Analysis of the Electromagnetic Field of Electric Machines Based on Object-oriented Design Principles

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Abstract - The authors propose to work out common principles of synthesis of electromagnetic field models for the main types of electrical machines. The main objective is to create field models of electric machines by inheritance of their characters from the original Maxwell's equations. An inverse problem also is solving - the synthesis of electric machines electromagnetic field equations using object-oriented design.

1. INTRODUCTION

Object-oriented design (OOD) implements the idea of solving problems using models based on the concepts of the real world. Fundamental element is an object that combines the data structure (parameters) to the behavior (calculating procedures). Philosophy of knowledge representation in the form of objects of prisoners in their data and procedures developed by G. Booch, but so far only been applied in the field of software development, as well as an ideological foundation of programming languages such as Java, C++, C# , Visual Basic, Delphi [1].

Consideration of OOD principles for the design and modeling of electrical machines (EM) at all stages, starting from analysis and finishing with the project implementation, will enable a new approach to the EM design methodology and apply object-oriented theory, which revolutionized the information technology in the field of electrical engineering, which will significantly improve the efficiency and quality of EM design.

We go out, assumes a basic class of generalized EM, descendants of which are well-known types of EM. By inheritance, using OOD principles, added or cut off those features which lead to the synthesis of a specific model of EM.

Any EM can be described by Maxwell's equations. Select by equations features of a certain type of EM it is possible to form a hierarchical structure of species and thus obtain the desired EM model species level. The resulting model, inherited from the base class of Maxwell's equations, having the status of nature law, will adequately reflect the specific features of EM [2].

2 ELECTROMAGNETIC FIELD EQUATIONS ANALYSIS OF AN INDUCTION MOTOR WITH SQUIRREL-CAGE ROTOR

As an example we shall describe obtaining electromagnetic field equations of an induction motor with squirrel-cage rotor (IM with SCR) through inheritance from equations of generalized EM. As the base class of generalized EM, in terms of OOD accepted Maxwell's equations of classical electrodynamics [3]:

$$\begin{cases} rot\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}, \\ div\vec{D} = \rho, \\ rot\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}, \\ div\vec{B} = 0, \end{cases} \tag{1}$$

In (1) vectors of the electric $\overline{E}$ and magnetic $\overline{H}$ fields are connected by following relations:

$$\vec{D} = \varepsilon\varepsilon_0\vec{E}\,,\ \vec{B} = \mu\mu_0\vec{H}\,,\ \vec{J} = \sigma\vec{E}\,, \tag{2}$$

with the vectors of the electric $\vec{D}$ and magnetic $\vec{B}$ inductions, the vector of the electric current density $\vec{J}$, which represent the response of the environment for the presence of electromagnetic fields. Accordingly, ρ -

the bulk density of the third-party charge; ε_0 and μ_0 - electric and magnetic constants; σ- the specific electrical conductivity; relative permittivity ε and magnetic permeability μ of the environment.

The mathematical description of the magnetic field of IM with SCR is making in a cylindrical coordinate system (R, φ, Z). To simplify the mathematical description standard assumptions and restrictions was made [4].

Performing mathematical transformations and projecting the resulting expression for the coordinate axis Z, we obtain the equation for the magnetic field of the ideal idling mode (rotor winding are open or have not current flow) in the area $R_{in} \leq R \leq R_{out}$; $0 \leq \varphi \leq 2\pi$:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{1}{\mu}\frac{\partial A}{\partial z}\right)+\frac{1}{R^2}\frac{\partial}{\partial \varphi}\left(\frac{1}{\mu}\frac{\partial A}{\partial \varphi}\right)=-J_{st}, \tag{3}$$

where R_{in}, R_{out} – the inner and outer radius of the IM with SCR stator accordingly.

