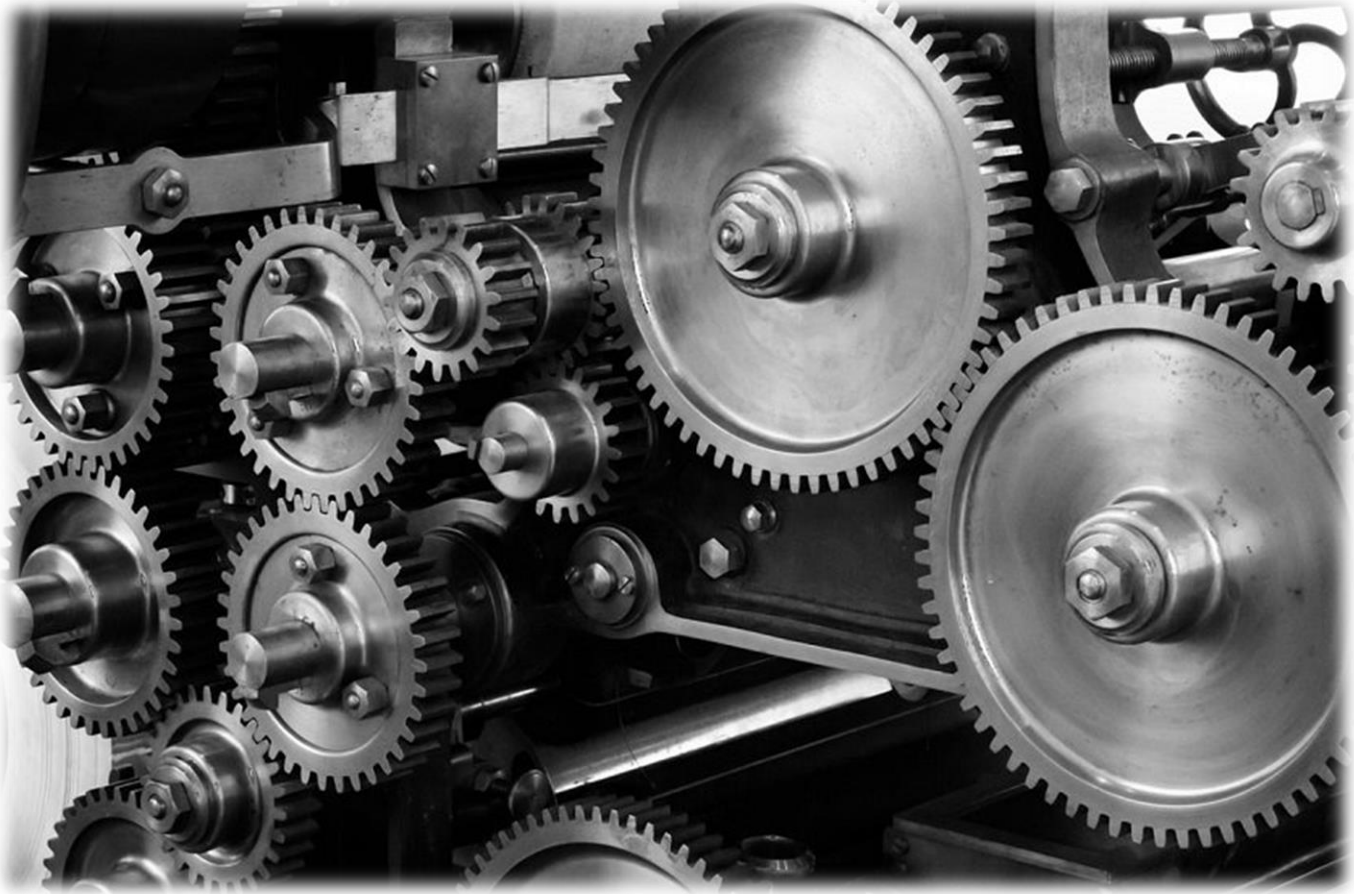
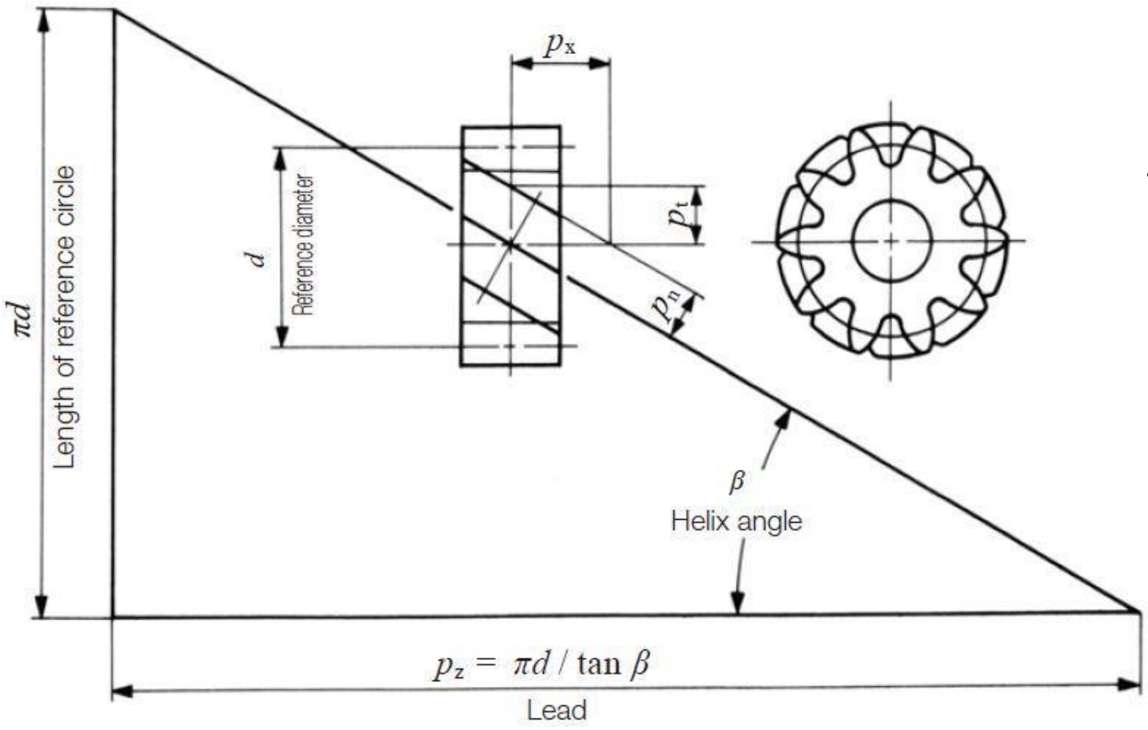
Gears



Helical gears



normal pitch

$$P_n$$

normal module

$$m_{\mathrm{n}} = \frac{P_n}{\pi}$$

The tooth profile of a helical gear with applied normal module, and normal pressure angle belongs to a normal system.

transverse pitch

$$P_t$$

transverse module

$$m_{\mathrm{t}} = \frac{P_t}{\pi}$$

These transverse module and transverse pressure angle at are the basic configuration of transverse system helicalgear.

$$P_{\mathrm{t}} = \frac{P_{\mathrm{n}}}{\cos(\beta)} \qquad m_{\mathrm{t}} = \frac{m_{\mathrm{n}}}{\cos(\beta)}$$

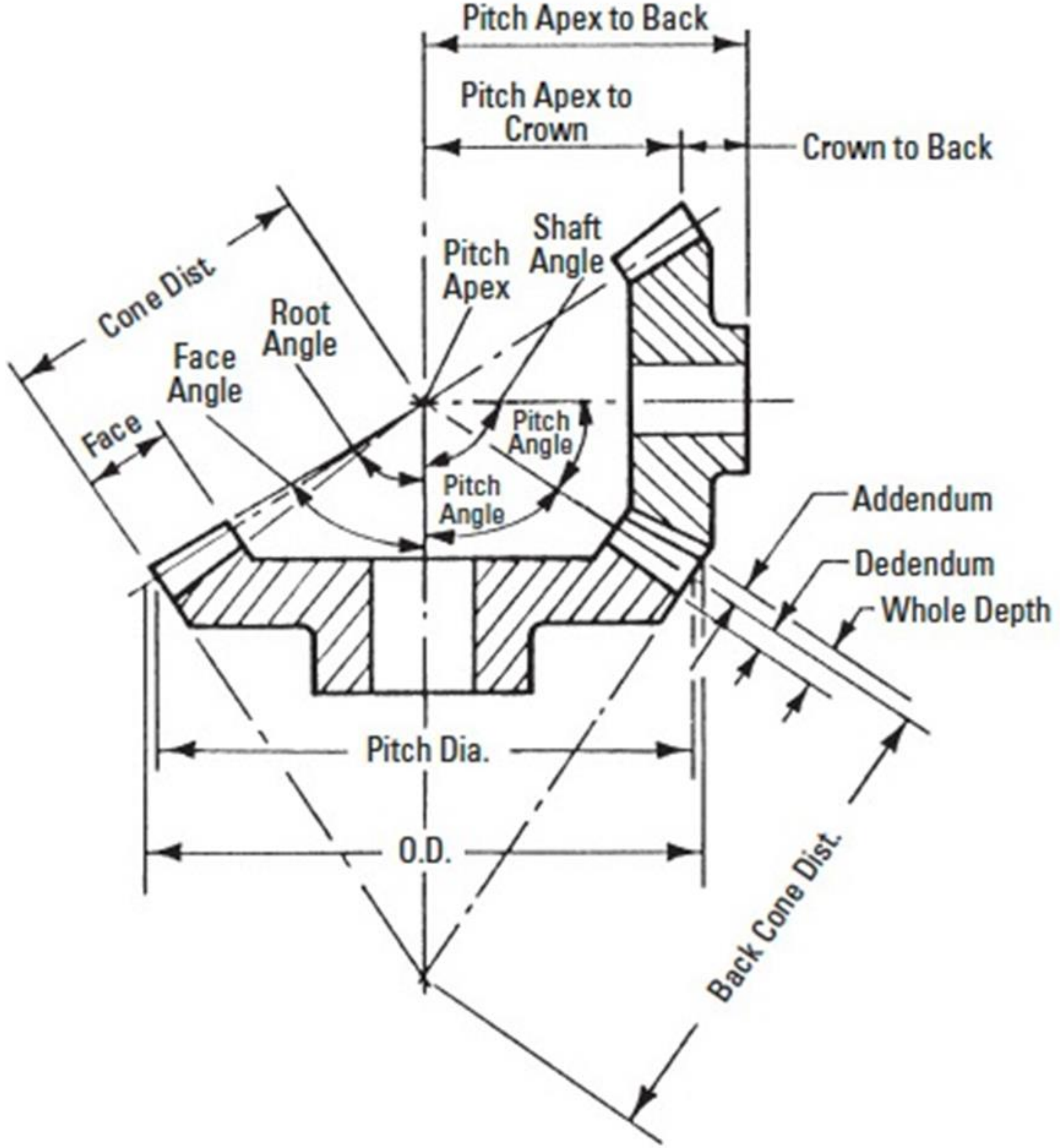
$$d_\omega = m_{\mathrm{t}} \cdot z = \frac{m_{\mathrm{n}} \cdot z}{\cos(\beta)}$$

$$c = 0.25 \cdot m$$

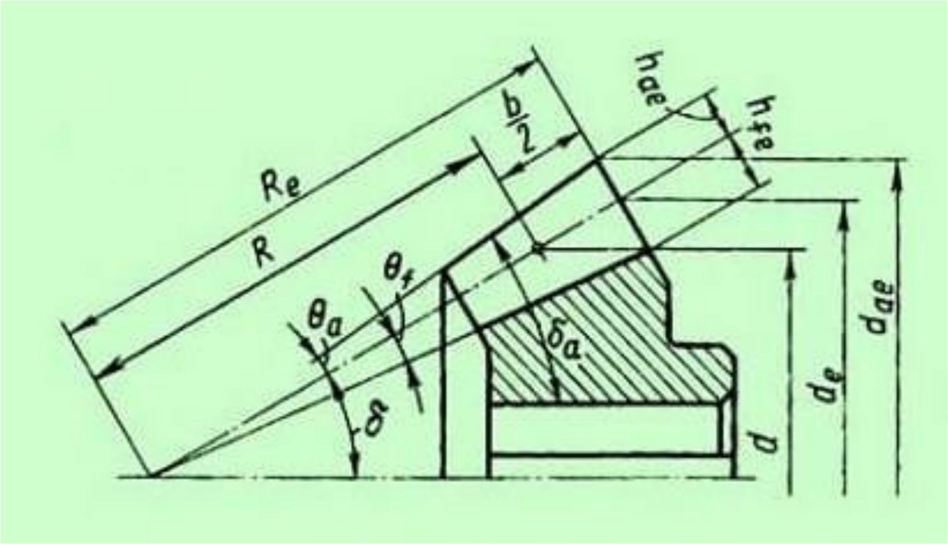
$$d_a = d_\omega + 2 \cdot h_a = m_{\mathrm{n}} \cdot \left(\frac{z}{\cos(\beta)} + 2\right)$$

