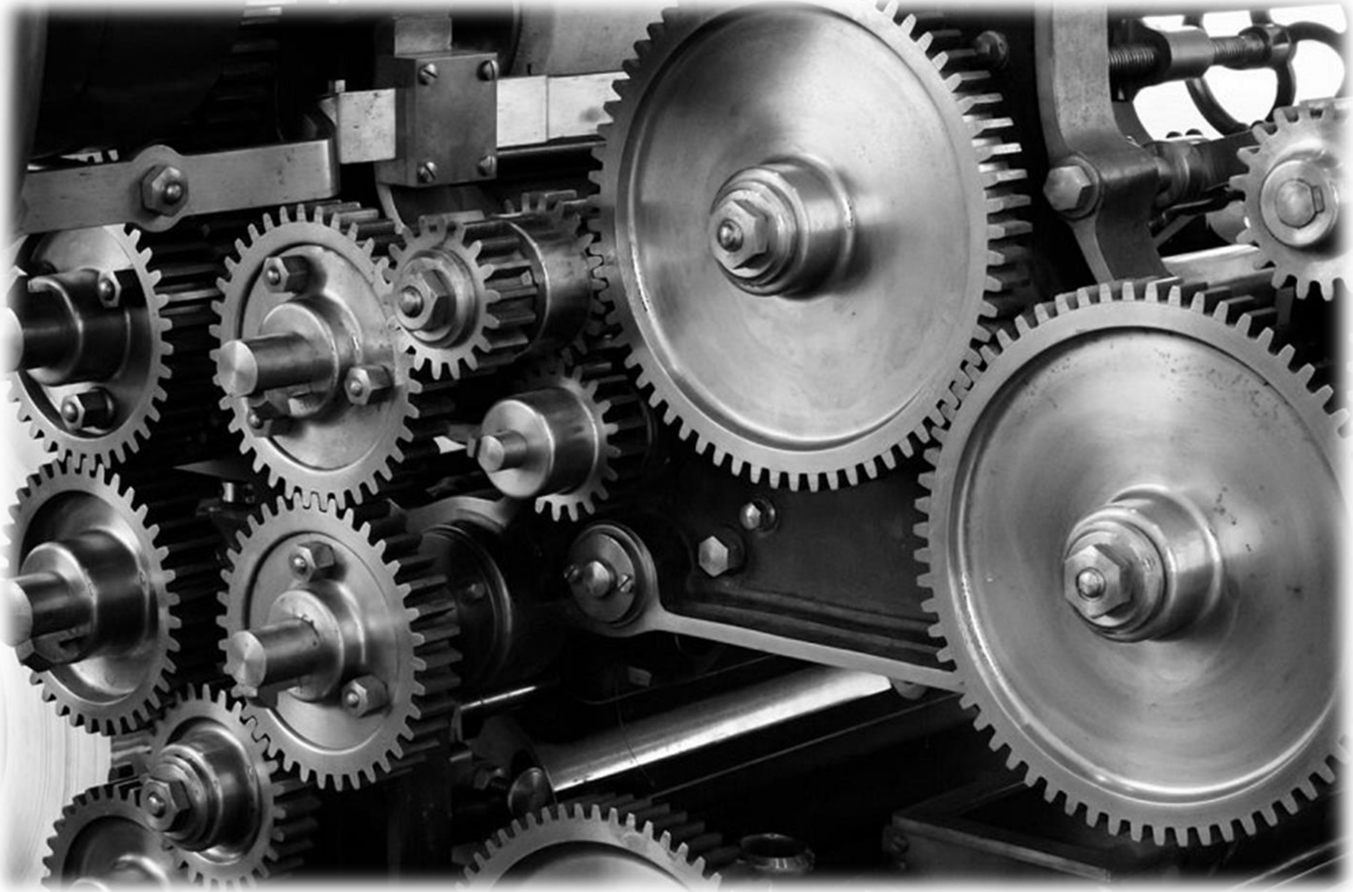
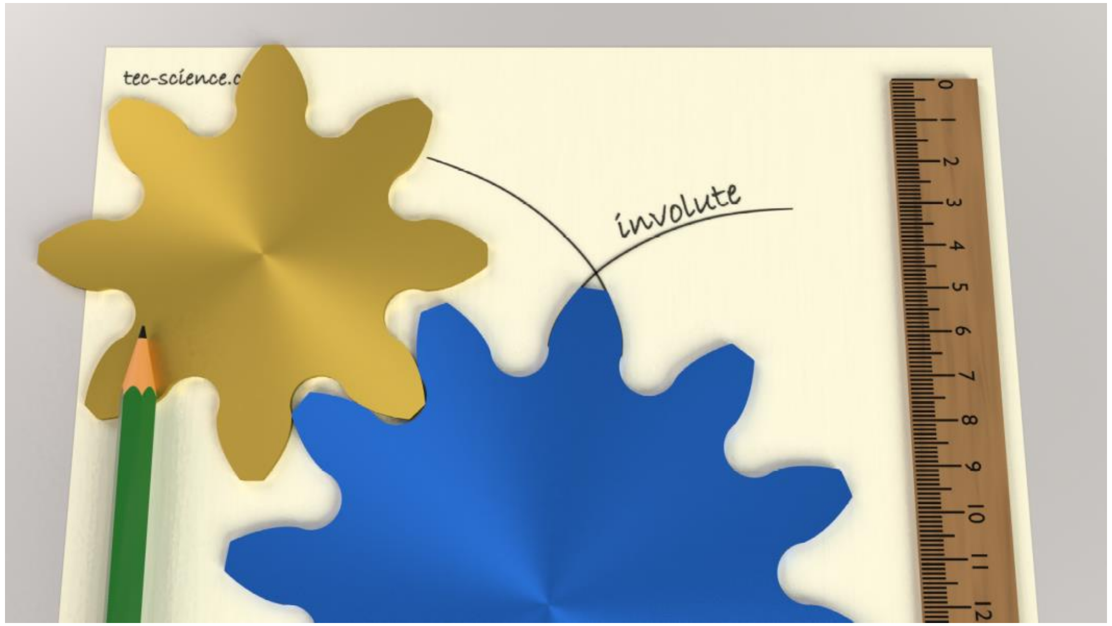
Gears. Involute profile parameters



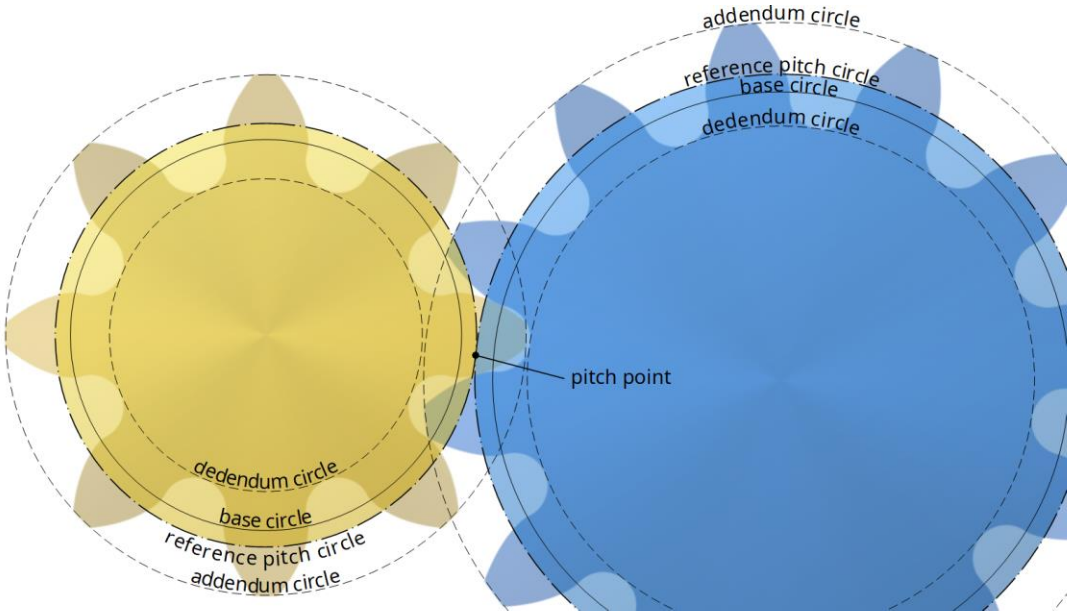
Involute profile



In mechanical engineering, the involute is used almost exclusively as a tooth form for gears. Such gears are called involute gears. The use of involute toothing is due on the one hand to the favorable meshing (engagement of two gearwheels). On the other hand, involute gears can be manufactured cost-effectively due to the relatively simple tool geometry.

