SCATTERING MATRIX APPROACH

TO NON-STATIONARY QUANTUM TRANSPORT

(Lecture notes)

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The plan of the course:

Lecture 1 (**20 May 2010**): Chapter 1, Section 2.2

Lecture 2 (**27 May 2010**): Chapter 3

Lecture 3 (**03 June 2010**): Chapters 4, 5, Sections 6.2, 6.3.2

Lecture 4 (**17 June 2010**): Appendix B

Other chapters are given for completeness.

The aim of this course is to present the basic elements of the scattering matrix approach to transport phenomena in dynamical quantum systems of non-interacting electrons. In particular, the generation and manipulation of a flow of separate electrons by the periodically driven mesoscopic systems, which was realized experimentally, and the energetics of such systems will be presented.

Contents

Chapter 1

Landauer-Büttiker formalism

The Landauer-Büttiker approach [1, 2, 3, 4, 5, 6] to transport phenomena in mesoscopic [7, 8] conducting systems consists in treating of a propagation of electrons through the system as a quantum-mechanical scattering problem. The mesoscopic system is assumed to be connected to macroscopic contacts playing a role of electron reservoirs. The contacts are source of equilibrium particles which are scattered by the mesoscopic sample. After scattering electrons return to the same or to the different contact. Thus the problem of calculating of such transport characteristics as, for example, an electrical conductivity or a thermal conductivity is reduced to solving of a quantum-mechanical scattering problem with a potential profile corresponding to a sample under consideration. All information concerning the transport properties of a sample is encoded in its scattering matrix, $\hat{S}$.

We concentrate on a single-particle scattering matrix but the multi-particle scattering matrix is also can be introduced. Thus we neglect electron-electron interactions and use the Schrödinger equation for spinless electrons as a basic equation. In principle interactions can be easily incorporated on the mean-field level.

1.1 Scattering matrix

Accordingly to the quantum mechanics an electron (or more precisely its state) is characterized by the wave function, $\Psi(t, \mathbf{r})$, dependent on a time t and on a spatial coordinate $\mathbf{r}$. If the wave function, $\Psi^{(in)}$, for an electron incident to the scatterer is known then solving the Schrödinger equation one can calculate the wave function, $\Psi^{(out)}$, for a scattered electron.

In principle one can prepare an electron in different initial states, $\Psi_j^{(in)}$. Therefore, one can ask whether we need to solve the Schrödinger equation for each $\Psi_j^{(in)}$. The answer is no. It is enough to solve the scattering problem for electrons in any of states $\psi_\alpha^{(in)}$ constituting the full orthonormal basis. After that using the superposition principle one can find the scattering state for an electron in an arbitrary initial state.

To this end we expand an incident electron wave function, $\Psi^{(in)}$, into the series in the basis functions $\psi_\alpha^{(in)}$,

$$\Psi^{(in)} = \sum_\alpha a_\alpha \, \psi_\alpha^{(in)} . \tag{1.1}$$

Then we expand a wave function for scattered electron, $\Psi^{(out)}$, into the series in the basis functions $\psi_\alpha^{(out)}$,

$$\Psi^{(out)} = \sum_\beta b_\beta \, \psi_\beta^{(out)} . \tag{1.2}$$

The set of functions $\psi_\alpha^{(in)}$ and $\psi_\beta^{(out)}$ constitutes the full orthonormal basis.

The problem is to find the coefficients b_β if the set of coefficients a_α is known. First we consider an auxiliary problem: Scattering of an electron with initial state $\Psi_1^{(in)} = \psi_1^{(in)}$. In this case the set of coefficients in Eq. (1.1) is the following: $(1, 0, 0, \dots)$. The solution for this scattering problem we write as Eq. (1.2) with coefficients $S_{\beta 1}$,

$$\Psi_1^{(out)} = \sum_\beta S_{\beta 1} \, \psi_\beta^{(out)} . \tag{1.3}$$

The coefficient $S_{\beta 1}$ is a quantum-mechanical transition amplitude from the initial state $\psi_1^{(in)}$ to the final state $\psi_\beta^{(out)}$. Note if the initial wave function is multiplied by the some constant factor A then the wave function for the scattered state is also multiplied by the same factor,

$$\Psi_1^{(in)} = A \, \psi_1^{(in)} \quad \Rightarrow \quad \Psi_1^{(out)} = A \sum_\beta S_{\beta 1} \, \psi_\beta^{(out)} . \tag{1.4}$$

After solving the scattering problem with initial state $\Psi_\gamma^{(in)} = \psi_\gamma^{(in)}$ we find the coefficients $S_{\beta\gamma}$,

$$\Psi_\gamma^{(out)} = \sum_\beta S_{\beta\gamma} \, \psi_\beta^{(out)} . \tag{1.5}$$

With coefficients $S_{\alpha\beta}$ we can solve the scattering problem with arbitrary initial state. Formally the corresponding algorithm is the following:

1. The wave function for an initial state is expanded into the series in basis functions $\psi_\alpha^{(in)}$, Eq. (1.1).

2. The scattered state wave function, $\Psi^{(out)}$, is represented as the sum of partial contributions, $\Psi_\alpha^{(out)}$, due to scattering of partial initial waves, $\Psi_\alpha^{(in)} = a_\alpha \psi_\alpha^{(in)}$,

$$\Psi^{(out)} = \sum_\alpha \Psi_\alpha^{(out)},$$
$$\Psi_\alpha^{(out)} = a_\alpha \sum_\beta S_{\beta\alpha} \psi_\beta^{(out)} . \tag{1.6}$$

3. The coefficients b_β for the scattered state of interest, $\Psi^{(out)} = \sum_\alpha a_\alpha \sum_\beta S_{\beta\alpha} \psi_\beta^{(out)} \equiv \sum_\beta b_\beta \psi_\beta^{(out)}$, are the following,

$$b_\beta = \sum_\alpha S_{\beta\alpha} a_\alpha . \tag{1.7}$$

The equation (1.7) solves the problem: It expresses the coefficients b_β for scattered wave function in terms of the coefficients a_α for incident wave function. It is convenient to treat the quantities, $S_{\beta\alpha}$, entering Eq. (1.7) as elements of some matrix, $\hat{S}$, which is referred to as *the scattering matrix*.

If the coefficients a_α and b_β are collected into the vector-columns,

$$\hat{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \end{pmatrix}, \quad \hat{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix}, \tag{1.8}$$

then the corresponding relations becomes short,

$$\hat{b} = \hat{S} \hat{a} . \tag{1.9}$$

As we already mentioned, the scattering matrix elements, $S_{\alpha\beta}$, are quantum-mechanical amplitudes for a particle in the state $\psi_\beta^{(in)}$ to pass into the state $\psi_\alpha^{(out)}$. The order of indices is important. We use such a convention that the first index (for the element $S_{\alpha\beta}$ it is an index α) corresponds to the final state while the second index corresponds to the initial state.

1.1.1 Scattering matrix properties

The general physical principles put some constraints onto the scattering matrix elements.

1.1.1.1 Unitarity

The particle number conservation during scattering requires the scattering matrix to be unitary,

$$\hat{S}^{\dagger}\hat{S} = \hat{S}\hat{S}^{\dagger} = \hat{I}\,. \tag{1.10}$$

Here $\hat{I}$ is a unit matrix of the same dimension as $\hat{S}$,

$$\hat{I} = \begin{pmatrix} 1\;0\;0\;\dots \\ 0\;1\;0\;\dots \\ 0\;0\;1\;\dots \\ \dots \end{pmatrix}. \tag{1.11}$$

The elements of the matrix $\hat{S}^{\dagger}$ are related to the elements of the scattering matrix $\hat{S}$ in the following way: $\left(\hat{S}^{\dagger}\right)_{\alpha\beta} = \left(\hat{S}\right)^{*}_{\beta\alpha}$. Therefore, the expanded equation (1.10) reads,

$$\sum_{\alpha=1}^{N_r} S^{*}_{\alpha\beta} S_{\alpha\gamma} = \delta_{\beta\gamma}\,, \tag{1.12}$$

$$\sum_{\beta=1}^{N_r} S_{\alpha\beta} S^{*}_{\delta\beta} = \delta_{\alpha\delta}\,. \tag{1.13}$$

To prove the unitarity, for instance, in the case if the wave function is normalized, i.e., it corresponds to scattering of a single particle, we use the integral over the space for both the initial wave function and the scattered wave function:

$$\int d^3r\,|\Psi^{(in)}|^2 = \int d^3r\,|\Psi^{(out)}|^2 = 1\,. \tag{1.14}$$

Then we use Eqs. (1.1) and (1.2). For instance, for $\Psi^{(in)}$ we get,

$$\begin{aligned}\int d^3r\,|\Psi^{(in)}|^2 &= \int d^3r \sum_\alpha a_\alpha\,\psi_\alpha^{(in)}\left(\sum_\beta a_\beta^*\,\psi_\beta^{(in)}\right)^* \\ &= \sum_\alpha\sum_\beta a_\alpha\,a_\beta^* \int d^3r\,\psi_\alpha^{(in)}\left(\psi_\beta^{(in)}\right)^* = \sum_\alpha\sum_\beta a_\alpha\,a_\beta^*\,\delta_{\alpha\beta} \\ &= \sum_\alpha |a_\alpha|^2 = 1\,.\end{aligned} \tag{1.15}$$

Here we took into account that the functions $\psi_\alpha^{(in)}$ are orthonormal,

$$\int d^3r\,\psi_\alpha^{(in)}\left(\psi_\beta^{(in)}\right)^* = \delta_{\alpha\beta}\,, \tag{1.16}$$

where $\delta_{\alpha\beta}$ is the Kronecker symbol,

$$\delta_{\alpha\beta} = \begin{cases} 1\,, & \alpha = \beta\,, \\ 0\,, & \alpha \neq \beta\,. \end{cases} \tag{1.17}$$

By analogy we find for $\Psi^{(out)}$:

$$\sum_\alpha |b_\alpha|^2 = 1\,. \tag{1.18}$$

Therefore, from Eqs. (1.15) and (1.18) it follows that,

$$\sum_\alpha |a_\alpha|^2 = \sum_\alpha |b_\alpha|^2\,. \tag{1.19}$$

Representing the coefficients a_α and b_α as vector-columns, $\hat{a}$ and $\hat{b}$, we write,

$$\sum_\alpha |a_\alpha|^2 = \hat{a}^\dagger \hat{a}\,,$$
$$\sum_\alpha |b_\alpha|^2 = \hat{b}^\dagger \hat{b}\,. \tag{1.20}$$

Next we take into account that $\hat{b} = \hat{S}\,\hat{a}$ and, correspondingly, $\hat{b}^\dagger = \hat{a}^\dagger \hat{S}^\dagger$ and finally calculate,

$$\hat{b}^\dagger \hat{b} = \hat{a}^\dagger \hat{S}^\dagger \hat{S}\,\hat{a} = \hat{a}^\dagger \hat{a}\,. \tag{1.21}$$

From the last equality the required relation, Eq. (1.10), follows directly.

Note, however, that for the particles with continuous spectrum, which we will consider, the wave function is normalized on the Dirac delta-function rather than on a unity. In such a case scattering of particles with fixed incoming flow is a more natural problem. For instance, a plane wave e^{ikx} corresponds to a flow of particles with intensity $v = \hbar k/m$ rather than to a single particle. The charge conservation in this case (under the stationary conditions) implies a current conservation. Therefore, it is convenient to choose the basis functions normalized to carry unit flux, see, e.g. [9, 5]. Then we can say more precisely:
The equation (1.9) defines the scattering matrix $\hat{S}$ if the vectors $\hat{b}$ and $\hat{a}$ are calculated using the unit flux basis.
The square of modulus of a scattering matrix element defines an intensity of a scattered flow if the intensity of an incident flow is unity. Then the unitarity of the scattering matrix reflects the particle flow conservation.

1.1.1.2 Micro-reversibility

The micro-reversibility is an invariance of the equations of motion under the time-reversal. Neither the classical physics nor the quantum physics makes distinction between the forward time and the backward time.

If to change simultaneously, $t \to -t$ and $\mathbf{v} \to -\mathbf{v}$, then the classical equations of motion predict that the particle will move along the same trajectory but in opposite direction. From the scattering theory point of view the movement in opposite direction means that the scattered particle becomes incoming and the incoming particle becomes scattered.

The quantum-mechanical formalism deals with states rather than with particles. The additional complication comes from the fact that the wave function is complex. To analyze the micro-reversibility in the quantum mechanics [10] we consider the Schrödinger equation,

$$i\hbar \frac{\partial \Psi}{\partial t} = \mathcal{H}\Psi \,, \tag{1.22}$$

where $\mathcal{H}$ is the Hamiltonian dependent on a momentum $\mathbf{p}$ of a particle. The velocity reversal within the classical physics is equivalent to a momentum reversal within the quantum physics. The Hamiltonian [and, correspondingly, Eq. (1.22)] is invariant under the momentum reversal, $\mathcal{H}(\mathbf{p}) = \mathcal{H}(-\mathbf{p})$. While under the time-reversal the sign on the left hand side (LHS) of Eq. (1.22) is changed. On the other hand if simultaneously with it we go over to the complex conjugate equation and take into account that the Hamiltonian is Hermitian, $\mathcal{H}^* = \mathcal{H}$, then we find that the transformed equation for the complex conjugate wave function $\Psi^*(-t)$ is identical to the original equation for $\Psi(t)$,

$$i\hbar \frac{\partial \left(\Psi^*\right)}{\partial(-t)} = \mathcal{H}\left(\Psi^*\right) . \tag{1.23}$$

We conclude: If the evolution in a forward time is described by the wave function $\Psi(t)$ then the evolution in a backward time is described by the complex conjugate function $\Psi^*(-t)$. For the scattering theory it means the following. If initially the incident particle is in the state $\Psi^{(in)}(t)$ and the scattered particle is in the state $\Psi^{(out)}(t)$ then under the time-reversal the state $\left(\Psi^{(out)}(-t)\right)^*$ is for an incident particle and the state $\left(\Psi^{(in)}(-t)\right)^*$ is for a scattered particle.

Such a symmetry results in some properties of the scattering matrix. To clarify them we consider scattering in forward and backward times in detail.