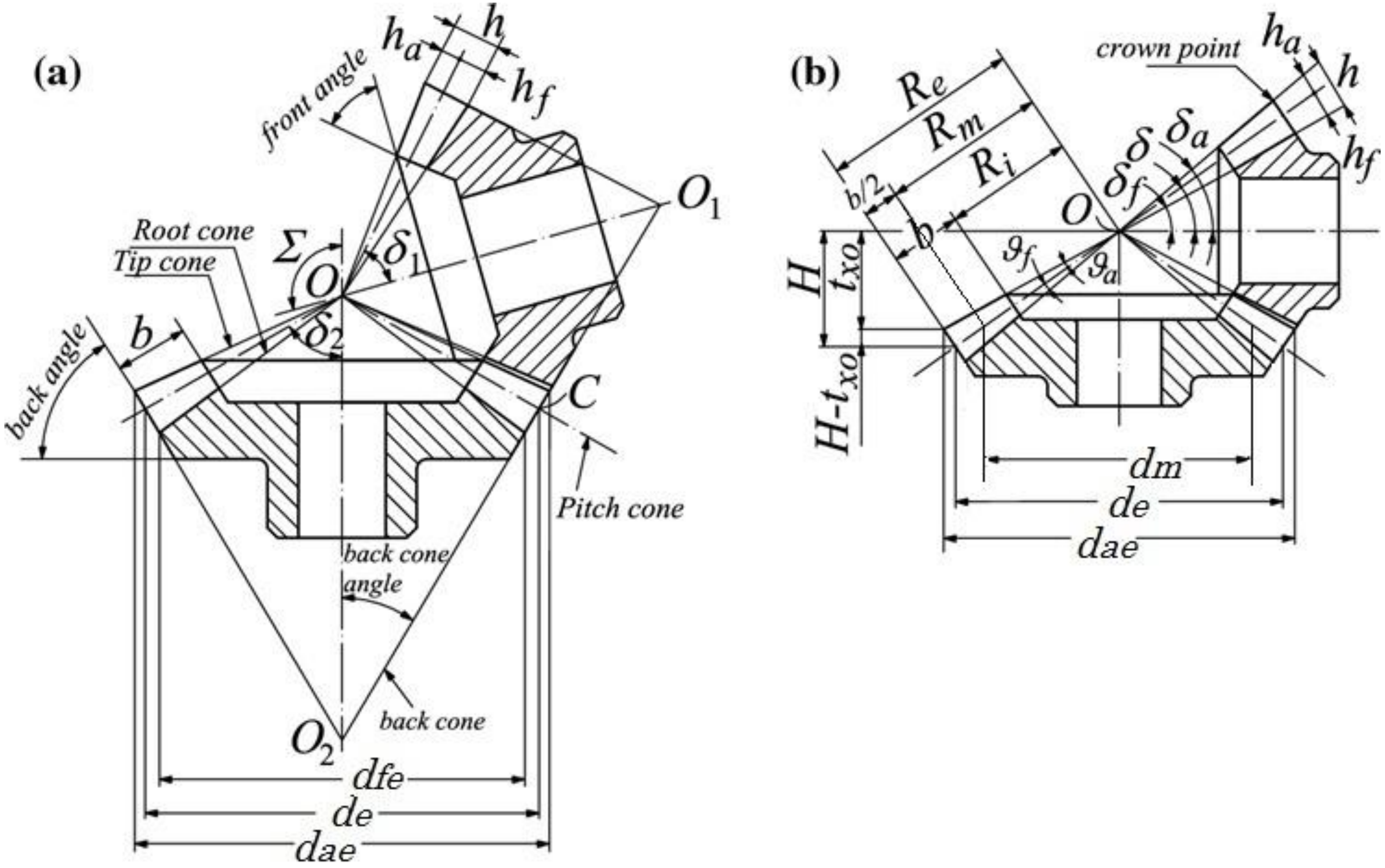
$$d_d = d_\omega - 2 \cdot h_d = m_{\mathrm{n}} \cdot \left(\frac{z}{\cos(\beta)} - 2.5\right)$$

Bevel gears



shaft angle	Σ	$\Sigma = \delta_1 + \delta_2$
pitch angle	δ_1 δ_2	$\delta_1 = \tan^{-1}\left(\frac{z_1}{z_2}\right)$
root angle	δ_{f1} δ_{f2}	
face angle	δ_{a1} δ_{a2}	$\delta_2 = \tan^{-1}\left(\frac{z_2}{z_1}\right)$
dedendum angle	θ_{f1} θ_{f2}	
addendum angle	θ_{a1} θ_{a2}	





face width b

mean cone distance R R_{m}

outer cone distance R_{e}

$$R_{\mathrm{m}} = R_{\mathrm{e}} - \frac{b}{2}$$

mean pitch diameter d_{m1} d_{m2}

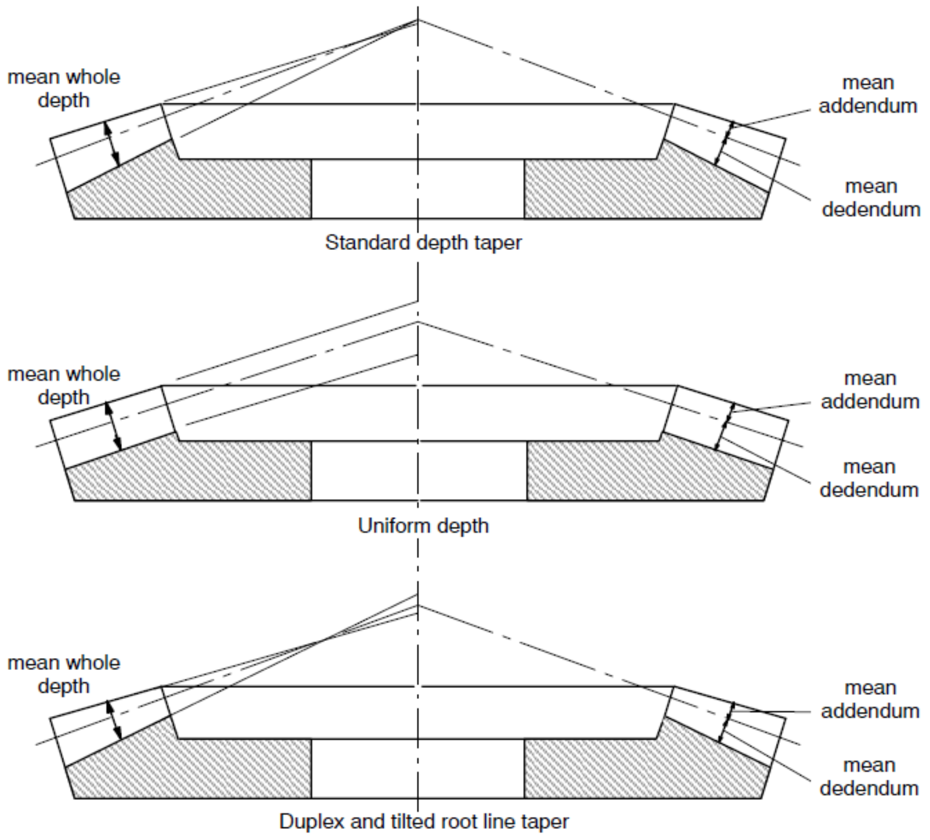
outer pitch diameter d_{e1} d_{e2}

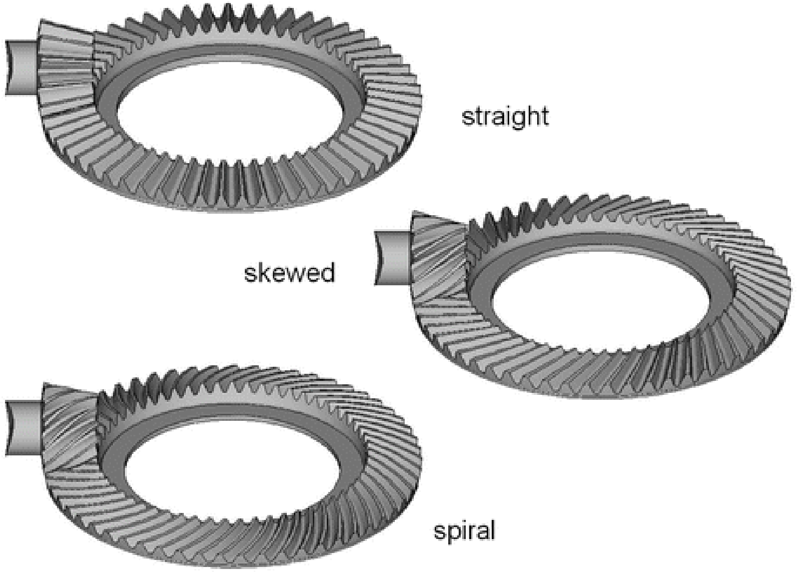
outside diameter d_{ae1} d_{ae2}

mean pitch module m_{m} $d_m = m_{\mathrm{m}} \cdot z = d_e\text{-b} \cdot \sin(\delta)$

outer pitch module m_{e} $d_e = m_{\mathrm{e}} \cdot z$

$$R_{\mathrm{e}} = \frac{\sqrt{(d_{e1}^2 + d_{e2}^2)}}{2} = m_{\mathrm{e}} \cdot \frac{\sqrt{(z_1^2 + z_2^2)}}{2}$$





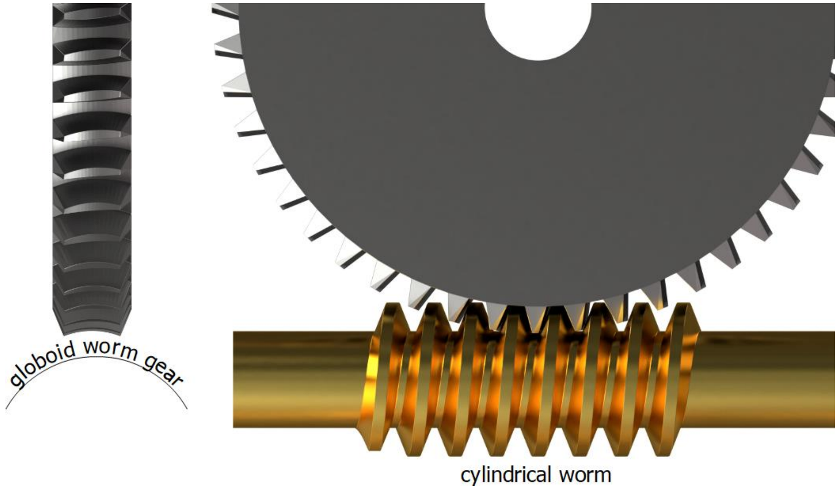
Standard depth taper

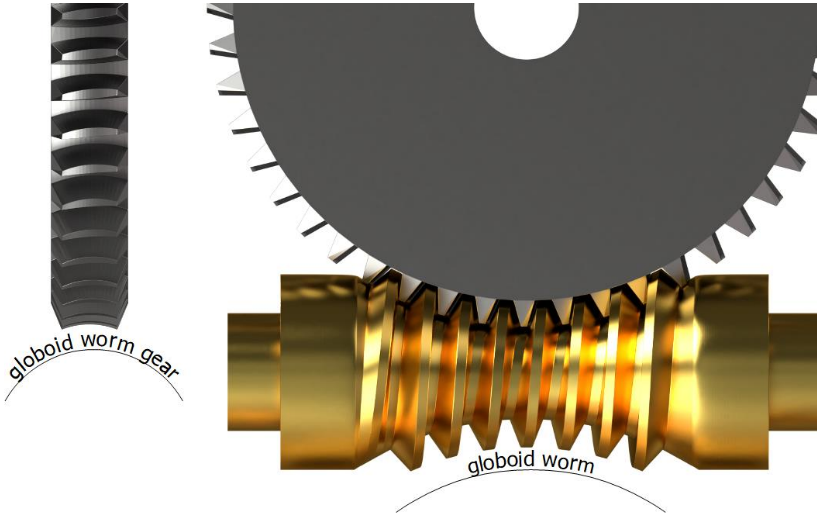
Uniform depth

Constant and modified slot width

Warm gears

Depending on the shape of the worm, worm drives can be classified differently.



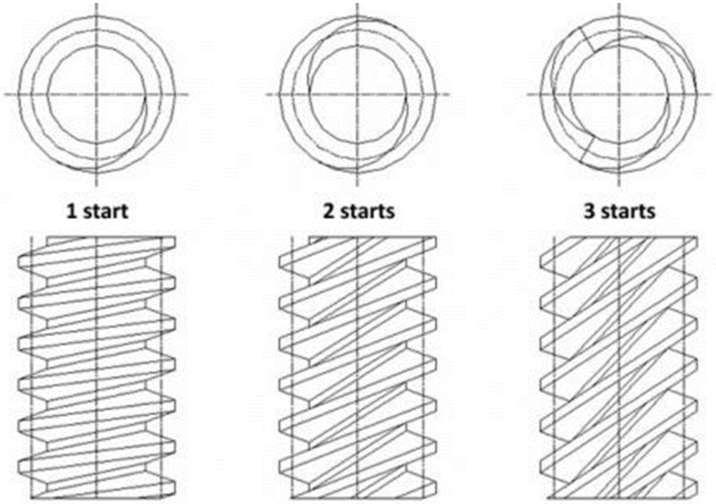


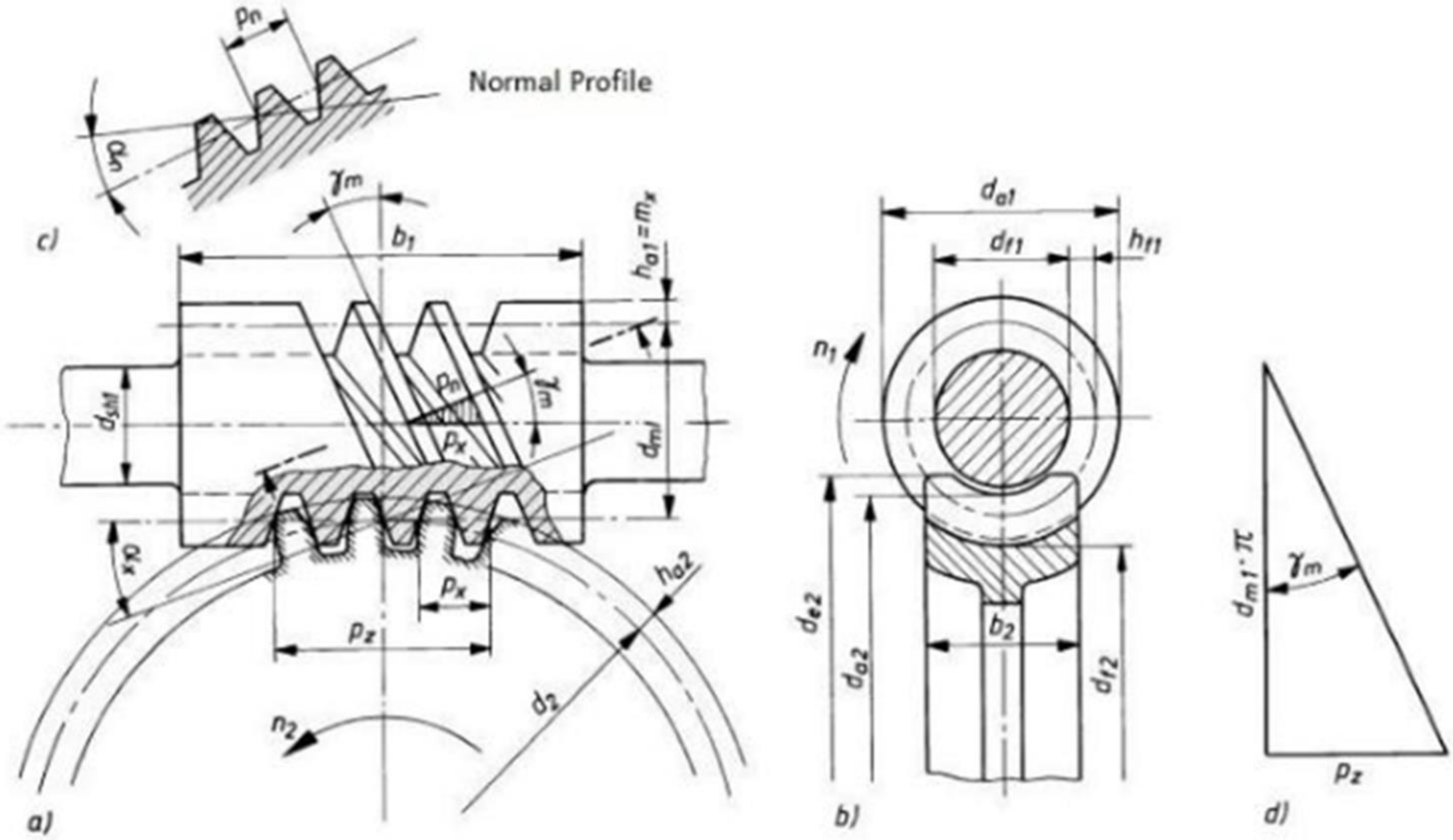
Cylindrical worms are relatively easy to produce and are preferred for cost reasons!