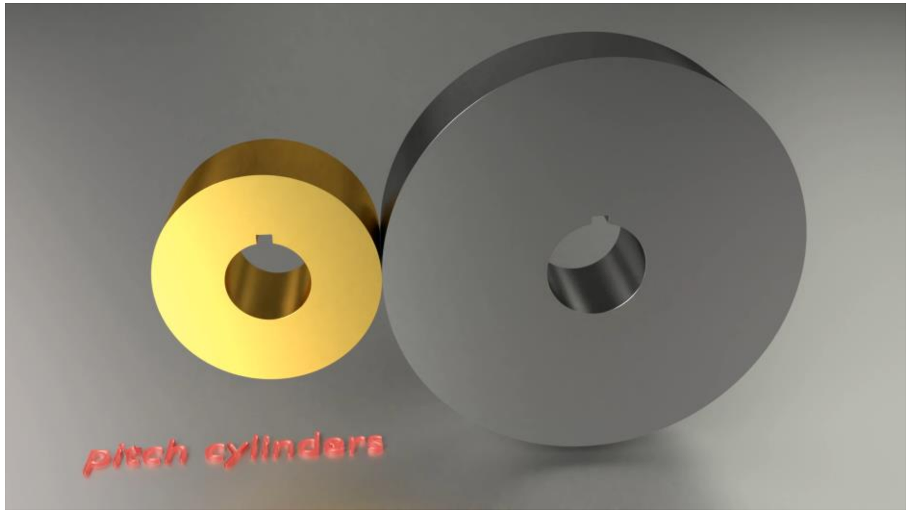
An involute is constructed by rolling a so-called rolling line around a base circle. The resulting trajectory curve describes the shape of the involute. Two mirror-inverted involutes then form the basic shape of a tooth.

Diameters



$d_{\omega 1}$ $d_{\omega 2}$ - pitch diameters (reference)

The size of a gearwheel is defined by the reference pitch diameter. Strongly simplified, this diameter corresponds to the diameters of imaginary cylinders that roll on each other.

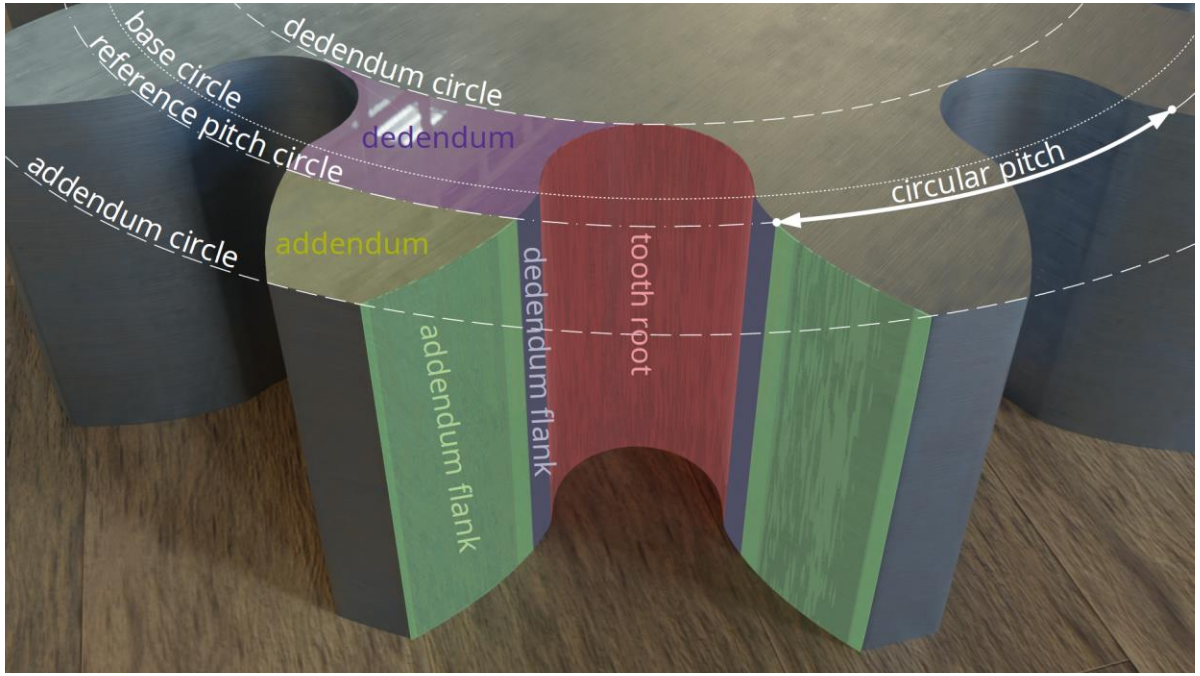


d_{b1} d_{b2} - diameters of the base circle

d_{a1} d_{a2} - diameters of addendum circles (top circles)

d_{f1} d_{f2} - diameters of dedendum circles (root circles)

Nomenclature



$$P_0 \cdot z = \pi \cdot d$$

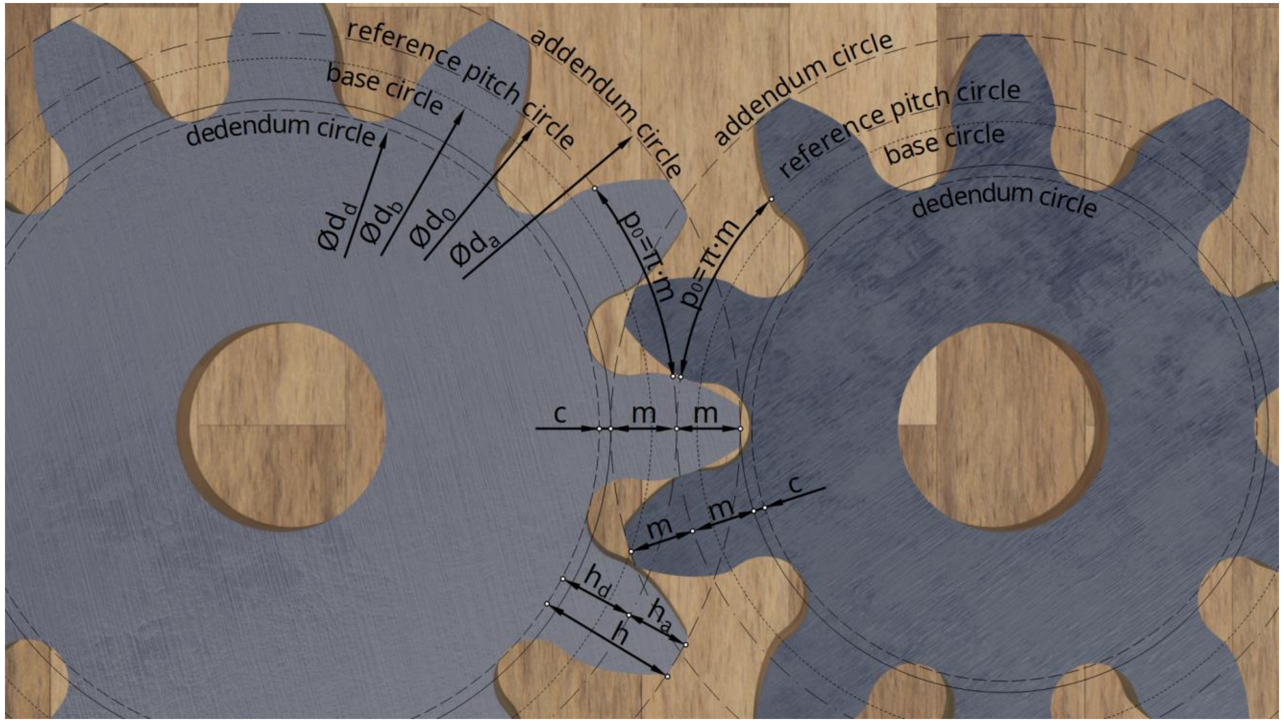
$$P_0 = \frac{\pi \cdot d}{z}$$

In order to avoid contact between the tip and the root of the tooth of two meshing gears, the root of the tooth is rounded out (called fillet).

The tooth spacing is related to pitch diameter and is referred to as the circumferential pitch or *circular pitch*.

The *circular pitch* is the arc distance between two tooth flanks of the same direction on the pitch circle. This circular pitch must be identical for all gears so that the teeth can mesh without interfering.