The initial scattering process: $\Psi^{(in)}(t) = \sum_{\alpha} a_{\alpha}\, \psi_{\alpha}^{(in)}(t)$ is an incident wave and $\Psi^{(out)}(t) = \sum_{\beta} b_{\beta}\, \psi_{\beta}^{(out)}(t)$ is a scattered wave. The coefficients a_{α} and b_{β} are related through Eq.(1.9). The scattering process after the time-reversal: $\left(\Psi^{(out)}(-t)\right)^{*} = \sum_{\beta} b_{\beta}^{*} \left(\psi_{\beta}^{(out)}(-t)\right)^{*}$ is an incident wave and $\left(\Psi^{(in)}(-t)\right)^{*} = \sum_{\alpha} a_{\alpha}^{*} \left(\psi_{\alpha}^{(in)}(-t)\right)^{*}$ is a scattered wave. Under both the time-reversal and the complex conjugation the basis functions for incident and scatterer states replace each other, $\left(\psi_{\beta}^{(out)}(-t)\right)^{*} = \psi_{\beta}^{(in)}(t)$. Therefore, we can write,

$$\begin{aligned} \left(\Psi^{(out)}(-t)\right)^{*} &= \left(\sum_{\beta} b_{\beta}\, \psi_{\beta}^{(out)}(-t)\right)^{*} = \sum_{\beta} b_{\beta}^{*}\, \psi_{\beta}^{(in)}(t)\,, \\ \left(\Psi^{(in)}(-t)\right)^{*} &= \left(\sum_{\alpha} a_{\alpha}\, \psi_{\alpha}^{(in)}(-t)\right)^{*} = \sum_{\alpha} a_{\alpha}^{*}\, \psi_{\alpha}^{(out)}(t)\,. \end{aligned} \tag{1.24}$$

Since the Hamiltonian and the basis functions remain invariant the scattering matrix is invariant as well. Therefore, the coefficients a_{α}^{*} and b_{β}^{*} in Eq. (1.24) are related in the same way as the corresponding coefficients (b_{β} and a_{α}) in Eqs. (1.1) and (1.2),

$$\hat{a}^{*} = \hat{S}\, \hat{b}^{*}\,. \tag{1.25}$$

Thus the sets of coefficients $\hat{a}$ and $\hat{b}$ have to fulfill two equations, (1.9) and (1.25). From Eq. (1.9) we find,

$$\hat{a} = \hat{S}^{-1}\hat{b}\,, \tag{1.26}$$

where $\hat{S}^{-1}$ is an inverse matrix, $\hat{S}\hat{S}^{-1} = \hat{S}^{-1}\hat{S} = \hat{I}$. Comparing Eqs. (1.26) and (1.25) we conclude that $\hat{S}^{*} = \hat{S}^{-1}$. Further, from the unitarity, Eq. (1.10), it follows that,

$$\left.\begin{aligned} \hat{S}^{\dagger}\hat{S} &= \hat{I} \\ \hat{S}^{-1}\hat{S} &= \hat{I} \end{aligned}\right\} \quad \Rightarrow \quad \hat{S}^{\dagger} = \hat{S}^{-1}\,. \tag{1.27}$$

Finally we arrive at the following. The micro-reversibility requires the scattering matrix to be invariant under the transposition operation. In other words, the scattering matrix elements are symmetric in their indices,

$$\hat{S} = \hat{S}^T \quad \Rightarrow \quad S_{\alpha\beta} = S_{\beta\alpha}\,. \tag{1.28}$$

Note the presence of a magnetic field H slightly changes the micro-reversibility condition: In addition to a time and a momentum reversal we need to inverse a magnetic field direction, $H \to -H$. It is clear, for instance, from the Hamiltonian of a free particle with mass m and charge e propagating along the axis x in the presence of a magnetic field,

$$\mathcal{H} = \frac{(p_x - eA_x)^2}{2m}\,,$$

where A_x is a vector-potential projection onto the axis x. Note that it is $H = \text{rot}\mathbf{A}$. Thus in the presence of a magnetic field Eq. (1.28) is transformed, [5]

$$\hat{S}(H) = \hat{S}^T(-H) \quad \Rightarrow \quad S_{\alpha\beta}(H) = S_{\beta\alpha}(-H)\,. \tag{1.29}$$

In particlar, the reflection amplitude, $\alpha = \beta$, is an even function of a magnetic field.

1.2 Current operator

Now we consider how the scattering matrix formalism can be applied to transport phenomena in mesoscopic samples. The scattering matrix relies on *the single-electron approximation*. Within this approximation the separate electrons are considered as independent particles whose interaction with other electrons, nuclei, impurities, quasi-particles, etc. can be described via the effective potential energy, $U_{eff}(t, \mathbf{r})$. Such an approach allows a simple and physically transparent descriptions of transport phenomena on the qualitative level and in many cases even on the quantitative level.

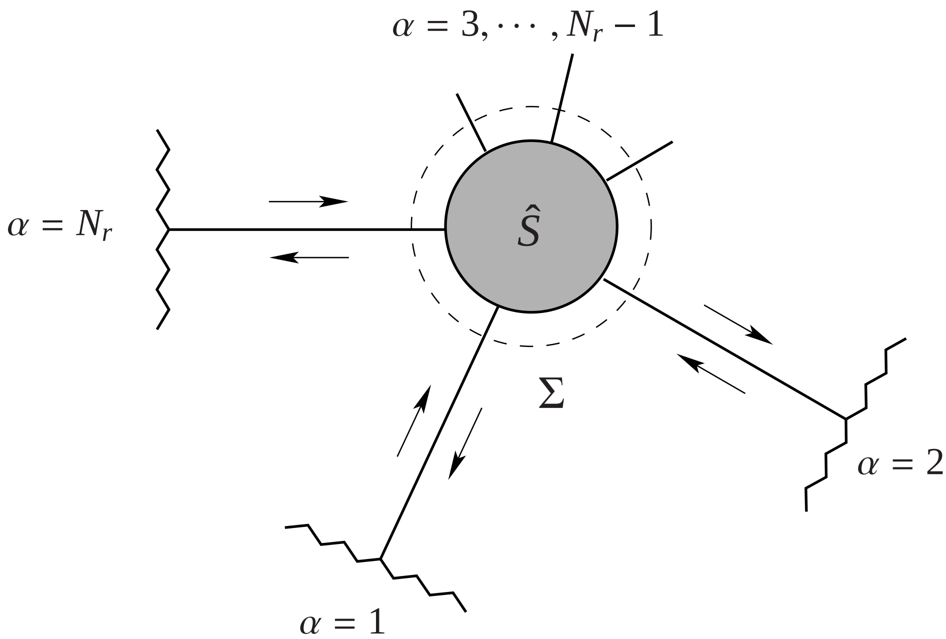

Figure 1.1: Mesoscopic sample with scattering matrix $\hat{S}$. The index $\alpha = 1, 2, \dots, N_r$ numbers electron reservoirs. The arrows directed to (from) the scatterer show a propagation direction for incident (scattered) electrons. An electron flow is calculated at the surface Σ shown as a dashed line.

Let us consider a mesoscopic sample connected to several, N_r, macroscopic contacts playing role of electrons reservoirs, Fig. 1.1. Electrons, propagating from some reservoir to the sample, enter it, are scattered inside it, and at the end leave it to the same or any other reservoirs. To calculate a current flowing between the sample and the reservoirs we do not need to watch what is happening with an electron inside the sample. It is enough to look at incoming and outgoing electron flows. To this end we enclose a sample by a fictitious surface Σ, see Fig. 1.1, and consider electron flows crossing this surface in the direction to the sample or back. In this case we, in fact, are dealing with the scattering problem: Electrons propagating to the sample are incident, or in-coming, particles [we denote them via an upper index (in)], while electrons propagating from the sample are scattered, or out-going, particles [upper index (out)]. We emphasize that we consider only elastic, i.e., energy conserving, scattering. To neglect inelastic scattering we assume enough low temperatures when the phase coherence length, L_φ, is much larger than the size of a sample, $L_\varphi(T) \gg L$.

It is convenient to choose the eigen-wave-functions for electrons in leads connecting a scatterer to the reservoirs as the basis functions for defining the scattering matrix elements. These wave functions can be represented as the product of longitudinal and transverse terms. For the sake of simplicity we assume the leads having only one conducting sub-band. Therefore, there is only one type of transverse wave functions in each lead. As the longitudinal wave functions we choose plane waves propagating to the scatterer (the wave number $-k$) or from the scatterer (the wave number k). The former (latter) wave functions comprise the basis for incident, $\psi_\alpha^{(in)}$, (scattered, $\psi_\alpha^{(out)}$) electrons.

To calculate a current flowing between the scatterer and the reservoirs we use the second quantization formalism. This formalism deals with operators creating/annihilating particles in some quantum state. We use different operators corresponding to incident electrons, $\hat{a}_\alpha^\dagger(E)/\hat{a}_\alpha(E)$, and to scatterer electrons, $\hat{b}_\alpha^\dagger(E)/\hat{b}_\alpha(E)$. The operator $\hat{a}_\alpha^\dagger(E)$ creates one electron in the state with wave function $\psi_\alpha^{(in)}(E)/\sqrt{v_\alpha(E)}$, while the operator $\hat{b}_\alpha^\dagger(E)$ creates one electron in the state with wave function $\psi_\alpha^{(out)}(E)/\sqrt{v_\alpha(E)}$. The factor $1/\sqrt{v_\alpha(E)}$ takes account of a unit flux normalization. Note the index α can be composite, i.e., it can include, apart from the reservoir's number, the additional sub-indices, for instance, the number of a sub-band, an electron spin, etc.

Introduced fermionic operators are subject to the following anti-commutation relations:

$$\begin{aligned} \hat{a}_\alpha^\dagger(E)\,\hat{a}_\beta(E') + \hat{a}_\beta(E')\,\hat{a}_\alpha^\dagger(E) &= \delta_{\alpha\beta}\,\delta(E-E')\,, \\ \hat{b}_\alpha^\dagger(E)\,\hat{b}_\beta(E') + \hat{b}_\beta(E')\,\hat{b}_\alpha^\dagger(E) &= \delta_{\alpha\beta}\,\delta(E-E')\,. \end{aligned} \tag{1.30}$$

Next we introduce the field operators for electrons in lead α,

$$\begin{aligned} \hat{\Psi}_\alpha(t,\mathbf{r}) &= \frac{1}{\sqrt{h}}\int_0^\infty dE\, e^{-i\frac{E}{\hbar}t}\left\{\hat{a}_\alpha(E)\,\frac{\psi_\alpha^{(in)}(E,\mathbf{r})}{\sqrt{v_\alpha(E)}} + \hat{b}_\alpha(E)\,\frac{\psi_\alpha^{(out)}(E,\mathbf{r})}{\sqrt{v_\alpha(E)}}\right\}, \\ \hat{\Psi}_\alpha^\dagger(t,\mathbf{r}) &= \frac{1}{\sqrt{h}}\int_0^\infty dE\, e^{i\frac{E}{\hbar}t}\left\{\hat{a}_\alpha^\dagger(E)\,\frac{\psi_\alpha^{(in)*}(E,\mathbf{r})}{\sqrt{v_\alpha(E)}} + \hat{b}_\alpha^\dagger(E)\,\frac{\psi_\alpha^{(out)*}(E,\mathbf{r})}{\sqrt{v_\alpha(E)}}\right\}. \end{aligned} \tag{1.31}$$

Here $v_\alpha(E) = \hbar k_\alpha(E)/m$ is an electrons velocity, $\mathbf{r} = (x, r_\perp)$, with x longitudinal and $r_\perp$ tranverse spatial coordinates in the lead α. Note that $1/(hv_\alpha(E))$ is the density of states, $(2\pi)^{-1}dk/dE$, for a one-dimensional conductor.

Using the field operators we write the operator, $\hat{I}_\alpha$, for a current flowing in the lead α,

$$\hat{I}_\alpha(t, x) = \frac{i\hbar e}{2m} \int dr_\perp \left\{ \frac{\partial \hat{\Psi}^\dagger_\alpha(t, \mathbf{r})}{\partial x} \hat{\Psi}_\alpha(t, \mathbf{r}) - \hat{\Psi}^\dagger_\alpha(t, \mathbf{r}) \frac{\partial \hat{\Psi}_\alpha(t, \mathbf{r})}{\partial x} \right\} . \tag{1.32}$$

Here the positive direction is from the scatterer to the reservoir.

Next we represent the basis wave functions as the product of transverse and longitudinal parts,

$$\begin{aligned} \psi^{(in)}(E, \mathbf{r}) &= \xi_E(r_\perp)\, e^{-ik(E)\,x} , \\ \psi^{(out)}(E, \mathbf{r}) &= \xi_E(r_\perp)\, e^{ik(E)\,x} , \end{aligned} \tag{1.33}$$

and take into account that the transverse wave functions are normalized,

$$\int dr_\perp\, |\xi_E(r_\perp)|^2 = 1 . \tag{1.34}$$

In what follows we are interested in currents flowing under the bias much smaller that the Fermi energy, μ_0. Therefore, in all the equations we use the main contribution comes from energies within the interval much smaller that the energy itself,

$$|E - E'| \ll E \sim \mu_0 . \tag{1.35}$$

The last inequality allows us to simplify strongly the equation for a current. We can put, $v(E) \approx v(E')$ and $k(E) \approx k(E')$. Moreover, within the same sub-band the transverse wave functions are the same, $\xi_E = \xi_{E'}$. Note if the functions ξ_E and $\xi_{E'}$ are from different sub-bands then they are orthogonal,

$\int dr_\perp\, \xi_E(r_\perp)\Big(\xi_{E'}(r_\perp)\Big)^* = 0$. That allows us to split the total current into the sum of contributions from different sub-bands. Remind that we assume each lead having only one sub-band.

