rotational effect, which can be estimated by the moment of the couple $M = F_1 \cdot h = F_2 \cdot h$, where h is the arm of the force couple. The torque direction M is perpendicular to the plane of the pair of forces and is considered positive if the rotation performed by a couple of forces is directed counter-clockwise (Fig. 4.4 – the moment of the couple is positive).

The action exerted by a pair of forces on the body, will not change if a couple of forces is carried to any point of the plane of the couple of forces or to a plane parallel to it. Consequently, the vector momentum of the couple of forces can be applied (or transferred) to any point of the solid body that a couple of forces are applied at.

Couples of forces can be added and subtracted.

4. Parallel transport of force to an arbitrary point.

To transfer the power $\vec{F}$ from point *A* of the solid body to point B, we'll apply in point *B* the system of two forces $\vec{F'}$ and $\vec{F''}$ - equivalent to zero, and let $|\vec{F}| = |\vec{F'}| = |\vec{F''}|$ (Fig. 4.5).

Then the force $\vec{F}$ is equivalent to the same in magnitude force $\vec{F'}$ applied at point B, and a couple of forces ($\vec{F}, \vec{F''}$), the moment of which is $M = F \cdot h$, where h - the length of the segment *AB*.

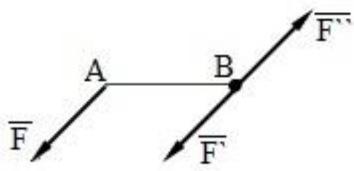


Fig. 4.5.

5. The moment of the force about a point.

The moment of the force about a point characterizes the rotation of the body relative to the point. If you know the radius vector $\vec{r}$ of the application point of the force $\vec{F}$ about point *O* (Fig. 4.6), the moment of this force about *O* is expressed as follows:

$$\overrightarrow{Mo}(\vec{F}) = \vec{F} \times \vec{r}\,. \qquad (4.3)$$

Numerically, the torque is equal to:

$$\left|\overrightarrow{Mo}\right| = \left|\vec{F}\right| \cdot \left|\vec{r}\right| \cdot \sin(\alpha) = \left|\vec{F}\right| \cdot h \qquad (4.4)$$

The vector $\overrightarrow{Mo}$, as a result of the cross product, is perpendicular to vectors $\vec{F}$ and $\vec{r}$, that belong to the plane *OAB* and directed so that if you look towards it, a force tending to rotate the plane in which it lies counter-clockwise can be seen.

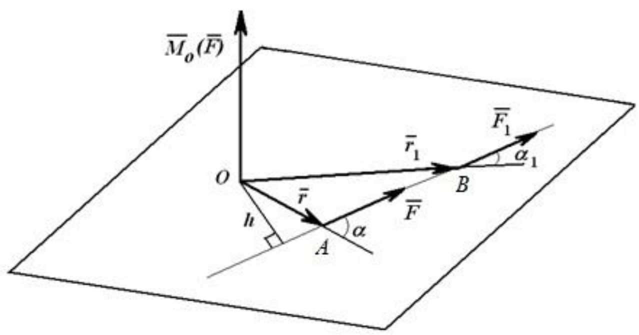


Figure 4.6.

In Fig. 4.6 one can see that if the force is moved along the line of action to another point, the magnitude and the sign of the moment will not change:

$$\left|\overrightarrow{Mo}\right| = \left|\vec{F}\right| \cdot \left|\vec{r}\right| \cdot \sin(\alpha) = \left|\vec{F_1}\right| \cdot \left|\vec{r_1}\right| \cdot \sin(\alpha_1) = \left|\vec{F}\right| \cdot h = \left|\vec{F_1}\right| \cdot h\,.$$

6. The moment of the force about the axis.

The moment of the force about the axis characterizes the rotation of the body relative to the axis and is calculated as the moment of the force projection $\overrightarrow{F_{II}}$ on the plane *XOY*, perpendicular to the axis, relative to the point of intersection of the axis with the plane (Figure 4.7):

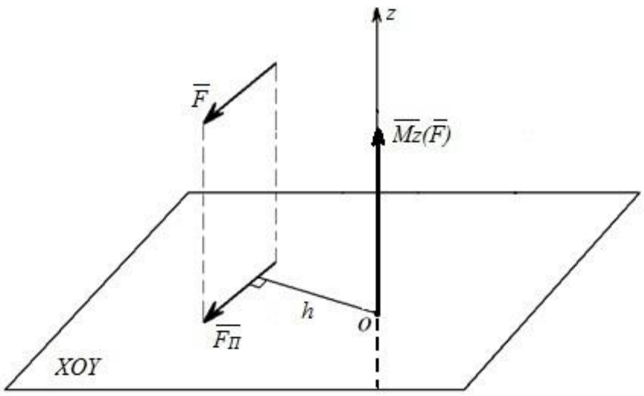


Figure 4.7.