With globoid worms (enveloping worms) higher powers can be transmitted; however, they are relatively expensive due to the complex production!

Self-locking worm drives can only be set in motion by the worm!

Note that the thread of a single start worm generally has a smaller lead angle than multi-start worms. Single start worms twist more and are therefore more often self-locking than multi start thread worms. Conversely, this means that self-locking can be prevented (if desired) with multi-thread worms.





Normal Profile
p_n
α_n
c)
γ_m
b_1
$h_{a1}=m_x$
d_{sh1}
p_n
γ_m
p_x
d_{m1}
α_x
p_x
p_z
h_{a2}
d_2
n_2
a)
d_{a1}
d_{f1}
h_{f1}
n_1
d_{e2}
d_{a2}
b_2
d_{f2}
b)
$d_{m1} \cdot \pi$
γ_m
p_z
d)

Axial pitch of worm threads and circular pitch of wheel teeth, the pitch between adjacent threads	P_x	Axial module	m
			$m_\mathrm{n} = m \cdot \cos(\gamma)$
Normal pitch of of worm threads and gear teeth	P_n	Normal module	m_n

Lead of worm P_z $P_\mathrm{z} = P_\mathrm{x} \cdot z_1$

Normal pressure angle $\alpha = 20°$

Length of worm thread b_1

Number of threads (starts) on worm z_1

Worm lead angle γ $\gamma = \tan\left(\frac{z_1}{q}\right)$

Effective face width of worm wheel b_2

Number of teeth on worm wheel z_2

Reference pitch diameter of worm	d_m1	$d_\mathrm{m1} = \mathrm{m} \cdot \mathrm{q}$
Reference pitch diameter of worm wheel	d_2	$d_2 = \mathrm{m} \cdot z_2$

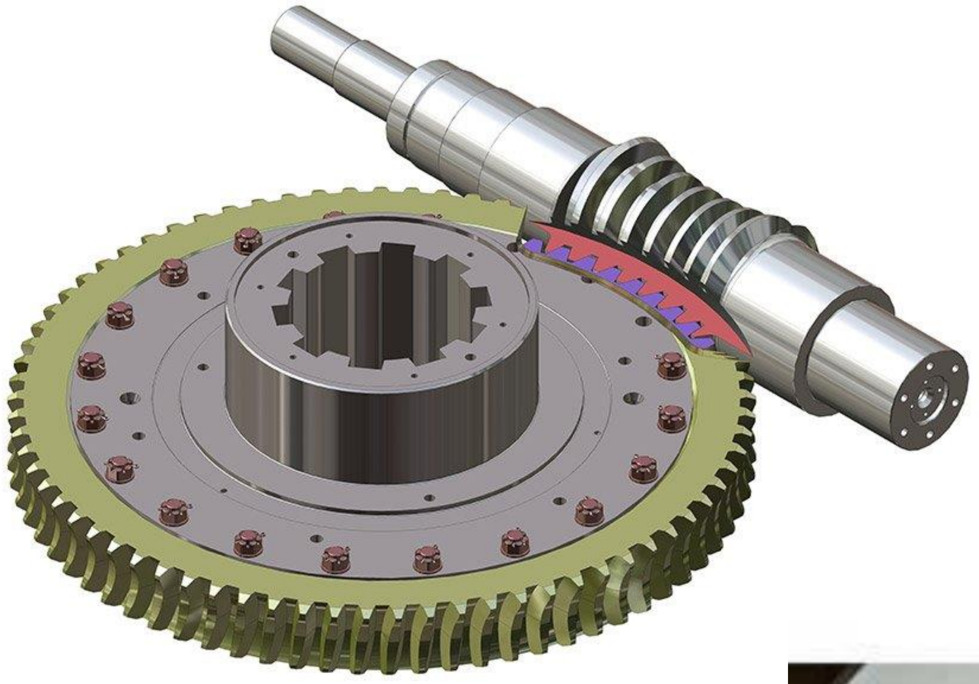
Worm diameter factor (standard) q

Tip diameter of worm	d_a1	$d_\mathrm{a1} = d_\mathrm{m1} + 2 \cdot h_\mathrm{a1} = \mathrm{m} \cdot (\mathrm{q}+2)$
Tip diameter of worm wheel	d_a2	$d_\mathrm{a2} = d_2 + 2 \cdot h_\mathrm{a2} = \mathrm{m} \cdot (z_2 + 2)$
Root diameter of worm	d_f1	$d_\mathrm{f1} = d_\mathrm{m1} - 2 \cdot h_\mathrm{f1} = \mathrm{m} \cdot (\mathrm{q} - 2.4)$
Root diameter of worm wheel	d_f2	$d_\mathrm{f2} = d_2 - 2 \cdot h_\mathrm{f2} = \mathrm{m} \cdot (z_2 - 2.4)$

Worm thread addendum $h_\mathrm{a1} = \mathrm{m}$

Worm thread dedendum $h_\mathrm{f1} = \mathrm{m} + \mathrm{c}$

$\mathrm{c} = 0.2 \cdot \mathrm{m}$

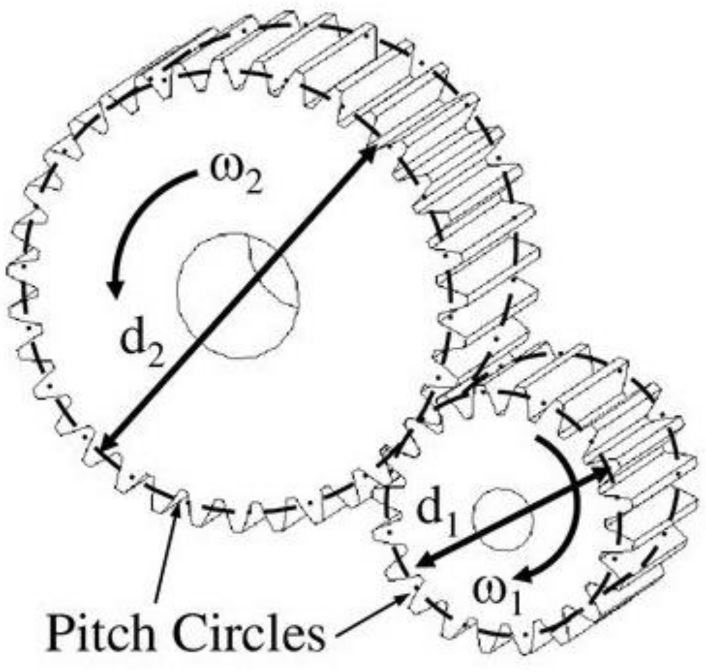


This sliding action causes friction and heat, which limits the efficiency of worm gears to 30 to 50 percent. In order to minimize friction (and therefore, heat), the worm and gear are made of dissimilar metals – for example, the worm may be made of hardened steel and the gear made of bronze or aluminum.

bolted connection

welding

Gear ratio



$$v = \omega \cdot \frac{d_\omega}{2}$$

$$v = \omega_1 \cdot \frac{d_{\omega 1}}{2} = \omega_2 \cdot \frac{d_{\omega 2}}{2}$$

$$\omega_1 \cdot \frac{m \cdot z_1}{2} = \omega_2 \cdot \frac{m \cdot z_2}{2}$$

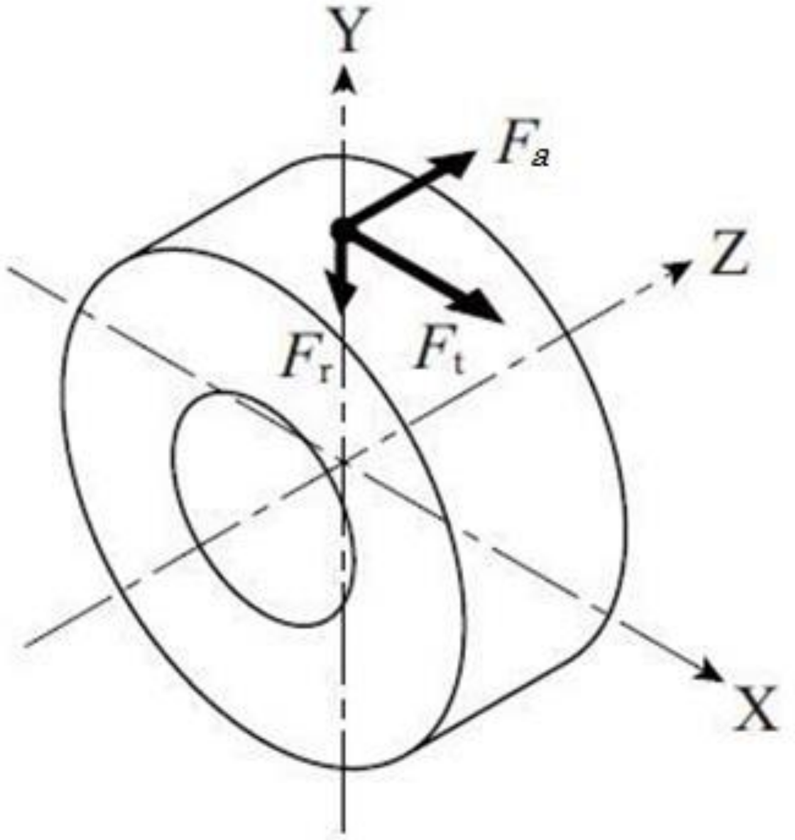
$$i = u = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1}$$

$$u = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{d_{e2}}{d_{e1}} = \frac{2 \cdot R_e \cdot \sin\delta_2}{2 \cdot R_e \cdot \sin\delta_1} = \frac{\sin\delta_2}{\sin\delta_1} = \frac{z_2}{z_1}$$

$$for \quad \sum = \delta_1 + \delta_2 = 90^\circ$$

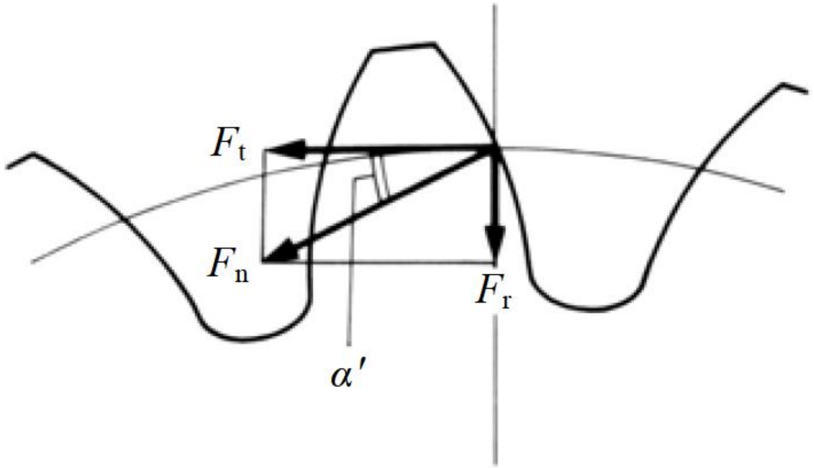
$$u = \frac{\sin(90 - \delta_1)}{\sin\delta_1} = \frac{\cos\delta_1}{\sin\delta_1} = ctg\ \delta_1 = tg\ \delta_2$$

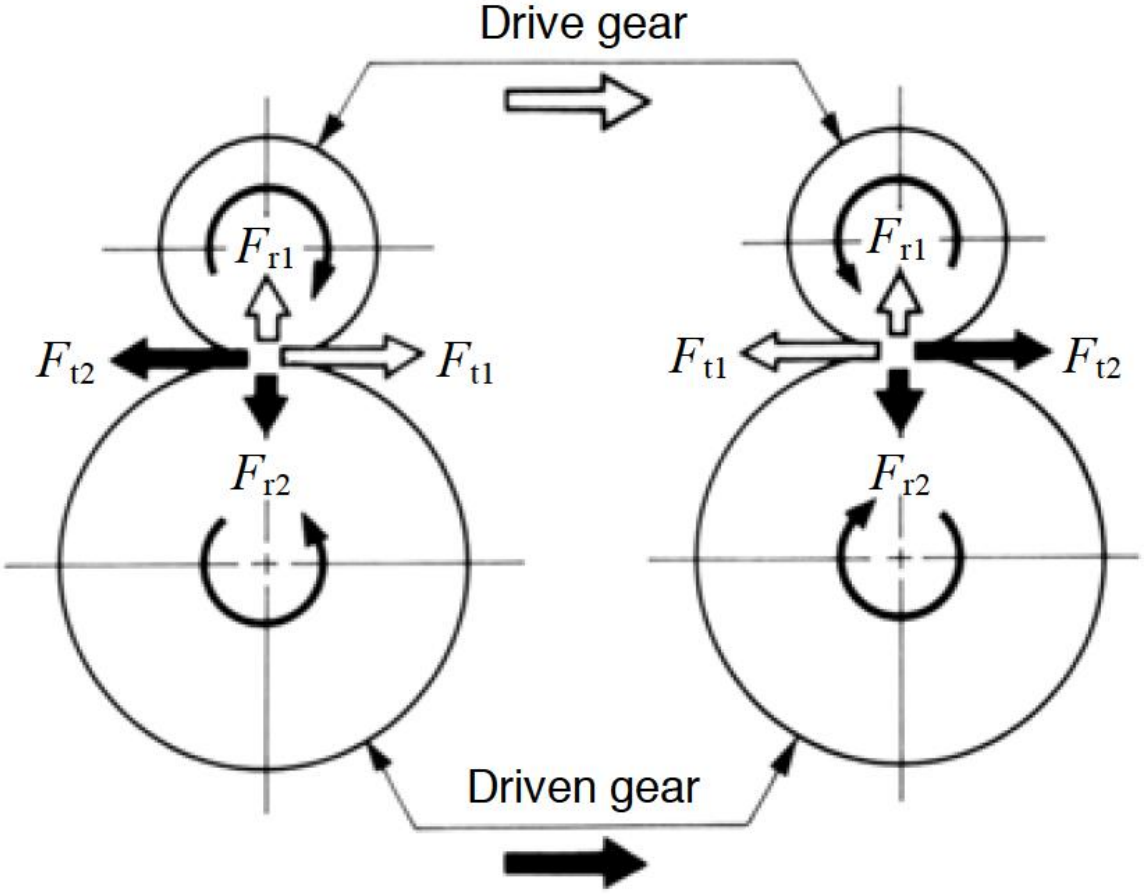
Forces



Tangential (circumferential) force	F_{t}
Radial (thrust) force	F_{r}
Axial force	F_{a}