Substituting Eq. (1.31) into Eq. (1.32) and taking into account Eq. (1.35) we calculate,

$$\hat{I}_\alpha(t,x) = \frac{i\hbar e}{2m} \iint dE\, dE' \frac{e^{i\frac{E-E'}{\hbar}t}}{h v_\alpha(E)} \int dr_\perp\, |\xi_{E,\alpha}(r_\perp)|^2$$

$$\times \left\{ \frac{\partial}{\partial x}\left[\hat{a}_\alpha^\dagger(E) e^{ik_\alpha(E)x} + \hat{b}_\alpha^\dagger(E) e^{-ik_\alpha(E)x}\right]\left(\hat{a}_\alpha(E') e^{-ik_\alpha(E)x} + \hat{b}_\alpha(E') e^{ik_\alpha(E)x}\right)\right.$$

$$\left. -\left(\hat{a}_\alpha^\dagger(E) e^{ik_\alpha(E)x} + \hat{b}_\alpha^\dagger(E) e^{-ik_\alpha(E)x}\right)\frac{\partial}{\partial x}\left[\hat{a}_\alpha(E') e^{-ik_\alpha(E)x} + \hat{b}_\alpha(E') e^{ik_\alpha(E)x}\right]\right\}.$$

Differentiating over x and combining the similar terms we finally arrive at the following equation for the current operator [5],

$$\hat{I}_\alpha(t) = \frac{e}{h} \iint dE\, dE'\, e^{i\frac{E-E'}{\hbar}t} \left\{\hat{b}_\alpha^\dagger(E)\, \hat{b}_\alpha(E') - \hat{a}_\alpha^\dagger(E)\, \hat{a}_\alpha(E')\right\}. \tag{1.36}$$

In what follows we use this equation and calculate, in particular, a measurable current, $I_\alpha = \langle \hat{I}_\alpha \rangle$, flowing into the lead α. Here $\langle \dots \rangle$ stands for quantum-statistical averaging over the state of incoming electrons. To calculate such an average for the products of $\hat{a}^\dagger \hat{a}$ and $\hat{b}^\dagger \hat{b}$ we take into account the following. The creation and annihilation operators, $\hat{a}_\alpha^\dagger$ and $\hat{a}_\alpha$, correspond to particles propagating from the reservoir. We suppose that the presence of a mesoscopic scatterer does not affect the equilibrium properties of reservoirs. Therefore, the in-coming particles are equilibrium particles of macroscopic reservoirs. And for them we can use the standard rules for calculating the quantum-statistical average of the product of creation and annihilation operators. In addition we suppose that electrons at different reservoirs, $\alpha \neq \beta$, are not correlated. Then we can write,

$$\begin{aligned}\langle \hat{a}_{\alpha}^{\dagger}(E)\, \hat{a}_{\beta}(E') \rangle &= \delta_{\alpha\beta}\, \delta(E - E')\, f_{\alpha}(E)\,, \\ \langle \hat{a}_{\alpha}(E)\, \hat{a}_{\beta}^{\dagger}(E') \rangle &= \delta_{\alpha\beta}\, \delta(E - E')\, \left\{1 - f_{\alpha}(E)\right\},\end{aligned} \tag{1.37}$$

where $f_{\alpha}(E)$ is the Fermi distribution function [11] for electrons in the reservoir α,

$$f_{\alpha}(E) = \frac{1}{1 + e^{\frac{E-\mu_{\alpha}}{k_B T_{\alpha}}}}\,. \tag{1.38}$$

Here k_B is the Boltzmann constant, μ_{α} is the Fermi energy (the electro-chemical potential) and T_{α} is the temperature of the reservoir α.

In contrast the operators $\hat{b}_{\alpha}^{\dagger}$ and $\hat{b}_{\alpha}$ correspond to scattered particles which, in general, are non-equilibrium particles. To calculate the quantum-statistical average for (the product of) them we need to express then in terms of the operators for in-coming particles for which we know how to calculate a corresponding average. To this end we consider both the field operator, $\hat{\Psi}^{(in)}$, corresponding to in-coming wave,

$$\hat{\Psi}^{(in)} = \sum_{\alpha=1}^{N_r} \hat{a}_{\alpha}\, \frac{\psi_{\alpha}^{(in)}}{\sqrt{v_{\alpha}}}\,,$$

and the field operator, $\hat{\Psi}^{(out)}$, corresponding to scattered wave,

$$\hat{\Psi}^{(out)} = \sum_{\beta=1}^{N_r} \hat{b}_{\beta}\, \frac{\psi_{\beta}^{(out)}}{\sqrt{v_{\beta}}}\,.$$

These equations are similar to Eqs. (1.1) and (1.2) excepting the coefficients being the second quantization operators now. Thus each of the operators $\hat{b}_{\beta}$ is expressed in terms of all the operators $\hat{a}_{\alpha}$ through the elements of the scattering matrix being $N_r \times N_r$ unitary matrix. By analogy with Eq. (1.7) we write, [5]

$$\begin{aligned} \hat{b}_\alpha &= \sum_{\beta=1}^{N_r} S_{\alpha\beta}\, \hat{a}_\beta\,, \\ \hat{b}_\alpha^\dagger &= \sum_{\beta=1}^{N_r} S_{\alpha\beta}^{*}\, \hat{a}_\beta^\dagger\,. \end{aligned} \tag{1.39}$$

The equations (1.36) - (1.39) constitute the basis of the scattering matrix approach to transport phenomena in mesoscopics.

1.3 DC current and the distribution function

Let us calculate a current, I_α,

$$I_\alpha = \langle \hat{I}_\alpha \rangle\,, \tag{1.40}$$

flowing into the lead α under the dc bias, $\Delta V_{\alpha\beta} = V_\alpha - V_\beta$. In this case the different reservoirs have different electro-chemical potentials,

$$\mu_\alpha = \mu_0 + eV_\alpha\,. \tag{1.41}$$

Note we include the potential energy eV_α into the μ_α. Then the energy E means a total (kinetic plus potential) energy of an electron. The use of a total energy (instead of a kinetic on) is convenient since it is conserved (in the stationary case) while an electron propagates from one reservoir through the scatterer to another reservoir.

The current operator, $\hat{I}_\alpha(t)$, is given in Eq. (1.36). After averaging Eq. (1.40) reads,

$$I_\alpha = \frac{e}{h} \int dE \left\{ f_\alpha^{(out)}(E) - f_\alpha^{(in)}(E) \right\}, \tag{1.42}$$

where we have introduced the distribution functions for incident electrons, $f_{\alpha}^{(in)}$, and for scattered electrons, $f_{\alpha}^{(out)}$,

$$\begin{aligned}\langle \hat{a}_{\alpha}^{\dagger}(E)\,\hat{a}_{\alpha}(E')\rangle &= \delta(E-E')\, f_{\alpha}^{(in)}(E)\,,\\ \langle \hat{b}_{\alpha}^{\dagger}(E)\,\hat{b}_{\alpha}(E')\rangle &= \delta(E-E')\, f_{\alpha}^{(out)}(E)\,.\end{aligned} \tag{1.43}$$

The physical meaning for introduced distribution functions is the following: The quantity $\frac{dE}{h} f_{\alpha}^{(in/out)}(E)$ defines the average number of electrons with energy within the interval dE near E crossing the cross-section of the lead α in unit time to/from the scatterer. The dc current is obviously the difference of flows times an electron charge e.

Accordingly to Eq. (1.37) the distribution function for in-coming electrons is the Fermi function for a corresponding reservoir,

$$f_{\alpha}^{(in)}(E) = f_{\alpha}(E)\,. \tag{1.44}$$

To calculate the distribution function for scattered electrons, $f_{\alpha}^{(out)}(E)$, we use Eqs. (1.39), (1.37) and find,

$$\begin{aligned}\delta(E-E')\, f_{\alpha}^{(out)}(E) &\equiv \langle \hat{b}_{\alpha}^{\dagger}(E)\,\hat{b}_{\alpha}(E')\rangle = \\ &= \sum_{\beta=1}^{N_r}\sum_{\gamma=1}^{N_r} S_{\alpha\beta}^{*}(E)\, S_{\alpha\gamma}^{*}(E')\,\langle \hat{a}_{\beta}^{\dagger}(E)\,\hat{a}_{\gamma}(E')\rangle = \\ &= \sum_{\beta=1}^{N_r}\sum_{\gamma=1}^{N_r} S_{\alpha\beta}^{*}(E)\, S_{\alpha\gamma}^{*}(E')\,\delta(E-E')\,\delta_{\beta\gamma}\, f_{\beta}(E)\,.\end{aligned}$$

Therefore, the distribution function, $f_{\alpha}^{(out)}(E)$, for electrons scattered into the lead α depends on the Fermi functions, $f_{\beta}(E)$, for all the reservoirs, $\beta = 1, 2, \dots, N_r$:

$$f_{\alpha}^{(out)}(E) = \sum_{\beta=1}^{N_r} \left|S_{\alpha\beta}(E)\right|^2 f_{\beta}(E)\,. \tag{1.45}$$

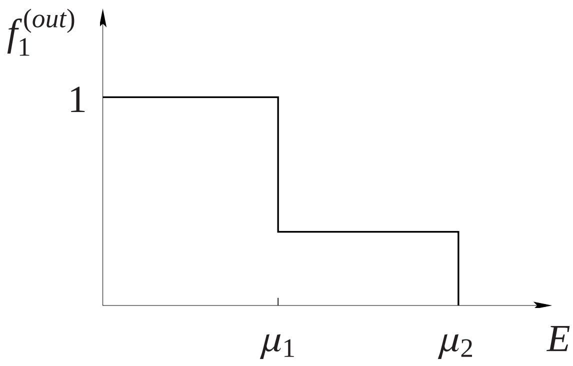

Figure 1.2: The distribution function for electrons scattered into the contact $\alpha = 1$. The height of a step is $|S_{12}|^2$. The scatterer is connected to two electron reservoirs being at zero temperature, $T_1 = T_2 = 0$, and having chemical potentials μ_1 and μ_2.

Note if all the reservoirs have the same electro-chemical potentials and temperatures (hence the same Fermi functions), $f_\beta = f_0$, $\forall\beta$, then the distribution function for scattered electrons is the Fermi function as well, i.e., the scattered elections are equilibrium. To show it we use the unitarity of the scattering matrix,

$$\hat{S}\hat{S}^\dagger = \hat{I} \quad \Rightarrow \quad \sum_{\beta=1}^{N_r} \left| S_{\alpha\beta}(E) \right|^2 = 1\,, \tag{1.46}$$

and find, $f_\alpha^{(out)}(E) = f_0(E) \sum_{\beta=1}^{N_r} \left| S_{\alpha\beta}(E) \right|^2 = f_0(E)$. In contrast, if the potentials and/or temperatures of different reservoirs are different then the scattered electrons are characterized by the non-equilibrium distribution function, Fig. 1.2.

Substituting Eqs.(1.44) and (1.45) into Eq.(1.42) and using Eq. (1.46) we finally calculate a dc current,

$$I_\alpha = \frac{e}{h} \int dE \sum_{\beta=1}^{N_r} \left| S_{\alpha\beta}(E) \right|^2 \left\{ f_\beta(E) - f_\alpha(E) \right\}. \tag{1.47}$$

We see that the current flowing into the lead α depends on the difference of the

Fermi functions times the corresponding square of the scattering matrix element modulus. If all the reservoirs have the same potentials and temperatures then the current is zero. Otherwise there is a current through the sample.

1.3.1 DC current conservation

Let us check whether Eq. (1.47) fulfills a dc current conservation law,

$$\sum_{\alpha=1}^{N_r} I_\alpha = 0\,, \tag{1.48}$$

which is a direct consequence of no charge accumulation inside the mesoscopic sample. This equation tells as that the sum of current flowing into all the leads is zero. To avoid misunderstanding we stress that in each lead the positive direction is chosen from the scatterer to the corresponding reservoir. Therefore, the current has a sign "+" or "−" if it is directed from or to the scatterer.

First of all we derive Eq. (1.48). To this end we use the electrical charge continuity equation,

$$\mathrm{div}\,\mathbf{j} + \frac{\partial\rho}{\partial t} = 0\,, \tag{1.49}$$

where $\mathbf{j}$ is a current density vector, ρ is a charge density. We integrate it over the volume enclosed by the surface Σ (see, Fig. 1.1). Then transforming the volume integral of a current density divergence into the surface integral of a current density and taking into account that the current flows into the leads only we arrive at the following,

$$\sum_{\alpha=1}^{N_r} I_\alpha(t) + \frac{\partial Q}{\partial t} = 0\,. \tag{1.50}$$

Here Q is the charge onto the scatterer. In the stationary case under consideration there are only dc currents in the leads and the charge Q is constant. Then Eq. (1.50) results in Eq. (1.48). In the non-stationary case we should average Eq. (1.50) over the time. With the following definition of a dc current,

$I_\alpha = \lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt\, I_\alpha(t)$, and assuming that the charge $Q(t)$ is bounded we again conclude that Eq. (1.48) is a consequence of Eq. (1.50).

Now we check whether Eq. (1.47) does satisfy to Eq. (1.48). We use the unitarity of the scattering matrix in the form slightly different but still equivalent to Eq. (1.46),

$$\hat{S}^\dagger \hat{S} = \hat{I} \quad \Rightarrow \quad \sum_{\alpha=1}^{N_r} \left| S_{\alpha\beta}(E) \right|^2 = 1\,. \tag{1.51}$$

Then from Eq. (1.47) we get,

$$\begin{aligned} \sum_{\alpha=1}^{N_r} I_\alpha &= \frac{e}{h} \int dE \sum_{\alpha=1}^{N_r} \sum_{\beta=1}^{N_r} \left| S_{\alpha\beta}(E) \right|^2 \left\{ f_\beta(E) - f_\alpha(E) \right\} = \\ = \frac{e}{h} \int dE &\left\{ \sum_{\beta=1}^{N_r} f_\beta(E) \sum_{\alpha=1}^{N_r} \left| S_{\alpha\beta}(E) \right|^2 - \sum_{\alpha=1}^{N_r} f_\alpha(E) \sum_{\beta=1}^{N_r} \left| S_{\alpha\beta}(E) \right|^2 \right\} \\ &= \frac{e}{h} \int dE \left\{ \sum_{\beta=1}^{N_r} f_\beta(E) - \sum_{\alpha=1}^{N_r} f_\alpha(E) \right\} = 0\,, \end{aligned}$$

as expected. Therefore, we illustrated the earlier mentioned connection between the unitarity and the current conservation. Further we use Eq. (1.47) and calculate a current in two simple but generic cases.

