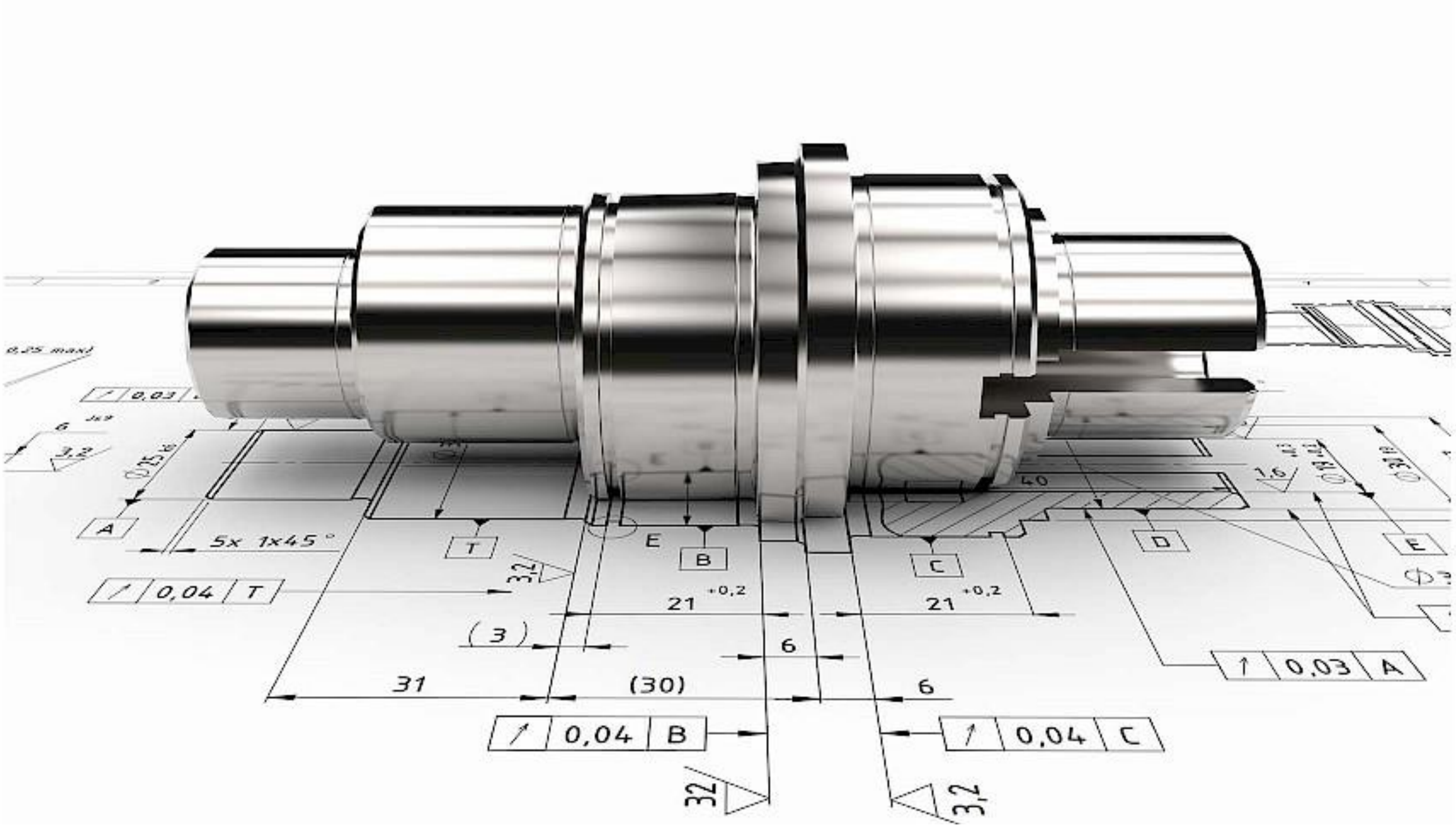
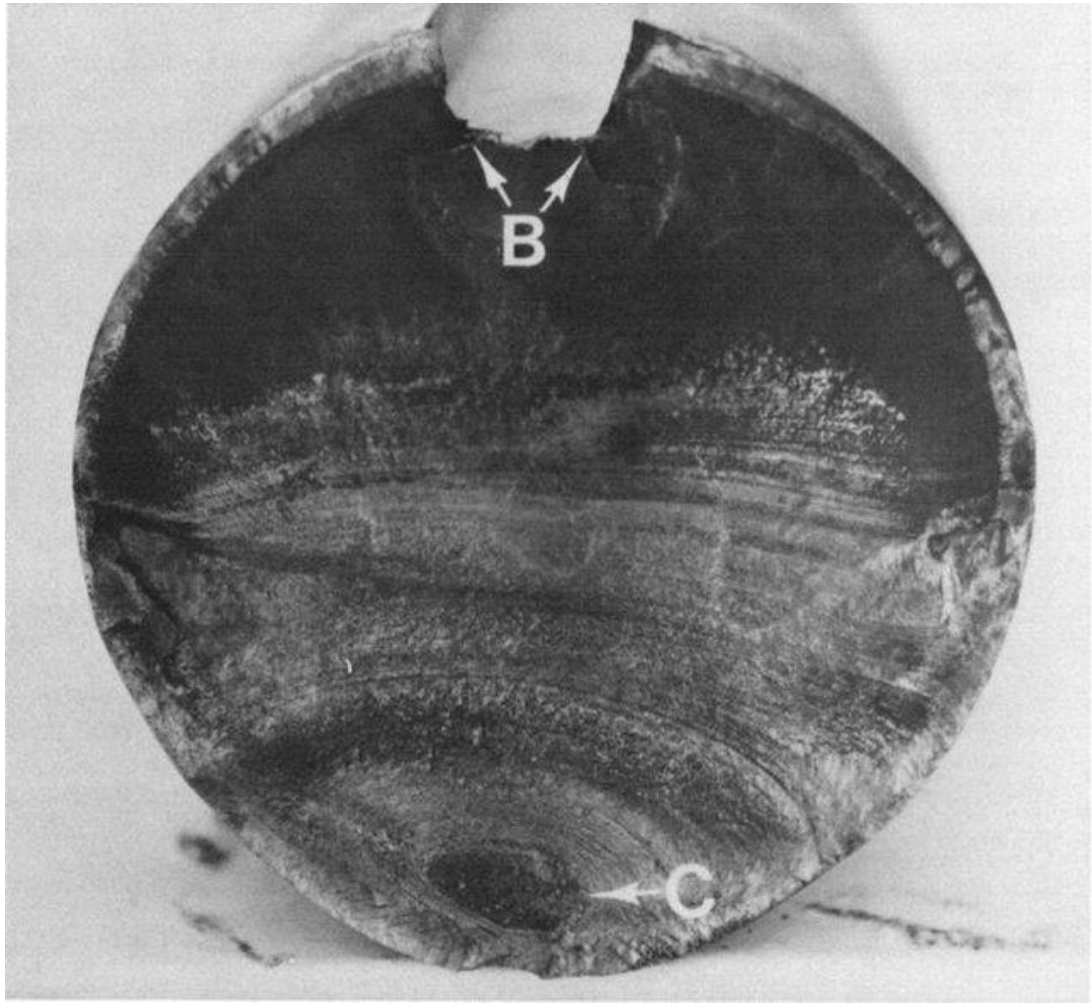
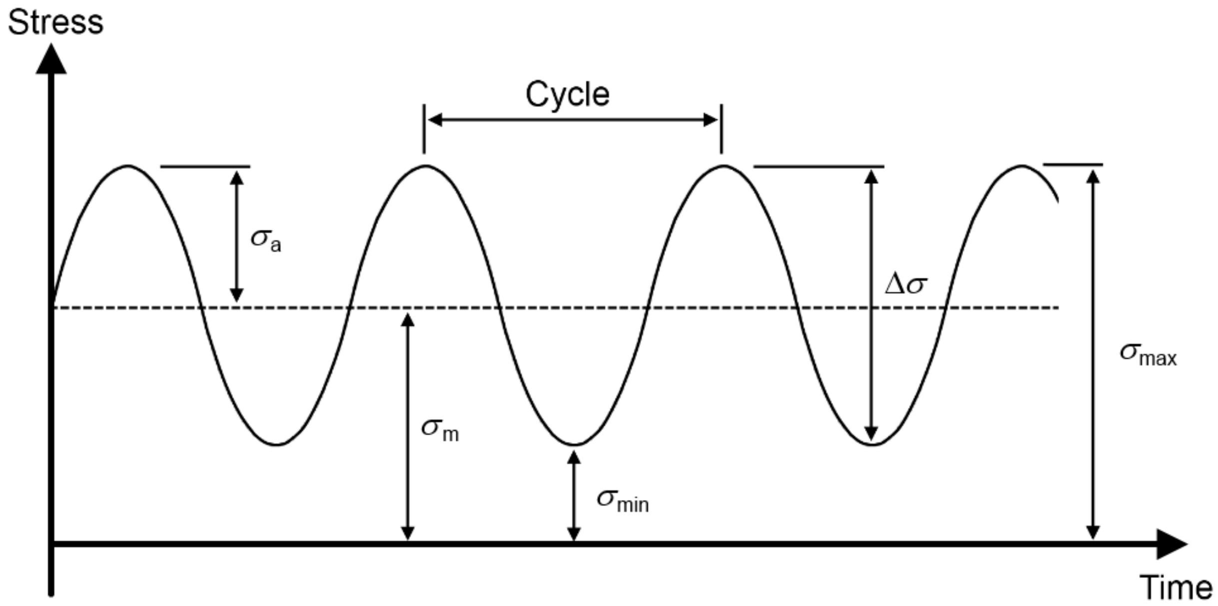
Shafts, axels_3