$$\overrightarrow{Mz}(\vec{F}) = \overrightarrow{Mz}(\overrightarrow{F_{П}}) = \left|\overrightarrow{F_{П}}\right| \cdot h\,. \qquad (4.5)$$

The sign of the moment is determined by the direction of rotation, which tends to give the body the force $\overrightarrow{F_{П}}$, similarly as for the moment of the force about a point.

4.2. The main vector and the main moment of forces. The equilibrium conditions of the body under the action of the arbitrary system of forces.

Let's assume that the body is acted on by an arbitrary system of forces ($\vec{F_1}$, $\vec{F_2}$ ..., $\vec{F_n}$). We give it to the center *O*. In this case, an arbitrary spatial system of forces is given to the main vector and the main point. The main vector of the system of forces $\vec{R}$ is called a vector sum of these forces, that is $\vec{R} = \sum_{i=1}^{n} \vec{F}_i$. The main moment of the system of forces about the point $\overrightarrow{M_O}$ is called the vector sum of moments of these forces with respect to this point: $\overrightarrow{M_O} = \sum_{i=1}^{n} \overrightarrow{M}_O\left(\vec{F}_i\right)$.

Analytically, the main vector and the main moment about a point are determined by its three projections on the coordinate axes:

$$\begin{cases} R_x = \sum_{i=1}^{n} F_{ix}; \\ R_y = \sum_{i=1}^{n} F_{iy}; \\ R_z = \sum_{i=1}^{n} F_{iz}. \end{cases} \qquad \begin{cases} M_{Ox} = \sum_{i=1}^{n} M_x(F_i); \\ M_{Oy} = \sum_{i=1}^{n} M_y(F_i); \\ M_{Oz} = \sum_{i=1}^{n} M_z(F_i). \end{cases} \qquad (4.6)$$

The absolute values of the principal vector and principal moment are determined by the formulas:

$$R = \sqrt{R_x^{\;2} + R_y^{\;2} + R_z^{\;2}}\;;\; M_O = \sqrt{M_{Ox}^{\;\;2} + M_{Oy}^{\;\;2} + M_{Oz}^{\;\;2}}\;\;.$$

In order to balance the body under the action of an arbitrary system of forces ($\vec{F_1}$, $\vec{F_2}$, ..., $\vec{F_n}$), it is necessary and sufficient for the principal vector and the principal moment of forces to vanish:

$$\vec{R} = \sum_{i=1}^{n} \vec{F}_i = 0\,; \qquad \overrightarrow{M_O} = \sum_{i=1}^{n} \vec{M}_O\left(\vec{F}_i\right) = 0\,.$$

We shall write the six equations of equilibrium in the analytical form:

$$\begin{cases} \sum_{i=1}^{n} F_{ix} = 0; \\ \sum_{i=1}^{n} F_{iy} = 0; \\ \sum_{i=1}^{n} F_{iz} = 0; \end{cases} \qquad \begin{cases} \sum_{i=1}^{n} M_x(F_i) = 0; \\ \sum_{i=1}^{n} M_y(F_i) = 0; \\ \sum_{i=1}^{n} M_z(F_i) = 0. \end{cases} \qquad (4.7)$$

The recorded equilibrium equations of the spatial system of forces correspond to the analytical representation of six vanishing independent possible movements of the body in space: three displacements along the coordinate axes and three rotations around these axes.

If the body is acted on by the system of converging forces, part of equilibrium equations (4.7) vanish identically:

$$\begin{cases} \sum_{i=1}^{n} F_{ix} = 0; \\ \sum_{i=1}^{n} F_{iy} = 0; \\ \sum_{i=1}^{n} F_{iz} = 0. \end{cases} \tag{4.8}$$

In this case, in order to balance the converging system of forces, it is necessary and sufficient for the sum of projections of all the forces on the coordinate axes according to three movements to vanish.

Under the action of the plane system of forces equations (4.7) will be simplified to the following form:

$$\begin{cases} \sum_{i=1}^{n} F_{ix} = 0; \\ \sum_{i=1}^{n} F_{iy} = 0; \\ \sum_{i=1}^{n} M_z(F_i) = 0. \end{cases} \tag{4.9}$$

Where, for compliance with the conditions of equilibrium under the action of the plane system of forces it is necessary and sufficient the vanishing of the sum of projections of all the forces on the coordinate axes according to two movements and the vanishing of the sum of the torques about any point located in the plane of their action.