Spur gear



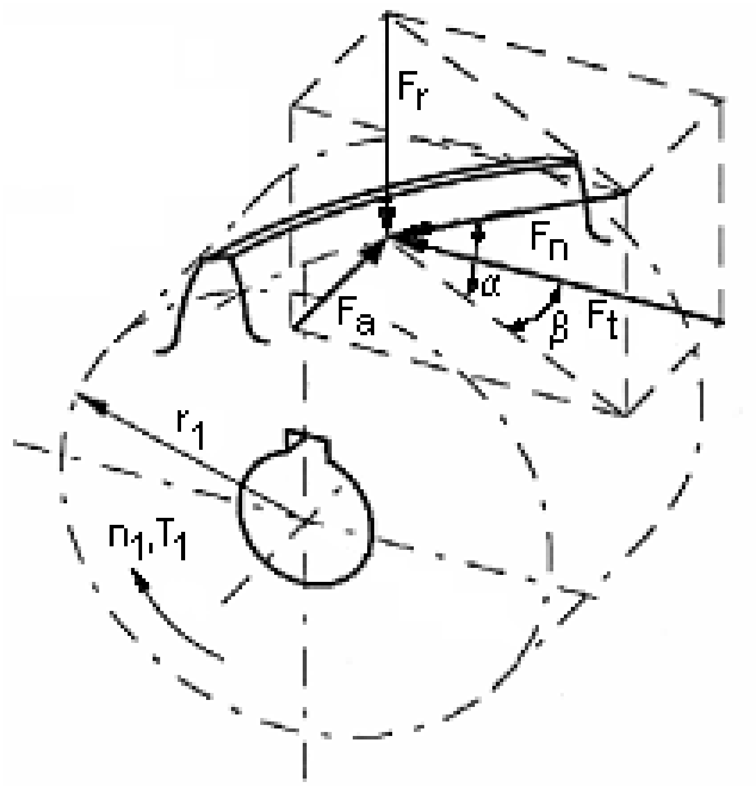


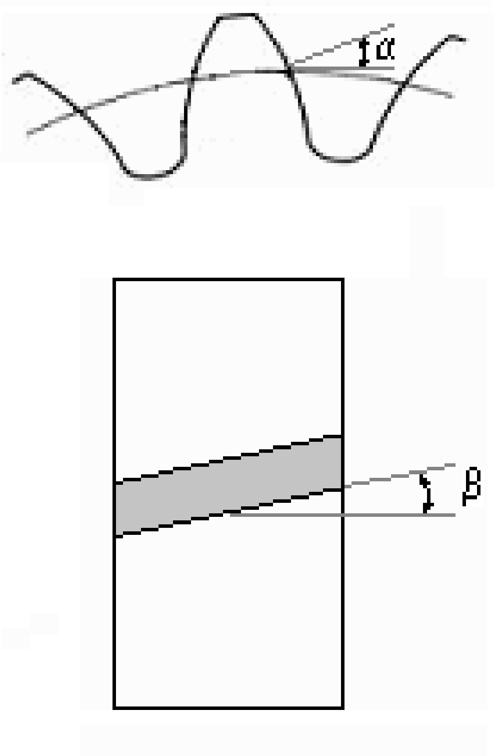
$$F_t = \frac{2 \cdot T}{d_\omega}$$

$$F_r = F_t \cdot \mathrm{tg}(\alpha)$$

$$F_n = \frac{F_t}{\cos(\alpha)}$$

Helical gear



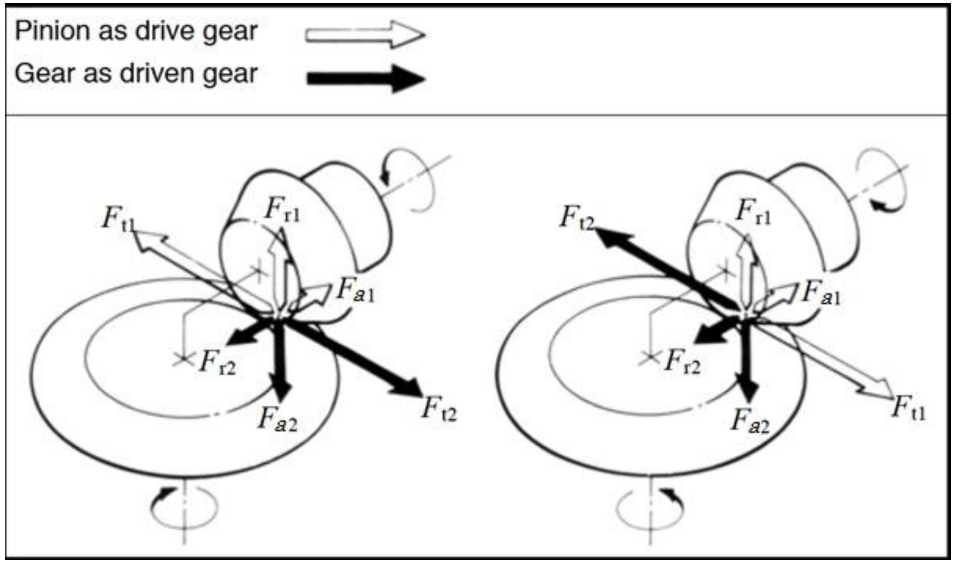


$$F_t = \frac{2 \cdot T}{d_\omega}$$

$$F_r = \frac{F_t}{\cos(\beta)} \cdot \mathrm{tg}(\alpha)$$

$$F_a = F_t \cdot \mathrm{tg}(\beta)$$

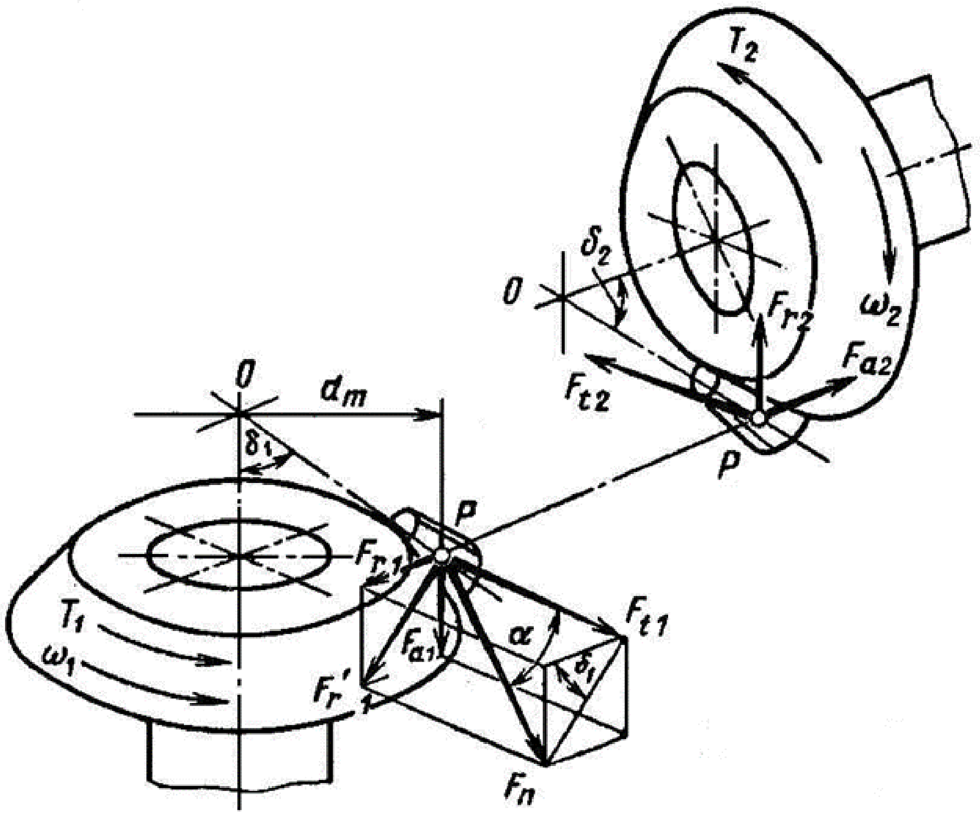
Bevel gear



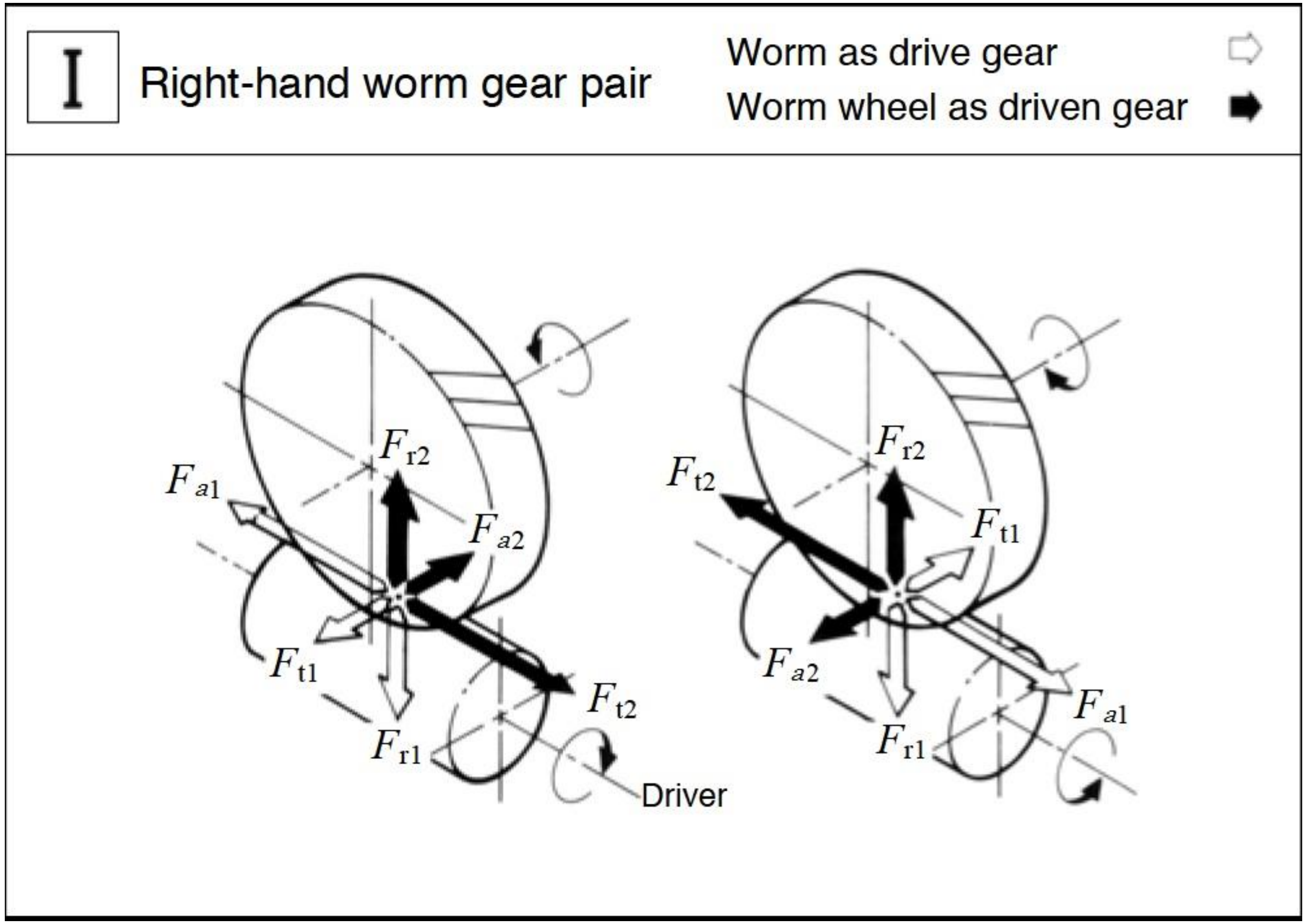
$$F_t = \frac{2 \cdot T}{d_\omega}$$

$$F_{r1} = F_{a2} = F_t \cdot \mathrm{tg}(\alpha) \cdot \cos(\delta_1)$$

$$F_{r2} = F_{a1} = F_t \cdot \mathrm{tg}(\alpha) \cdot \sin(\delta_1)$$



Warm gear



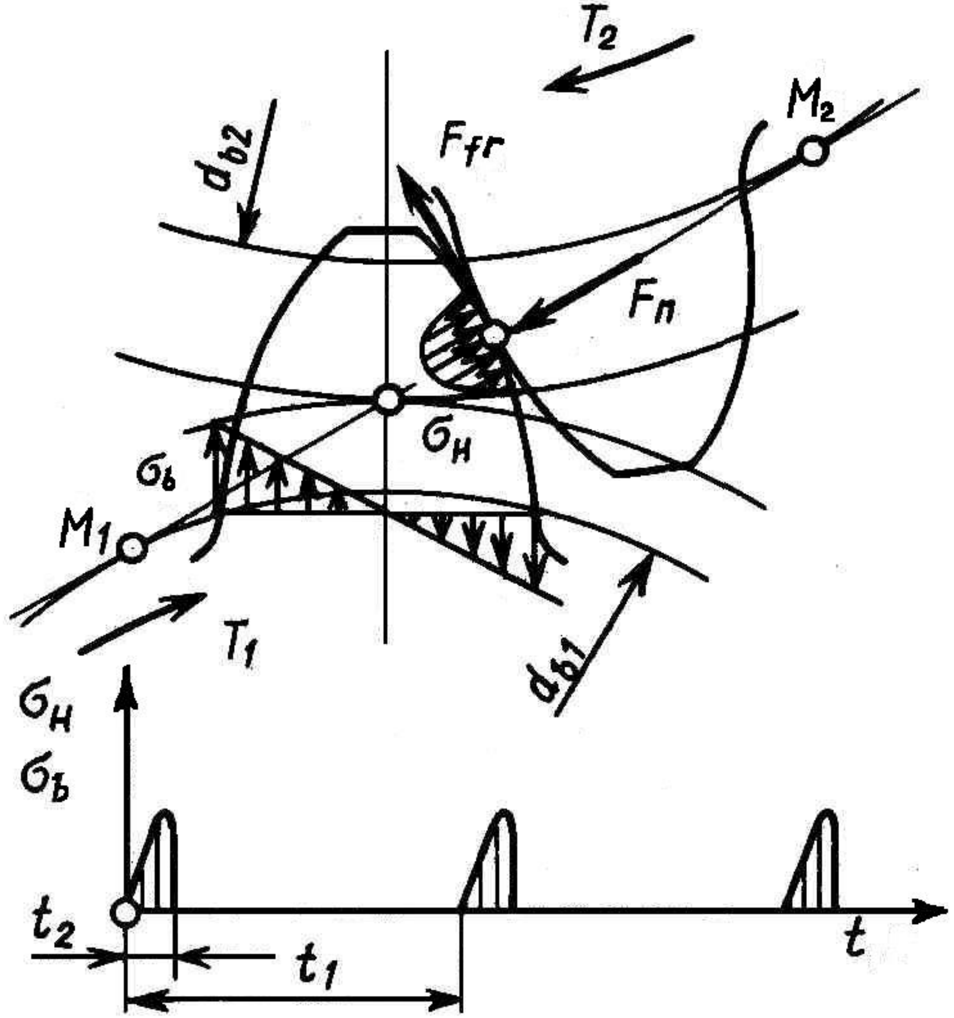
$$F_{t1} = \frac{2 \cdot T_1}{d_{m1}} \qquad F_{t2} = \frac{2 \cdot T_2}{d_2}$$