1.3.2 Potential difference

Let the reservoirs have different potentials but the same temperatures,

$$\begin{aligned} &\mu_\alpha = \mu_0 + eV_\alpha\,, \quad eV_\alpha \ll \mu_0\,, \\ &T_\alpha = T_0\,, \quad \forall \alpha\,. \end{aligned} \tag{1.52}$$

If $|eV_\alpha| \ll k_B T_0$ we can expand,

$$f_\alpha = f_0 - eV_\alpha \frac{\partial f_0}{\partial E} + \mathcal{O}(V_\alpha^2)\,,$$

where f_0 is the Fermi function with a chemical potential μ_0 and a temperature T_0. Using this expansion in Eq. (1.47) we calculate a current,

$$I_\alpha = \sum_{\beta=1}^{N_r} G_{\alpha\beta} \left\{ V_\beta - V_\alpha \right\}\,, \tag{1.53}$$

where we introduce the elements of the conductance matrix,

$$G_{\alpha\beta} = G_0 \int dE \left(-\frac{\partial f_0}{\partial E} \right) \left| S_{\alpha\beta}(E) \right|^2\,, \tag{1.54}$$

with $G_0 = e^2/h$ the conductance quantum (for spinless electrons). Taking into account an electron spin the conductance quantum should be doubled.

At zero temperature, $T_0 = 0$, it is,

$$-\frac{\partial f_0}{\partial E} = \delta(E - \mu_0)\,,$$

and the integration over energy in Eq. (1.54) becomes trivial. In this case the conductance matrix elements become especially simple, [5]

$$G_{\alpha\beta} = G_0 \left| S_{\alpha\beta}(\mu_0) \right|^2\,. \tag{1.55}$$

It is clear that the linear dependence of a current on the potential difference is kept at a relatively small bias. The corresponding scale is dictated by the energy dependence of the scattering matrix elements, $S_{\alpha\beta}(E)$. To illustrate it we calculate a dc current at zero temperature, $T_0 = 0$, but finite potentials, $eV_\alpha \neq 0$. In this case we can not expand the Fermi function in powers of a potential, therefore, Eq. (1.47) becomes,

$$I_\alpha = \frac{G_0}{e} \sum_{\beta=1}^{N_r} \int_{\mu_0+eV_\alpha}^{\mu_0+eV_\beta} dE \, \left| S_{\alpha\beta}(E) \right|^2 . \tag{1.56}$$

If the quantity $G_{\alpha\beta}$ changes only a little within the energy interval $\sim |eV_\beta - eV_\alpha|$ near the Fermi energy μ_0 then we can use $S_{\alpha\beta}(E) \approx S_{\alpha\beta}(\mu_0)$ in Eq. (1.56) that results in a linear $I - V$ characteristics, Eq. (1.53).

On the other hand if one can not ignore the energy dependence of $S_{\alpha\beta}(E)$ then the current becomes a non-linear function of a bias. As a simple example we consider a sample with two leads ($\alpha = 1, 2$) whose scattering properties are governed by the resonance level of a width Γ located at the energy E_1:

$$|S_{12}(E)|^2 = \frac{\Gamma^2}{(E - E_1)^2 + \Gamma^2}. \tag{1.57}$$

For simplicity suppose that $E_1 = \mu_0$. Then substituting equation above into Eq. (1.56) we find a current,

$$I_1 = \frac{e}{h} \Gamma \left\{ \mathrm{arctg} \left(\frac{eV_2}{\Gamma} \right) - \mathrm{arctg} \left(\frac{eV_1}{\Gamma} \right) \right\}. \tag{1.58}$$

If the potentials are small compared to the resonance level width, $|eV_1|, |eV_2| \ll \Gamma$, we recover the Ohm law, $I_{12} = G_0 \left(V_1 - V_2 \right)$. While in the opposite case, $|eV_1|, |eV_2| \gg \Gamma$, the current is an essentially non-linear function of potentials, $I_1 = (\Gamma^2/h) \left(V_1^{-1} - V_2^{-1} \right)$. Therefore, we see that in this problem the level width Γ is a relevant energy scale.

1.3.3 Temperature difference

The temperature difference also can result in a current. This is so called *the thermoelectric current*. To calculate it we suppose that the reservoirs have the same potentials but their temperatures are different,

$$\begin{aligned} \mu_\alpha &= \mu_0\,, \quad \forall\alpha\,, \\ T_\alpha &= T_0 + \mathcal{T}_\alpha, \quad \mathcal{T}_\alpha \ll T_0\,. \end{aligned} \tag{1.59}$$

Expanding the Fermi functions in Eq. (1.47) in powers of $\mathcal{T}_\alpha$,

$$f_\alpha = f_0 + \mathcal{T}_\alpha \frac{\partial f_0}{\partial T} + \mathcal{O}(\mathcal{T}_\alpha^2)\,,$$

and taking into account that,

$$\frac{\partial f_0}{\partial T} = -\frac{E-\mu_0}{T_0}\frac{\partial f_0}{\partial E}\,,$$

we calculate the thermoelectric current flowing into the lead α,

$$I_\alpha = \sum_{\beta=1}^{N_r} G_{\alpha\beta}^{(T)} \left\{\mathcal{T}_\beta - \mathcal{T}_\alpha\right\}. \tag{1.60}$$

Here we have introduced the thermoelectric conductance matrix elements,

$$G_{\alpha\beta}^{(T)}(E) = \frac{\pi^2 e}{3h} k_B T_0 \frac{\partial \left|S_{\alpha\beta}(E)\right|^2}{\partial E}\,, \tag{1.61}$$

and used the following integral,

$$\int_0^\infty dE \frac{e^{\frac{E-\mu_0}{k_B T_0}}}{\left(1 + e^{\frac{E-\mu_0}{k_B T_0}}\right)^2} \left(\frac{E-\mu_0}{k_B T_0}\right)^2 = \frac{\pi^2}{3} k_B T_0\,.$$

From Eq. (1.61) it follows that if the conductance is energy independent, $G_{\alpha\beta}(E) = const$, then the thermoelectric conductance (and the thermoelectric current) is zero.

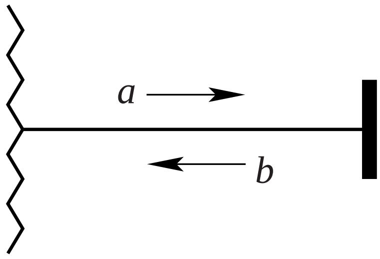

Figure 1.3: Single-channel scatterer. a is an amplitude of an incoming wave; b is an amplitude of a reflected wave. The wave line denotes an electron reservoir.

1.4 Examples

Now we consider several examples to clarify the physical meaning of the scattering matrix elements. The scattering matrix is a square matrix $N_r \times N_r$, where N_r is a number of one-dimensional conducting sub-bands in all the leads, connecting a mesoscopic sample to the reservoirs. We call N_r as a number of *the scattering channels*.

1.4.1 Scattering matrix 1×1

Such a matrix has only one element, S_{11}, and it describes a sample connected to a single reservoirs via a one-dimensional lead, Fig. 1.3. Sometimes such a sample is referred to as *a mesoscopic capacitor*.[1] The unitarity, Eq. (1.10), requires, $|S_{11}|^2 = 1$. Therefore, quite generally the scattering matrix 1×1 reads:

$$\hat{S} = e^{i\gamma}, \tag{1.62}$$

where i is an imaginary unity, γ is real. Scattering in this case is reduced to a total reflection of an incident wave. Therefore, the element S_{11} is *the reflection coefficient*. Generally speaking any diagonal element, $S_{\alpha\alpha}$, of the scattering matrix of a higher dimension is a reflection coefficient, since it defines both the

[1] More precisely it is one of the capacitor's plates.

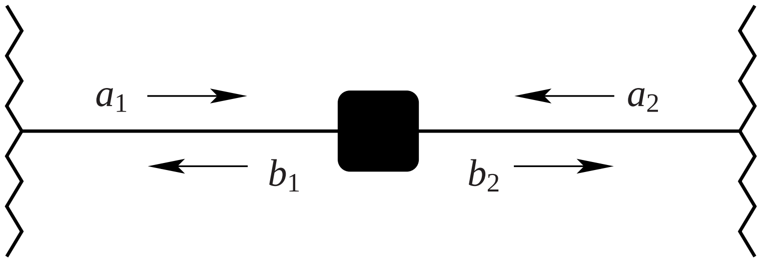

Figure 1.4: Two-channel scatterer. a_α (b_α) are amplitudes of incoming (scattered) waves, $\alpha = 1, 2$.

amplitude and the phase of a wave coming back to the same reservoir where the incident wave is originated from. In the case under consideration (1×1) the amplitude of a wave remains the same, while the phase is changed by γ which only the quantity encoding information about the properties of a mesoscopic sample. For instance, if the wave is reflected by the hard and infinite potential well then the phase is changed by $\gamma = \pi$, while if the scatterer is a ring then γ depends on the magnetic flux threading the ring, and so on.

1.4.2 Scattering matrix 2 × 2

This matrix has four in general complex elements, hence there are eight real parameters. However the unitarity, Eq. (1.10), puts four constraints. As a result there are only four independent parameters. It is convenient to choose the following independent parameters:

1) $R = |S_{11}|^2$ – a reflection probability.

2) γ – a phase relating to an effective charge, Q, of a scatterer via the Friedel sum rule, $Q = e/(2\pi i)\ln(\det \hat{S}) = e\gamma/\pi$.[12]

3) θ – a phase characterizing the reflection asymmetry, $\theta = i\ln\left(S_{11}/S_{22}\right)/2$.

4) ϕ – a phase characterizing the transmission asymmetry, $\phi = i\ln\left(S_{12}/S_{21}\right)/2$. This phase depends on an external magnetic field or on an internal magnetic moment of a scatterer.

Therefore, the general expression for the scattering matrix 2×2, describing a sample connected to two electron reservoirs, Fig. 1.4, can be written as follows,

$$\hat{S} = e^{i\gamma} \begin{pmatrix} \sqrt{R}\, e^{-i\theta} & i\sqrt{1-R}\, e^{-i\phi} \\ i\sqrt{1-R}\, e^{i\phi} & \sqrt{R}\, e^{i\theta} \end{pmatrix}. \tag{1.63}$$

Note the reflection probability is the same in both scattering channels,

$$|S_{11}|^2 = |S_{22}|^2 = R\,. \tag{1.64}$$

The same is valid with respect to the transmission probabilities: They are independent of the direction of movement,

$$|S_{12}|^2 = |S_{21}|^2\,. \tag{1.65}$$

In addition the symmetry condition, Eq. (1.29), puts some restrictions onto a possible dependence of the parameters chosen on the magnetic field. Easy to see that $\gamma(H)$, $R(H)$, and $\theta(H)$ are even functions, while $\phi(H)$ is an odd function, $\phi(H) = -\phi(-H)$. In particular, if $H = 0$ then it is $\phi = 0$ and, correspondingly, the transmission amplitude is independent of a movement direction,

$$S_{12}(H = 0) = S_{21}(H = 0)\,. \tag{1.66}$$

Stress that Eq. (1.65) holds also in the presence of a magnetic field.

Turning to the transport properties, we say that the conductance, $G \equiv G_{12} = G_{21}$, of a sample with two leads is an even function of a magnetic field,

$$G(H) = G(-H)\,. \tag{1.67}$$

As we will show this property holds also for a sample with two quasi-one-dimensional leads. This symmetry is a consequence of micro-reversibility of quantum-mechanical equations of motion which are valid in the absence of inelastic interactions breaking the phase coherence.

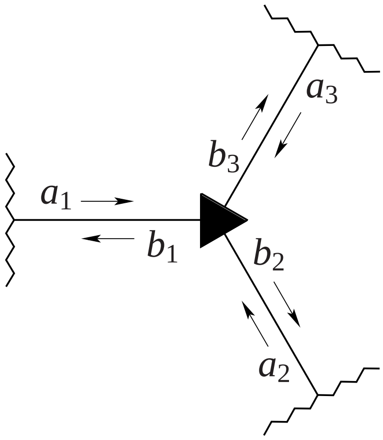

Figure 1.5: Three-channel scatterer. a_α (b_α) are amplitudes of incoming (scattered) waves, $\alpha = 1, 2, 3$.

1.4.3 Scattering matrix 3 × 3

Such a matrix describes a scatterer connected to three reservoirs, Fig. 1.5. It has already many, namely nine, independent real parameters, that makes it difficult to find a general expression. Usually the particular expressions for the scattering matrix elements are used. For instance, following to Ref. [13] one can write a one-parametric scattering matrix,

$$\hat{S} = \begin{pmatrix} -(a+b) & \sqrt{\epsilon} & \sqrt{\epsilon} \\ \sqrt{\epsilon} & a & b \\ \sqrt{\epsilon} & b & a \end{pmatrix}, \tag{1.68}$$

where $a = (\sqrt{1-2\epsilon} - 1)/2$, $b = (\sqrt{1-2\epsilon} + 1)/2$, and the real parameter ϵ changes within the following interval, $0 \le \epsilon \le 0.5$. The parameter ϵ characterizes a strength of coupling between the lead $\alpha = 1$ and the scatterer. At $\epsilon = 0$ this lead is decoupled completely from the scatterer, $S_{11} = -1$, while electrons freely propagate from the lead $\alpha = 2$ into the lead $\alpha = 3$ and back, $S_{32} = S_{23} = 1$. The limit $\epsilon = 0.5$ corresponds to a reflectionless coupling between the sample and the lead $\alpha = 1$: $S_{11} = 0$.

Sometimes, solving the Schrödinger equation for the junction of three one-dimensional leads, the Griffith boundary conditions are used [14]. These conditions include both the continuity of a wave function and the current conservation at crossing point. Then the scattering matrix of a type given in Eq. (1.68) with parameter $\epsilon = 4/9$ arises. Other values of a parameter ϵ, for instance, can be understood as related to the presence of some tunnel barrier at crossing point.

It should be noted that in contrast to the two-lead case, see, Eq. (1.64), in the case of thee leads, the reflection probabilities, $R_{\alpha\alpha} \equiv |S_{\alpha\alpha}|^2$, $\alpha = 1, 2, 3$, for different scattering channels can be different. Moreover the current flowing between some two leads depends not only on the corresponding transmission probability, $T_{\alpha\beta} \equiv |S_{\alpha\beta}|^2$, $\alpha \neq \beta$, but also on the transmission probabilities to the third lead, $T_{\gamma\alpha}$ and $T_{\gamma\beta}$, $\gamma \neq \alpha, \beta$.