Tooth size: the module



In order to characterize gears and above all to ensure that the teeth of two gears can mesh properly, one of the most important parameters is the so-called *module m*.

$$m = \frac{P_0}{\pi}$$

h_a	addendum
h_d	dedendum
h	depth of the tooth

All geometric parameters of gears are expressed through the module *m.*

$$h_a = m$$

$$h_d = h_a + c = m + c$$

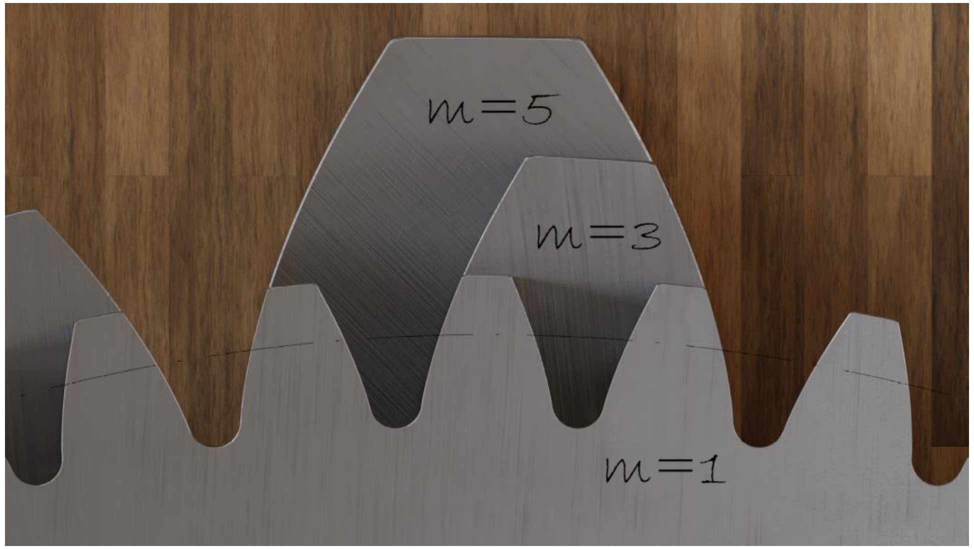
$$h = h_a + h_d = 2 \cdot m + c$$

Depending on the application, the clearance is typically 10% to 30% of the module (for cylindrical gears $c = 0.25 \cdot m$)

c	clearance

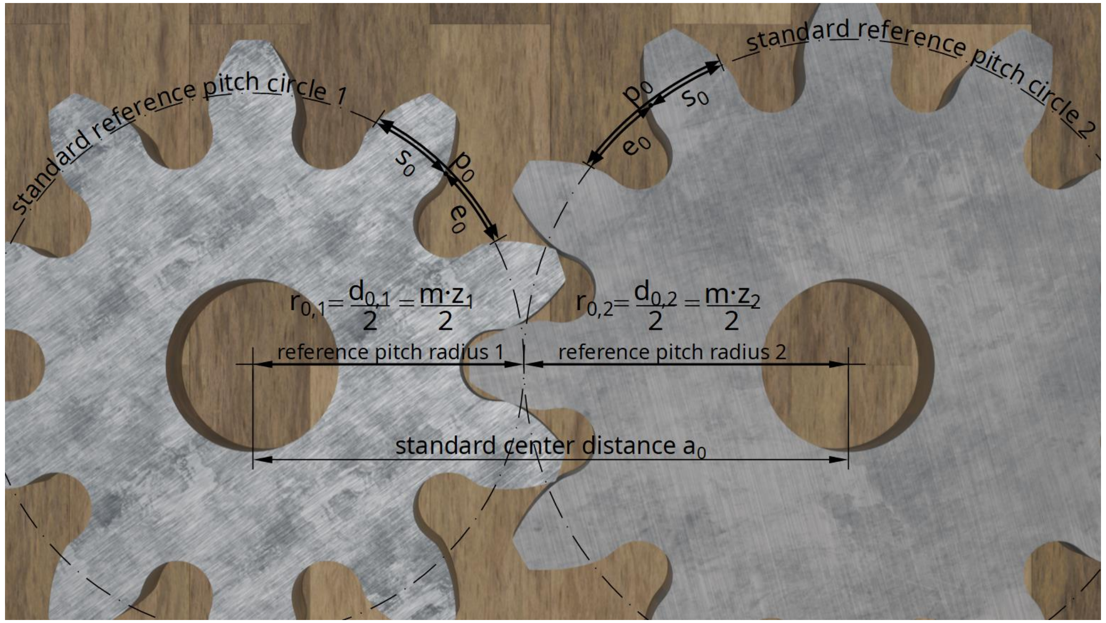
Influence of the module on the tooth size

Three gears of the same size (i.e. identical reference pitch circles), but manufactured with different modules.



The module is a measure of the tooth size: the larger the module, the larger the tooth! Only gears with the same module can be paired!

Gear size: the standard reference pitch diameter



For gears that are not profile-shifted, the circular tooth thickness s_0 and the tooth space width e_0 on the reference pitch circle are identical in length and thus correspond to half the circular pitch p_0.

$$s_0 = e_0 = \frac{p_0}{2} = \frac{m}{2} \cdot \pi$$

The standard reference pitch diameter d_0 results from the product of module *m* and number of teeth *z* and is a measure for the size of the gear:

$$d_0 = m \cdot z$$

The circular pitch of the teeth is related to this diameter.

The standard reference pitch diameter d_0 can now be used to determine the tip diameter d_a (addendum circle) and the root diameter d_d (dedendum circle):

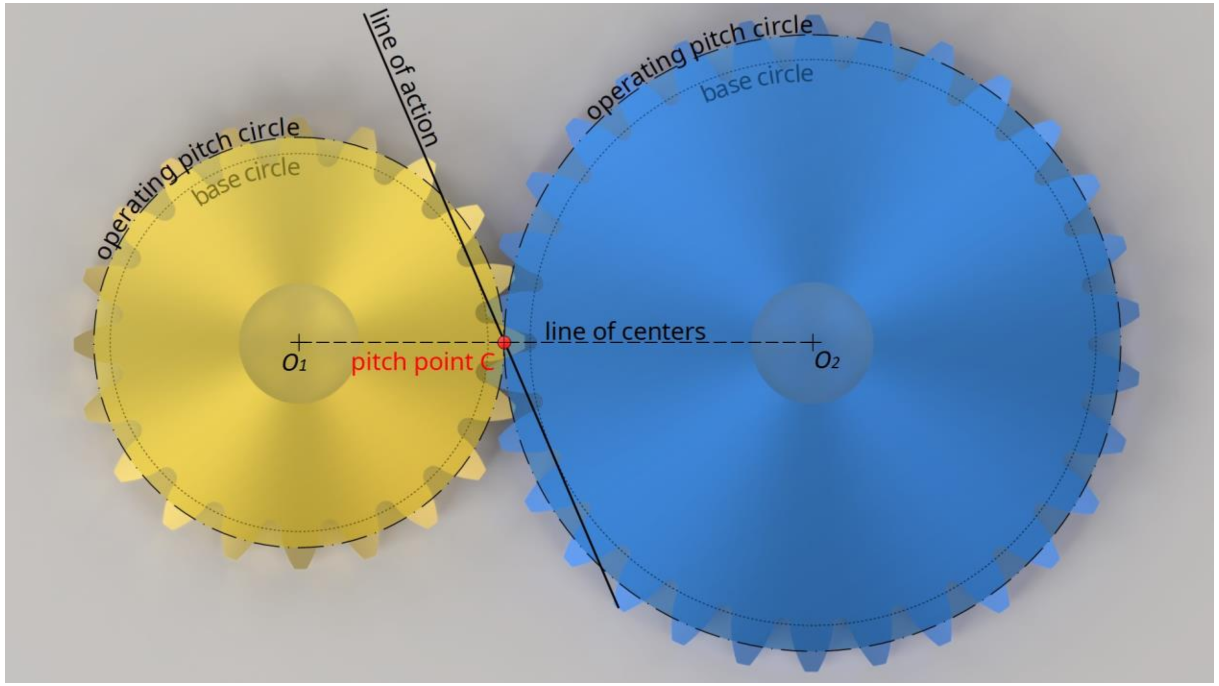
$$d_a = d_0 + 2 \cdot h_a = m \cdot (z + 2)$$

$$d_d = d_0 - 2 \cdot h_d = m \cdot (z - 2) - 2 \cdot c$$

The standard center distance

$$a_\omega = \frac{d_{01} + d_{02}}{2} = m \cdot \frac{(z_1 + z_2)}{2}$$

Fundamental law of gearing

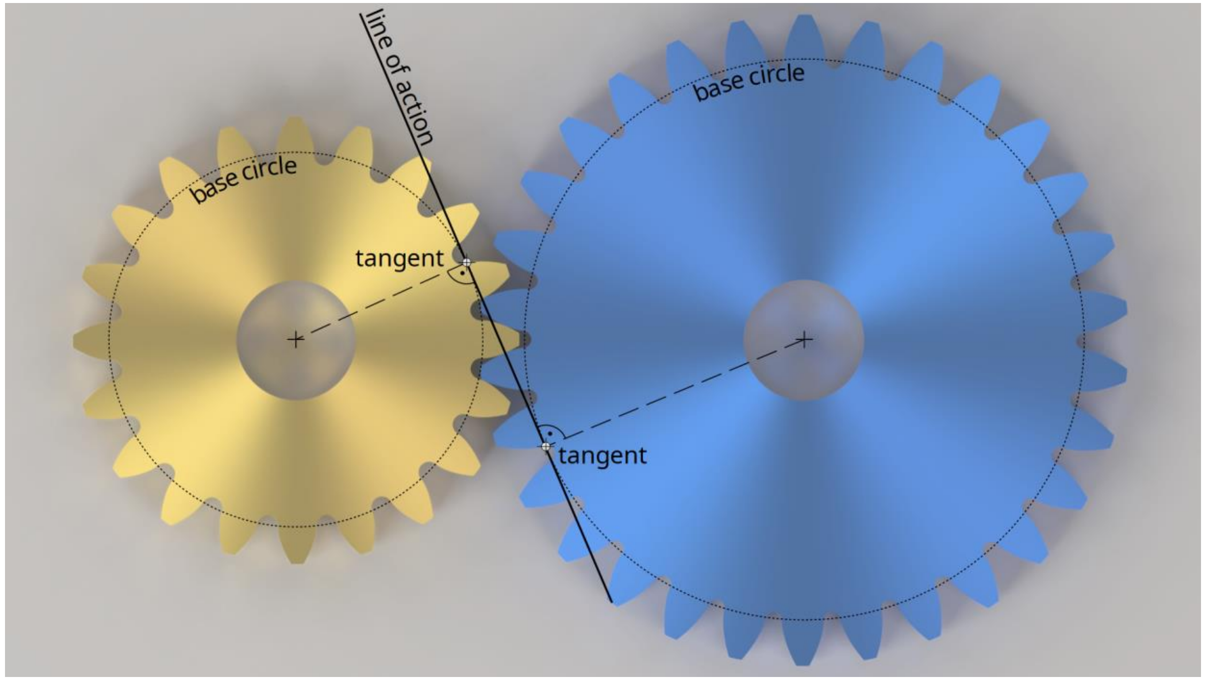


The normal at the point of contact of two tooth flanks must run at any time through the pitch point *C,* which will divide the center distance O_1 and O_2 in the inverse ratio of angular velocity.