$$F_{r1} = F_{r2} = F_{t2} \cdot \mathrm{tg}(\alpha)$$

$$F_{t2} = F_{a1}$$

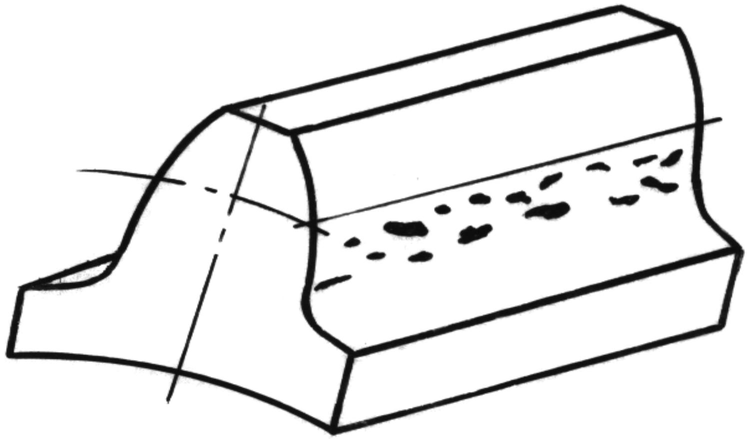
$$F_{t1} = F_{a2}$$

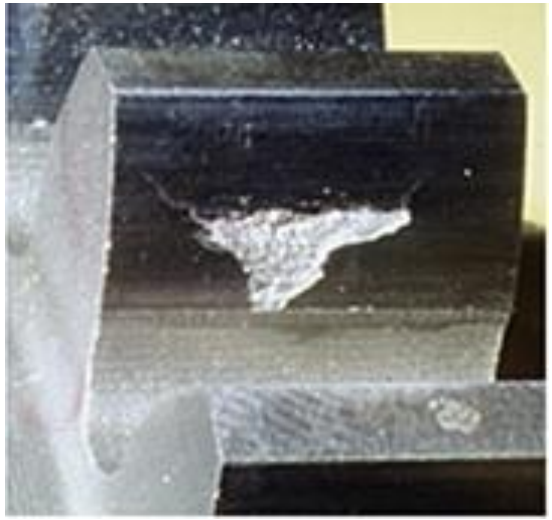
Stresses

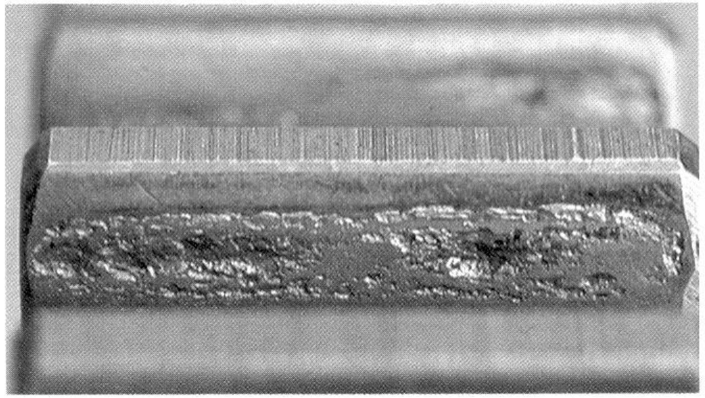


F_{fr}	friction force
σ_H	contact stress
σ_b	bending stress

Gear Tooth Failures





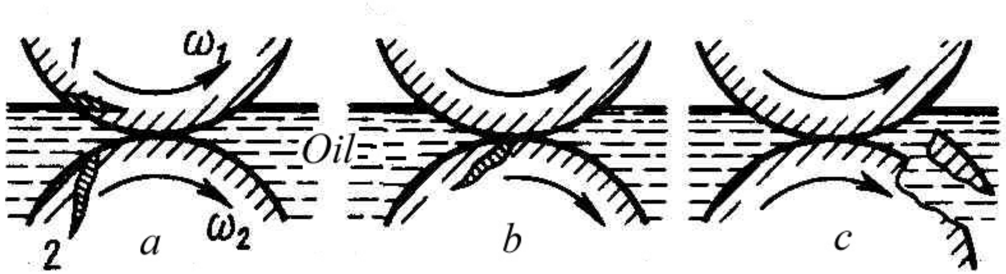


Surface fatigue (pitting)

Is a process of the removal of small pieces of metal, leaving cavities or pits on the surface.

This is caused by repeated loads that produce stresses higher than the endurance limit of the material. It usually progresses over a long period of time.

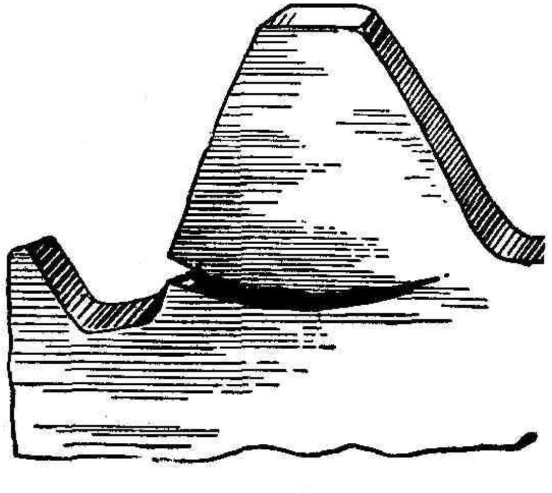
A severe form, in which large pits occur in considerable area is called **spalling**.

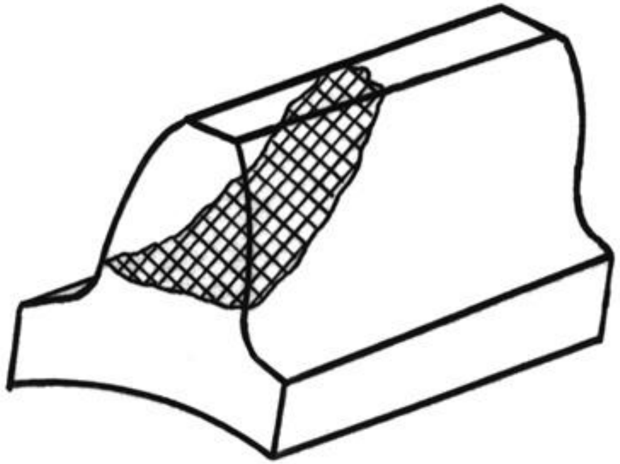


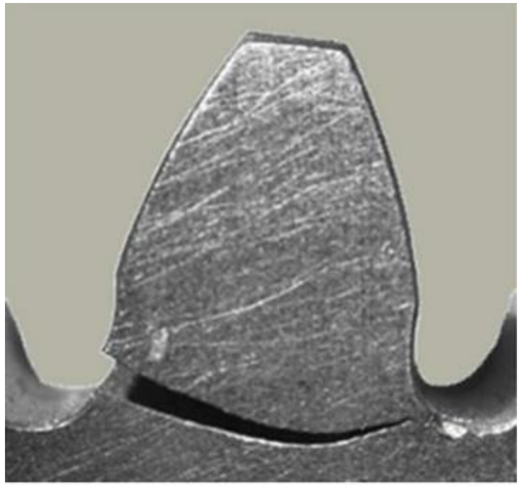
Breakage

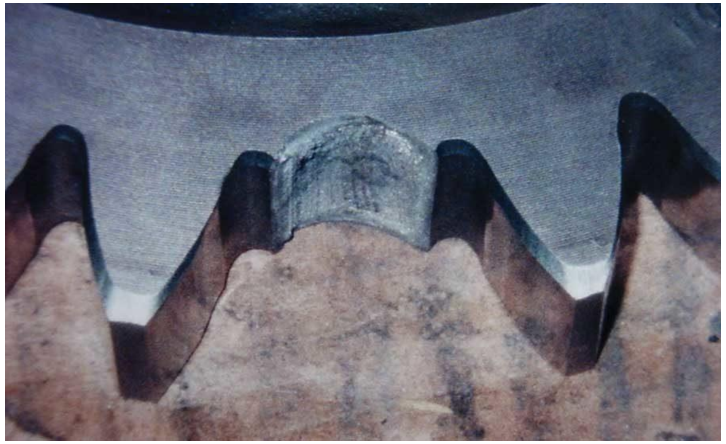
Breakage is a fracture of the entire tooth or substantial part of it due to overload or repeated over-stressing of the tooth material.

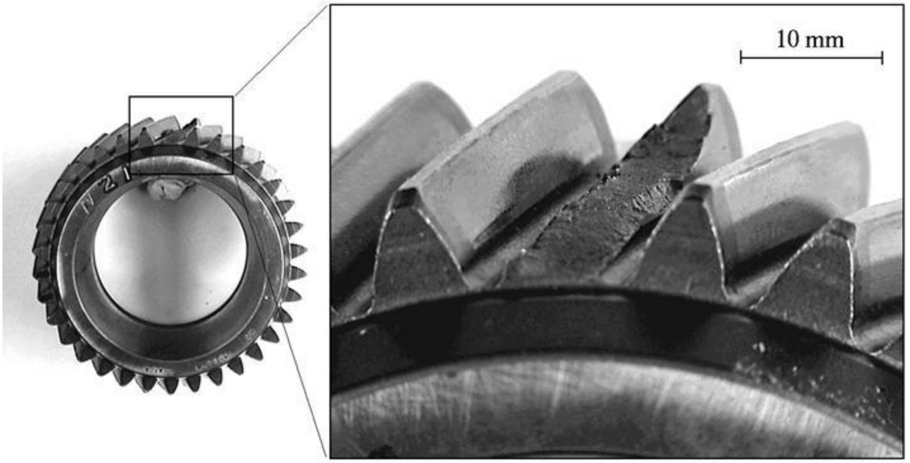
Fractures generally happen due to high bending stresses in the tooth root or fillet radius, sometimes emphasized by cracks or notches.



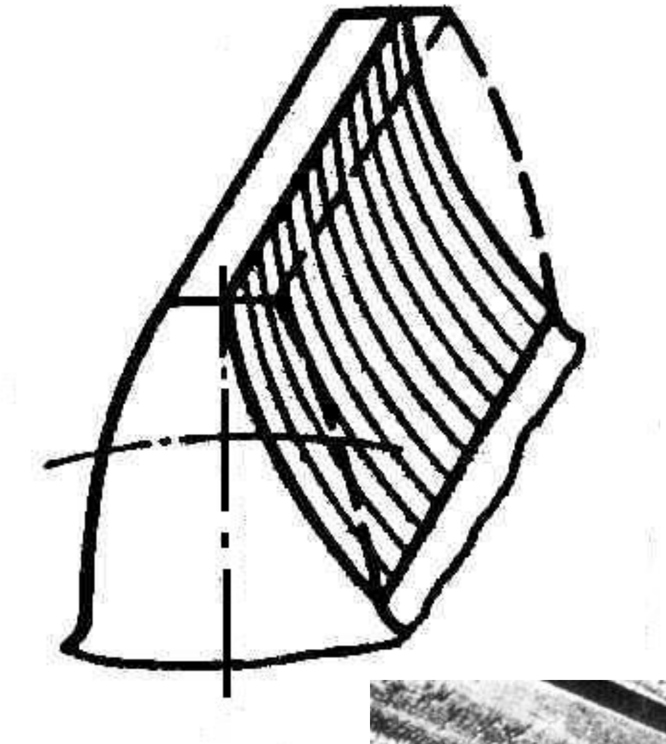








Wear

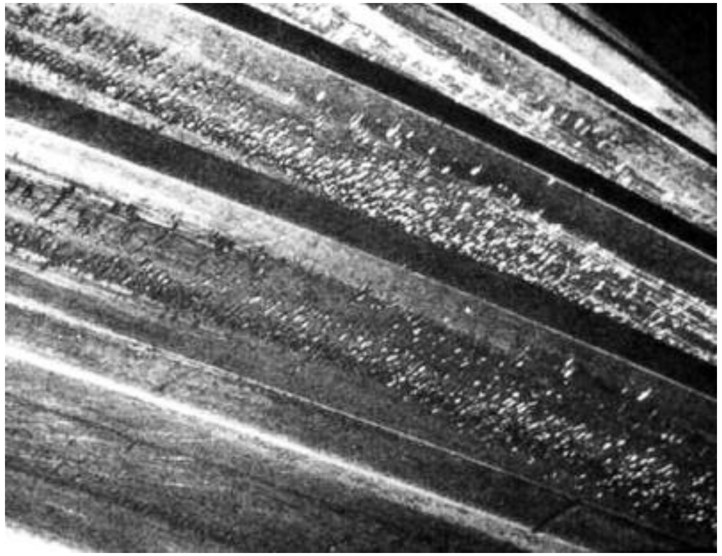


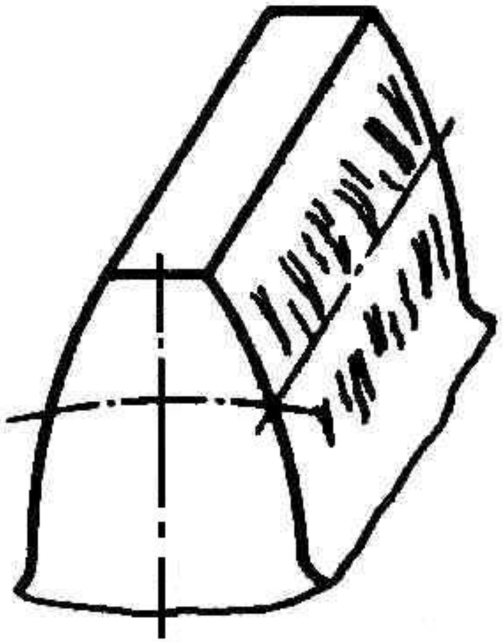
Wear is a more or less gradual removal of the material from the contact surfaces of the meshing teeth.