FATIGUE STRENGTH



The failure began at the end of a keyway that was machined without fillets (B) and progressed to final rupture at (C). The final rupture zone is small, indicating that loads were low.



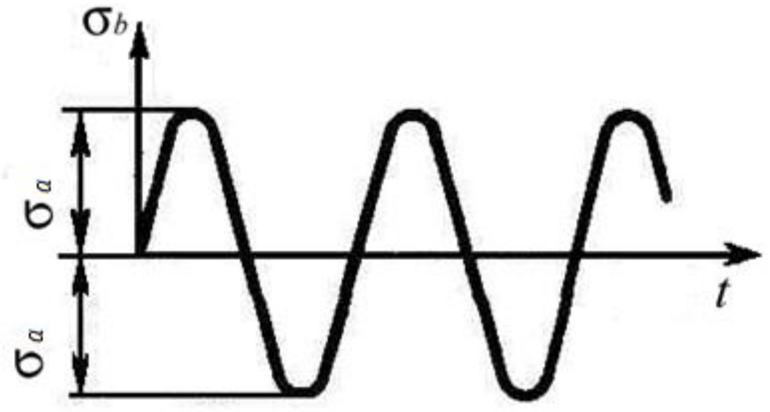
$$\sigma = \sigma_c + \sigma_a \sin \omega t$$