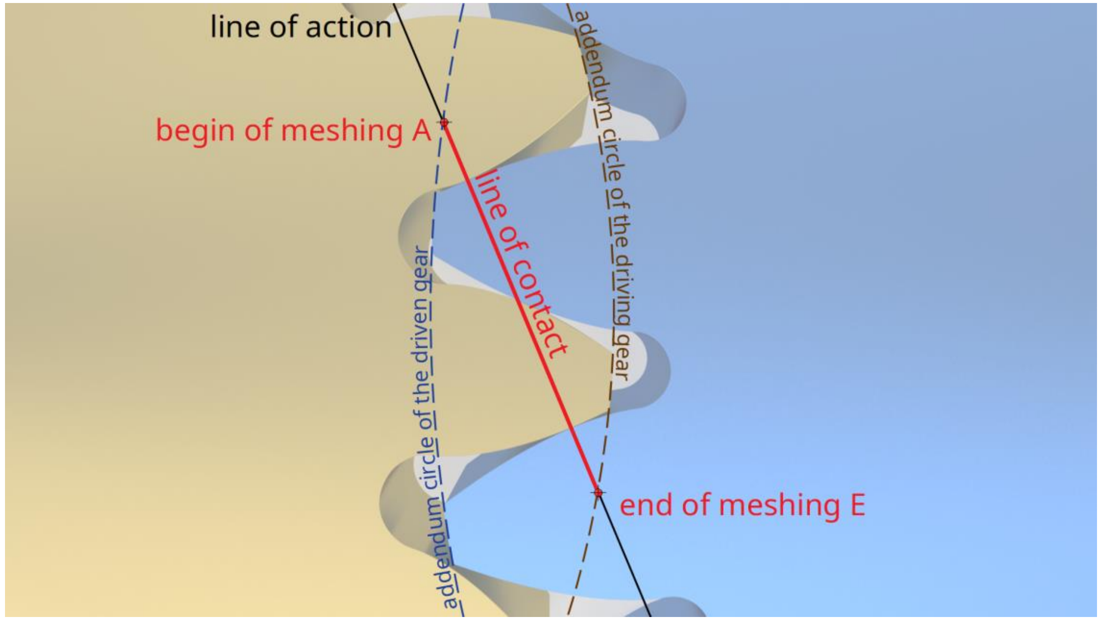
$$\frac{O_2C}{O_1C} = \frac{\omega_1}{\omega_2} = i$$

Hence this is the condition to have constant angular velocity ratio (constant transmission ratio) for all positions of the wheel which must be satisfied and it is known as the *law of gearing*.

Line of action. Line of contact



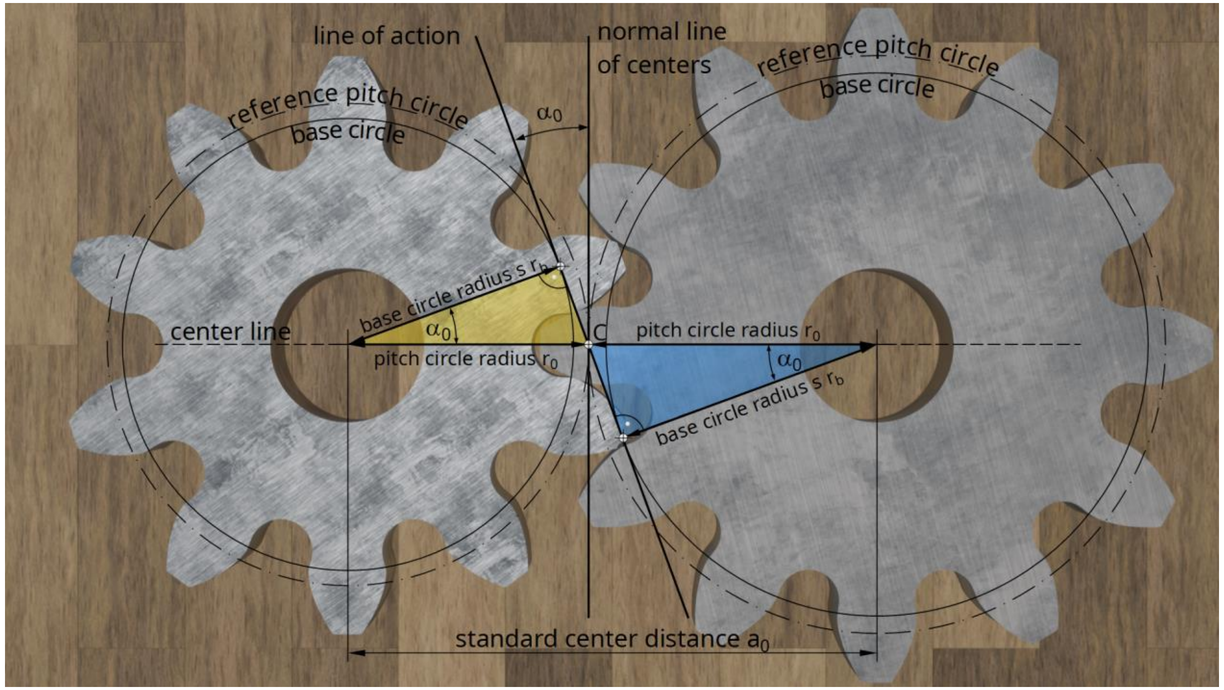
Due to the special design of the tooth shape of an involute gear (rolling a straight line on a circle), the intersection of two involutes rolling off each other describes a straight line. This situation occurs when two gears are meshing. The involute tooth flanks then slide along a straight line. This straight line is also referred to as the line of action or line of engagement.



The line of action corresponds to the tangent applied on the base circles of the gears.

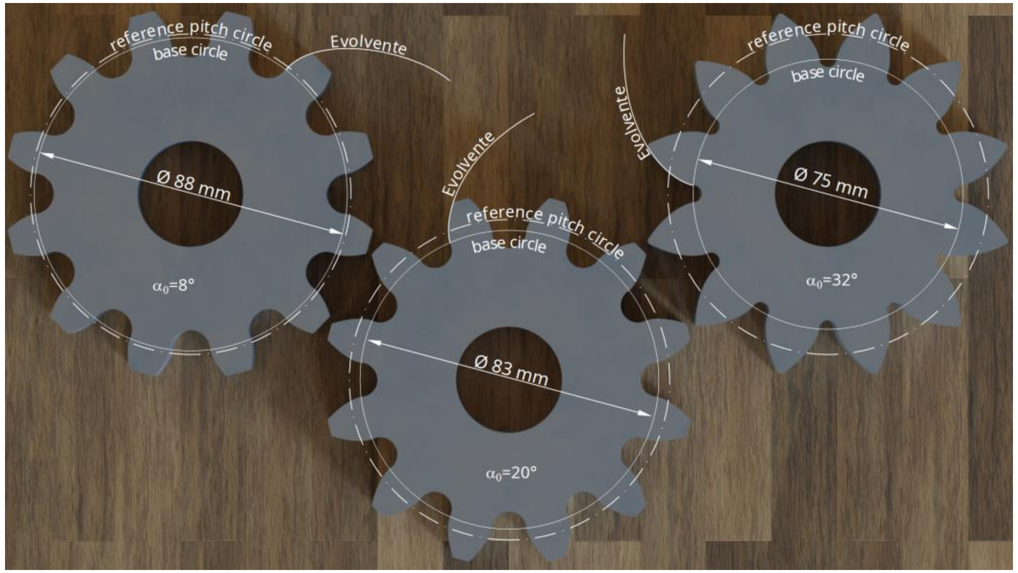
The distance actually covered on the line of action is called line of contact (red line in the animation above).