Factors that contributor to wear include load, improper lubrication, abrasive particles, corrosion.

Abrasive wear is the principal reason for the failure of open gearing and closed gearing of machinery operating in media polluted by abrasive materials.

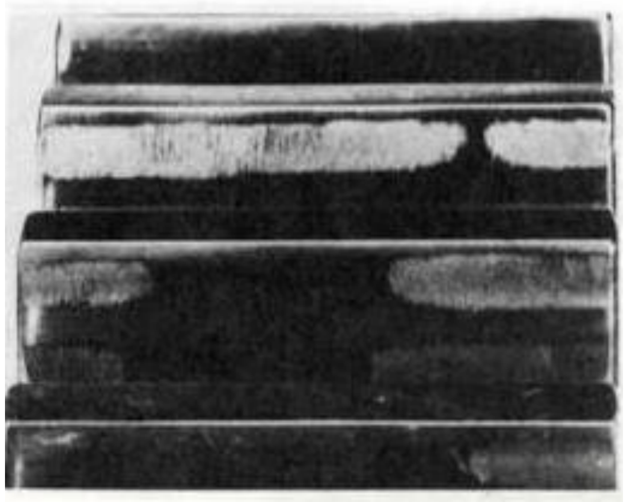
(mining machinery; cement mills; road laying, building construction, agricultural and transportation machinery, etc)

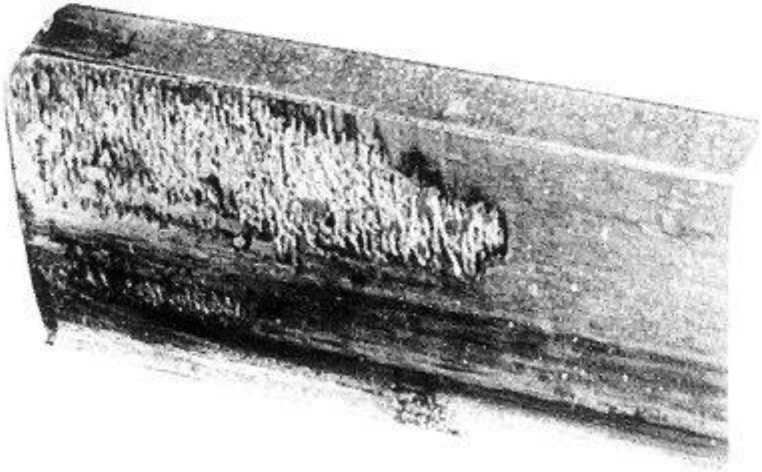




Scoring (seizure, scuffing)

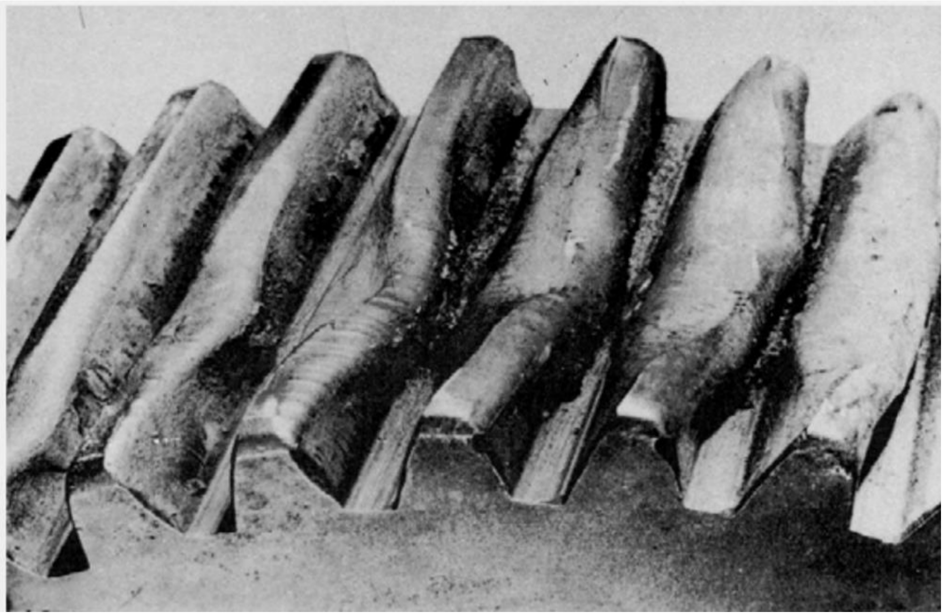
Scoring is due to combination of two distinct activities: First, lubrication failure in the contact region and second, establishment of metal to metal contact. Later on, welding and tearing action resulting from metallic contact removes the metal rapidly and continuously so far the load, speed and oil temperature remain at the same level. The scoring is classified into initial, moderate and destructive.



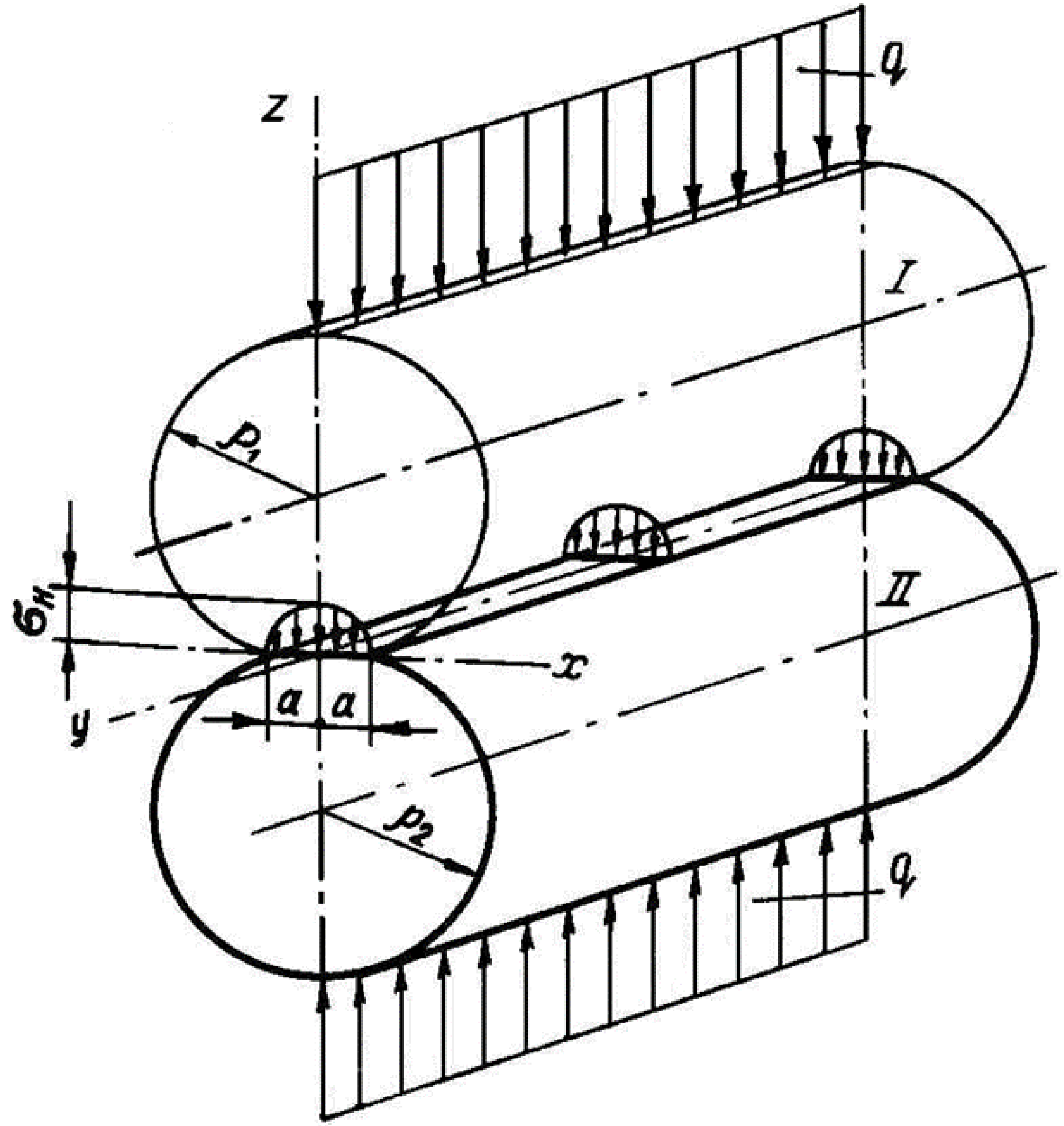


Plastic Deformation

This happens when the stress is sufficient to permanently deform the metal. Heavy loads, in combination with the rolling and sliding action of mashing teeth causes the contact surfaces to yield and deform permanently.



Contact Strength



Hertz formula

$$\sigma_H = \sqrt{\frac{q}{\rho_{tr}} \cdot \frac{E_1 \cdot E_2}{\pi \cdot \left[E_2 \cdot \left(1-\nu_1^{\,2}\right) + E_1 \cdot \left(1-\nu_2^{\,2}\right)\right]}}$$

Poisson's ratio $\nu_1 = \nu_2 = 0.3$

$$\sigma_H = 0.418 \cdot \sqrt{\frac{q \cdot E_{tr}}{\rho_{tr}}}$$

Design load $q = \dfrac{F_n \cdot K_H}{l_\Sigma}$

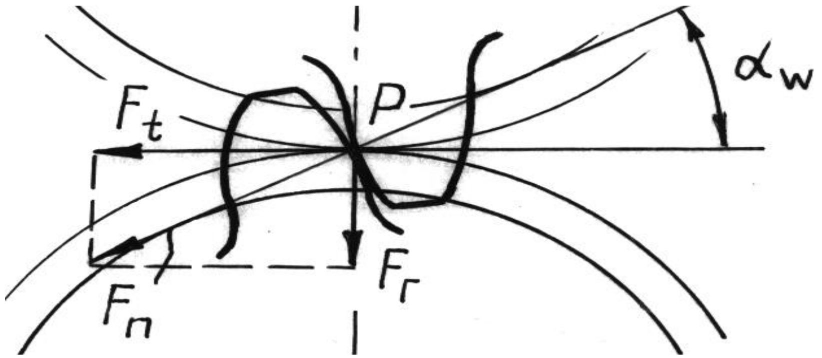
K_H Load factor

Transformed modulus of elasticity $E_{tr} = \dfrac{2 \cdot E_1 \cdot E_2}{E_1 + E_2}$

Transformed radius of curvature $\rho_{tr} = \dfrac{\rho_1 \cdot \rho_2}{\rho_2 \pm \rho_1}$

CILINDRICAL GEARS

Spur gears



$$F_n = \frac{F_t}{\cos\alpha_w} = \frac{2 \cdot T_1}{d_{\omega 1} \cdot \cos\alpha_w}$$

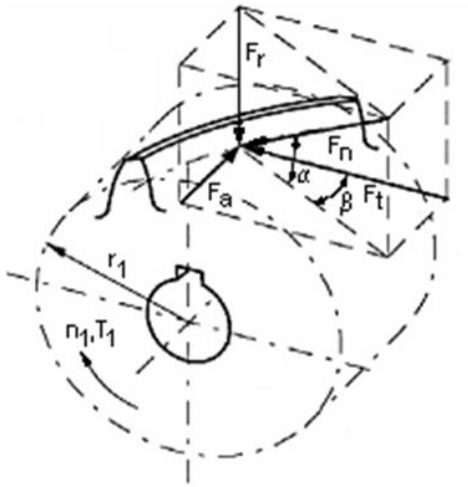
$$\boldsymbol{l_\Sigma = b_w}$$

$$\boldsymbol{K_H = K_{H\beta} \cdot K_{HV}}$$

$\boldsymbol{K_{H\beta}}$ Face load factor for Contact stress

$\boldsymbol{K_{HV}}$ Dynamic factor

Helical gears



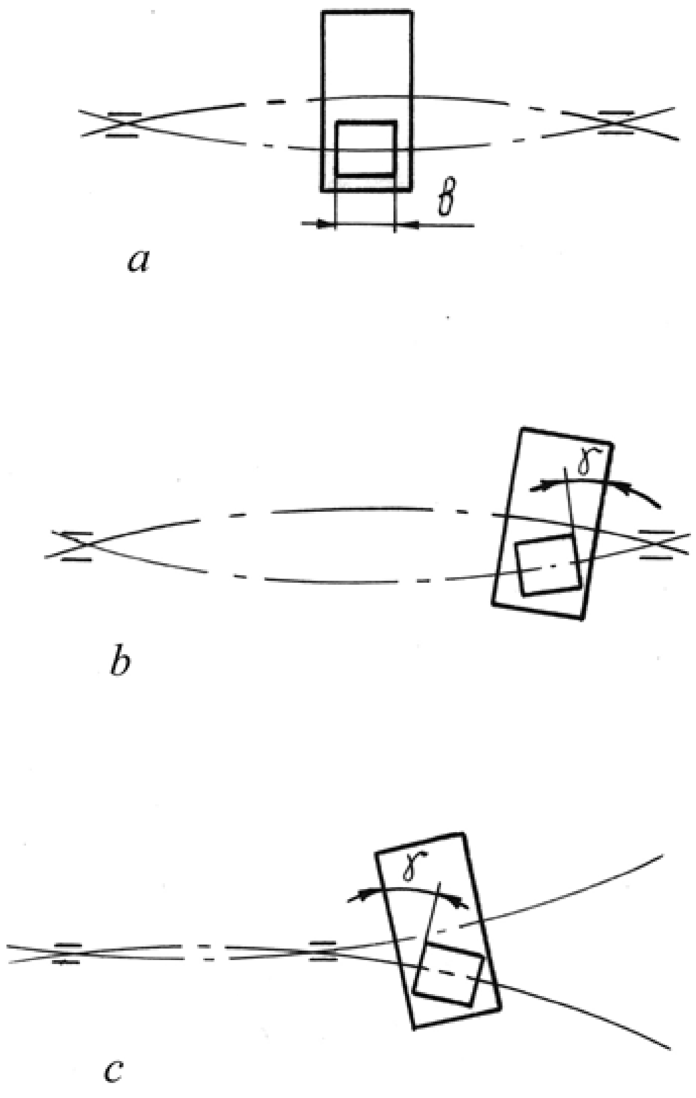
$$F_n = \frac{F_t}{\cos\alpha_w \cdot \cos\beta} = \frac{2 \cdot T_1}{d_{\omega 1} \cdot \cos\alpha_w \cdot \cos\beta}$$