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$ Mean stresse

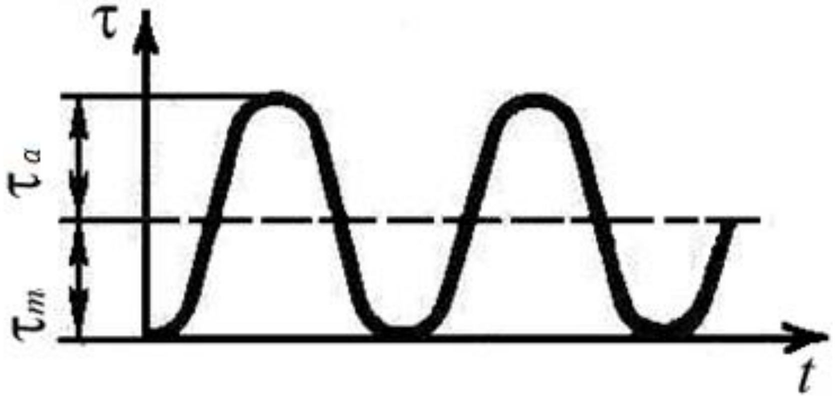
$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$ Stress Amplitude (Alternating stress)

$$r = \frac{\sigma_{\min}}{\sigma_{\max}}$$ Stress ratio

Changing of bending stresses



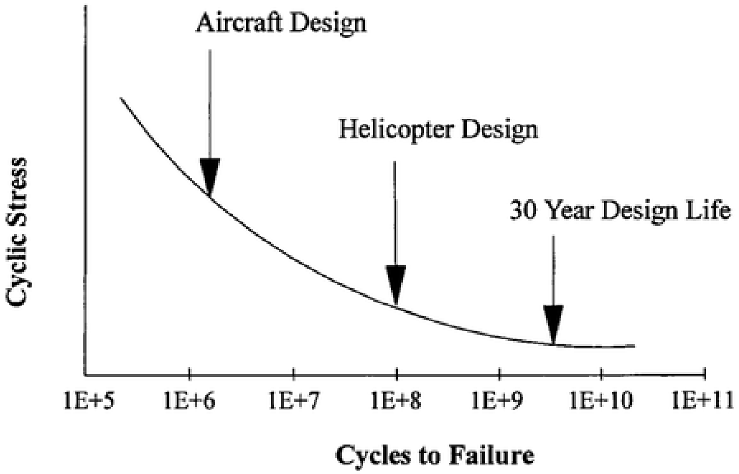
Changing of torsion stresses

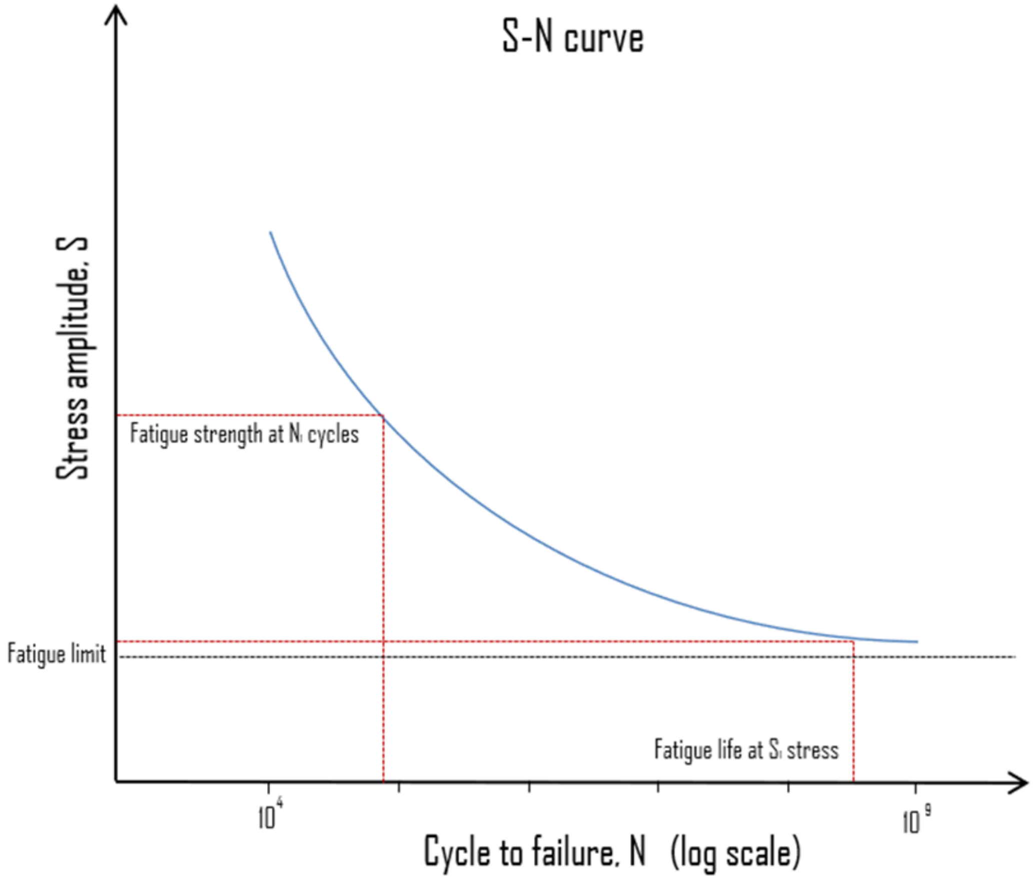


Fatigue Limit (**endurance limit**) is the stress level, below which fatigue failure does not occur. This limit exists only for some ferrous (iron-base) and titanium alloys, for which the S–N curve becomes horizontal at higher N values. Other structural metals, such as aluminium and copper, do not have a distinct limit and will eventually fail even from small stress amplitudes.