1.4.4 Scatterer with two leads

Let us show that the conductance of a mesoscopic sample with two quasi-one-dimensional leads is an even function of a magnetic field. Before we showed it, see Eq. (1.67), for the case of two one-dimensional leads when the scattering matrix is a 2×2 unitary matrix. Now we generalize this result onto the case when each lead has several conducting sub-bands. [15]

Let one of the leads, say left, has N_L conducting sub-bands while another one, right, has N_R conducting sub-bands. The total number of scattering channels is $N_r = N_L + N_R$, therefore, the scattering matrix is an $N_r \times N_r$ unitary matrix. It is convenient to number the scattering channels in such a way that the first N_L scattering channels, $1 \leq \alpha \leq N_L$, correspond to the left lead, while the last N_R scattering channels, $N_L + 1 \leq \alpha \leq N_r$, correspond to the right lead. We assume that the left reservoir has a potential $-V/2$ while the right reservoir has a potential $V/2$. Note for all the sub-bands belonging to the same lead the corresponding potential V_α is the same,

$$V_\alpha = \begin{cases} -\frac{V}{2}, & 1 \leq \alpha \leq N_L\,, \\ \frac{V}{2}, & N_L \leq \alpha \leq N_r\,. \end{cases} \tag{1.69}$$

The current, I_α, carried by the electrons of the sub-band α is given in Eq. (1.53). For simplicity we consider a zero temperature case while the conclusion remains valid at finite temperatures also. So we write,

$$I_\alpha = G_0 \sum_{\beta=1}^{N_r} \left| S_{\alpha\beta} \right|^2 \left\{ V_\beta - V_\alpha \right\} . \tag{1.70}$$

Here and below the scattering matrix elements are calculated at $E = \mu_0$. To calculate a current, I_L, flowing within the left lead we need to sum up the contributions from all the sub-bands belonging to the left lead. These are sub-bands with numbers from 1 until N_L. Therefore, the current I_L is,

$$I_L = \sum_{\alpha=1}^{N_L} I_\alpha . \tag{1.71}$$

Substituting Eq. (1.70) into Eq. (1.71), we find,

$$I_L = V G_0 \sum_{\alpha=1}^{N_L} \sum_{\beta=N_L+1}^{N_r} \left| S_{\alpha\beta} \right|^2 . \tag{1.72}$$

Calculating in the same way a current I_R flowing into the right lead it is easy to check that, $I_R = -I_L$, as it should be. Note the equations for currents $I_{L/R}$ depends only on the transmission probabilities, $|S_{\alpha\beta}|^2$, between the scattering channels belonging to the different leads. Neither intra-sub-bands reflections nor inter-sub-bands transitions within the same lead do affect a current.

The conductance, $G = I_L/V$, is,

$$G = G_0 \sum_{\alpha=1}^{N_L} \sum_{\beta=N_L+1}^{N_r} \left| S_{\alpha\beta} \right|^2 . \tag{1.73}$$

Our aim is to show that this quantity is an even function of a magnetic field, $G(H) = G(-H)$. To this end we introduce some generalized reflection coefficients to the reservoirs,

$$R_{LL} = \sum_{\alpha=1}^{N_L} \sum_{\beta=1}^{N_L} |S_{\alpha\beta}|^2 , \qquad R_{RR} = \sum_{\alpha=N_L+1}^{N_r} \sum_{\beta=N_L+1}^{N_r} |S_{\alpha\beta}|^2 , \tag{1.74}$$

and transmission coefficients between the reservoirs,

$$T_{LR} = \sum_{\alpha=1}^{N_L} \sum_{\beta=N_L+1}^{N_r} |S_{\alpha\beta}|^2 \,, \qquad T_{RL} = \sum_{\alpha=N_L+1}^{N_r} \sum_{\beta=1}^{N_L} |S_{\alpha\beta}|^2 \,. \tag{1.75}$$

These coefficients satisfy the following identities,

$$R_{LL} + T_{LR} = \sum_{\alpha=1}^{N_L} \sum_{\beta=1}^{N_L} |S_{\alpha\beta}|^2 + \sum_{\alpha=1}^{N_L} \sum_{\beta=N_L+1}^{N_r} |S_{\alpha\beta}|^2 = \sum_{\alpha=1}^{N_L} \sum_{\beta=1}^{N_r} |S_{\alpha\beta}|^2 = \sum_{\alpha=1}^{N_L} 1 = N_L \,,$$

$$R_{LL} + T_{RL} = \sum_{\alpha=1}^{N_L} \sum_{\beta=1}^{N_L} |S_{\alpha\beta}|^2 + \sum_{\alpha=N_L+1}^{N_r} \sum_{\beta=1}^{N_L} |S_{\alpha\beta}|^2 = \sum_{\beta=1}^{N_L} \sum_{\alpha=1}^{N_r} |S_{\alpha\beta}|^2 = \sum_{\beta=1}^{N_L} 1 = N_L \,,$$

where we used the unitarity of the scattering matrix, $\sum_{\alpha=1}^{N_r} |S_{\alpha\beta}|^2 = 1$, $\sum_{\beta=1}^{N_r} |S_{\alpha\beta}|^2 = 1$. From given above identities it also follows that,

$$T_{LR} = T_{RL} \,. \tag{1.76}$$

Next we use the symmetry conditions, Eq. (1.29), for the scattering matrix elements in the magnetic field and find,

$$T_{LR}(-H) = \sum_{\alpha=1}^{N_L} \sum_{\beta=N_L+1}^{N_r} |S_{\alpha\beta}(-H)|^2 = \sum_{\alpha=1}^{N_L} \sum_{\beta=N_L+1}^{N_r} |S_{\beta\alpha}(H)|^2$$

$$= \sum_{\beta=N_L+1}^{N_r} \sum_{\alpha=1}^{N_L} |S_{\beta\alpha}(H)|^2 = T_{RL}(H) \,.$$

Therefore, we have

$$T_{LR}(-H) = T_{RL}(H) \,. \tag{1.77}$$

Combining together Eqs.(1.76) and (1.77) we finally arrive at the required relation,

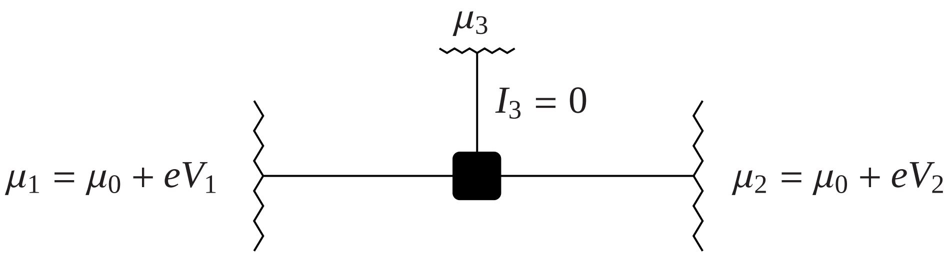

Figure 1.6: Mesoscopic scatterer with potential lead.

$$\left.\begin{array}{l} T_{LR} = T_{RL} \\ \\ T_{LR}(-H) = T_{RL}(H) \end{array}\right\} \quad \Rightarrow T_{LR}(H) = T_{LR}(-H),$$

which shows that the conductance, $G = G_0\, T_{LR}$, of a sample with two quasi-one-dimensional leads is an even function of a magnetic field.

1.4.5 Current in the presence of a potential contact

The phase coherent system represents itself some entity whose properties sometimes quite sensitive to the measurement procedure. If one attaches an additional contact, for instance to measure an electric potential inside the mesoscopic sample, then the current, flowing through the sample, is changed. [16]

Let us consider a sample connected to three leads, Fig. 1.6. Two of them, having different electrochemical potentials, $\mu_1 = \mu_0 + eV_1$ and $\mu_2 = \mu_0 + eV_2$, are used to let pass a current through the system. In contrast the third lead plays a role of a potential contact. As for any potential contacts the current, flowing into it, is zero, $I_3 = 0$. This condition defines the electrochemical potential, $\mu_3 = \mu_0 + eV_3$, of the third reservoir (which the third lead is connected to) as a function of the bias between the first and the second reservoirs, $V = V_2 - V_1$. One can say that V_3 is a potential of a mesoscopic sample at the point where the third lead is attached to.

Now we calculate a current through the sample. Since, $I_3 = 0$, then it is $I_1 = -I_2$ like for the sample with two leads. Following this analogy we would say that at a given bias V the current depends only on the probability for an electron to go from the first lead to the second lead. However this is not the case. In the presence of a potential contact (the third lead) the conductance, $G_{12} = I_1/V$, in addition depends on the probability for an electron to be scattered between the current-carrying and the potential leads,

$$I_1 \neq G_0 T_{12} V \quad \Rightarrow \quad G_{12} \neq G_0 T_{12}\,.$$

Using Eq. (1.53) we write,

$$I_1 = G_0 \Big(T_{12}(V_2 - V_1) + T_{13}(V_3 - V_1) \Big)\,,$$

$$I_2 = G_0 \Big(T_{21}(V_1 - V_2) + T_{23}(V_3 - V_2) \Big)\,,$$

$$I_3 = G_0 \Big(T_{31}(V_1 - V_3) + T_{32}(V_2 - V_3) \Big)\,.$$

From the condition $I_3 = 0$ we find,

$$V_3 = \frac{T_{31}V_1 + T_{32}V_2}{T_{31} + T_{32}}\,.$$

Note the potential $V_3 = 0$ in the symmetric case, namely, if $V_1 = -V_2$ and $T_{31} = T_{32}$. Using equation for V_3, we can find a conductance, $G_{12} = I_1/(V_2 - V_1)$:

$$G_{12} = G_0 \left\{ T_{12} + \frac{T_{13}T_{32}}{T_{31} + T_{32}} \right\}\,.$$

In the case of a weak coupling between the potential contact and the sample, $T_{31}, T_{32} \ll T_{12}$, we recover a result for the sample with two leads, $G_{12} \approx G_0 T_{12}$.

1.4.6 Scatterer embedded in a ring

We consider two generic case: (i) the ring with a magnetic flux Φ and (ii) the ring with scatterer having different transmission amplitudes to the left and to the right. For simplicity we suppose the scatterer located at $x = 0$ to be very thin: Its width w is small compared to the length L of a ring. Then we can choose a wave function on a ring threaded by the magnetic flux Φ, Fig. 1.7, as follows,

$$\psi(x) = \left(Ae^{ik(x-L)} + Be^{-ikx}\right)e^{i2\pi\frac{x}{L}\frac{\Phi}{\Phi_0}}, \quad 0 \leq x < L. \tag{1.78}$$

The scattering matrix is,

$$\hat{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}. \tag{1.79}$$

The scatterer introduces the following boundary conditions ($\alpha = 1$ for $x \to L-0$ and $\alpha = 2$ for $x \to +0$),

$$\begin{aligned} Be^{-ikL}e^{i\phi} &= Ae^{i\phi}S_{11} + BS_{12}, \\ Ae^{-ikL} &= Ae^{i\phi}S_{21} + BS_{22}. \end{aligned} \tag{1.80}$$

Here we have introduced $\phi = 2\pi\Phi/\Phi_0$. We see that the magnetic flux can be fully incorporated into the non-diagonal scattering matrix elements,

$$S'_{12} = S_{12}\,e^{-i\phi}, \quad S'_{21} = S_{21}\,e^{i\phi}. \tag{1.81}$$

Therefore, in what follow we will ignore any magnetic flux and mere consider the scattering matrix, Eq. (1.79), with $S_{12} \to S'_{12}$ and $S_{21} \to S'_{21}$.

1.4.6.1 Spectrum

Now we consider the spectrum of free electrons in a ring with embedded scatterer. The dispersion equation is defined by the consistency condition for Eq. (1.80). We rewrite this equation as follows (note that we incorporated ϕ into $S'_{\alpha\beta}$, $\alpha \neq \beta$),

$$\begin{aligned} AS_{11} - B\left(e^{-ikL} - S'_{12}\right) &= 0, \\ A\left(e^{-ikL} - S'_{21}\right) - BS_{22} &= 0. \end{aligned} \tag{1.82}$$

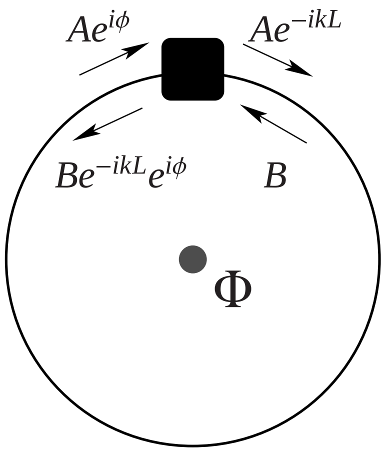

Figure 1.7: One-dimensional ring with scatterer.

The consistency condition means that the corresponding determinant is zero,

$$\det \equiv \left(e^{-ikL} - S'_{21}\right)\left(e^{-ikL} - S'_{12}\right) - S_{11}S_{22} = 0\,. \tag{1.83}$$

To solve it we make the following substitution,

$$S'_{12} = te^{-i\phi}\,, \quad S'_{21} = te^{i\phi}\,. \tag{1.84}$$

Next we divide Eq. (1.83) by $S'_{12}S'_{21} = t^2$ and use the equality, $S_{11}S'^{*}_{21} = -S'_{12}S^{*}_{22}$, following from the unitarity of the scattering matrix. Then we arrive at the following,

$$\left(\frac{e^{-ikL}}{t} - e^{i\phi}\right)\left(\frac{e^{-ikL}}{t} - e^{-i\phi}\right) = -\frac{|S_{22}|^2}{|S'_{21}|^2}\,. \tag{1.85}$$

Note the amplitude t can be complex.