$$\boldsymbol{l_\Sigma = \varepsilon_\alpha \cdot b_w}$$

$$\boldsymbol{K_H = K_{H\alpha} \cdot K_{H\beta} \cdot K_{HV}}$$

$\boldsymbol{K_{H\alpha}}$ Distributed load factor

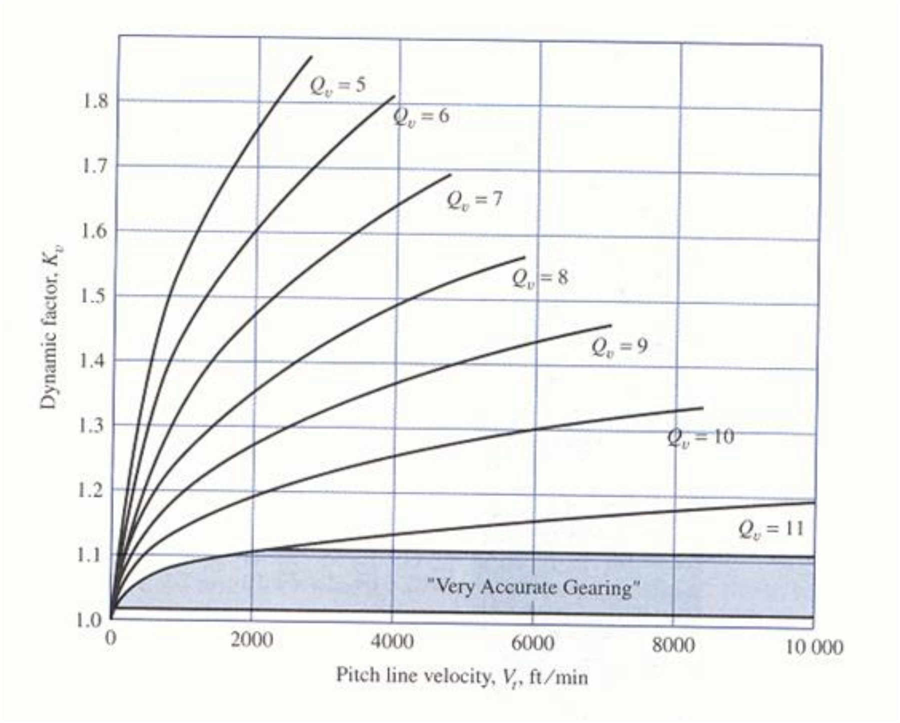
Face load factor for Contact stress $K_{H\beta}$

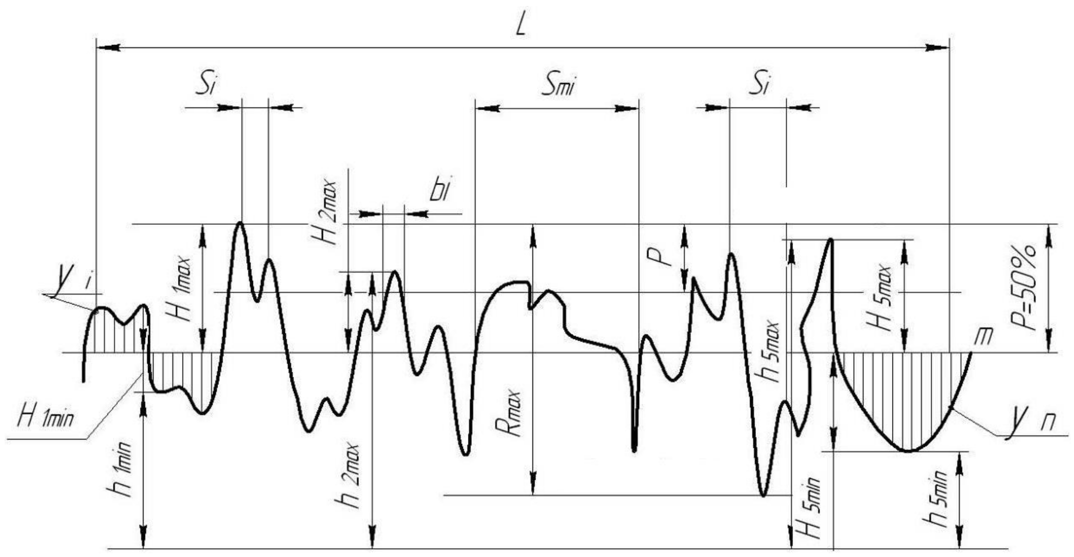


$$\psi_{bd} = \frac{b_w}{d_{w1}}$$

$\frac{b}{d_1}$	Supporting method			
	Support on both end			Unbalanced support
	Balanced to both bearings	Bearing is on one side and stiffness of axis is increased.	Bearing is on one side and less stiffness of axis.	
0.2	1.0	1.0	1.1	1.2
0.4	1.0	1.1	1.3	1.45
0.6	1.05	1.2	1.5	1.65
0.8	1.1	1.3	1.7	1.85
1.0	1.2	1.45	1.85	2.0
1.2	1.3	1.6	2.0	2.15
1.4	1.4	1.8	2.1	-
1.6	1.5	2.05	2.2	-
1.8	1.8	-	-	-
2.0	2.1	-	-	-

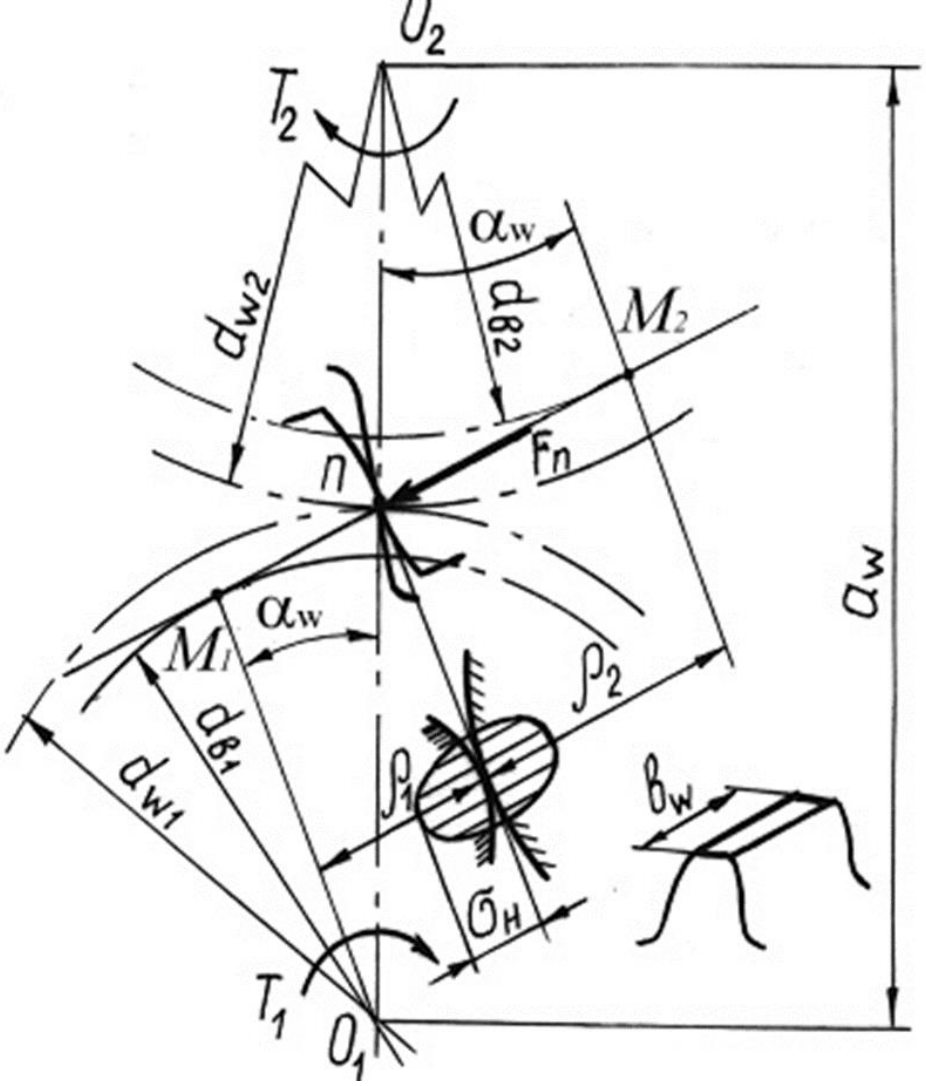
Dynamic factor K_{HV}





System of accuracy from JIS B 1702		Circumferential speed on the Reference pitch circle (m/s)						
Tooth profile		Below 1	Above 1.0 to below 3.0	Above 3.0 to below 5.0	Above 5.0 to below 8.0	Above 8.0 to below 12.0	Above 12.0 to below 18.0	Above 18.0 to below 25.0
Normal	Modified							
	1	-	-	1.0	1.0	1.1	1.2	1.3
1	2	-	1.0	1.05	1.1	1.2	1.3	1.5
2	3	1.0	1.1	1.15	1.2	1.3	1.5	-
3	4	1.0	1.2	1.3	1.4	1.5	-	-
4	-	1.0	1.3	1.4	1.5	-	-	-
5	-	1.1	1.4	1.5	-	-	-	-
6	-	1.2	1.5	-	-	-	-	-

Spur gears

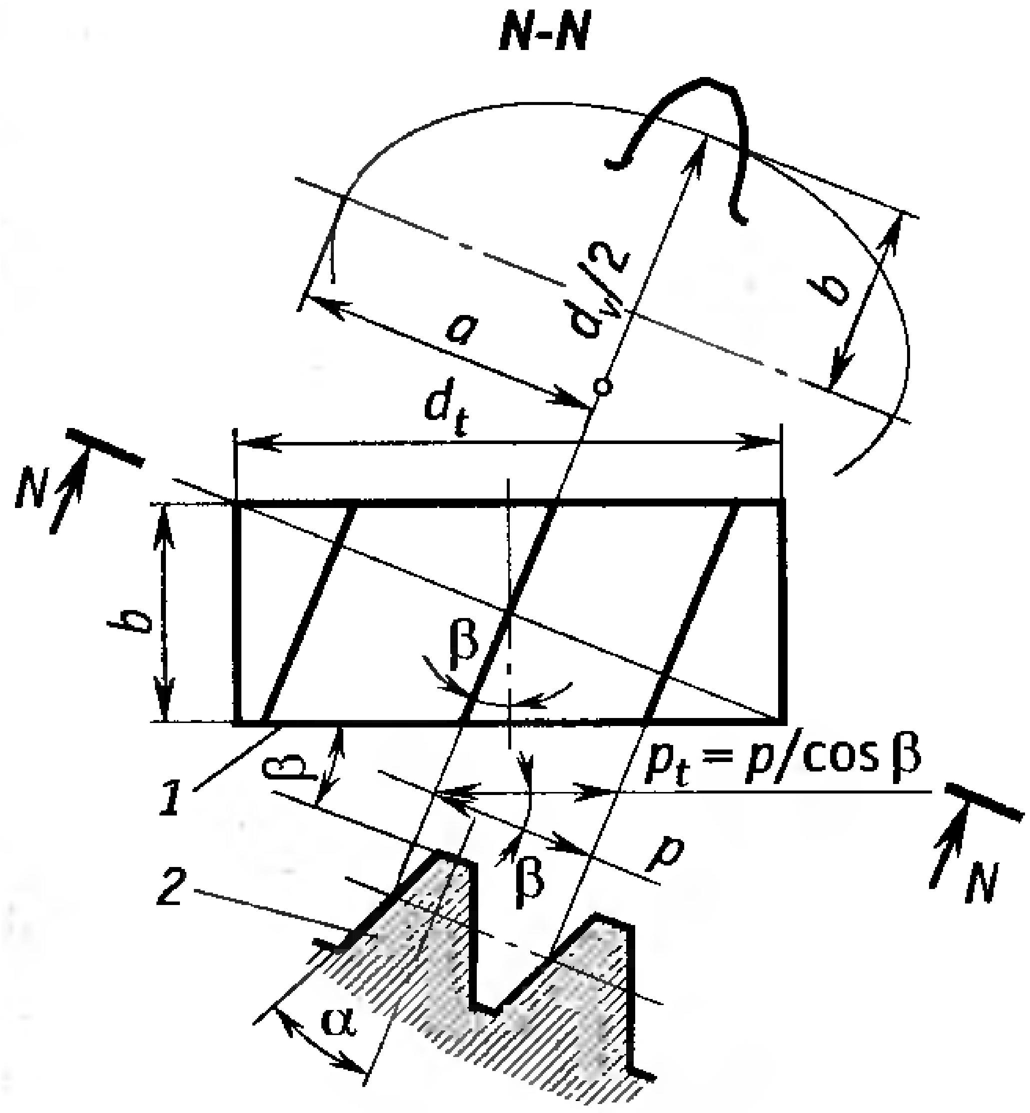


$$\rho_1 = \frac{d_{\omega 1}}{2} \cdot \sin\alpha_w \qquad \rho_2 = \frac{d_{\omega 2}}{2} \cdot \sin\alpha_w$$

$$\frac{1}{\rho_{tr}} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{2}{d_{\omega 2} \cdot \sin\alpha_w} + \frac{2}{d_{\omega 1} \cdot \sin\alpha_w} =$$

$$= \frac{2}{d_{\omega 1} \cdot \sin\alpha_w} \cdot \left(\frac{u \pm 1}{u} \right)$$