Fatigue Strength - the value of stress at which failure occurs after some specified number of cycles

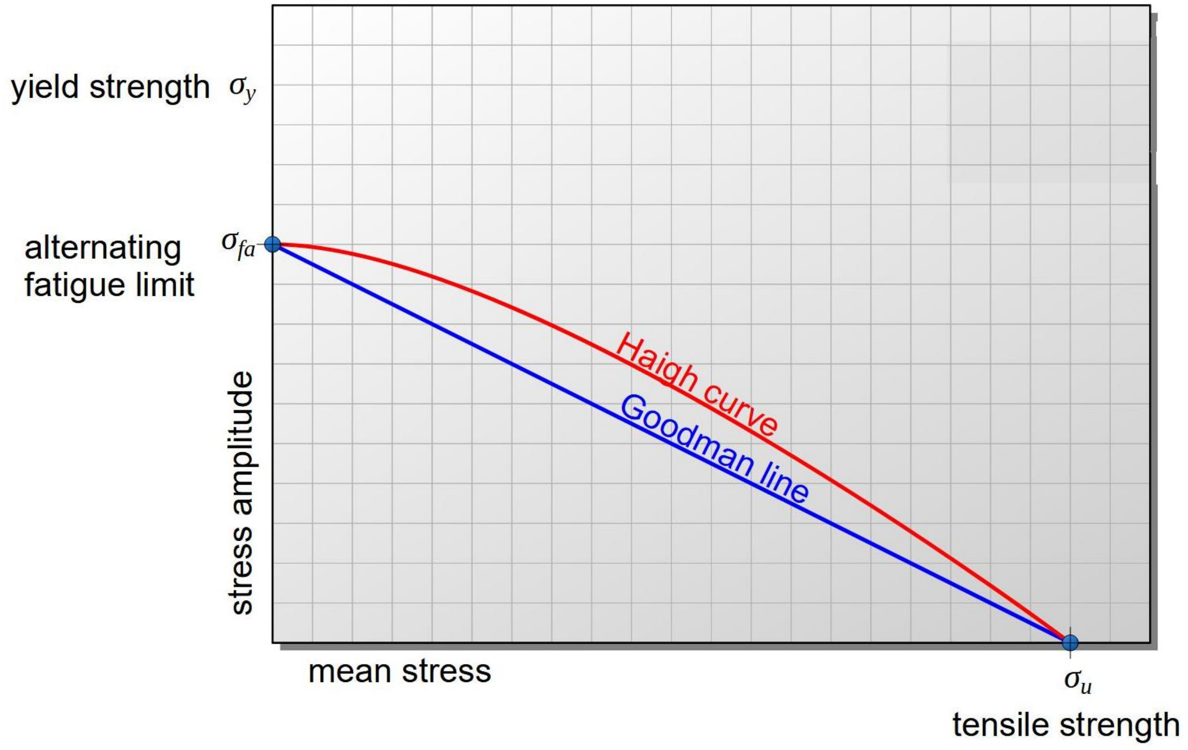
Fatigue Life characterizes a material’s fatigue behavior. It is the number of cycles to cause failure at a specified stress level

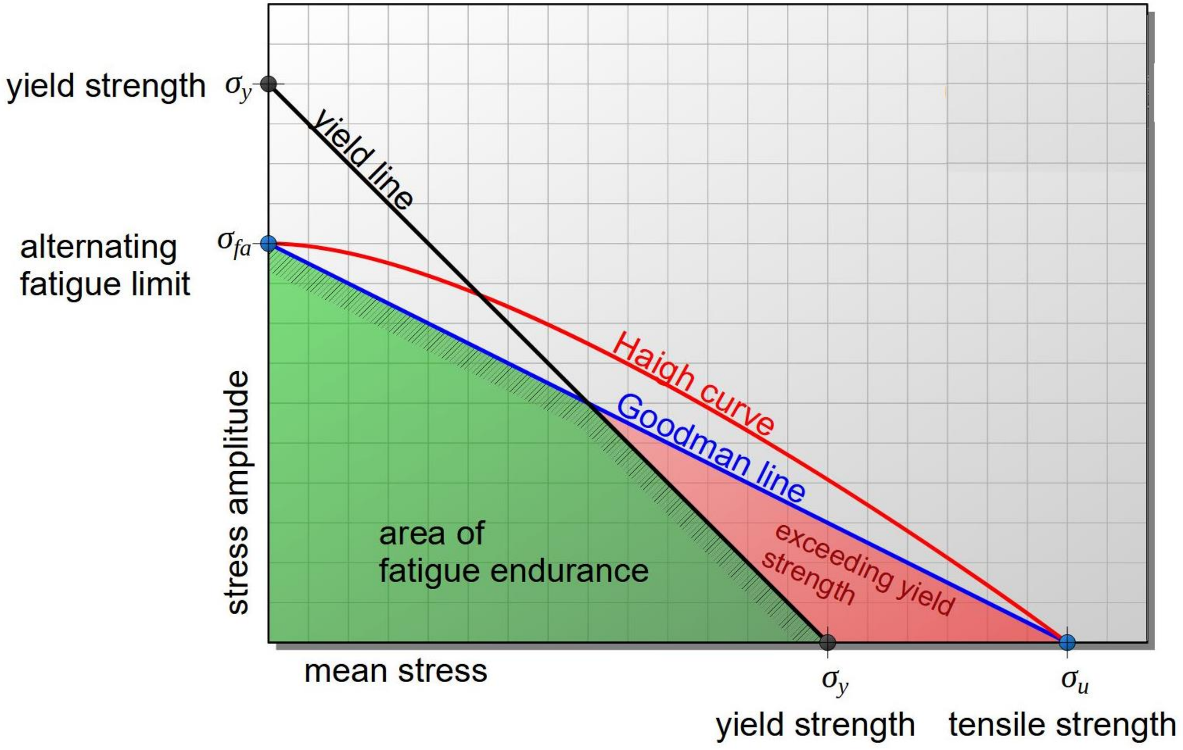




Wöhler curve

Haigh Diagram (Goodman diagram)



