Syllabus
Course Program
Course Program

FEM Algorithms

Specialty
113 – Applied Mathematics

Institute
Institute of Computer Modeling, Applied Physics and Mathematics

Educational program
Computer and Mathematical Modeling

Department
Mathematical Modelling and Intelligent Computing in Engineering (161)

Level of education
Master's level (1 year 4 months)

Course type
Special (professional), Elective

Semester
1

Language of instruction
English

Lecturers and course developers

Oleksiy Larin (responsible lecturer)

Oleksiy.Larin@khpi.edu.ua

Doctor of Technical Sciences, Professor, work experience - 15 years
Specialist in the field of computational mechanics, probabilistic modeling and reliability prediction. The main focus of scientific works is devoted to the development of models, methods, approaches and algorithms of computer modeling and statistical analysis of engineering systems, in particular with random parameters. Author of more than 150 scientific and methodical works

More information about the teacher on the website of the department

General information

Summary

The course is a comprehensive study of computational methods for solving complex engineering and physical problems using the concept of the finite element method (FEM). The course covers a number of topics, including variational methods, theoretical features of building finite element (FEs) formulations in mechanics, numerical integration and meshing algorithms. Students will gain practical experience in programming for individual elements of a complex FEM-based computer modelling problem.

Course objectives and goals

The course aims to provide students with a solid mathematical and computational foundation for conducting advanced simulations in engineering and physics, equipping them with the knowledge and skills needed for effective problem-solving and model development.
The course objectivities:
1) to foster a deep understanding of mathematical principles underpinning variational techniques, finite element methods, and numerical integration
2) to develop the ability to apply mathematical concepts to engineering and physical problems effectively.

3) enable students to critically evaluate the suitability of simulation approaches for specific challenges
4) to train students to critically assess simulation results, interpreting them within the context of engineering and physical systems
5) to develop the ability to determine the accuracy and reliability of simulation outcomes and communicate findings effectively to stakeholders

Format of classes

Lectures, practical classes. Final control in the form of a credit.

Competencies

GC1. Ability to generate new ideas (creativity) and non- standard approaches to their implementation.
GC7. Ability to think abstractly, analyse and synthesise..
PC1. Ability to solve tasks and problems that can be formalised, require updating and integrating knowledge, in particular in conditions of incomplete information.
PC2. Ability to conduct scientific research aimed to develop new and adapt existing mathematical and computer models to study various processes, phenomena and systems, conduct appropriate experiments and analyse the results.
PC3. Ability to develop methods and algorithms for the
construction, research and software implementation of mathematical models in engineering, physics, biology, medicine and other fields and to analyse them.
PC7. Ability to design and develop software to solve formalised problems, including systems with large amounts of data.

Learning outcomes

LO4. Build mathematical models of complex systems and choose methods of their research, implement the built models in software and check their adequacy using computer technologies.
LO5. Justify and, if necessary, develop new algorithms and software tools for solving scientific and applied problems, apply, modify and investigate analytical and computational methods for solving them.
LO8. Develop and implement algorithms for solving applied problems, system and application software of information systems and technologies.

Student workload

The total volume of the course is 90 hours (3 ECTS credits): lectures - 16 hours, laboratory classes - 16 hours, self-study - 58 hours.

Course prerequisites

For a general understanding of lectures, it is assumed that students have a background in the following disciplines: Mathematical Analysis (Differential, Integral calculus and Series);
Linear Algebra (matrix, vector algebra) and Tensor analysis (basics);
Differential Equations and Mathematical Physics (Differential Equations in partial derivatives);
Numerical methods and Theory of Optimization basics.
The knowledge of Theoretical and Analytical Mechanics;
Theory of Materials Elasticity or Strength of Materials;
Theory of Linear Dynamics of Discrete Systems
in general, are assumed to be in a basic level of understanding form mathematical, physical and engineering points of view;
Skills in basic Python coding is welcomed but not mandatory.

Features of the course, teaching and learning methods, and technologies

The course teaching and learning methods include traditional lectures for theoretical foundations and practice. The practice is structured in such a way that students are given a task for self-study in advance of the practical assignment. It can be a task or a series of mathematical tasks to solve or a task for independent programming. The practical lesson itself is devoted to consultations on problems that have arisen during the independent solution of the tasks and the defense of the results obtained. It is important to answer theoretical questions about the topic from the given practice, as well as to interpret the results and make conclusions.

