Syllabus
Course Program

Mathematical logic, theory of algorithms and data structures

Specialty
113 - Applied mathematics

Institute
Institute of Computer Modelling, Applied Physics and Mathematics

Educational program
Computer and mathematical modelling

Department
Computer modelling of processes and systems (162)

Level of education
Bachelor's degree

Course type
Special (professional), Compulsory

Semester
3

Language of instruction
English

Lecturers and course developers

First name and surname
oksana.tatarinova@khpi.edu.ua
Candidate of Technical Sciences, Associate Professor

Specialist in mathematical and computer modelling of nonlinear processes. Author of more than 60 scientific articles and conference papers, co-author of copyright certificates, monographs, and textbooks

More about the lecturer on the department's website

Yulia Andriyivna Vyazovychenko
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Candidate of Technical Sciences, Associate Professor

More about the lecturer on the department's website

General information

Summary

Mathematical Logic, Algorithmic and Data Structure Theory is a compulsory discipline in the applied mathematics curriculum that covers the basics of formal language, algorithmic procedures and data organisation. It provides a fundamental understanding of the mathematical mechanisms and strategies that underpin computer programming and computational analysis.
As part of the course, students study the basics of mathematical logic in depth, including the study of statements, predicates, logical operations and proofs. An important aspect of the course is the theory of

algorithms, which covers methods of designing, analysing and optimising algorithmic procedures. Students study classical algorithms, search and sorting algorithms, and modern approaches to algorithmic problems.
Special attention is paid to data structures, from simple arrays and lists to more complex structures such as trees, graphs, and hash tables.

Course objectives and goals

The aim of the course "Mathematical Logic, Algorithmic Theory and Data Structures" is to provide students with in-depth knowledge and practical skills in applying mathematical logic and information analysis methodology, as well as in studying algorithmic and structural approaches that are critical for designing, developing and evaluating the effectiveness of algorithms in applied mathematics.

Format of classes

Lectures, practical classes, independent work, consultations. The final control is an exam.

Competencies

GC06. Ability to think abstractly, analyse and synthesise.
PC04. Ability to develop algorithms and data structures, software tools and software documentation.

Learning outcomes

PO04. Perform mathematical description, analysis and synthesis of discrete objects and systems using the concepts and methods of discrete mathematics and algorithm theory.
PO05. Be able to develop and apply in practice algorithms related to the approximation of functional dependencies, numerical differentiation and integration, solving systems of algebraic, differential and integral equations, solving boundary value problems, finding optimal solutions.

Student workload

The total volume of the discipline is 150 hours (5 ECTS credits): lectures - 32 hours, practical classes - 32 hours, independent work - 86 hours.

Course prerequisites

The course requires knowledge and skills acquired in Linear Algebra to understand mathematical structures and methods used in algorithms and data structures. It is also important to understand concepts from Discrete Mathematics, especially in the context of logical structures and algorithmic principles.

Features of the course, teaching and learning methods, and technologies

Lectures are conducted interactively with the use of multimedia technologies, using a demonstrative and illustrative approach, analysis of specific examples, systematisation and generalisation of theoretical concepts, and critical thinking discussions. Practical classes use a partially searching method and a discussion method, with an emphasis on the application of practical problems in the field of applied mathematics. Group work and peer-to-peer learning are actively used in the individual assignment.

Learning materials are available to students on Microsoft OneDrive

Program of the course

Topics of the lectures

Topic 1. The number of statements

Subject, purpose, objectives and content of the course.
The alphabet of the logic of statements. Formulas of the logic of statements. The concept of a logically general formula (tautology), identical to false, executable, refutable formulas. Equivalence of formulas of logic of statements. Dual formulas. The problem of decidability in the logic of statements.

Different methods of proving the truth of formulas. The logical sequence. The method of resolutions. Formal theories. Alphabet and formulas of the number of statements. Axioms, theorems and rules for deriving the number of statements. Metatheorem of deduction. Completeness, decidability, consistency, independence of the system of axioms and rules for deriving the number of statements.

Topic 2. Calculus of predicates

The concept of a predicate. Operations on predicates. Quantifiers. Signature of the language. Terms. Decision problems in predicate logic. Logical following in predicate logic. Reduction of formulas to the Skolemian standard form. Unification of disjuncts. Universal unifier. The method of resolutions.
First-order language. Terms and formulas. Logical and special axioms. Rules of derivation. Examples of mathematical theories. Proofs in first-order theory. Deduction theorem.
A theorem on the non-contradiction of the enumeration of first-order predicates. Interpretations. Executability and truth. Models. Model isomorphism and categoricality. Completeness of the number of first-order predicates

Topic 3. Algorithm theory

Basic concepts, requirements for algorithms. Historical overview. Tasks of the theory of algorithms. Practical application of the results of the theory of algorithms. Formalisation of the concept of algorithm. Church's thesis, Turing's thesis, Markov's thesis.
Analysis of algorithms. Comparative evaluation of algorithms. Notation system in the analysis of algorithms. Classification of algorithms by type of labour intensity function. Asymptotic analysis of functions.
Labour intensity of algorithms and their time estimates. Elementary operations in the algorithmic language. Examples of analysis of simple algorithms. Transition to time estimates. An example of an operational time analysis.
Theories of computational complexity and classes of task complexity. Theoretical limit of task complexity. Classes of task complexity. The problem P = NP. Class NPC (NP-complete problems). Examples of NP-complete problems.
The task of sorting. Sorting by selection. Sorting by simple selection. Bubble sorting. Shell sorting. Sorting by inclusion. Sorting by simple inclusion. Tournament sorting. Sorting by an ordered binary tree. Sorting by distribution. Quick Hoare sort. Sorting by merging. Comparison of sorting algorithms by asymptotic complexity.
Sequential linear search. Binary search. Method of interpolation. Sequence search algorithms. Knuth, Morris, Pratt algorithm. Bowyer and Moore's algorithm.