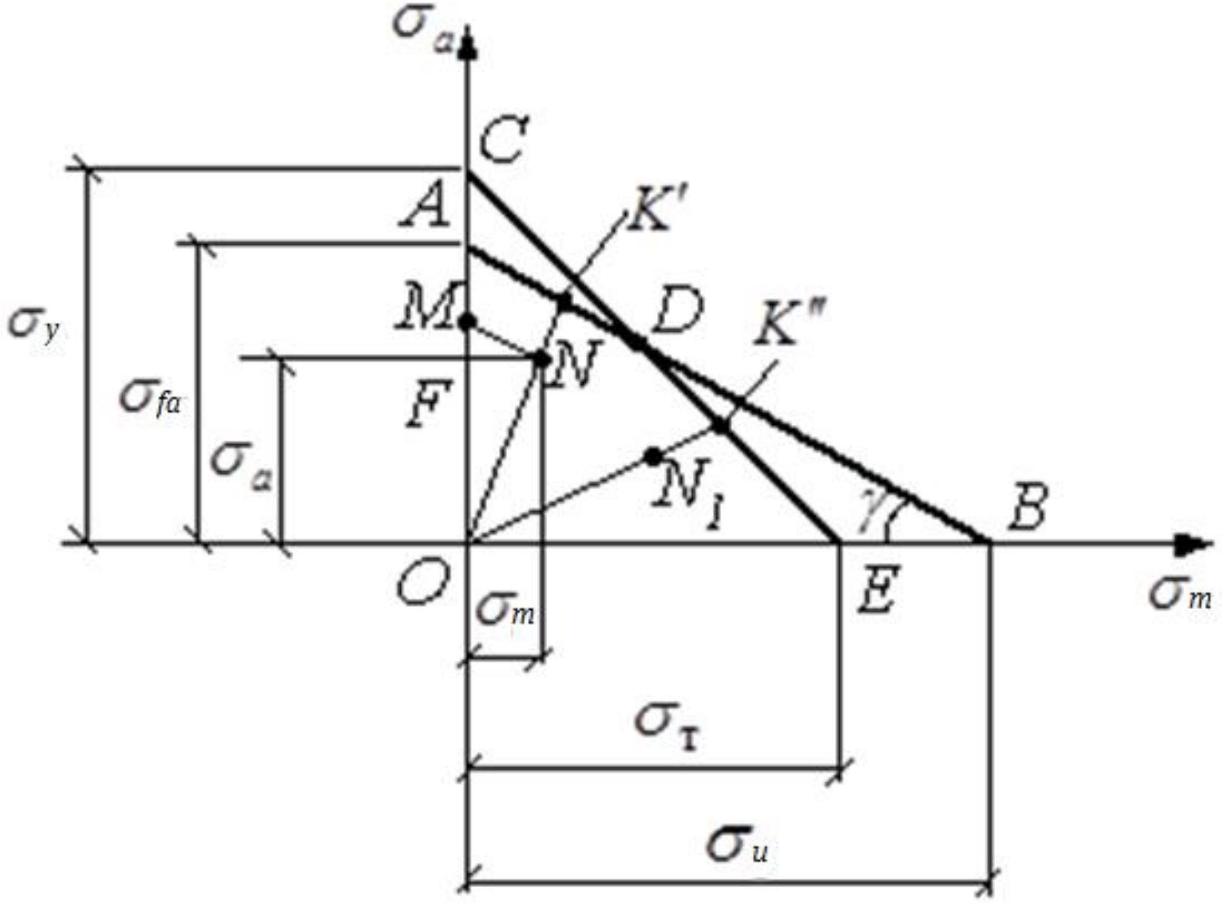


Goodman line $$\frac{\sigma_a}{\sigma_{fa}} + \frac{\sigma_m}{\sigma_u} = 1$$

Yield line $$\frac{\sigma_a}{\sigma_y} + \frac{\sigma_m}{\sigma_y} = 1$$

$$S \geq [S]$$ ***Safety factor***

$$S_\sigma = \frac{\sigma_{fa}}{K_{\sigma d} \cdot \sigma_{\max}} \qquad S_\tau = \frac{\tau_{fa}}{K_{\tau d} \cdot \tau_{\max}}$$



$$S_\sigma = \frac{\sigma'_{\max}}{\sigma_{\max}} = \frac{\sigma'_m + \sigma'_a}{\sigma_m + \sigma_a} = \frac{OK'}{ON}$$

$$\frac{OK'}{ON} = \frac{OA}{OM}$$

$$OA = \sigma_{fa}$$

$$OM = OF + FM = \sigma_a + \sigma_m \cdot tg\gamma$$

$$\psi_\sigma = tg\gamma = \frac{\sigma_{fa}}{\sigma_u}$$

$$S_\sigma = \frac{\sigma_{fa}}{K_{\sigma d}\sigma_a + \psi_\sigma \sigma_m}$$

$$K_{\sigma d} = \frac{K_\sigma}{K_b \cdot K_a} \qquad K_{\tau d} = \frac{K_\tau}{K_b \cdot K_a}$$

the total factors of fatigue strength reducing

K_σ, K_τ stress concentration factors

K_a surface roughness factor

K_b scale factor

Obtained experimentally, analytically, etc

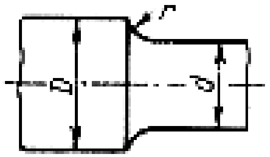
Published in charts and tables

Diameter (d) (mm)	K_b
$d \le 7.5$	1.00
$7.5 < d \le 50$	0.85
$d > 50$	0.75

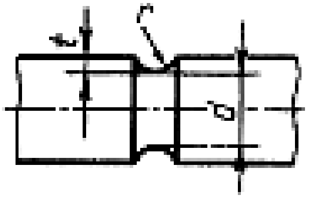
Values of size factor

The most typical stress concentrations of the shaft

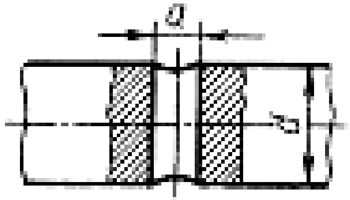
- Filleted transition regions;