The equation of the vector potential in the gap of the machine has the following form:

$$rot\left(\frac{1}{\mu}rot\vec{A}\right)=\vec{J}_{st}+\vec{J}_{rot}. \tag{4}$$

Performing operations *rot*, converting the resulting expression like (3) and projecting equation on the coordinate axis Z, we obtain:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{1}{\mu}\frac{\partial A_z}{\partial z}\right)+\frac{1}{R^2}\frac{\partial}{\partial \varphi}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial \varphi}\right)=-J_{st}-J_{rot}. \tag{5}$$

The first term of this expression shows the distribution of the tangential component of the magnetic induction B_φ in the radial coordinate. The tangential component B_φ takes place in the stator yoke and the rotor, and also in stator and rotor slots defining there leakage fluxes. Represent this component as the sum (here and in the further indexes of the only component of current density and vector potential will be omitted)

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{1}{\mu}\frac{\partial A}{\partial R}\right)=\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{1}{\mu}\frac{\partial A}{\partial R}\right)\right]_a+\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{1}{\mu}\frac{\partial A}{\partial R}\right)\right]_\sigma, \tag{6}$$

first term represents the distribution of the magnetic induction in the yoke of the machine, and the second - the rotor leakage currents.

The first component (3) after transformation can be represented as

$$\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{1}{\mu}\frac{\partial A}{\partial R}\right)\right]_a=-qA. \tag{7}$$

Leakage flux in the rotor slots is proportional to the rotor current. Therefore, the second component (3) can be written as:

$$\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{1}{\mu}\frac{\partial A}{\partial R}\right)\right]_\sigma=K_{\sigma 2}J_{rot}. \tag{8}$$

$K_{\sigma 2}$ factor in this expression can be regarded as the ratio of rotor inductive reactance to resistance of magnetizing circuit $X_{\sigma 2}/X_\mu$. Substituting (7) and (8) into (5):

$$\frac{1}{R_0^2}\frac{\partial^2 A}{\partial \varphi^2}-qA=-\mu_0 J_{st}-\mu_0(1+K_{\sigma 2})J_{rot}. \tag{9}$$

Thus, simple equations for the vector magnetic potential, taking into account the influence of the magnetic resistance of the yoke of the stator and rotor and stator leakage is obtained. The solution of this equation together with boundary conditions of periodic type allows to specify the value of the vector potential in the gap of IM, radial and tangential components of the magnetic induction.

3 SYNTHESIS OF THE GENERALIZED MODEL OF ELECTRIC MACHINES

Electromagnetic field equations of non-considered types of EM can be obtained by the same way as was described for IM with SCR. In further we can make conclusions which will help to systematize common features of their equations and synthesize equations of any EM (descendant classes) from the base equations of generalized EM (parent class). From analysis (Tab. 1) of the electromagnetic field equations, written for some types of cylindrical machines, we can retrieve common features [5].

Electric Machine	Left side of equation	Right side of equation
IM with SCR	$\frac{1}{R_0^2}\frac{\partial^2 A}{\partial \varphi^2} - qA$	$-\mu_0 J_{st} - \mu_0 (1 + K_{\sigma 2}) J_{rot}$
IM with phase rotor	$\frac{1}{R_0^2}\frac{\partial^2 A}{\partial \varphi^2} - qA$	$-\mu_0 J_{st} - \mu_0 (1 + K_{\sigma 2}) J_{rot}$
Condensing IM	$\frac{1}{R_0^2}\frac{\partial^2 A}{\partial \varphi^2} - qA$	$-\mu_0 J_{work} - \mu_0 J_{cond} - \mu_0 J_{rot}$
IM with ferromagnetic rotor	$\frac{1}{R_0^2}\frac{\partial^2 A}{\partial \varphi^2} - qA$	$-\mu_0 (J_{st} + J_{rot})$
DC motor	$\frac{1}{R_0^2}\frac{\partial^2 A}{\partial \varphi^2} - qA$	$-\mu_0 (J_{exc} + J_{arm} + J_{comp} + J_{add.pole})$
Synchronous generator	$\frac{1}{R_0^2}\frac{\partial^2 A}{\partial \varphi^2} - qA$	$-\mu_0 (J_{exc} + J_{arm})$

Table 1. Maxwell's equations for cylindrical EM

According to Tab. 1, we can make the following conclusions:

1) primary (stator) circuit is common for different EM and determined by the topology of cylindrical machines;

2) the power supply system (right side of the equations) does not affect the matrix calculation model;

3) secondary (rotor) circuit is determined by specific characters of EM.