Helical gears



$$b = \frac{d_\omega}{2} \qquad a = \frac{d_\omega}{2 \cdot \cos\beta}$$

$$d_v = \frac{2 \cdot a^2}{b} = \frac{d_\omega}{\cos^2 \beta}$$

$$\frac{1}{\rho_{tr}} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{2}{d_{v1} \cdot \sin\alpha_w} + \frac{2}{d_{v2} \cdot \sin\alpha_w} =$$

$$= \frac{2 \cdot \cos^2 \beta}{d_{\omega 1} \cdot \sin\alpha_w} \cdot \left(\frac{u \pm 1}{u} \right)$$

Hertz (contact) stress

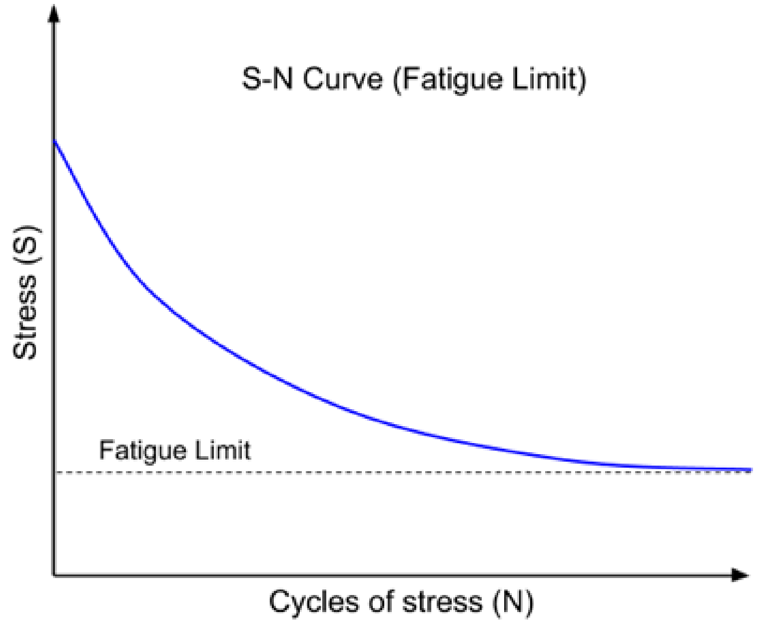
$$\sigma_H = Z_H \cdot Z_M \cdot Z_\varepsilon \cdot \sqrt{\frac{2 \cdot T_1 \cdot K_{H\alpha} \cdot K_{H\beta} \cdot K_{Hv}}{b_w \cdot d_{\omega 1}{}^2} \cdot \frac{(u \pm 1)}{u}} \le [\sigma_H]$$

	Spur gears	Helical gears
Zone factor	$Z_H = \sqrt{\frac{2}{\sin 2 \cdot \alpha_w}} = 1.76$	$Z_H = \sqrt{\frac{2 \cdot \cos^2 \beta}{\sin 2 \cdot \alpha_w}} = 1.76 \cdot \cos \beta$
Elasticity factor	$Z_M = \sqrt{\frac{E_1 \cdot E_2}{\pi \cdot \left[E_2 \cdot \left(1 - \nu_1{}^2\right) + E_1 \cdot \left(1 - \nu_2{}^2\right) \right]}} = 275$	
Contact ratio factor	$Z_\varepsilon = 1$	$Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}}$

Allowable hertz stress

$$[\sigma_H] = \frac{\sigma_{H\lim b} \cdot K_{HL}}{S_H} \cdot Z_R Z_V$$

Endurance limit for surface stress $\sigma_{H\lim b}$



Safety factor for Surface durability S_H

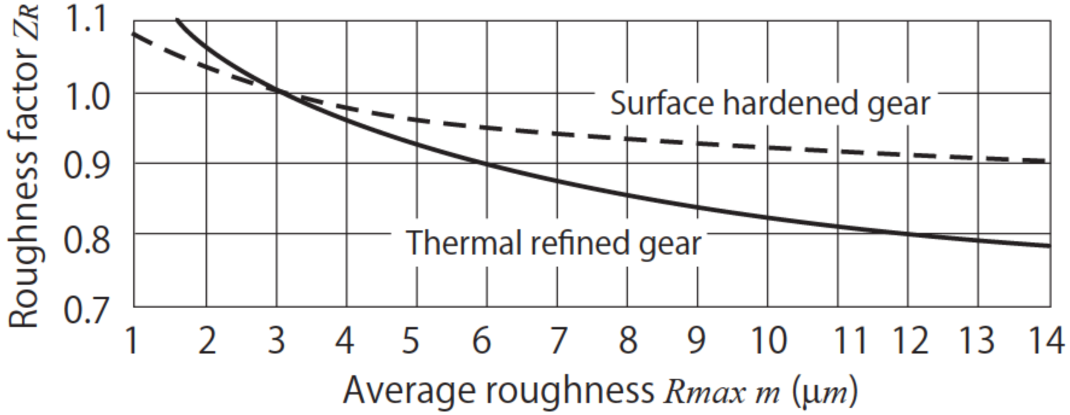
A minimum Safety factor for flank damage (Pitting) $S_H = 1.15$

Life Factor $$K_{HL} = \sqrt[6]{\frac{N_{H0}}{N_{HE}}}$$

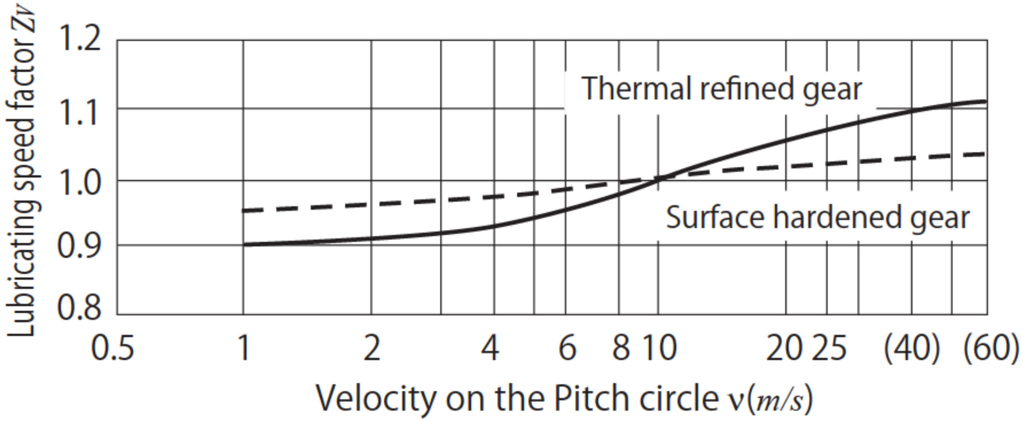
N_{H0} base number of cycling repetition

N_{HE} duty cycles

Number of repeated	Life factor for Surface durability
Below 10,000	1.5
About 100,000	1.3
About 10^6	1.15
Above 10^7	1.0



Roughness factor Z_R



Lubricating speed factor Z_V

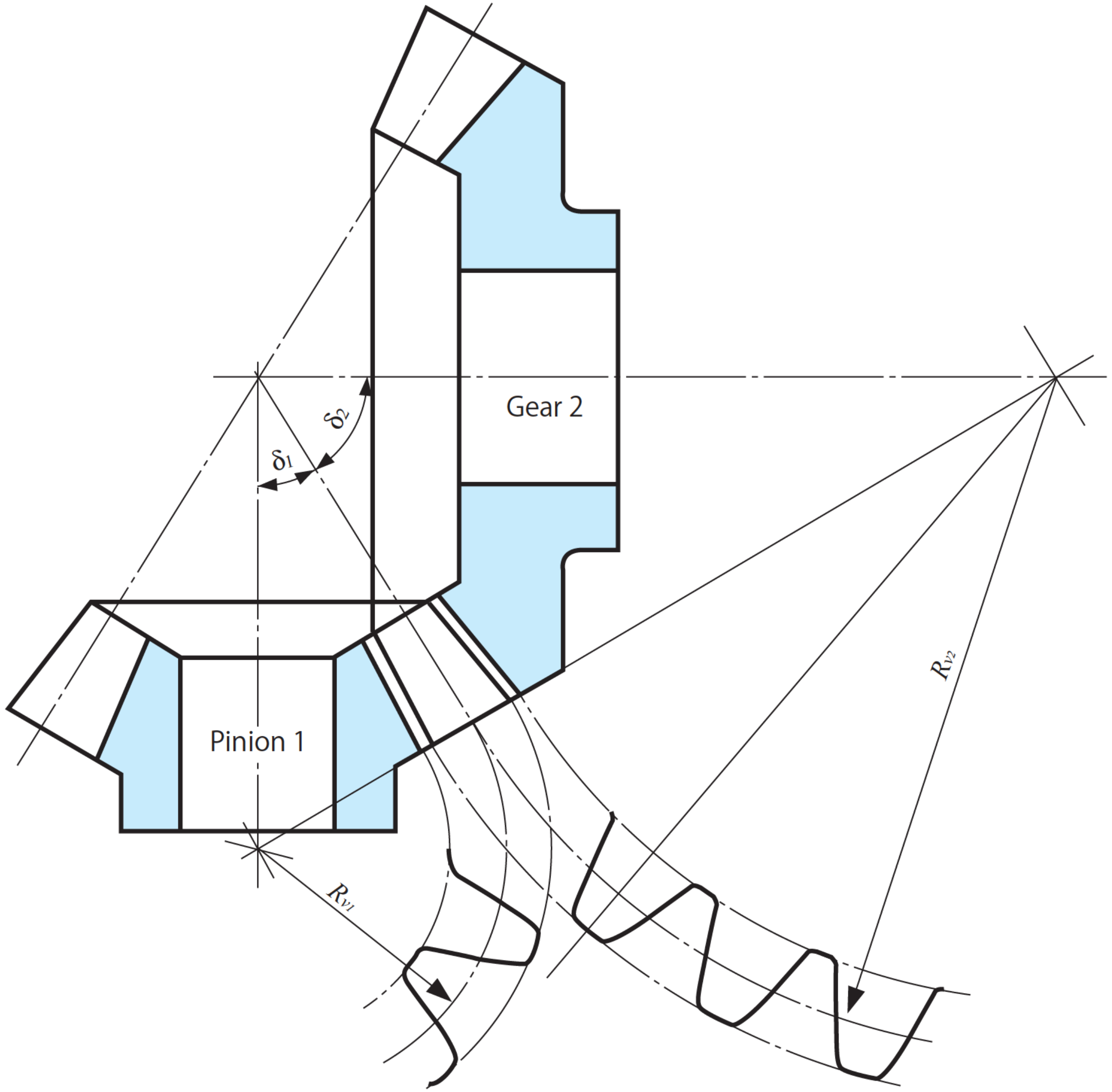
Design calculation

$$d_{\omega 1} = \sqrt[3]{\frac{2 \cdot T_1 \cdot K_{H\alpha} \cdot K_{H\beta} \cdot K_{Hv} \cdot \left(Z_H \cdot Z_M \cdot Z_\varepsilon\right)^2}{\Psi_d \cdot \left[\sigma_H\right]^2} \cdot \frac{(u \pm 1)}{u}}$$

z_1 β

$$m = \frac{d_{\omega 1}}{z_1} \qquad m_n = \frac{d_{\omega 1}}{z_1} \cdot \cos\beta$$

BEVEL GEARS



Equivalent straight spur gear

$$d_{ve} = \frac{d_e}{\cos \delta}$$

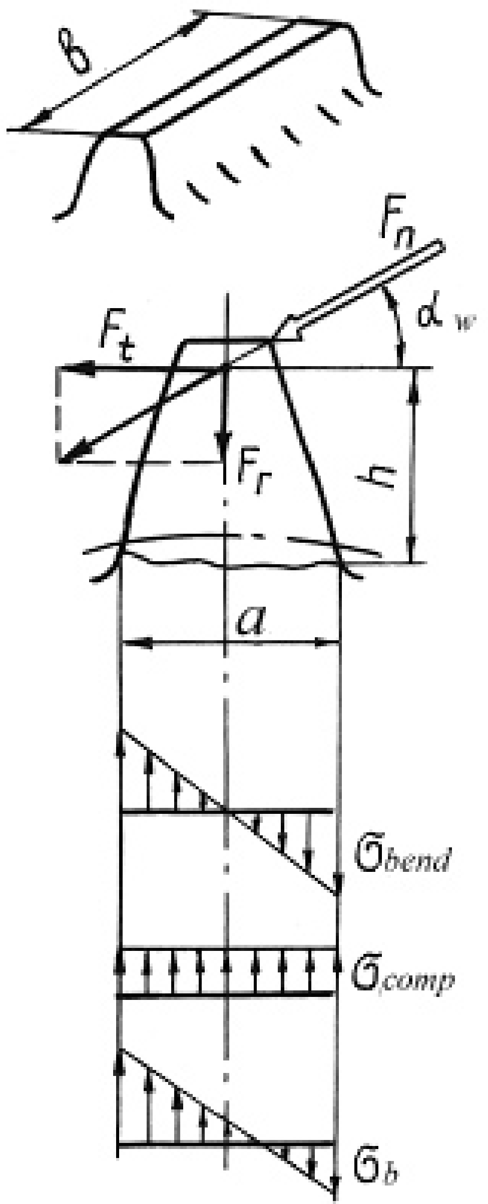
$$z_v = \frac{z}{\cos \delta}$$

Transformed radius of curvature

$$\frac{1}{\rho_{tr}} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{2}{d_{v1} \cdot \sin\alpha_w} + \frac{2}{d_{v2} \cdot \sin\alpha_w} =$$

$$= \frac{2}{d_{m1} \sin \alpha_w}\left(\cos \delta_1 + \frac{\cos \delta_2}{u}\right) = \frac{2}{d_{m1} \sin \alpha_w}\left(\frac{\sqrt{u^2+1}}{u}\right)$$

Bending strength



$$\sigma_b = \sigma_{bend} - \sigma_{comp};$$

$$\sigma_{bend} = \frac{M_{bend}}{W} = \frac{6 \cdot F_t \cdot h}{b \cdot a^2};$$

$$\sigma_{comp} = \frac{F_r}{A} = \frac{F_t \cdot tg\,\alpha_w}{a \cdot b};$$

$$h = \gamma \cdot m; \qquad a = \beta \cdot m;$$

$$\sigma_b = \frac{F_t}{b \cdot m} \cdot \left(\frac{6 \cdot \gamma}{\beta^2} - \frac{tg\,\alpha_w}{\beta} \right).$$

CALCULATION OF STRAIGHT SPUR GEARS FOR BENDING STRENGTH

$$\sigma_b = \frac{F_t \cdot K_b \cdot Y_b}{b \cdot m} \leq [\sigma_b]$$

$$m = \frac{2 \cdot T_2 \cdot Y_b \cdot K_b}{d_2 \cdot b \cdot [\sigma_b]}$$