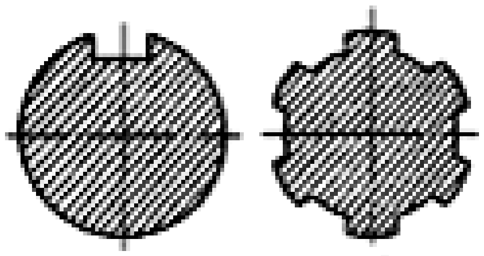
- Grooves;



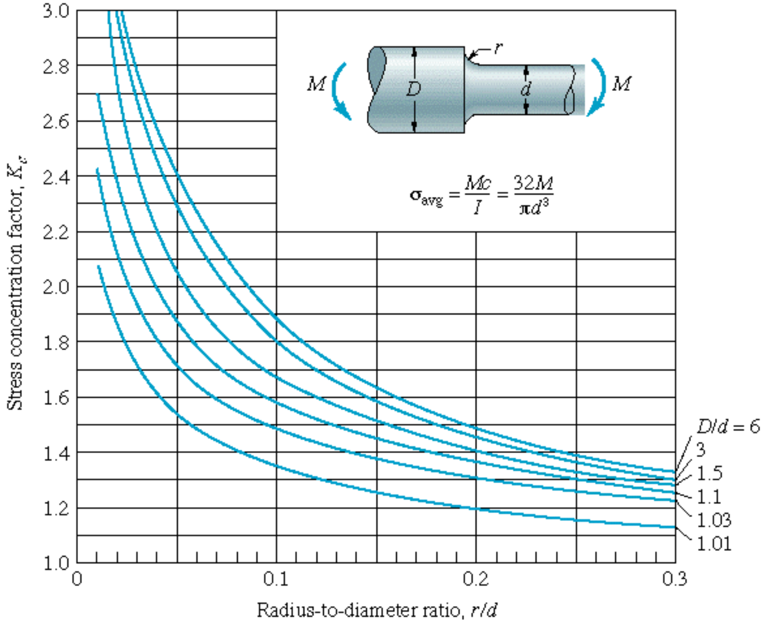
- Radial holes;



- Keyed and splined portions;

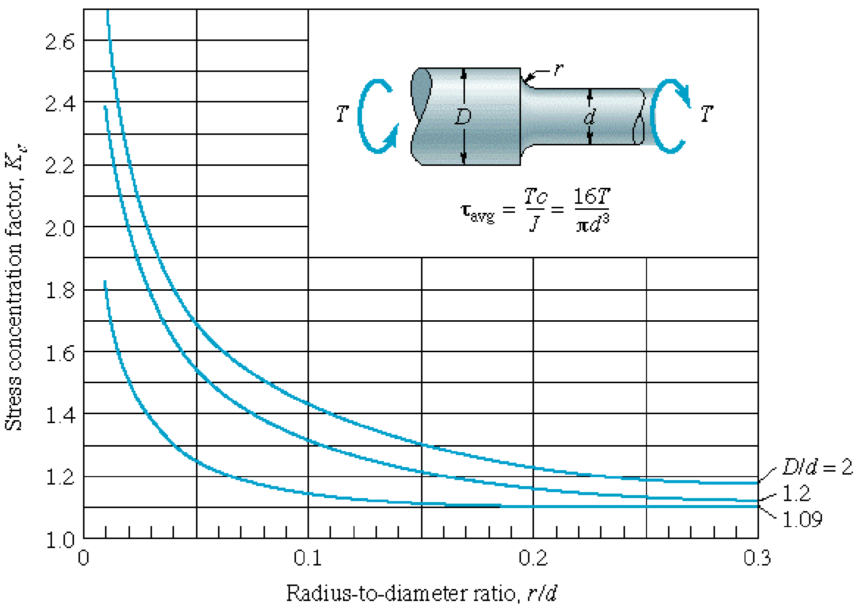


- Threaded portions;
- Interference fits.

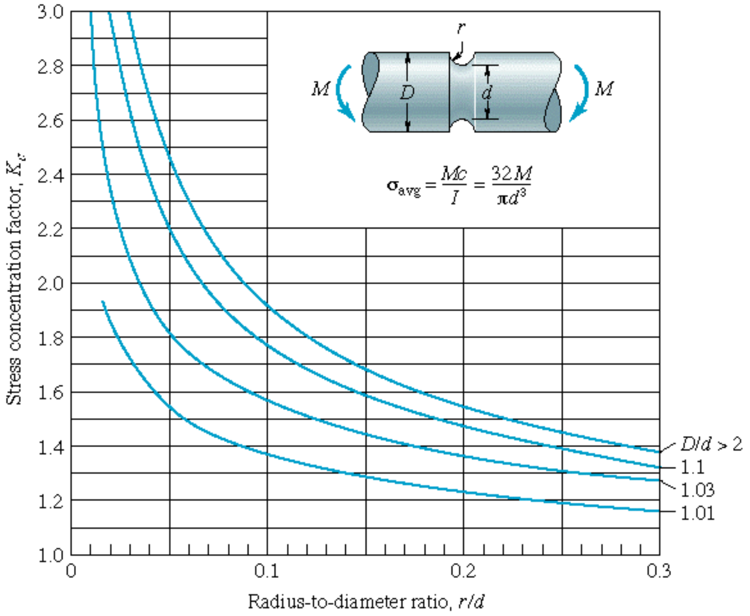


M
D
d
r
M
$\sigma_{avg} = \frac{Mc}{I} = \frac{32M}{\pi d^3}$
Stress concentration factor, K_c
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
D/d = 6
3
1.5
1.1
1.03
1.01
Radius-to-diameter ratio, r/d