Program of the course

Topics of the lectures

Topic 1. The main questions of the finite element method (FEM).
Main ideas of FEM, approaches, concepts and applications.
Topic 2. Variational principles and their application to the general formulation of FEM.
Short information about variational calculus. The principle of virtual work.
Topic 3. Solution strategies.
The Ritz method. The Galerkin method. Weighted residuals method. The point collocation method.
Topic 4. Building shape functions of FE.
The principle of construction and types of shape functions. 1D, 2D and 3D options. Approaches of creating isoparametric FE.
Topic 5. Schemes of FE integration and construction of the stiffness matrix.
Different methods of numerical integration are especially used in FEM formulations. Gauss method.
Topic 6. Algorithms for mesh generation.
Methods and algorithms for generating mapped and free meshes in plane and three-dimensional statements. Delaunay triangulation method. Indicators and criteria for checking the quality of FE.
Topic 7 FEM in dynamics problems.
Formulation of dynamics problems within the framework of the FEM concept. Approaches of explicit dynamics and direct integration of equations in time.
Topic 8 Nonlinear FEM problems.
Models and formulations leading to nonlinearity. Formulation of physically nonlinear equations and algorithm for solving the corresponding problem.

Topics of the workshops

Not appliable

Topics of the laboratory classes

Practical class 1.
Consideration of the procedure for assembling a global stiffness matrix. Examples for plane problems. Converting local and global coordinates.
Practical class 2
Application of the Galerkin method, Weighted residuals and point collocation methods to a 1-dimensional mechanical deformation problem of a rod.
Practical class 3
Creating FE shape functions in the 1D case.
Practical class 4.
Introduction to Colab in Python.
Practical class 5.
Creating shape functions and their software visualisation.
Practical class 6
Delaunay triangulations.
Practical class 7
Software implementation of automatic mesh generation.
Practical class 8
Solution of the transient heat conduction problem for the one-dimensional case within the framework of the FEM approach.

Self-study

Processing of lecture material.
Completion of practical and programming (coding) tasks, which are given after each lecture and are expected to be completed in advance of the practical class. It should be noted here that the practice time

is mainly allocated for consultations on problems that arose during the previous performance of tasks and for individual defense of reports on them, but not for their performance as such, which is expected within the framework of self-study.

Course materials and recommended reading

[1] Zienkiewicz, Olek C., Robert Leroy Taylor, and Jian Z. Zhu. The finite element method: its basis and fundamentals. Elsevier, 2005.
[2] Hughes, Thomas JR. The finite element method: linear static and dynamic finite element analysis. Courier Corporation, 2012.
[3] Reddy, Junuthula Narasimha. Introduction to the finite element method. McGraw-Hill Education, 2019.
[4] Belytschko, Ted, et al. Nonlinear finite elements for continua and structures. John Wiley & sons, 2014

Assessment and grading

Criteria for assessment of student performance, and the final score structure

Theoretical part: **30 points**
in the form of an oral exam (2 questions in the exam paper for 12 points and additional oral questions for 6 points)

Practical part of the course: **70 points**
The work is formed from 7 individually completed tasks that are presented and defended in practical classes. Each practical class is worth 10 points. Practical class №4 is not graded.
Completion of all practical assignments is a prerequisite for the theoretical colloquium at the exam.

Grading scale

Total points	National	ECTS
90–100	Excellent	A
82–89	Good	B
75–81	Good	C
64–74	Satisfactory	D
60–63	Satisfactory	E
35–59	Unsatisfactory (requires additional learning)	FX
1–34	Unsatisfactory (requires repetition of the course)	F

Norms of academic integrity and course policy

The student must adhere to the Code of Ethics of Academic Relations and Integrity of NTU "KhPI": to demonstrate discipline, good manners, kindness, honesty, and responsibility. Conflict situations should be openly discussed in academic groups with a lecturer, and if it is impossible to resolve the conflict, they should be brought to the attention of the Institute's management.
Regulatory and legal documents related to the implementation of the principles of academic integrity at NTU "KhPI" are available on the website: http://blogs.kpi.kharkov.ua/v2/nv/akademichna-dobrochesnist/.

Approval

Approved by	Date	Head of the department
	August 30, 2023	Oleksii VODKA
	Date August 30, 2023	Guarantor of the educational and professional program (1 year 4 months) Oleksiy LARIN