Further, since the right hand side (RHS) of Eq. (1.85) is definitely real the left hand side (LHS) of the same equation has to be real as well. After decoupling of the real part from the imaginary part we obtain two equations,

$$\left[Re\left(\frac{e^{-ikL}}{t}\right)-\cos(\phi)\right]^2+\sin^2(\phi)-\left[Im\left(\frac{e^{-ikL}}{t}\right)\right]^2=-\frac{R}{T}\,, \tag{1.86a}$$

$$Im\left(\frac{e^{-ikL}}{t}\right)\left[Re\left(\frac{e^{-ikL}}{t}\right)-\cos(\phi)\right]=0\,. \tag{1.86b}$$

where we introduced $|S_{22}|^2 = R \geq 0$, $|S'_{12}|^2 \equiv |t|^2 = T \geq 0$. From Eq. (1.86a) we conclude that $Im\left(e^{-ikL}/t\right) \neq 0$ otherwise the LHS of Eq. (1.86a) would be positively defined but the RHS is strictly negative. Therefore, from Eq. (1.86b) we conclude that the dispersion equation is the following,

$$Re\left(\frac{e^{-ikL}}{t}\right)=\cos(\phi)\,, \tag{1.87}$$

as it is well known from the literature. [17, 18]

One can check directly that Eq. (1.86a) does consistent with Eq. (1.87).

1.4.6.2 Circulating current

The current carried by an electron in the state with a wave function given in Eq. (1.78) is the following,

$$I=\frac{e\hbar k}{m}\left(|A|^2-|B|^2\right)\,. \tag{1.88}$$

Note the magnetic flux Φ does not enter this equation. Therefore, this equation can be used no matter whether there is a magnetic flux through the ring or the scattering matrix is merely asymmetric, $S'_{12} \neq S'_{21}$.

To calculate a current, Eq. (1.88), we use both the normalization condition,

$$\int_0^L dx|\psi|^2 \equiv |A|^2+|B|^2=1\,, \tag{1.89}$$

and one of the equations of the system (1.82), say, the second one,

$$B = A\frac{e^{-ikL} - S'_{21}}{S_{22}} \equiv A\frac{e^{-ikL} - te^{i\phi}}{S_{22}}\,. \tag{1.90}$$

Substituting Eqs. (1.89) and (1.90) into Eq. (1.88) we find,

$$I = \frac{e\hbar k}{mL}\frac{1-|F|^2}{1+|F|^2}\,, \quad |F|^2 = \frac{T}{R}\left|\frac{e^{-ikL}}{t} - e^{i\phi}\right|^2\,. \tag{1.91}$$

Note at $\phi = 0$, i.e., in a symmetric case $S'_{12} = S'_{21}$, the current, Eq. (1.91), is identically zero, because $|F|^2 = 1$. The latter follows from Eqs. (1.86) and (1.87). The dispersion equation, Eq. (1.87), gives, $Re\left(e^{-ikL}/t\right) = 1$. Then at $\phi = 0$ we find from Eq. (1.86a), $\left[Im\left(e^{-ikL}/t\right)\right]^2 = R/T$. Therefore, $|F|^2 = T\left[Im\left(e^{-ikL}/t\right)\right]^2/R = TR/(TR) = 1$.

If the scatterer is not symmetric, $S'_{12} \neq S'_{21}$, (i.e., $\phi \neq 0$), then the current is not zero. Using the dispersion equation (1.87), $Re\left(e^{-ikL}/t\right) = \cos(\phi)$, we calculate $|F|^2$:

$$\frac{R}{T}|F|^2 = \left[Im\left(\frac{e^{-ikL}}{t}\right)\right]^2 + \sin^2(\phi) - 2Im\left(\frac{e^{-ikL}}{t}\right)\sin(\phi)\,. \tag{1.92}$$

Then from Eqs.(1.86) we find,

$$\left[Im\left(\frac{e^{-ikL}}{t}\right)\right]^2 = \sin^2(\phi) + \frac{R}{T}\,,$$

Substituting equation above into Eq. (1.92) and then into Eq. (1.91) we calculate a current,

$$I = -\frac{e\hbar k}{mL}\frac{T\sin(\phi)}{T\sin(\phi) + \dfrac{R}{\sin(\phi) - Im\left(\frac{e^{-ikL}}{t}\right)}}\,. \tag{1.93}$$

If we denote $t = it_0e^{i\chi}$ then the dispersion equation gives: $\sin(kL + \chi) =$

$-t_0 \cos(\phi)$. We write a solution as follows, $k_n L + \chi = \pi n + (-1)^n \arcsin[t_0 \cos(\phi)]$. In this case we calculate, $Im\left(e^{-ik_nL}/t\right) = -\cos(k_n L + \chi)/t_0$. Then the current, Eq. (1.93), reads,

$$I_n = -\frac{e\hbar k_n}{mL} \frac{\sqrt{T}\sin(\phi)}{\sqrt{T}\sin(\phi) + \dfrac{R}{\sqrt{T}\sin(\phi) + \cos(k_n L + \chi)}}, \tag{1.94}$$

where we use $t_0 = \sqrt{T}$.

Note in equation above ϕ is either an enclosed magnetic flux or an asymmetry in transmission to the left and to the right, Eq. (1.84), caused, for instance, by the internal magnetic moment. In general R and $T = 1 - R$ can depend on k_n.

Chapter 2

Current noise

One of the manifestations of a change quantization is fluctuating of a current, that is a deviation of an instant value of a current, I, from its average value, $\langle I\rangle$. The magnitude of fluctuations, or the noise value, is characterized by the mean square fluctuations,

$$\langle\delta I^2\rangle = \left\langle\left(I - \langle I\rangle\right)^2\right\rangle . \tag{2.1}$$

On the other hand this quantity can be represented as the difference between the average square current, $\langle I^2\rangle$, and the square of an average current,

$$\langle\delta I^2\rangle = \langle I^2\rangle - \langle I\rangle^2 . \tag{2.2}$$

Below we concentrate on two sources of noise in mesoscopics. First, it is a thermal noise, or the Nyquist-Johnson noise, due to finite temperature, $T_0 > 0$, of reservoirs, see e.g., [11, 19]. This noise exists even in equilibrium. If the sample is connected to the reservoirs with the same potentials then the average current through such a sample is zero, $\langle I\rangle = 0$. Nevertheless there is a fluctuating current with non-zero mean square fluctuations,

$$\frac{\langle\delta I^2\rangle^{(th)}}{\Delta\nu} = 2k_B T_0 G\,, \tag{2.3}$$

where G is the conductance of a sample, $\Delta\nu$ is a frequency band-width within which the current fluctuations are measured. This noise is due to fluctuations of the occupation numbers of quantum states in the macroscopic reservoirs, see e.g., Ref. [11], that results in fluctuating of electron flows incident to the scatterer. At zero temperature the quantum state occupation numbers do not fluctuate and, therefore, the thermal noise is absent.

Second, it is a shot noise, see Ref. [20]. As it was first shown by Schottky [21], who investigated the current flow in electronic lamps, the probabilistic character of a propagation of electrons through the system results in current fluctuations. In mesoscopics samples the shot noise arises due to quantum-mechanical probabilistic nature of scattering. The shot noise arises only in non-equilibrium case, if the current flows through the sample. If the bias V is applied to the sample then the average current is, $\langle I\rangle = GV$. While this current fluctuates even at zero temperature,

$$\frac{\langle \delta I^2\rangle^{(sh)}}{\Delta\nu} = |e\langle I\rangle|\,(1 - T_{12})\,, \tag{2.4}$$

The term T_{12}, a probability for an electron came from one reservoir to be scattered into another one, reflects the probabilistic nature of the shot noise. Moreover, taking into account that $\langle I\rangle = VG$ and $G \sim T_{12}$, one can easily show that the shot noise is maximum if the reflection and the transmission probabilities are equal, $R_{11} = T_{12} = 1/2$. Then we conclude: The larger uncertainty in the scattering outcome the larger the shot noise is. If the outcome of scattering is definite, i.e., an electron is always either transmitted through the sample, $T_{12} = 1$, or reflected from the sample, $R_{11} = 1$, the shot noise is zero [22].

We stress the two mentioned sources of noise are not independent. The presence of a current changes a thermal noise and the shot noise is modified at finite temperatures. This fact points out that the physics underlying the thermal noise and the shot noise is of the same nature. Before we present a formal theory of current fluctuations we give simple physical arguments illustrating appearance of a current noise in mesoscopic systems.

2.1 Nature of a current noise

We consider the extremely simplified model, a sample transmitting only electrons with energy E. To clarify physics we first consider separately cases with either thermal or shot noise present.

2.1.1 Thermal noise

Let the sample be a channel connecting two reservoirs. Electrons with energy E propagate ballistically, $T_{12}(E) = T_{21}(E) = 1$, while electrons with any other energy do not propagate at all, $T_{12}(E') = T_{21}(E') = 0$, $\forall E' \neq E$. Then the electrons propagating in a channel, say, from the first reservoirs to the second one, carry a current,

$$\langle I_{\rightarrow} \rangle = I_0 \, P_{\rightarrow} \,, \tag{2.5}$$

where $I_0 = ev/\mathcal{L}$ is a current supporting by the state $\Psi_{\rightarrow}(E)$ of an electron with energy E in the channel, e is an electron charge, v is an electron velocity, $\mathcal{L}^{-1}$ is an electron density for a unite length, $P_{\rightarrow}$ is a probability that the state $\Psi_{\rightarrow}(E)$ is occupied. Since in the ballistic case any electron propagating to the second reservoir came from the first reservoir, the probability $P_{\rightarrow}$ is equal to the occupation probability for electrons with energy E within the first reservoir. The latter is given by the Fermi distribution function, $f_1(E)$, see Eq. (1.38),

$$P_{\rightarrow} = f_1(E) \,. \tag{2.6}$$

The occupation probability can be defined as the ratio of a time, $\Delta t_{\rightarrow}$, when the state $\Psi_{\rightarrow}(E)$ is occupied and the total time (the observation time), $\mathcal{T} \rightarrow \infty$,

$$P_{\rightarrow} = \lim_{\mathcal{T}\rightarrow\infty} \frac{\Delta t_{\rightarrow}}{\mathcal{T}} \,. \tag{2.7}$$

Using this definition we can say that during a time $\Delta t_{\rightarrow}$ there is a current $I_{\rightarrow}(t) = I_0$ in a channel, while during the rest time, $\mathcal{T} - \delta t_{\rightarrow}$, there is no current, $I_{\rightarrow}(t) = 0$. Therefore, the current varies in time. With Eq. (2.7) we calculate the mean current,

$$\langle I_{\rightarrow} \rangle = \lim_{\mathcal{T}\rightarrow\infty} \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt \, I_{\rightarrow}(t) = \lim_{\mathcal{T}\rightarrow\infty} \frac{I_0 \Delta t_{\rightarrow}}{\mathcal{T}} = I_0 P_{\rightarrow} \,, \tag{2.8}$$

that coincides with Eq. (2.5). The mean square current is,

$$\langle I_{\rightarrow}^2 \rangle = \lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt\, I_{\rightarrow}^2(t) = \lim_{\mathcal{T}\to\infty} \frac{I_0^2 \Delta t_{\rightarrow}}{\mathcal{T}} = I_0^2 P_{\rightarrow}\,. \tag{2.9}$$

And finally, using Eq. (2.2), we calculate the mean square current fluctuations,

$$\langle \delta I_{\rightarrow}^2 \rangle = I_0^2 P_{\rightarrow} \left(1 - P_{\rightarrow}\right)\,. \tag{2.10}$$

We see the current fluctuations are absent, $\langle \delta I_{\rightarrow}^2 \rangle = 0$, in those cases when the state of interest, $\Psi_{\rightarrow}(E)$, is either always occupied, $P_{\rightarrow} = 1$, or always empty, $P_{\rightarrow} = 0$. In contrast if the presence of an electron in a current-carrying state has a probabilistic character, $0 < P_{\rightarrow} < 1$, the current fluctuates.

Let us express $\langle \delta I_{\rightarrow}^2 \rangle$, Eq. (2.10), in terms of a temperature T_1 of a reservoirs where electrons come from. To this end we use Eq. (2.6) and take into account the following identity for the Fermi function,

$$f_1(E)\left(1 - f_1(E)\right) = \left(-\frac{\partial f_1(E)}{\partial E}\right) k_B T_1\,. \tag{2.11}$$

As a result we get,

$$\langle \delta I_{\rightarrow}^2 \rangle = I_0^2 \left(-\frac{\partial f_1(E)}{\partial E}\right) k_B T_1\,. \tag{2.12}$$

Thus the fluctuations under consideration vanish at zero temperature, $T_1 = 0$, as it should be for the thermal noise, see Eq. (2.3).

Next we take into account electrons propagating in opposite direction, i.e., from the second reservoir to the first one. Then we calculate the mean total current, $\langle I \rangle$, and current fluctuations, $\langle \delta I^2 \rangle$ of the total current, $I(t) = I_{\rightarrow}(t) - I_{\leftarrow}(t)$,

$$
\begin{aligned}
\langle I\rangle &= \langle I_{\rightarrow}\rangle - \langle I_{\leftarrow}\rangle = I_0\left\{f_1(E) - f_2(E)\right\}, \\
\langle \delta I^2\rangle &= \langle \delta I_{\rightarrow}^2\rangle + \langle \delta I_{\leftarrow}^2\rangle \\
&= I_0^2\Big[f_1(E)\big\{1 - f_1(E)\big\} + f_2(E)\big\{1 - f_2(E)\big\}\Big],
\end{aligned}
\tag{2.13}
$$

where $f_2(E)$ is the Fermi distribution function for electrons in the second reservoir. Calculating $\langle \delta I^2\rangle$ we took into account that in the ballistic case electrons propagating from the first reservoir to the second one and back originate from different reservoirs, which are assumed to be uncorrelated. Therefore, the corresponding fluctuating currents, $I_{\rightarrow}(t)$ and $I_{\leftarrow}(t)$, are statistically independent and have to be averaged independently, $\langle I_{\rightarrow}(t)I_{\leftarrow}(t)\rangle = \langle I_{\rightarrow}(t)\rangle\langle I_{\leftarrow}(t)\rangle$.