(b)

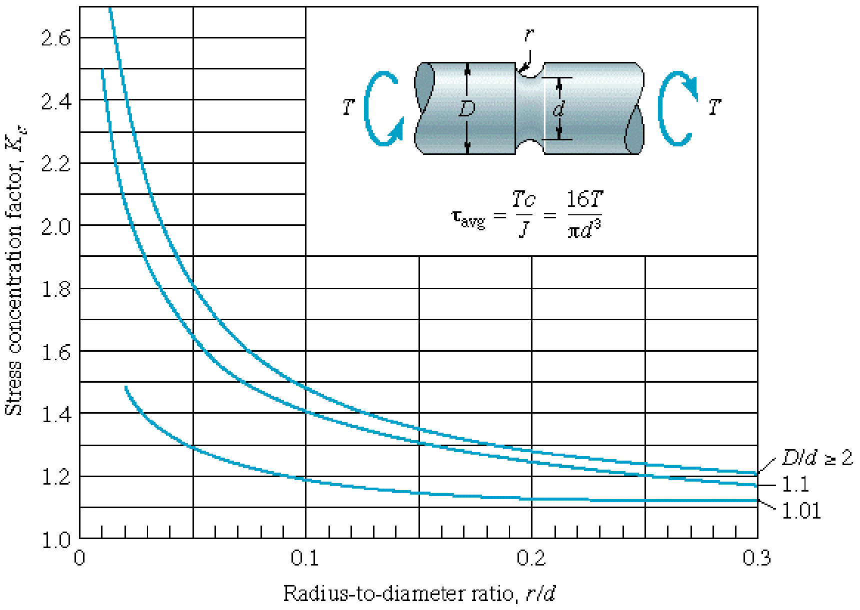


T
D
d
r
T
$\tau_{avg} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$
Stress concentration factor, K_c
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
D/d = 2
1.2
1.09
Radius-to-diameter ratio, r/d



M
D
r
d
M
$\sigma_{avg} = \frac{Mc}{I} = \frac{32M}{\pi d^3}$
Stress concentration factor, K_c
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
D/d > 2
1.1
1.03
1.01
Radius-to-diameter ratio, r/d

(b)



T
D
r
d
T
$\tau_{avg} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$
Stress concentration factor, K_c
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
D/d ≥ 2
1.1
1.01
Radius-to-diameter ratio, r/d

(c)

Safety factor for bending

$$S_\sigma = \frac{\sigma_{\mathrm{fa}}}{\frac{K_\sigma}{K_{\mathrm{b}} \cdot K_{\mathrm{a}}} \cdot \sigma_{\mathrm{a}} + \psi_\sigma \cdot \sigma_m}$$

Safety factor for torsion

$$S_\tau = \frac{\tau_{\mathrm{fa}}}{\frac{K_\tau}{K_{\mathrm{b}} \cdot K_{\mathrm{a}}} \cdot \tau_{\mathrm{a}} + \psi_\tau \cdot \tau_m}$$

Safety factor

$$S = \frac{S_\sigma \cdot S_\tau}{\sqrt{S_\sigma^2 + S_\tau^2}} \geq [S] = 1.5...2.5.$$

$$S_\sigma = \frac{\sigma_{\mathrm{fa}}}{\frac{K_\sigma}{K_{\mathrm{b}} \cdot K_{\mathrm{a}}} \cdot \sigma_{\mathrm{a}} + \psi_\sigma \cdot \sigma_m}$$

$$S_\tau = \frac{\tau_{\mathrm{fa}}}{\frac{K_\tau}{K_{\mathrm{b}} \cdot K_{\mathrm{a}}} \cdot \tau_{\mathrm{a}} + \psi_\tau \cdot \tau_m}$$

σ_{fa}, τ_{fa} – limit of endurance in bending and in torsion (table-material)

$$\sigma_{\mathrm{fa}} = 0.43 \cdot \sigma_{ult}$$ - for carbon steels;

$$\sigma_{\mathrm{fa}} = 0.35 \cdot \sigma_{ult} + 120$$ - for alloy steels;

$$\tau_{\mathrm{fa}} = (0.2...0.3) \cdot \sigma_{ult}.$$

σ_a, τ_a – alternating bending and torsion stresses

$$\sigma_{\mathrm{a}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \sigma_{max} = \frac{M_\Sigma}{W} = \frac{M_\Sigma}{0.1 \cdot d^3};$$

$$\tau_{\mathrm{a}} = \frac{\tau_{max} - \tau_{min}}{2} = \frac{\tau_{max}}{2} = 0.5 \cdot \frac{T}{W_p} = 0.5 \cdot \frac{T}{0.2 \cdot d^3}.$$

σ_m, τ_m – mean bending and torsion stresses

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 0;$$

$$\tau_{mean} = \frac{\tau_{max} + \tau_{min}}{2} = \frac{\tau_{max}}{2} = 0.5 \cdot \frac{T}{W_p} = 0.5 \cdot \frac{T}{0.2 \cdot d^3}.$$

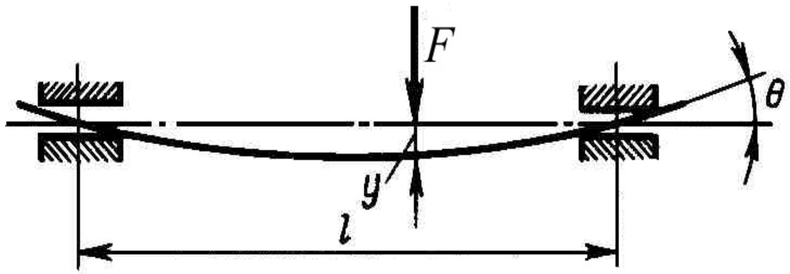
ψ_σ, ψ_τ – factors of constant components of bending and torsion stresses (table-material)

$\psi_\sigma = 0.1;\ \psi_\tau = 0.05$ – for carbon steels;

$\psi_\sigma = 0.15;\ \psi_\tau = 0.1$ – for alloy steels.