We can apply the principles of OOD to obtain the equations of the electromagnetic field for different types of EM. The base class will represent the system of Maxwell's equations (1), which generalizes the representation of the electromagnetic field distribution in an electromechanical system.

This class is abstract because it is impossible the existence of such a product as a generalized electric machine. A descendant of the base class is a concrete class of EM obtained by inheritance of certain characteristics from the base class. The decision of what kind of signs should inherit depending on the set of conditions, decides subclass of electromechanical object modifiers.

The result of this inheritance, taking into account the conditions and modifiers will be a concrete embodiment of EM, ready to create an object - a physical model, endowed with parameters, energy sources, geometric design.

The classes diagram (Fig. 1), developed with UML (Unified Modeling Language) blocks [6], show the hierarchical structure of the electromagnetic field equations for EM. Here, the block *attribute* is a system of Maxwell's equations for the generalized EM. The class is abstract because there is no real performance of the machine "in general". Classical system of field equations, common for all types of EM, refined by blocks located below the block "*Abstract Class*".

Block *attributes* is a set of parameters, equations, symbolic expressions or logical conditions. Class "*Machine*" is a descendant of EM "*AbstractClass*" and its concrete implementation.

The block attribute of this class have modifiers include boundary and environment conditions. For a two-dimensional model, this will be assumptions, the estimated radius of the circle and environment the permeability.

Block "*Current Density*" is responsible for the power supply primary and secondary circuits of the EM.

Related blocks "*Function*" and "*Event*" is a modifier of rotation of the secondary element and the power supply system. Finally, "*arguments set*" with "*associations subset*" in blocks "*Selected Item*" and "*Item Set*" forms requirements to the rotor circuit of block "*Machine*".

Using a template class of generalized EM (Fig. 1) and selecting certain features, we can go to an object of a particular EM. Thus, the UML-diagram of IM with SCR (Fig. 2) is an object structure generated from the class diagram.

Let's consider how on UML-diagram (Fig. 2) making synthesis of the electromagnetic field equations for IM with CDR.

Block 1 contains the basic system of Maxwell's equations, from which, under certain assumptions, we can obtain the equation of any EM. Below this block is written the name of the machine-descendant in order to indicate the direction of the inheritance of attributes and properties of the base machine.