Tooth shape: the pressure angle



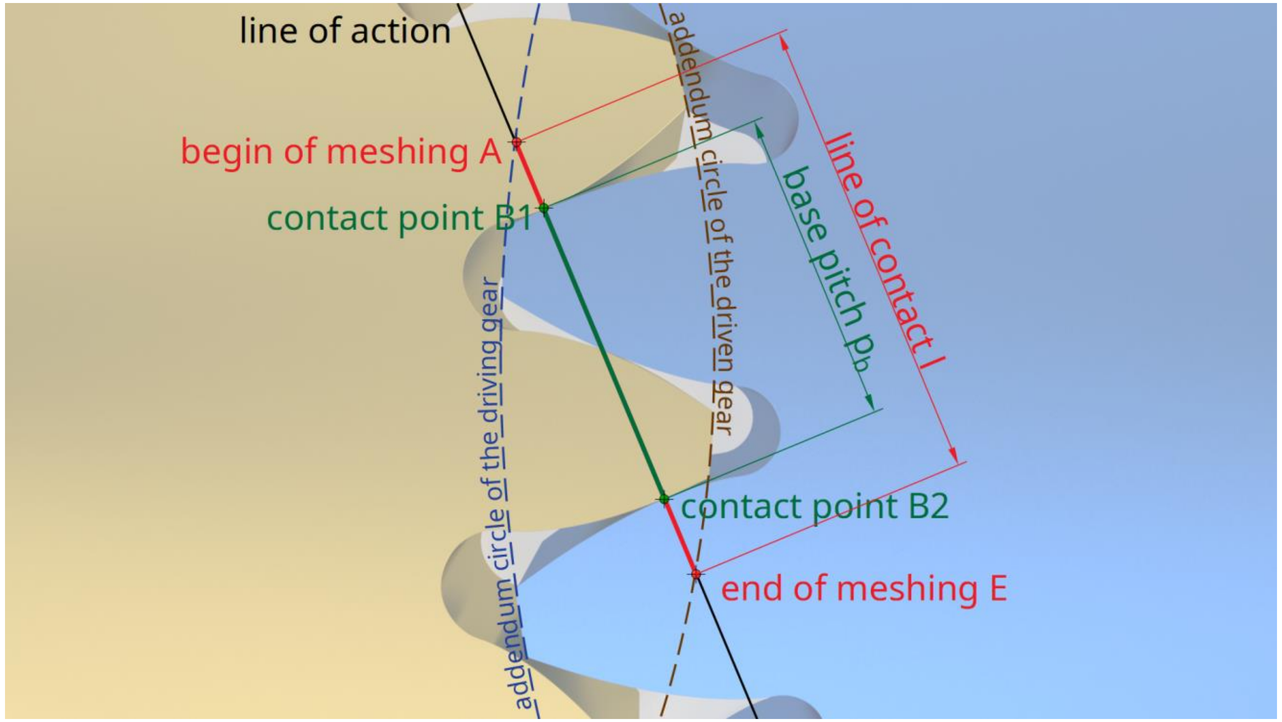
The pressure angle refers to the angle between the normal of the line of centers and the line of action. For a standard gear, this pressure angle is set to 20°. In this standard state, the pressure angle is also referred to as the standard pressure angle.

The tooth shape is thus decisively determined by the standard pressure angle.

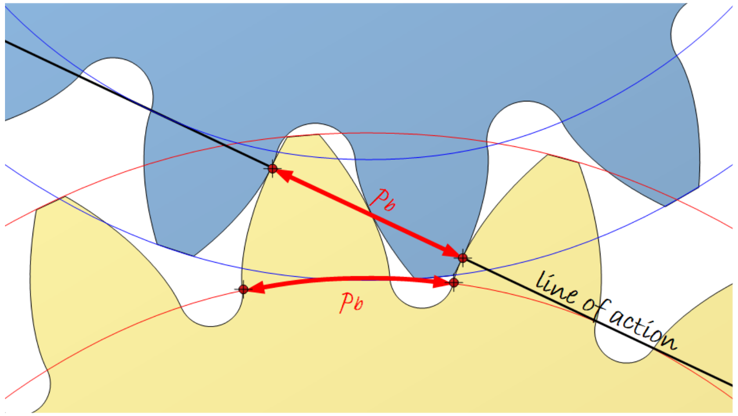


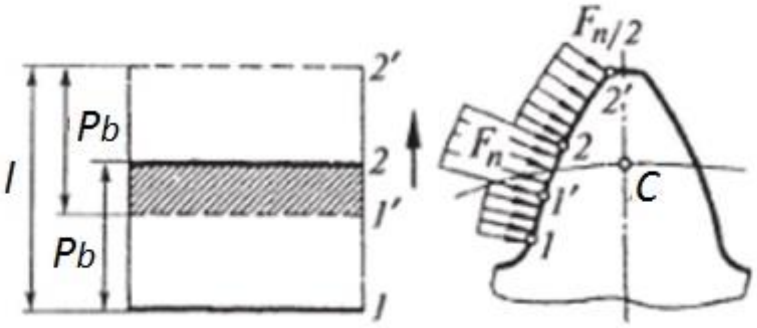
$$\cos(\alpha_0) = \frac{r_b}{r_0} = \frac{d_b}{d_0}$$

Base pitch and contact ratio



The distance between two adjacent contact points *B1* and *B2* (the distance between two flanks) on the line of contact *l* is called the base pitch *Pb*.

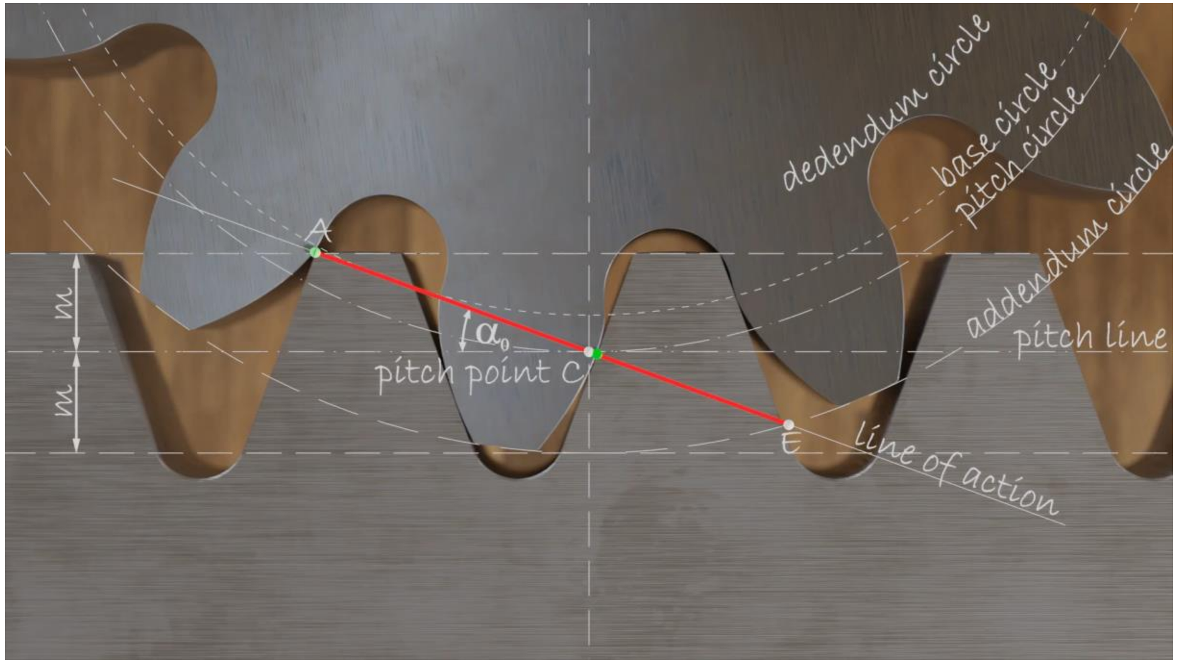




$$\varepsilon_\alpha = \frac{l}{P_b} \geq 1$$

The contact ratio indicates how many teeth are in mesh simultaneously on the line of contact. The greater the contact ratio, the greater the forces that can be transmitted and the lower the noise level!

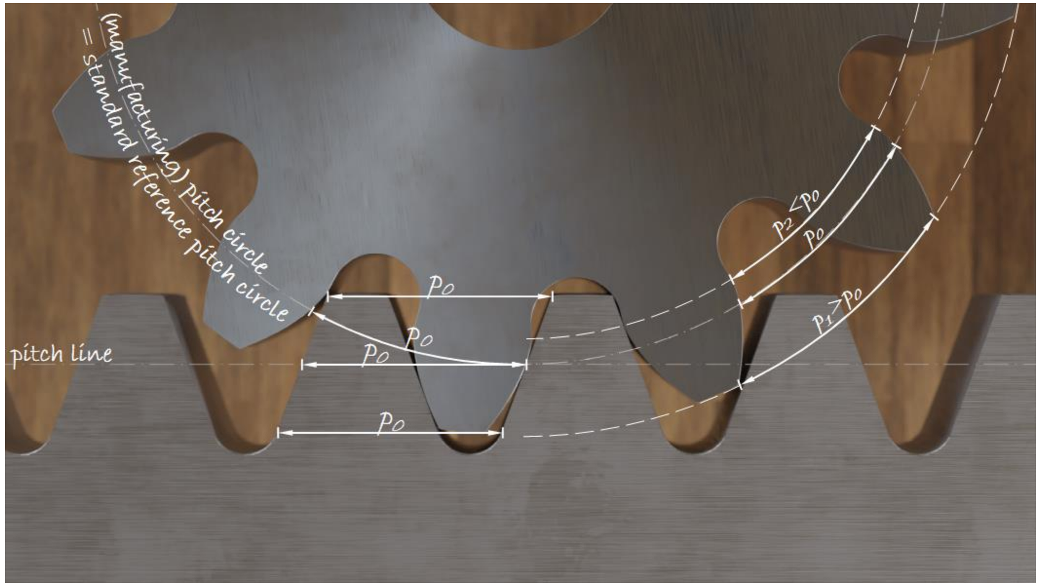
Rack



With the increase in the diameter of the base circle d_b the curvature of the involute is reduced and in case if $d_b \to \infty$ a toothed profile is transformed into a rack with a trapezoidal profile - the gear rack.