STIFFNESS ANALYSIS OF THE SHAFT

Flexural stiffness



Basic criteria of flexural stiffness are:

- Maximum deflection (sag) ***y*** of the shaft;
- Angle of rotation θ of support sections.

Flexural stiffness conditions

$$y \le [y]; \qquad \theta \le [\theta],$$

where

[y] is the maximum safe sag; ***[θ]*** is the maximum safe angle of rotation.

[y]= 0.01m – for shafts of spur gears and worm gear drives;

[y]= 0.005m – for shafts of bevel gear, hypoid gear and hourglass worm gear drives;

[y]= (0.0002…0.0003)l – for general purpose shafts used in machine tools;

[θ]= 0.001 rad – for shafts mounted in sliding contact bearings;

[θ]= 0.005 rad – for shafts mounted in radial ball bearings.

Torsional stiffness

Basic criterion of torsional rigidity is the angle of twist.

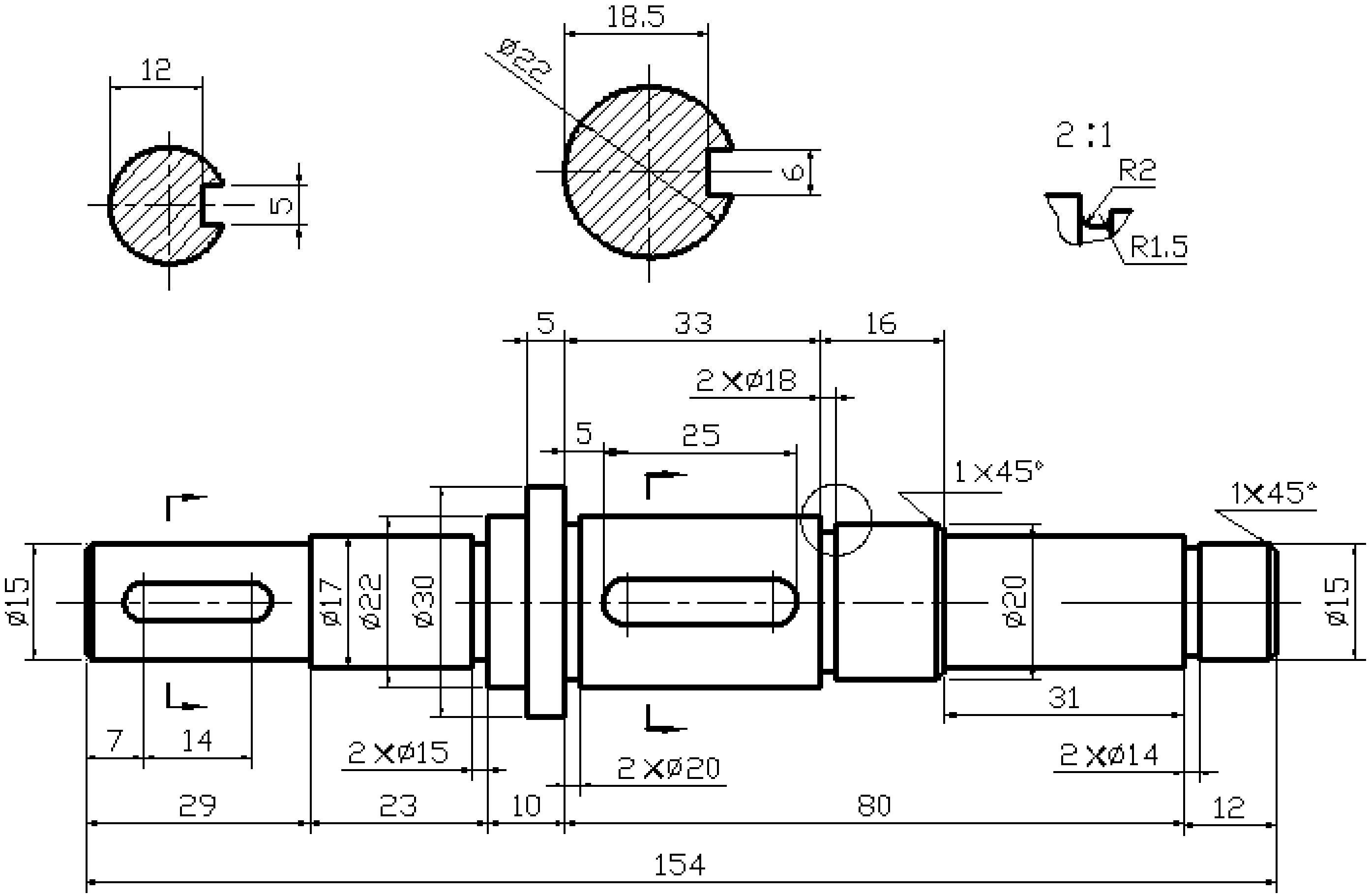
Torsional stiffness condition

$$\varphi \leq [\varphi],$$

where ***[φ]*** is the maximum safe angle of twist.

$$\varphi = \frac{T \cdot l}{G \cdot J_p},$$

where ***T*** is torque; ***l*** is length of the shaft; ***G*** is shear modulus;
$J_p = \pi d^4/32$ is polar moment of inertia.



12
5
18.5
ø22
6
2 :1
R2
R1.5
5
33
16
2×ø18
5
25
1×45°
1×45°
ø15
ø17
ø22
ø30
ø20
ø15
31
7
14
2×ø15
2×ø20
2×ø14
29
23
10
80
12
154