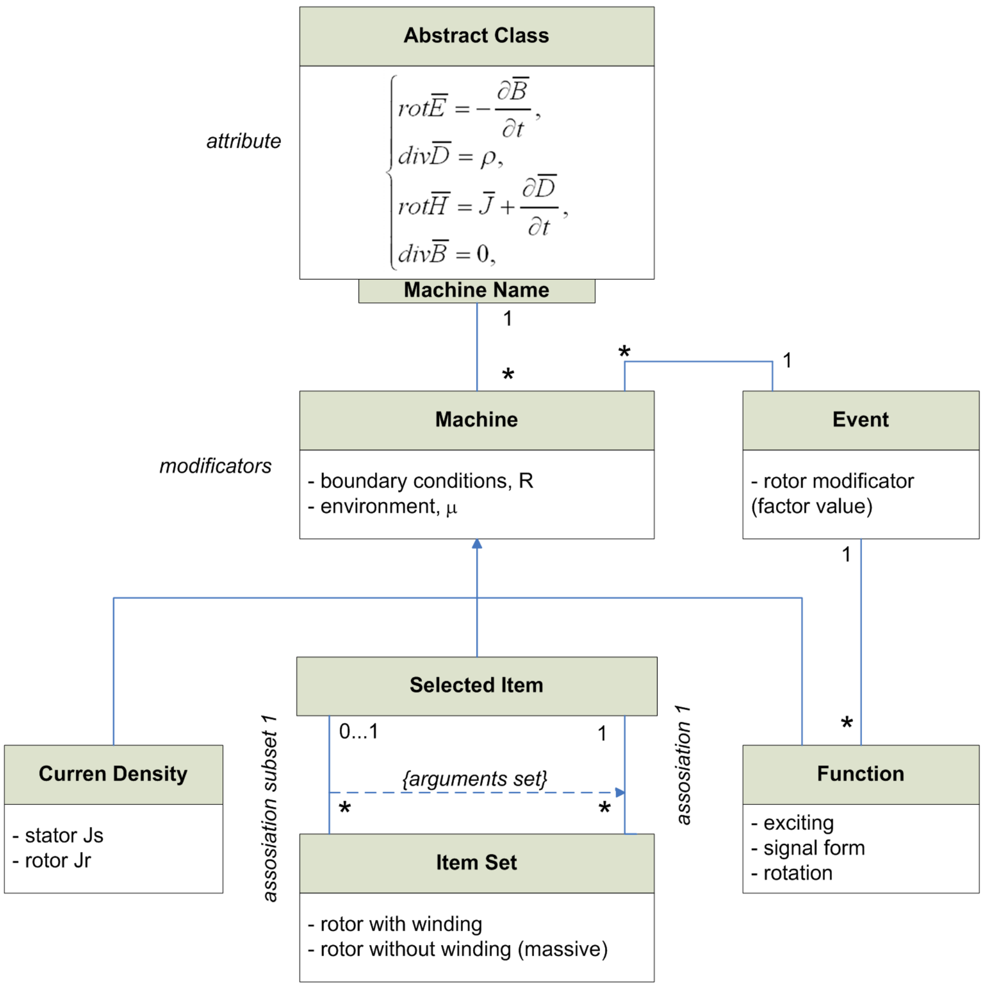

Figure 1. The UML class diagram based on the EM field equations

Block 2 represents the left side of Maxwell's equations (Tab. 1), which also indicates the specific values of resistance and magnetic permeability of the environment. In this example field calculation environment is the area of the IM air gap. Modifying the geometry of the block is adaptive and allows to apply an arbitrary grid computational domain, described by the equations, invariant to the type of EM.

Modifier of parameters required for consideration of the physical properties of the object, as well as to select a region in the case of simulation of one-dimensional models.

Block 3 and *Block 4* defines a set of parameters of the EM, as well as the type of rotor. In this case - a rotor winding with its corresponding parameters.

Block 5 allows to set the current density in the stator windings and in the rotor (the right side of the equations system). Current modifier defines a set of current densities of the stator and rotor, which is the source of the electromagnetic field.

Block 6 contains factors influencing to rotor current density on the right side of the equation. Also this block activates the logical event responsible for the rotor rotation.

Block 7 specifies the character of the IM with SCR stator and rotor excitations. Thus, there is no excitation for SCR winding and the stator winding is given by the harmonic character of the current change. Also, from the output of Block 7 to the input of Block 6 transmitted an event flag of rotor rotation.

Thus, on the output of the class generator we obtain the equations of the electromagnetic field of the IM that have been selected by modifiers.

Different combinations of current sources and the laws of their changes allow to obtain models for various types of EM. Object structure (Fig. 2) is a direct implementation of IM with CDR form the abstract pattern (Fig. 1). Similarly, it is possible to generate object models for all known types of EM, and in first approach for EM have not yet created, giving them new features.

Moreover, the presence of object-oriented model eliminates the need to perform preparatory operations for the transition from theoretical studies to practical realization of the model in numerical form.

