Syllabus

Course Program

Discrete mathematics

Specialty
113 - Applied mathematics

Institute
Institute of Computer Modeling, Applied Physics and Mathematics

Educational program
Computer and mathematical modelling

Department
Computer modelling of processes and systems (162)

Level of education
Bachelor's degree

Course type
Special (professional), Compulsory

Semester
2

Language of instruction
English

Lecturers and course developers

Oksana Tatarinova

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Candidate of Technical Sciences, Associate Professor

Specialist in mathematical and computer modelling of nonlinear processes. Author of more than 60 scientific articles and conference papers, co-author of copyright certificates, monographs, and textbooks

More about the lecturer on the department's website

Viktor Alexandrovich Fedorov

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Candidate of Technical Sciences, Associate Professor

General information, number of publications, main courses, etc.

More about the lecturer on the department's website

General information

Summary

Discrete mathematics is a fundamental discipline in the computer science curriculum that focuses on the study of discrete mathematical structures and algorithms. It includes such topics as set theory, relation theory, Boolean algebra, graph theory, and is important for understanding the concepts underlying applied mathematics, data analysis, and practical problem solving.

Course objectives and goals

Formation of skills in modelling computer processes using discrete mathematical objects and development of the ability to analyse and determine the properties of discrete mathematical structures that are key in the field of mathematics. The main focus is on the formation of a scientific outlook on discrete mathematics as an instrumental basis for the analysis, modelling and optimisation of complex systems and processes.

Format of classes

Lectures, practical classes, independent work, consultations. The final control is an exam.

Competencies

GC06. Ability to think abstractly, analyse and synthesise.
PC01. Ability to use and adapt mathematical theories, methods and techniques to prove mathematical statements and theorems.

Learning outcomes

ELO01. Demonstrate knowledge and understanding of the basic concepts, principles, theories of applied mathematics and apply them in practice.
PO04. Perform mathematical description, analysis and synthesis of discrete objects and systems using the concepts and methods of discrete mathematics and algorithm theory.

Student workload

The total volume of the discipline is 180 hours (6 ECTS credits): lectures - 32 hours, practical classes - 48 hours, independent work - 100 hours.

Course prerequisites

Knowledge acquired in the disciplines Mathematical Analysis, Analytical Geometry, Linear Algebra to the extent sufficient to perceive the categorical apparatus of models and methods of discrete mathematics, as well as C++ programming skills to perform an individual task acquired in the discipline Algorithmisation and Programming.

Features of the course, teaching and learning methods, and technologies

Lectures are conducted interactively with the use of multimedia technologies, using a demonstration and illustration approach, case study analysis, systematisation and generalisation of theoretical concepts, and critical thinking discussions. Practical classes use a partially searching and discussion method, with an emphasis on the application of practical tasks in the field of computer science.
Teaching materials are available to students on Microsoft OneDrive.

Program of the course

Topics of the lectures

Topic 1. Set theory

History and problems of discrete mathematics. The concept of a set. Elements of a set. Ways to define sets. Sets and subsets. The set of subsets.
Operations on sets The power of sets. Inclusion and exclusion formulas.
Properties of operations on sets
Methods for solving equations with sets. Cartesian product of sets

Topic 2. Relationship theory

Binary relationships. Ways to set relationships. Operations on binary relationships. Properties of relations
Functional relationships. Mapping
The most common binary relations. Equivalence and tolerance relations

Topic 3. Boolean algebra

The emergence of Boolean functions. Table assignment of Boolean functions. Boolean functions of one variable. Boolean functions of two variables Elementary Boolean functions and their properties.
Geometric problem of Boolean functions. Boolean cube. Carrier of a Boolean function and its properties.
Special Boolean functions. Elementary conjunction. Disjunctive normal form (DNF). Elementary disjunction. Conjunctive normal form (CNF). DNF and CNF.
Properties of operations on Boolean functions. The completeness of the system of Boolean functions. Monotonically increasing Boolean functions. Post's classes. Post's criterion
Minimisation of Boolean functions in the class of DNFs. Minimisation of Boolean functions using the Quine-McCluskey, uncertain coefficients, and Blake algorithms
Implicants of Boolean functions and their properties. Reduced and minimal Boolean functions and their properties. Minimisation of Boolean functions using Carnot maps. Minimisation of Boolean functions in the class of KNFs

Topic 4. Graph theory

Basic concepts of graph theory. Connection of graph theory with the subject area. . Ways to define graphs. Adjacency and incidence matrices, their properties.
The main parts of the graph. Routes, paths, chains, cycles.
Graph connectivity. Identification of components of strong connectivity. Construction of a reachability matrix: determination of the availability of paths of a given length, the Warshall algorithm.
Operations on graphs. Metric characteristics of graphs. Isomorphism of graphs.
Types of graphs. Eulerian and Hamiltonian graphs. Flory and H. Tui's algorithms for finding an Eulerian cycle
Definition of a tree and a forest. Spanning tree, cyclic rank. Construction of the shortest spanning tree: Prim and Kraskal algorithms. Steiner's problem.
Strategies for traversing the graph in depth and in width. Determining the number of paths of a given length. Shortest and minimum paths in a loaded graph. Finding the shortest path: wave algorithm. Finding the minimum path: Dijkstra, Ford, and Bellman-Moore algorithms.
Flows in networks, algorithm for constructing a flow. The Ford-Falkerson theorem and the algorithm for constructing the maximum flow. Construction of a minimum cost flow: Ford-Falkerson algorithm, algorithms based on the selection of cycles of negative weight and on the search for a minimum path.
Graph stacking, the graph stacking theorem in three-dimensional space. Flat and planar graphs, theorem about graphs K5 and K3,3. Algorithms for graph layout on the plane

Topics of the workshops

Topic 1. Set theory

Ways to define sets. Set of subsets
Operations on sets Inclusion and exclusion formulas.
Proof of identities using the properties of set algebra. Cartesian product

Topic 2. Relationship theory

Relationship theory.