Topic 4. Data structures

The concept of data structure. Representation of data in computer systems. Classification of structures. Operations on structures. Data structure and programming technology.
Elementary data types. Data of numeric types (integer, real). Symbolic and logical data types. Data of the pointer type. Linear data structures: array, string, structures, sets, tables. Linear lists: unidirectional, bidirectional, cyclic. Sparse matrices. Stack. Queue. Deque. Operations on data.
Tables with direct addressing. Hash tables. Avoiding collisions with chains
(open hashing). Hash functions. Open addressing (closed hashing).
Binary trees. Ordered binary trees. Building a tree. Traversing the tree. Searching in a tree. Sorting using a tree. Ways to represent binary trees: using arrays, nonlinear branched lists.
Application of binary trees to search. Perfectly balanced binary trees. ABL trees. Inserting and removing nodes in AVL trees. Optimal search trees. Red and black trees.

Topics of the workshops

Topic 1. Counting statements.

Algebra of statements.
Logic of statements.
Calculus of statements.
Withdrawal rules.

Topic 2. Calculus of predicates.

Algebra of predicates.
The logic of predicates.
Calculus of predicates.
The resolution method

Topic 3. Theory of algorithms and data structures.
Primitive recursive functions.
Building Turing machine programs.
Normal algorithms.
Analysis of the time complexity of simple and sorting algorithms and their comparative efficiency.
Analysing search algorithms based on different input conditions.

Topics of the laboratory classes

Not covered by the curriculum

Self-study

The individual task is intended to consolidate the knowledge, skills and abilities acquired by students in the course of mastering the lecture material of the course, in practical classes and laboratory work. It is performed by a group of 3 students. The structure of the task has three parts, according to which the roles in the group are distributed - mathematical (mathematician), algorithmic (algorithmist) and software (programmer). To pass the task, a public defence is held.
Independent study of topics and issues that are not covered in lectures:
1. Higher-order logic.
2. Unconventional logic.
3. The algorithm for ordering inserts with a watchdog element.
4. 4. The ordering algorithm by counting.
5. Bitwise ordering algorithm.
6. Working with multi-lists.
7. Dictionaries as a data structure.
8. Fundamental algorithms on graphs and trees. Array ordering with a binary tree.
Graph processing tasks. Methods and algorithms for graph traversal: DSF-method, BFS-method.
Determination of the characteristics of graph traversal algorithms, their analysis.

Course materials and recommended reading

1. Matvienko M.P. Mathematical logic and theory of algorithms. Study guide. / M.P. Matvienko, S.P. Shapovalov. - K.: Lira-K Publishing House, 2021. - 212 p.
2. Zubenko V.V. Fundamentals of mathematical logic: a textbook / V.V. Zubenko, S.S. Shkilnyak. - Kyiv: NUBiP of Ukraine, 2020. 102 p.
3. Algorithms and data structures. Textbook. - Kyiv: VPC "Kyiv University", 2021. - 200 p.
4. Shakhovska N.B. Algorithms and data structures. - 2021. - 216 c.
5. Mathematical logic, theory of algorithms and data structures [Electronic resource]: methodical instructions for students of speciality 122 "Computer Science" / compiled by A. O. Tatarinova, Y. M. Andreev ; National Technical University "Kharkiv Polytechnic Institute." - Electronic text data - Kharkiv, 2023. - 104 p. - https://repository.kpi.kharkov.ua/handle/KhPI-Press/66909.

Assessment and grading

Criteria for assessment of student performance, and the final score structure

Semester control is carried out orally on the basis of examination tickets. The results of the current control are taken into account as auxiliary information for assigning a grade in this discipline. A student is considered admitted to the semester exam if he or she has defended all practical works and individual assignments. The exam is compulsory. Points are awarded as follows:
- exam - 100% of the semester grade

Grading scale

Total points	National	ECTS
90–100	Excellent	A
82–89	Good	B
75–81	Good	C
64–74	Satisfactory	D
60–63	Satisfactory	E
35–59	Unsatisfactory (requires additional learning)	FX
1–34	Unsatisfactory (requires repetition of the course)	F

Norms of academic integrity and course policy

The student must adhere to the Code of Ethics of Academic Relations and Integrity of NTU "KhPI": to demonstrate discipline, good manners, kindness, honesty, and responsibility. Conflict situations should be openly discussed in academic groups with a lecturer, and if it is impossible to resolve the conflict, they should be brought to the attention of the Institute's management.
Regulatory and legal documents related to the implementation of the principles of academic integrity at NTU "KhPI" are available on the website: http://blogs.kpi.kharkov.ua/v2/nv/akademichna-dobrochesnist/

Approval

Approved by	Date, signature	Head of the department Dmitry BRESLAVSKY
	Date, signature	Guarantor of the educational program Gennadiy LVOV