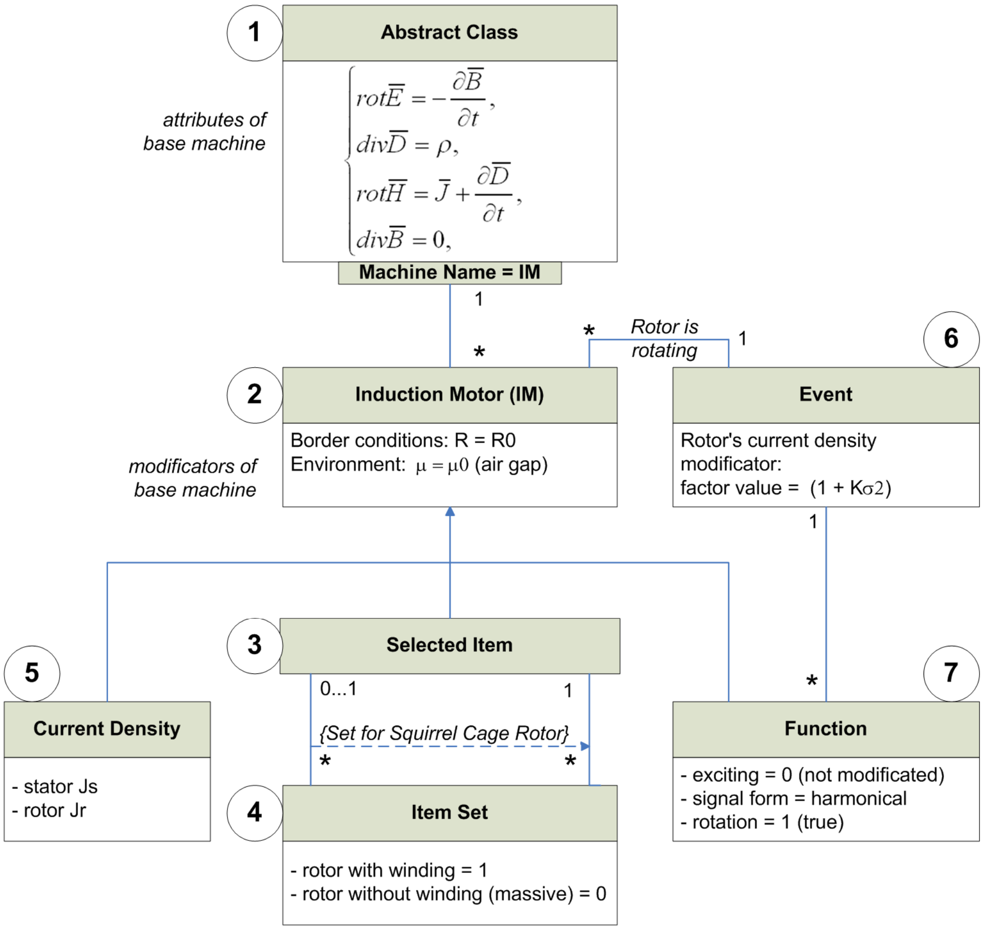

Figure 2. The UML object diagram of an IM with SCR based on field equations

Represented object-oriented model was used for automatic synthesis of equations for EM (Tab. 1) during electromagnetic field analysis in developed CAD program and for the forming methodology of EM classification [2], [5].

4. CONCLUSIONS

Applying the principles of object-oriented design model allowed to form the classification structure of different types of electrical machines and practically used in electric machines simulation in developed CAD program. The method of structural and system organization of electric machines design and mathematical modeling based on the inheritance of the design characteristics of their derived classes at the same time with the evolutionary synthesis of their mathematical representation was proposed. This makes possible not only to perform the systematization of electric machines, but also using the principles of object-oriented design, to generate mathematical models and design methods for electric machines automatically in CAD program.

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