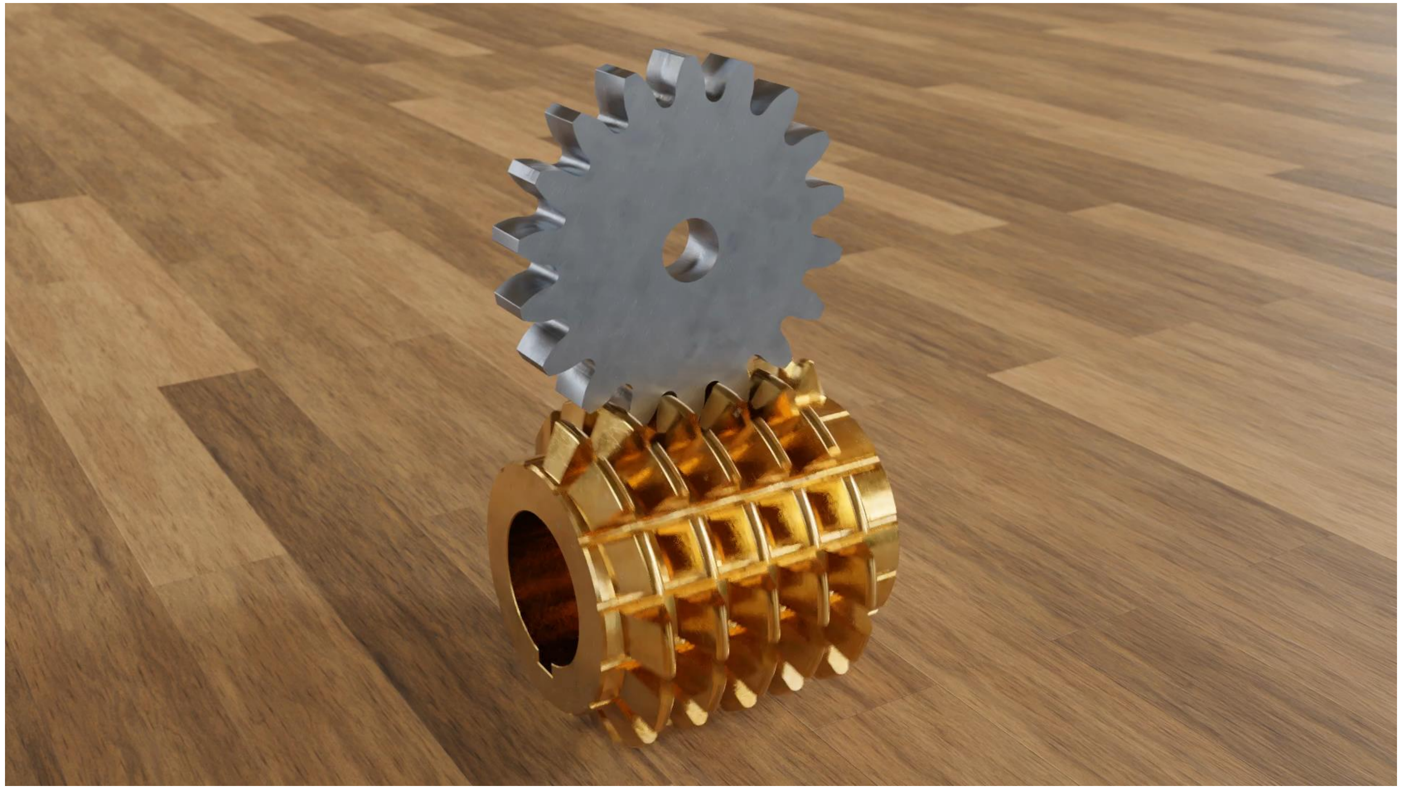
A rack is basically a special case of a gear with an infinitely large diameter.

In contrast to a gear, rack-shaped tools (or racks) have identical tooth spacings p_0 at every point.



The standard reference pitch circle is a fixed (unchangeable) parameter of a gear, which is determined solely by the rack-shaped cutting tool! Therefore, a rack can actually be used as a definition of the standard reference pitch circle of a gear

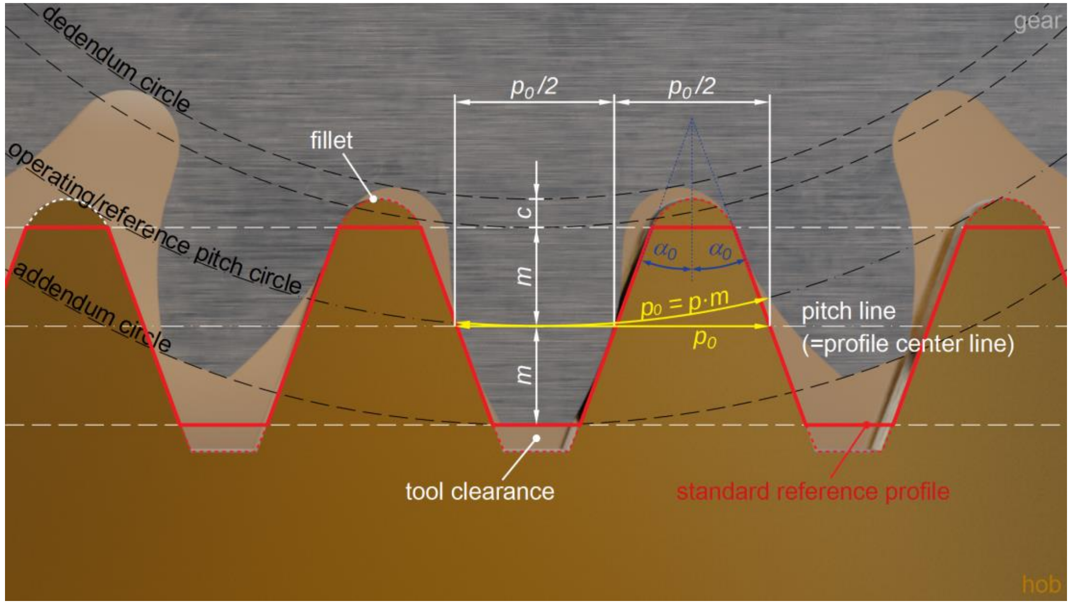
Gear cutting.Gear hobbing



Gear cutting with a hob (hobbing)

One single rack-type hob can be used to produce gears with any number of teeth and then paired with each other.

The cross-sectional profile of a hob is equal to that of a rack!



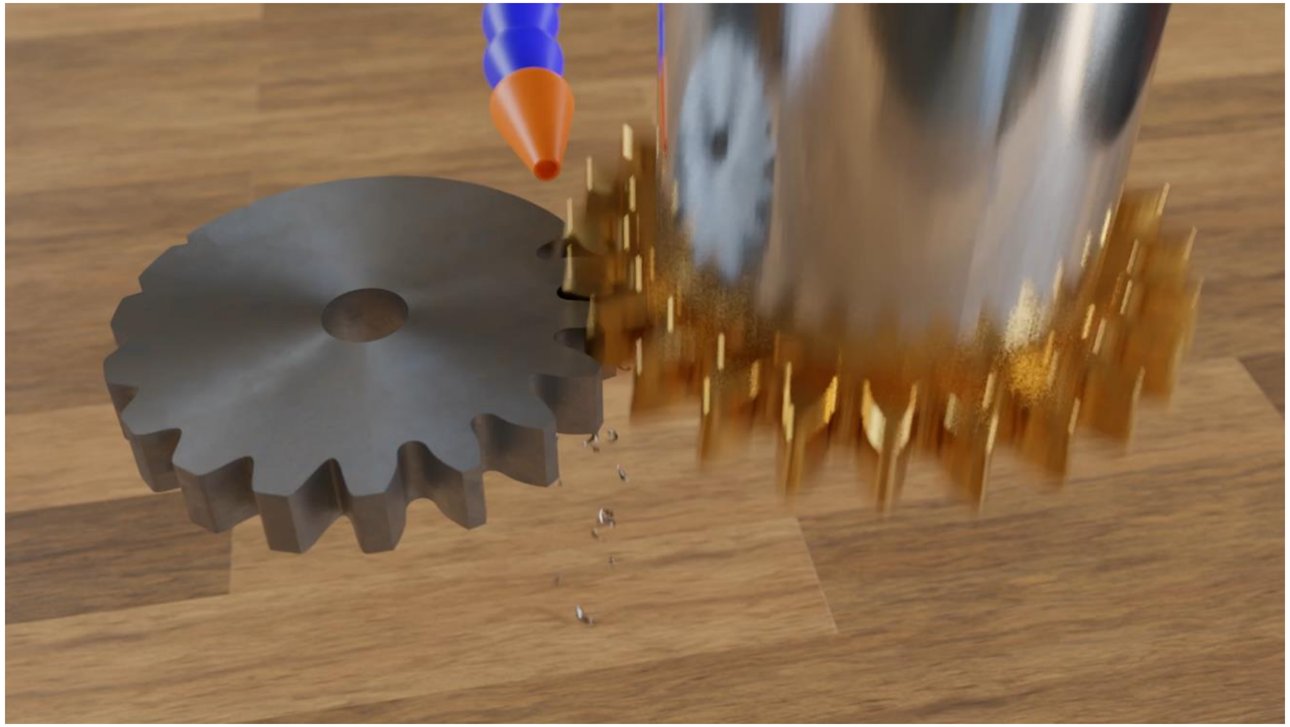
The cutting performance of gear hobbing is very high, so that especially very thick gears can be produced in a relatively short time. However, this process cannot be used to produce internal gears.