Topic 3. Boolean algebra

Identical transformations. Boolean cube. Construction of DNF and KNF
Dual Boolean functions. Properties of operations on Boolean functions. Construction of Boolean functions and Boolean functions
The completeness of the system of Boolean functions. Monotonically increasing Boolean functions. Post's classes. Post's criterion.
Minimisation of Boolean functions using the Quine-McCluskey algorithm, uncertain coefficients.
Minimisation of Boolean functions using the Blake algorithm, Carnot maps in the class of DNFs and KNFs.

Topic 4. Graph theory
Graphs, their classification and methods of assignment. Connectivity of undirected graphs. Connectivity of graphs. The game "Two bishops' advantage"
Operations on graphs. Metric characteristics of graphs. Isomorphism of graphs
Search for Eulerian and Hamiltonian paths (cycles)
Construction of the shortest spine: Prim and Kraskal algorithms
Minimum path search: Dijkstra, Ford and Bellman-Moore algorithms.
Building a minimum cost stream: the Ford-Falkerson algorithm
Algorithms for stacking a graph on a plane.

Topics of the laboratory classes

Not covered by the curriculum

Self-study

Individual assignment is designed to consolidate the knowledge, skills and abilities acquired by students in the course of mastering the lecture material of the course and in practical classes. It is issued after completing the topic "Boolean Algebra" and consists of three parts. The first part is devoted to the basic concepts of Boolean algebra, the second part - to the methods of minimising Boolean functions, and the third part - to writing Boolean algebra programs.
Independent study of topics and issues that are not covered in lectures:
1. The power of a set. The concept of countable set and continuum. Cantor's diagonal procedure. Examples of countable sets. Proof of countability of a set of algebraic numbers. Properties of countable sets. Necessary and sufficient conditions for infinity of a set. Examples of continuous sets. The Cantor-Bernstein theorem. Proof of the existence of irrational and transcendental numbers. Cardinal operations on sets.
2. Transient short circuit. Algorithms for finding a transient circuit. The problem of minimum coverage. Algorithms for solving the minimum coverage problem.
3. Proof of the Post criterion.
4. Application of the functions of logic algebra to the analysis and synthesis of relay-contact circuits
5. Methods for minimising partially specified Boolean functions
6. Separating set, cut, bridge. Measures of connectivity: vertex and edge connectivity. Menger's theorem in vertex form. Estimating the number of edges in a simple graph.
7. The salesman's problem. The method of branches and boundaries for solving the traveling salesman problem. Some heuristic algorithms: nearest neighbour, nearest insertion, local optimisation, Euler, Christofides
8. Finding k minimum paths: the double search algorithm. Finding k minimal paths between all pairs of vertices: generalised Floyd and Danzig algorithms. Finding k simple minimum paths: Yen's algorithm. Modification of search algorithms for graphs without contours. Latin properties, searching for paths with given properties by the method of Latin composition.

Course materials and recommended reading

1. Журавчак Л.М. Дискретна математика для програмістів :навчальний посібник / Л.М. Журавчак, Н.І. Мельникова, П.В. Сердюк ; Міністерство освіти і науки України, Національний університет "Львівська політехніка". - Львів: Видавництво Львівської політехніки, 2019.- 417 с.
2. Журавчак Л.М. Практикум з комп'ютерної дискретної математики: навчальний посібник / Л.М. Журавчак, Н.І. Мельникова, П.В. Сердюк ; Міністерство освіти і науки України, Національний університет "Львівська політехніка". - Львів: Видавництво Львівської політехніки, 2020.- 313 с.

3. Гнатів Б. В. Дискретна математика. / Б. В. Гнатів, В. Р. Гладун, Л. Б. Гнатів. – Львів: Видавництво Львівської політехніки, 2021. – 400 с.
4. Висоцька В.А. Дискретна математика: практикум (збірник задач з дискретної математики): навчальний посібник / В.А. Висоцька, В.В. Литвин, О.В. Лозинська; Міністерство освіти і науки України, Національний університет "Львівська політехніка". - Львів: "Новий світ-2000", 2020. – 575с.

Assessment and grading

Criteria for assessment of student performance, and the final score structure

Semester control is carried out orally on the basis of examination tickets. The results of the current control are taken into account as auxiliary information for assigning a grade in this discipline. A student is considered admitted to the semester exam if he/she has passed all control papers (tests) and individual assignments. If a control work (test) is missed for a valid reason, it can be written without reducing the number of points for it. The exam is compulsory.
Points are awarded as follows:
-Exam - 100% of the semester grade

Grading scale

Total points	National	ECTS
90–100	Excellent	A
82–89	Good	B
75–81	Good	C
64–74	Satisfactory	D
60–63	Satisfactory	E
35–59	Unsatisfactory (requires additional learning)	FX
1–34	Unsatisfactory (requires repetition of the course)	F

Norms of academic integrity and course policy

The student must adhere to the Code of Ethics of Academic Relations and Integrity of NTU "KhPI": show discipline, good manners, goodwill, honesty, responsibility. Conflict situations should be openly discussed in study groups with the teacher, and if it is impossible to resolve the conflict, they should be brought to the attention of the staff of the Institute's directorate.
Regulatory and legal support for the implementation of the principles of academic integrity of NTU "KhPI" is available on the website: http://blogs.kpi.kharkov.ua/v2/nv/akademichna-dobrochesnist/

Approval

Approved by	Date, signature	Head of the department Dmitry BRESLAVSKY
	Date, signature	Guarantor of the educational program Gennadiy LVOV