If both reservoirs have the same temperatures, $T_1 = T_2 \equiv T_0$, and potentials, then the corresponding distribution functions are the same as well, $f_1(E) = f_2(E) \equiv f_0(E)$. In this case Eq. (2.13) gives,

$$
\begin{aligned}
\langle I\rangle &= 0, \\
\langle \delta I_{\rightarrow}^2\rangle &= 2I_0^2\left(-\frac{\partial f_0(E)}{\partial E}\right)k_B T_0 .
\end{aligned}
\tag{2.14}
$$

We see that current is zero, as it should be without bias. While the mean square current fluctuations is not zero due to fluctuations of occupation of quantum states in the macroscopic reservoirs with finite temperature, $T_0 > 0$.

2.1.2 Shot noise

Now we analyze a zero temperature case when the thermal noise vanishes. However additionally we assume that there is scatterer in the otherwise ballistic channel, see Fig. 1.4. This scatterer is characterized by the same probabilities to transmit electrons with energy E from one side to another and back, $T_{12}(E) = T_{21}(E)$. Let us assume also that the reservoirs have different potentials. More precisely, we assume that electrons with energy E are present in the first reservoir only: $\mu_2 + eV_2 < E < \mu_1 + eV_1 \Rightarrow f_1(E) = 1,\ f_2(E) = 0$.

From the first reservoirs electrons with velocity v and linear density $1/\mathcal{L}$ fall onto the scatterer. They hit the scatterer with frequency $v/\mathcal{L}$. Each electron can be either transmitted or reflected. In the former case an electron reaches the second reservoir and causes a current pulse, $I_{\rightarrow}(t) = I_0$. While there is no current in the latter case, $I_{\rightarrow}(t) = 0$, since an electron returns to original reservoir. The quantity, $T_{21}(E)$, being a probability for electron to tunnel through the scatterer, defines a relative time period, $\Delta t_{\rightarrow}$, when the current flows between the reservoirs,

$$T_{21} = \lim_{\mathcal{T}\to\infty} \frac{\Delta t_{\rightarrow}}{\mathcal{T}} . \tag{2.15}$$

Repeating reasoning of Sec. 2.1.1 we can calculate a mean current and a mean square current fluctuations, see Eqs. (2.7) - (2.10):

$$\begin{aligned} \langle I \rangle &= I_0\, T_{21}(E) , \\ \langle \delta I^2 \rangle &= I_0 \langle I \rangle \left\{ 1 - T_{21}(E) \right\} . \end{aligned} \tag{2.16}$$

Comparing Eq. (2.10) with Eq. (2.16) we conclude that the structure of expressions for the thermal noise and for the shot noise is the same. The difference is only in the source of stochasticity: In the former case it comes from the distribution function of electrons in macroscopic reservoirs, while in the latter case it comes from the quantum-mechanical scattering processes.

2.1.3 Mixed noise

Finally we consider a case when both the thermal noise and the shot noise are present. We assume that the channel with a scatterer is connected to the reservoirs having non-zero temperatures and different potentials. In this case the probability, $P_{\rightarrow}$, that an electron, moving from the first reservoir to the second one, does contribute to a current, is a product of two factors, namely, a probability, $f_1(E)$, that the state with energy E is occupied in the first reservoirs and a probability, $T_{21}(E)$, that an electron tunnel through the scatterer,

$$P_{\rightarrow} = T_{21}(E)\, f_1(E) . \tag{2.17}$$

In the same way, it is,

$$P_{\leftarrow} = T_{12}(E)\, f_2(E)\,. \tag{2.18}$$

Thus the total current, $\langle I\rangle = \langle I_{\rightarrow}\rangle - \langle I_{\leftarrow}\rangle$, flowing through the channel is equal to

$$\langle I\rangle = I_0\, T_{12}(E)\left\{ f_1(E) - f_2(E)\right\}, \tag{2.19}$$

where we used, $T_{12}(E) = T_{21}(E)$.

Next we consider current fluctuations. If the currents $I_{\rightarrow}(t)$ and $I_{\leftarrow}(t)$ would be statistically independent then we could say, see Eq. (2.13), that $\langle \delta I^2\rangle$ is equal to the sum of $\langle \delta I_{\rightarrow}^2\rangle$ and $\langle \delta I_{\leftarrow}^2\rangle$, where

$$\begin{aligned} \langle \delta I_{\rightarrow}^2\rangle &= I_0^2\, P_{\rightarrow}\left(1 - P_{\rightarrow}\right) = I_0^2\, T_{12}(E)\, f_1(E)\left\{1 - T_{12}(E)\, f_1(E)\right\}, \\ \langle \delta I_{\leftarrow}^2\rangle &= I_0^2\, P_{\leftarrow}\left(1 - P_{\leftarrow}\right) = I_0^2\, T_{12}(E)\, f_2(E)\left\{1 - T_{12}(E)\, f_2(E)\right\}. \end{aligned} \tag{2.20}$$

However, as we show, this is not the case,

$$\langle \delta I^2\rangle \neq \langle \delta I_{\rightarrow}^2\rangle + \langle \delta I_{\leftarrow}^2\rangle\,. \tag{2.21}$$

This is because the currents $I_{\rightarrow}(t)$ and $I_{\leftarrow}(t)$ are correlated. These correlations arising between the scattered electrons are a manifestation of the Pauli exclusion principle. Due to this principle two electrons can not be in the same state. Let us consider the state corresponding to an electron propagating from the scatterer to the first reservoir. There are two ways to arrive at this state: Either an electron incident from the first reservoir is reflected, or an electron incident from the second reservoir is transmitted. Since this state can not be occupied by two electrons we conclude that the result of scattering of an electron came from the first reservoir depends on the result of scattering of an electron came from the second reservoir. Therefore, the initially uncorrelated electrons at two reservoirs after scattering at the same obstacle become correlated. Hence the currents carrying scattered electrons are correlated. In particular, these correlations result in vanishing of the shot noise in the case if there are equal electrons flow falling upon the scatterer from both sides.

To take into account mentioned correlations due the Pauli principle we should describe electrons quantum-mechanically. We use the second-quantization formalism and introduce creation/annihilation operators, $\hat{a}_\alpha^\dagger/\hat{a}_\alpha$, for electrons with energy E incident from the reservoir $\alpha = 1, 2$, and operators $\hat{b}_\alpha^\dagger/\hat{b}_1$ for electrons scattered into the reservoir α. The reflection and transmission at the obstacle we describe with the help of the unitary 2×2 scattering matrix $\hat{S}$. As we showed before, the operators for scattered and for incident electrons are related as follows,

$$\hat{b}_\alpha = \sum_{\beta=1}^{2} S_{\alpha\beta}\hat{a}_\beta\,, \quad \hat{b}_\alpha^\dagger = \sum_{\beta=1}^{2} S_{\alpha\beta}^{*}\hat{a}_\beta^\dagger\,. \tag{2.22}$$

For definiteness we calculate a current and its fluctuations on the left from the scatterer. As positive we choose a direction from the scatterer to the first reservoir. Then the current operator, $\hat{I}_1$, reads,

$$\hat{I}_1 = I_0(\hat{b}_1^\dagger\hat{b}_1 - \hat{a}_1^\dagger\hat{a}_1)\,. \tag{2.23}$$

The measured current, I_1, and its mean square fluctuations, $\langle\delta I_1^2\rangle$, are the following,

$$I_1 = \langle\hat{I}_1\rangle\,, \quad \langle\delta I_1^2\rangle = \langle\hat{I}_1^2\rangle - \langle\hat{I}_1\rangle^2\,. \tag{2.24}$$

where $\langle\dots\rangle$ stands for a quantum-statistical average over the incoming state with energy E we consider. To calculate it we take into account that the product $\hat{n}_\alpha = \hat{a}_\alpha^\dagger\hat{a}_\alpha$ is a particle number density operator. Averaging quantum-mechanically $\hat{n}_\alpha$ over the state with energy E we get a particle number density, n_α in this state in the reservoir α. While after statistical averaging of the particle number density we arrive at the Fermi distribution function, f_α, of the reservoir $\alpha = 1, 2$, where an incident electron (describing by the operator a_α) came from. Taking into account that electrons at different reservoirs are statistically independent, i.e., $\langle a_\alpha^\dagger a_\beta \rangle = 0$, $\alpha \neq \beta$, we have,

$$\langle \hat{a}_\alpha^\dagger \hat{a}_\beta \rangle = \delta_{\alpha\beta} f_\alpha \,, \quad f_\alpha = \frac{1}{1 + e^{\frac{E-\mu_\alpha}{k_B T_\alpha}}} \,, \quad \alpha = 1, 2 \,. \tag{2.25}$$

Also we take into account the anti-commutation relation for the Fermi particle operators,

$$\hat{a}_\alpha^\dagger \hat{a}_\beta + \hat{a}_\beta \hat{a}_\alpha^\dagger = \delta_{\alpha\beta} \,. \tag{2.26}$$

First, we calculate a current,

$$\langle \hat{I}_1 \rangle = I_0 \langle \hat{b}_1^\dagger \hat{b}_1 - \hat{a}_1^\dagger \hat{a}_1 \rangle = I_0 \left\langle \sum_{\beta=1}^{2} S_{1\beta}^* \hat{a}_\beta^\dagger \sum_{\gamma=1}^{2} S_{1\gamma} \hat{a}_\gamma - \hat{a}_1^\dagger \hat{a}_1 \right\rangle$$

$$= I_0 \left\{ \sum_{\beta=1}^{2} \sum_{\gamma=1}^{2} S_{1\beta}^* S_{1\gamma} \langle \hat{a}_\beta^\dagger \hat{a}_\gamma \rangle - \langle \hat{a}_1^\dagger \hat{a}_1 \rangle \right\} = I_0 \left\{ \sum_{\beta=1}^{2} |S_{1\beta}|^2 f_\beta - f_1 \right\} .$$

Using the unitarity of the scattering matrix, $|S_{11}|^2 + |S_{12}|^2 = 1$, and introducing the transmission probability, $T_{12} = |S_{12}|^2$, we finally find,

$$\langle \hat{I}_1 \rangle = I_0 T_{12} (f_2 - f_1) \,. \tag{2.27}$$

This equation is different form the current in a ballistic case, Eq. (2.13), by the evident factor $T_{12} < 1$, which reduces a current due to a partial reflection of an electron flow from the scatterer.

Next we calculate the mean square current fluctuations, $\langle \delta I_1^2 \rangle$. To simplify calculations we write the current operator, $\hat{I}_1$ directly in terms of operators for incident electrons,

$$\hat{b}_1 = S_{11} \hat{a}_1 + S_{12} \hat{a}_2 \,, \quad \hat{b}_1^\dagger = S_{11}^* \hat{a}_1^\dagger + S_{12}^* \hat{a}_2^\dagger \,,$$

$$\hat{I}_1/I_0 = \hat{b}_1^\dagger \hat{b}_1 - \hat{a}_1^\dagger \hat{a}_1 = \left(S_{11}^* \hat{a}_1^\dagger + S_{12}^* \hat{a}_2^\dagger\right) (S_{11}\hat{a}_1 + S_{12}\hat{a}_2) - \hat{a}_1^\dagger \hat{a}_1 =$$

$$= T_{12}(\hat{a}_2^\dagger \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_1) + S_{11}^* S_{12} \hat{a}_1^\dagger \hat{a}_2 + S_{12}^* S_{11} \hat{a}_2^\dagger \hat{a}_1 \,.$$

Note the last two term do not contribute to a measured current, $I_1 = \langle \hat{I}_1 \rangle$, since after averaging they give zero, see Eq. (2.25). However namely these terms are responsible for current fluctuations.

The square of the current operator is the following, $\hat{I}_1^2 = \left(\hat{I}_1\right)^2$:

$$\hat{I}_1^2/I_0^2 = \left(T_{12}(\hat{a}_2^\dagger \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_1) + S_{11}^* S_{12} \hat{a}_1^\dagger \hat{a}_2 + S_{12}^* S_{11} \hat{a}_2^\dagger \hat{a}_1\right)^2 =$$

$$= T_{12}^2 \left(\hat{a}_2^\dagger \hat{a}_2 \hat{a}_2^\dagger \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2 \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2\right)$$

$$+ R_{11} T_{12} \left(\hat{a}_1^\dagger \hat{a}_2 \hat{a}_2^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_1 \hat{a}_1^\dagger \hat{a}_2\right)$$

$$+ T_{12} S_{11}^* S_{12} \left(\hat{a}_2^\dagger \hat{a}_2 \hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2 \hat{a}_2^\dagger \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1^\dagger \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_2 \hat{a}_1^\dagger \hat{a}_1\right)$$

$$+ T_{12} S_{12}^* S_{11} \left(\hat{a}_2^\dagger \hat{a}_2 \hat{a}_2^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_1 \hat{a}_1^\dagger \hat{a}_1\right)$$

$$+ \left(S_{11}^* S_{12}\right)^2 \hat{a}_1^\dagger \hat{a}_2 \hat{a}_1^\dagger \hat{a}_2 + \left(S_{12}^* S_{11}\right)^2 \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_1 \,.$$

Here the reflection coefficient, $R_{11} = |S_{11}|^2$, was introduced. Note the terms in the last three lines give zero after averaging since they include a different number of creation and annihilation operators with the same indices. To average remaining terms we use Eq. (2.26),

$$\langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle = \langle \hat{a}_\alpha^\dagger \left(1 - \hat{a}_\alpha^\dagger \hat{a}_\alpha\right) \hat{a}_\alpha \rangle = \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle - \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha^\dagger \hat{a}_\alpha \hat{a}_\alpha \rangle = f_\alpha - 0 = f_\alpha \,,$$

$$\langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \hat{a}_\beta^\dagger \hat{a}_\beta \rangle = \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle \langle \hat{a}_\beta^\dagger \hat{a}_\beta \rangle = f_\alpha f_\beta, \quad \alpha \neq \beta \,,$$

$$\langle \hat{a}_\alpha^\dagger \hat{a}_\beta \hat{a}_\beta^\dagger \hat{a}_\alpha \rangle = \langle \hat{a}_\alpha^\dagger \left(1 - \hat{a}_\beta^\dagger \hat{a}_\beta\right) \hat{a}_\alpha \rangle = \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle - \langle \hat{a}_\alpha^\dagger \hat{a}_\beta^\dagger \hat{a}_\beta \hat{a}_\alpha \rangle =$$

$$= f_\alpha - \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \hat{a}_\beta^\dagger \hat{a}_\beta \rangle = f_\alpha - \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle \langle \hat{a}_\beta^\dagger \hat{a}_\beta \rangle = f_\alpha (1 - f_\beta) \,, \; \alpha \neq \beta \,.$$

With these equations we calculate,

$$\langle \hat{I}_1^2 \rangle / I_0^2 = T_{12}^2 (f_2 + f_1 - 2 f_1 f_2) + R_{11} T_{12} \left\{ f_1 (1 - f_2) + f_2 (1 - f_1) \right\} .$$

And finally we find the mean square current fluctuations,

$$\begin{aligned} \langle \delta I_1^2 \rangle / I_0^2 &= \langle I_1^2 \rangle / I_0^2 - \langle I_1 \rangle^2 / I_0^2 \\ &= T_{12}^2 (f_2 + f_1 - 2 f_1 f_2) + R_{11} T_{12} \left\{ f_1 (1 - f_2) + f_2 (1 - f_1) \right\} - T_{12}^2 (f_2 - f_1)^2 \\ &= T_{12}^2 \left\{ f_1 (1 - f_1) + f_2 (1 - f_2) \right\} + R_{11} T_{12} \left\{ f_1 (1 - f_2) + f_2 (1 - f_1) \right\} . \end{aligned} \quad (2.28)$$

Let us analyze where the different terms in this equation originate from.