Gear shaping

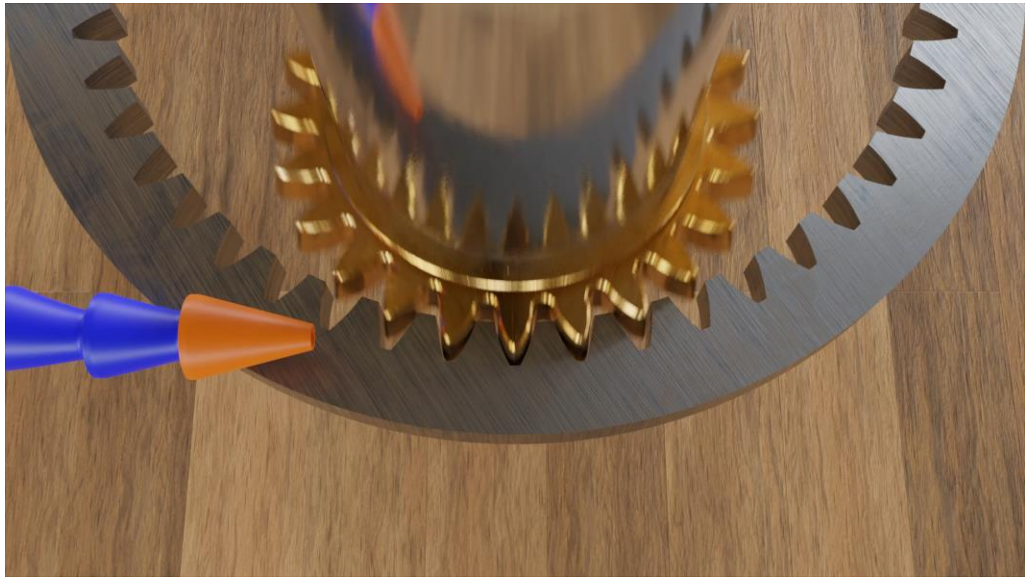
In contrast to hobbing, shaping is performed by a translational reciprocating motion of the tool.

The tool can actually be regarded as a “gear with cutting edges”.

In contrast to gear hobbing, gear shaping can also be used to manufacture internal gears.



Shaping can also be used to produce helical gears. For this purpose, the tool is simply inclined by the amount of the helix angle.



Gear planing with a rack type cutter

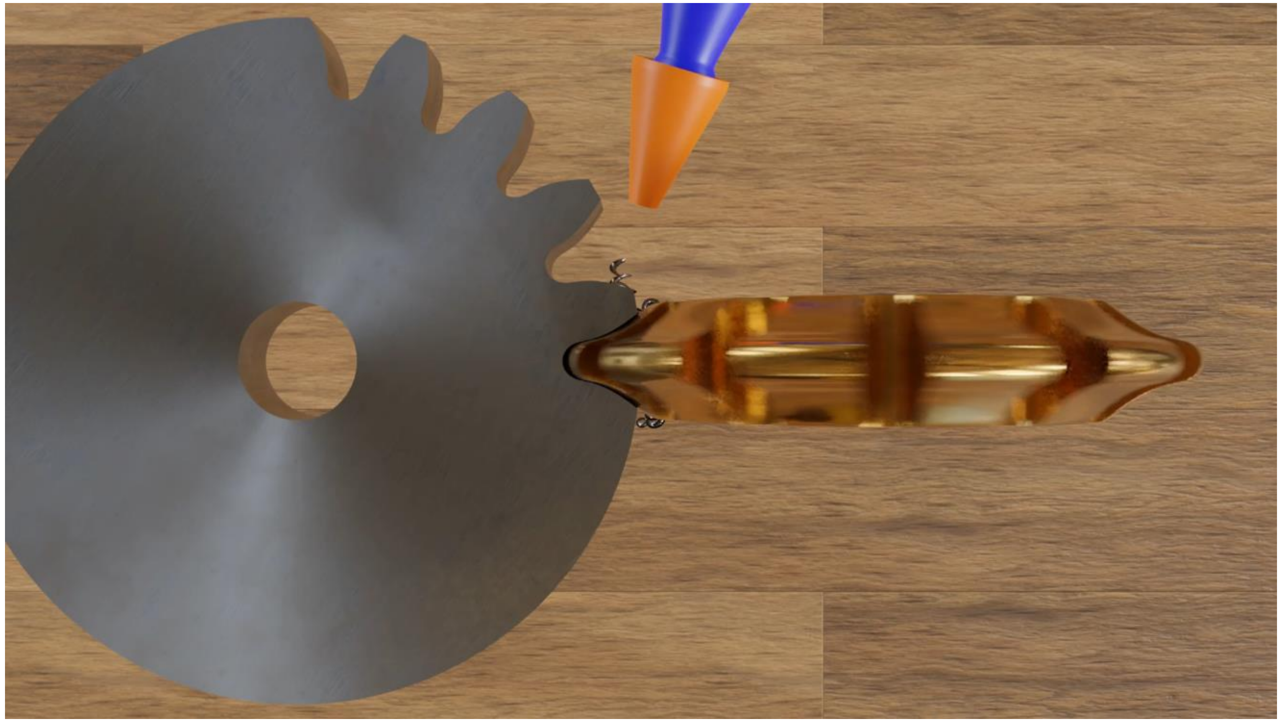
Instead of a “pinion-shaped” tool as is the case with gear shaping, a rack-shaped tools can also be used.

The rack shaped cutter can also be used to produce helical gears.

In contrast to gear shaping with a pinion type cutter, whose cutting edges are involute-shaped, the rack type cutter has straight flank cutting edges. Rack type cutters can therefore be produced much more easily and thus more cost-effectively. However, internal gears cannot be produced.

Planing with a rack type cutter is mainly used for very large gears. The cutting performance is relatively low compared to gear hobbing.

Form cutting with a disc cutter

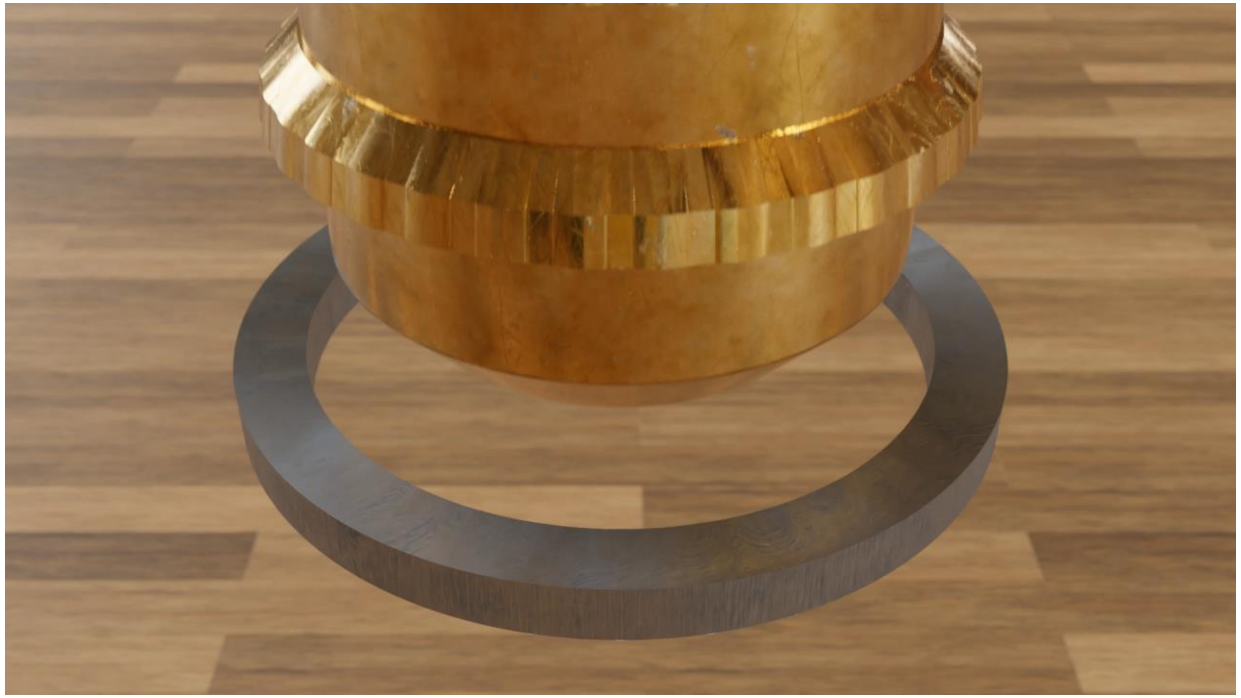


In contrast to hobbing, shaping or planing, the cutting motion of the tool does not have to be matched to the motion of the workpiece.