4.3. The forces acting on the mechanism.

For further calculation circuit, first, it is necessary to study the forces acting on the links of the mechanism. From the point of view of solving the problems of dynamics, forces can be classified as follows:

1. External and internal forces.

External – the forces of interaction of the mechanism link with other bodies, which are not part of the mechanism;

Internal forces - the forces of interaction between the links of the mechanism (the reaction of constraints in kinematic couples);

2. *Driving forces.*

Driving forces - the forces that are applied to the drive link, facilitate the movement of the link and develop a positive power. The driving forces overcome the resistance of the rest of the forces applied to the machine.

3. *Forces of useful resistance.*

Forces of useful resistance- the forces (to overcome which there was created the mechanism proper) that are applied to the master link.

Overcoming the forces of useful resistance, the mechanism provides useful work.

4. *The forces of parasitic drag.*

Parasitic drag forces - the forces to overcome which there is spent power and this power cannot be recuperated. Typically, the friction forces act as the forces of parasitic drag (forces resisting the relative displacement of mating surfaces when the units are in relative motion to each other).

In power calculations, the frictional forces are neglected.

5. The forces of inertia, the moments of inertia.

The inertial forces - forces arising from the irregular motion of the unit and resisting its acceleration (deceleration). The inertial force acts on the body, which causes the given link to accelerate (decelerate). In general, when there is non-uniform motion, the force of inertia and the moment of inertia occur:

$$\overrightarrow{F_{И}} = -m \cdot \overrightarrow{a_S} \qquad \overrightarrow{M_{И}} = -I_S \cdot \vec{\varepsilon} \tag{4.10}$$

where m - the mass of the unit (concentrated in the center of masses); I_S - the moment of inertia of the link; $\overrightarrow{a_S}$ - the absolute acceleration of the mass center of the unit; $\vec{\varepsilon}$ - the angular acceleration of the link.

The minus sign in (4.10) shows that the inertia force is directed opposite to the acceleration of the center of the unit mass, and the moment of inertia is directed opposite to the angular unit acceleration.

6. The force of gravity.

Gravity - is the force of interaction of mechanism links to the gravitational field of the earth. They are applied to the center of the mass and directed towards the center of the earth (vertically downward).

4.4. Power analysis of planar mechanisms.

The power analysis is reduced to the determination of unknown forces acting on the links of the mechanism. To determine the unknown forces they use static equilibrium equations. But the mechanism is a non-equilibrium system, as most of its links have uneven movement, and the points that belong to these links, move in complex curved paths. Therefore, to solve the posed problem they use the method of kinetostatics, which is based on the D'Alembert principle: if we add inertia forces and the moments of inertia to all external forces acting on the links of the mechanism, then the mechanism will be in a state of static equilibrium. So this is a workaround that results the unbalanced system to equilibrium.

Applying this technique, we can produce a power calculation using the equations of statics. In order to solve such equation systems there must be satisfied the condition of the kinetostatic definable - the number of kinetostatics equations must equal the number of unknown reactions.

For a planar kinematic chain the condition of kinetostatic definable can be represented as follows:

$$X = 3(n-1) - 2P_H - P_B \tag{4.11}$$

where X - the number of remaining equations to determine the unknown external forces, n - the number of links of the mechanism; P_H - the number of lower kinematic pairs; P_B - the number of higher kinematic pairs.

Equation (4.11) coincides with the Chebyshev's formula for determining the number of degrees of freedom of the plane kinematic chain. As a result, one can formulate the condition of the kinetostatic definable of the kinematic chain as follows: the kinematic chain is kinetostaticaly definable in that case when the number of unknown external forces, acting on its links, do not exceed the number of degrees of freedom of the chain.

In solving kinetostatics equations, most often there is required to determine the balancing force (determines the power of the desired source of energy), which is attached to the master link. Based on this, the calculation procedure of the power mechanism is as follows:

- a mechanism is broken into dyads, taking as a starting one the unit, which is acted on by an unknown external force (traction);
- solving is started with the last dyad and finished with the initial link.

With this approach the dyad will always be acted on only by known external forces, and from their balance consideration there will be identified the responses in kinematic pairs, but from consideration of equilibrium conditions of the initial links there will be determined the remaining reactions as well as unknown external forces.

Educational edition

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