Form cutting can also be carried out on ordinary milling machines.

The tooth spaces have different geometries depending on the size of the gear, a single form cutter can only be used to produce a specific gear. For each module (or diametral pitch) and each number of teeth, separate disc cutters are required, which are relatively expensive due to the individual shape.

Gear broaching

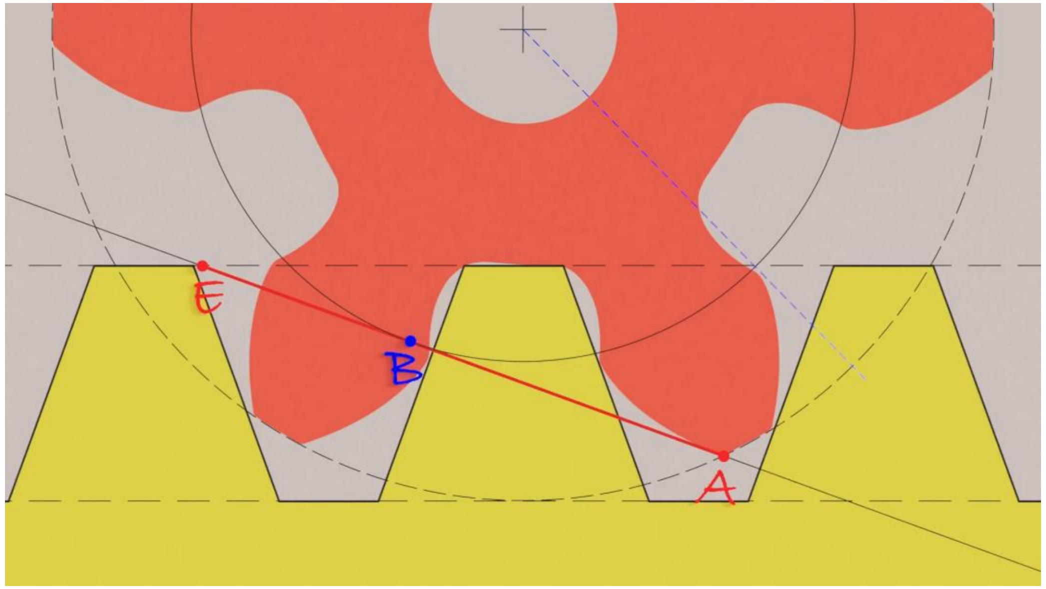


Broaching is similar to shaping or planing in terms of motion.

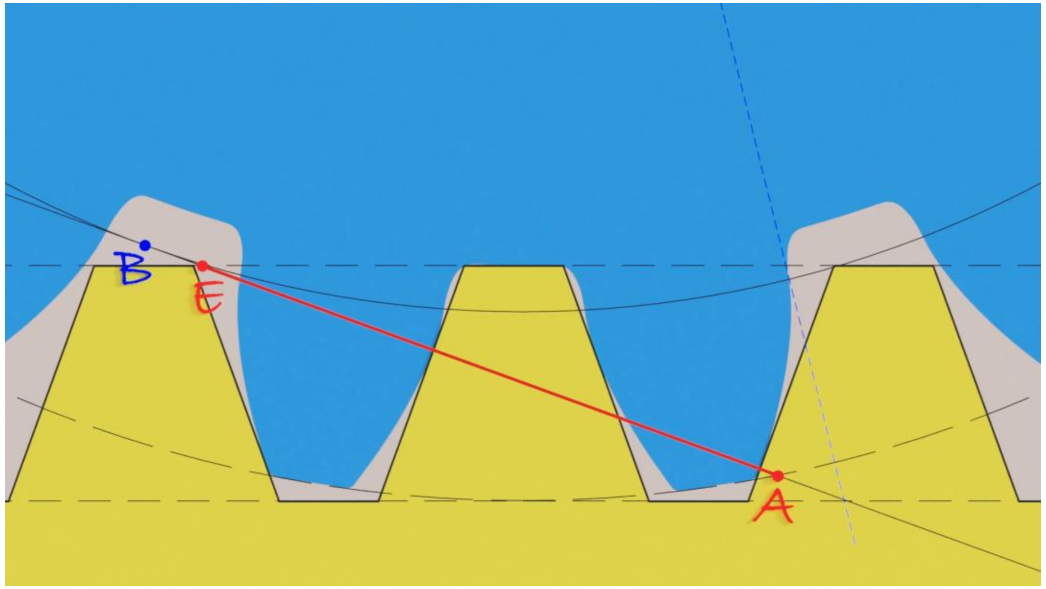
Broaching thus has a high cutting efficiency and is used above all for internal gears.

Only one specific gear can be produced with one broaching tool. For each number of teeth or module, a specific broaching tool is required.
Gear broaching is therefore only used in mass production.

Undercut



Undercutting of a gear wheel with 6 teeth



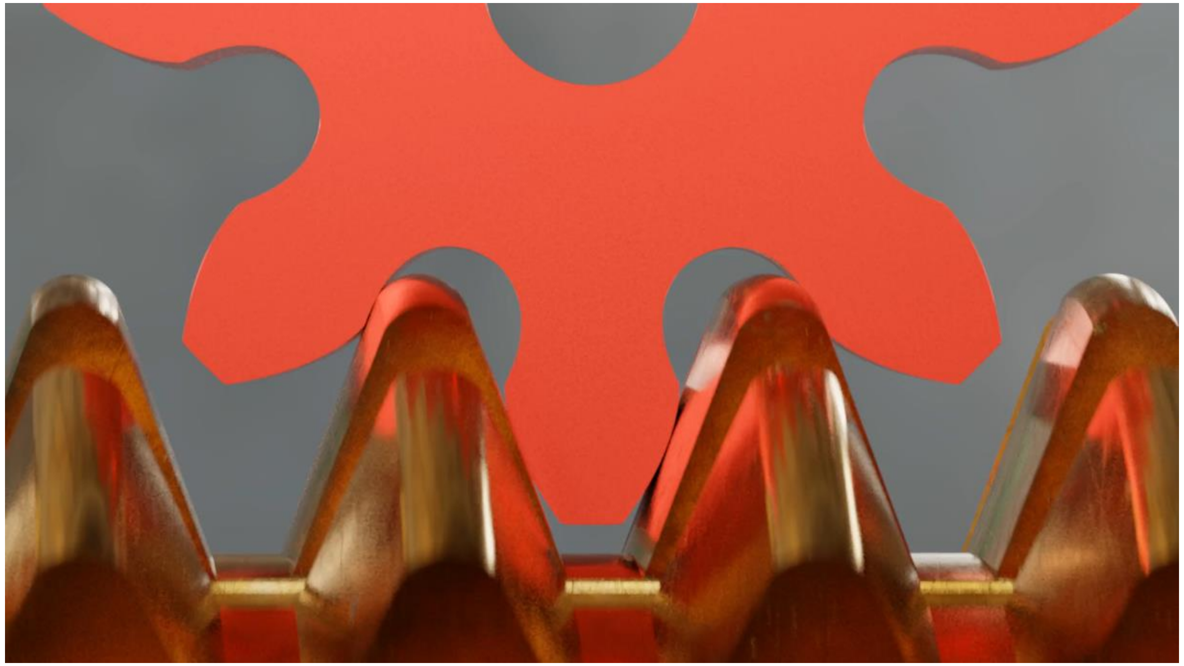
Gear with 20 teeth without undercut

Undercut occurs when the number of teeth of a gear is too small. An undercut leads to a weakening of the strength of the tooth!

An undercut leads not only to a weakening of the tooth but also to a shortening of the line of contact!

The theoretical minimum number of teeth above which no undercut occurs is 17 for a standard pressure angle of 20°. In practice, a minimum number of teeth of 14 is usually assumed!

Profile shift



It is also possible to produce gears below the minimum number of 17 teeth, without an undercut! For this, the manufacturing process must be specially adapted with a so-called profile shift.

Profile shifts are often used to adjust the centre distance!

With a profile shift, the tool profile is shifted outwards by a certain amount during gear cutting.
The animation shows the effects of a profile shift on the tooth form of a gear with 8 teeth. It becomes clear that as the profile shift increases, the undercut becomes smaller and can even be completely avoided.