First we consider the term with squared transmission probability, $T_{12}^2 \left\{ f_1 (1 - f_1) + f_2 (1 - f_2) \right\}$. This term originates from averaging those pairs of creation and annihilation operators which do contribute to current. Since the current is due to electrons transmitted from one reservoir to another one, we can attribute this part of a noise to fluctuations in incident electron flows. The effect of scattering in this case is rather trivial: It reduces an electron flow by the factor T_{12} and, correspondingly, it reduces a noise (a squared current) by the factor T_{12}^2. This is evident for electrons flowing from the second reservoir and transmitted through the scatterer before we calculated their contribution to the current I_1. However the same is also true for electrons flowing from the first reservoir, since their current is reduced by the factor $T_{21} = T_{12} = 1 - R_{11}$ due to reflection at the scatterer. As a result the (part of the) mean square current fluctuations due to fluctuating of the occupation numbers of states in the reservoirs are proportional to the transmission probability square. Since these fluctuations are present at non-zero temperature only, this part could be considered as the thermal noise in the system under consideration (the scatterer connected to reservoirs). Comparing it to Eq. (2.13) we see that these two results are consistent at $T_{12} = 1$. However at $T_{12} < 1$ this part of a noise is different from what we called as the thermal noise, Eq. (2.3), since the conductance G is proportional to the transmission probability, $G = G_0 T_{12}$, not its square.

To resolve a seemingly contradiction and to find a correct expression for the thermal noise, i.e., for the part of a noise vanishing at zero temperature, we should consider the remaining part in Eq. (2.28) due to reflection at the scatterer, $R_{11}T_{12}\left\{f_1(1-f_2)+f_2(1-f_1)\right\}$. This part originates from averaging those operators which do not contribute to a current and, therefore, they do not correspond to any real single-particle processes. While they correspond to some two-particle processes. To clarify them we introduce a notion of a hole whose distribution function is $1 - f_\alpha$. Then one can say that there are two kind of particles incident to the scatterer: There is incoming either an electrons (with probability f_α) or a hole (with probability $1 - f_\alpha$). Then the corresponding part of a noise is due to following two-particle processes: An electron/hole incoming from the first reservoir is reflected (with probability R_{11}) while a hole/electron incoming from the second reservoir is transmitted (with probability T_{12}). Apparently these processes do not contribute to current. Notice the fluctuations in reservoirs and fluctuations due to scattering are statistically independent, therefore, they contribute additively into the mean square current fluctuations. This fact justifies splitting present in Eq. (2.28). On the other hand one can rearrange these terms in another way,

$$\langle \delta I_1^2 \rangle / I_0^2 = T_{12}^2 \left\{ f_1(1-f_1) + f_2(1-f_2) \right\}$$

$$+ R_{11}T_{12}\left\{ f_1(1 - f_1 + f_1 - f_2) + f_2(1 - f_2 + f_2 - f_1) \right\} =$$

$$= \left(T_{12}^2 + R_{11}T_{12}\right)\left\{ f_1(1-f_1) + f_2(1-f_2) \right\}$$

$$+ R_{11}T_{12}\left\{ f_1(f_1 - f_2) + f_2(f_2 - f_1) \right\} =$$

$$= T_{12}\left\{ f_1(1-f_1) + f_2(1-f_2) \right\} + R_{11}T_{12}(f_2 - f_1)^2 .$$

One can see that the first terms vanishes at zero temperature, therefore, we call it as the thermal noise. The second term vanishes with vanishing of a current, Eq. (2.27), flowing through the scatterer. Therefore, following to Schottky one can attribute it to the stochasticity in scattering of indivisible particles at the obstacle. We call such a noise as the shot noise. Thus we write,

$$\langle \delta I_1^2 \rangle / I_0^2 = \langle \delta I_1^2 \rangle^{(th)} / I_0^2 + \langle \delta I_1^2 \rangle^{(sh)} / I_0^2 \,, \tag{2.29}$$

where

$$\langle \delta I_1^2 \rangle^{(th)} / I_0^2 = T_{12} \left\{ f_2 (1 - f_2) + f_1 (1 - f_1) \right\} ,$$

$$\langle \delta I_1^2 \rangle^{(sh)} / I_0^2 = R_{11} T_{12} (f_2 - f_1)^2 \,.$$

Notice the given above equation for the thermal noise is proportional to the fist power of a transmission probability, T_{12}, in agreement with Eq. (2.3). The shot noise equation is proportional to the product of a transmission and reflection probabilities, that by virtue of Eq. (2.27) is consistent with Eq. (2.4). Moreover, the equation (2.29) reproduces correctly equations for the thermal noise and for the shot noise in all particular cases we considered earlier.

2.2 Sample with continuous spectrum

Now using the scattering matrix approach we present a formal theory for current fluctuations in mesoscopic sample connected via one-dimensional leads to N_r reservoirs. The essential difference from a simple model considered above is that the incident electrons are particles with continuous spectrum. This fact complicates calculations but qualitatively the answer remains the same.

2.2.1 Current correlator

The mathematical quantity which is usually considered in connection with noise is a correlation function of currents,

$$P_{\alpha\beta}(t_1, t_2) = \frac{1}{2} \left\langle \Delta \hat{I}_\alpha(t_1) \Delta \hat{I}_\beta(t_2) + \Delta \hat{I}_\beta(t_2) \Delta \hat{I}_\alpha(t_1) \right\rangle . \tag{2.30}$$

The operator $\Delta \hat{I}_\alpha = \hat{I}_\alpha - \left\langle \hat{I}_\alpha \right\rangle$ describes a deviation of an instant current, $\hat{I}_\alpha$, from its mean value, $\langle \hat{I}_\alpha \rangle$. The quantity $P_{\alpha\alpha}$ is referred to as *the current auto-correlator*, while the quantity $P_{\alpha\beta}$, $\alpha \neq \beta$, is referred to as *the current cross-correlator*.

At $t_1 = t_2$ and $\alpha = \beta$ the equation (2.30) defines the mean square fluctuations of a current within a lead α, $P_{\alpha\alpha}(t_1, t_1) = \langle \Delta \hat{I}_\alpha^2 \rangle$, which strictly speaking diverges due to quantum fluctuations in the system with continuous unbounded spectrum. To overcome this difficulty usually in an experiments the fluctuations are measured within some frequency window $\Delta\omega$.

To calculate the spectral contents of fluctuations we go over from the real-time to the frequency representation,

$$P_{\alpha\beta}(\omega_1, \omega_2) = \int_{-\infty}^{\infty} dt_1\, e^{i\omega_1 t_1} \int_{-\infty}^{\infty} dt_2\, e^{i\omega_2 t_2} P_{\alpha\beta}(t_1, t_2)\,, \tag{2.31}$$

$$P_{\alpha\beta}(t_1, t_2) = \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi}\, e^{-i\omega_1 t_1} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi}\, e^{-i\omega_2 t_2} P_{\alpha\beta}(\omega_1, \omega_2)\,. \tag{2.32}$$

Note in the stationary case the correlation function depends on the difference of times only, $P_{\alpha\beta}(t_1, t_2) = P_{\alpha\beta}(t_1 - t_2)$, that reads in frequency representation as follows:

$$P_{\alpha\beta}(\omega_1, \omega_2) = 2\pi\, \delta(\omega_1 + \omega_2)\, \mathcal{P}_{\alpha\beta}(\omega_1)\,, \tag{2.33}$$

where $\delta(X)$ is the Dirac delta-function. The spectral noise power, $\mathcal{P}_{\alpha\beta}(\omega_1)$, is related to the correlator $P_{\alpha\beta}(t_1 - t_2) = P_{\alpha\beta}(t)$ in the following way:

$$\mathcal{P}_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\, P_{\alpha\beta}(t)\,, \tag{2.34}$$

$$P_{\alpha\beta}(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t}\, \mathcal{P}_{\alpha\beta}(\omega)\,. \tag{2.35}$$

As we already mentioned the quantity $P_{\alpha\alpha}(t=0)$, defining the mean square current fluctuations, diverges. However if we restrict the frequency interval, $\pm\Delta\omega/2$, where the current fluctuations are measured, then we obtain a finite quantity,

$$\left\langle \delta I_{\alpha}^{2} \right\rangle = \int_{-\Delta\omega/2}^{\Delta\omega/2} \frac{d\omega}{2\pi} \mathcal{P}_{\alpha\alpha}(\omega)\,. \tag{2.36}$$

Further simplification arises if the scattering properties of a sample depend on energy only a little. In this case the spectral noise power, $\mathcal{P}_{\alpha\beta}(\omega)$, depends weakly on frequency, $\mathcal{P}_{\alpha\beta}(\omega) \approx \mathcal{P}_{\alpha\beta}(0)$, and we can evaluate Eq. (2.36),

$$\frac{\langle \delta I_{\alpha}^{2} \rangle}{\Delta\nu} = \mathcal{P}_{\alpha\alpha}(0)\,, \tag{2.37}$$

where $\Delta\nu = \Delta\omega/(2\pi)$. In the same way the cross-correlator of currents flowing into the leads α and β measured within the frequency window $\Delta\omega$ becomes,

$$\frac{\langle \delta I_{\alpha} \delta I_{\beta} \rangle}{\Delta\nu} = \mathcal{P}_{\alpha\beta}(0)\,, \tag{2.38}$$

We see that the mean square current fluctuations is defined by the zero frequency noise power. Below we calculate $\mathcal{P}_{\alpha\beta}(0)$ and confirm announced earlier Eqs. (2.3) and (2.4).

2.2.2 Current correlator in frequency domain

Let us calculate a quantity $P_{\alpha\beta}(\omega_1, \omega_2)$ and show that indeed it can be represented as Eq. (2.33).

Substituting Eq. (2.30) into Eq. (2.31) we get,

$$P_{\alpha\beta}(\omega_1, \omega_2) = \frac{1}{2} \left\langle \Delta\hat{I}_{\alpha}(\omega_1)\Delta\hat{I}_{\beta}(\omega_2) + \Delta\hat{I}_{\beta}(\omega_2)\Delta\hat{I}_{\alpha}(\omega_1) \right\rangle\,, \tag{2.39}$$

where $\Delta\hat{I}_\alpha(\omega) = \hat{I}_\alpha(\omega) - \left\langle \hat{I}_\alpha(\omega) \right\rangle$, and $\hat{I}_\alpha(\omega)$ is a current operator in frequency representation. To calculate it we apply the Fourier transformation to Eq. (1.36) and find the following,

$$\hat{I}_\alpha(\omega) = e \int\limits_0^\infty dE \left\{ \hat{b}_\alpha^\dagger(E)\hat{b}_\alpha(E + \hbar\omega) - \hat{a}_\alpha^\dagger(E)a_\alpha(E + \hbar\omega) \right\} . \tag{2.40}$$

For convenience we represent a current as the sum of two contributions. The first one is due to scattered electrons, while the second one is due to incident electrons. To distinguish these contributions we use the upper indices (*out*) and (*in*) for the former and latter contributions, respectively. So, the total current is, $\hat{I}_\alpha(\omega) = \hat{I}_\alpha^{(out)}(\omega) + \hat{I}_\alpha^{(in)}(\omega)$, where

$$\hat{I}_\alpha^{(out)}(\omega) = e \int\limits_0^\infty dE\, \hat{b}_\alpha^\dagger(E)\hat{b}_\alpha(E + \hbar\omega)\,, \tag{2.41}$$

$$\hat{I}_\alpha^{(in)}(\omega) = -e \int\limits_0^\infty dE\, \hat{a}_\alpha^\dagger(E)\hat{a}_\alpha(E + \hbar\omega)\,. \tag{2.42}$$

Then the current correlator, $P_{\alpha\beta}(\omega_1, \omega_2)$, is the sum of four terms,

$$\begin{aligned} P_{\alpha\beta}(\omega_1, \omega_2) &= \sum_{i,j=in,out} P_{\alpha\beta}^{(i,j)}(\omega_1, \omega_2)\,, \\ P_{\alpha\beta}^{(i,j)}(\omega_1, \omega_2) &= \frac{1}{2}\left\langle \Delta\hat{I}_\alpha^{(i)}(\omega_1)\Delta\hat{I}_\beta^{(j)}(\omega_2) + \Delta\hat{I}_\beta^{(j)}(\omega_2)\Delta\hat{I}_\alpha^{(i)}(\omega_1) \right\rangle . \end{aligned} \tag{2.43}$$

We calculate each of these terms separately.

2.2.2.1 Correlator for incoming currents

The part of a current correlation function dependent only on incoming currents is:

$$P_{\alpha\beta}^{(in,in)}(\omega_1,\omega_2) = e^2 \iint_0^\infty dE_1\, dE_2\, \frac{J_{\alpha\beta}^{(in,in)}(E_{1,2},\omega_{1,2}) + J_{\beta\alpha}^{(in,in)}(E_{2,1},\omega_{2,1})}{2}\,, \tag{2.44}$$

where

$$\begin{aligned} J_{\alpha\beta}^{(in,in)}(E_{1,2},\omega_{1,2}) &= \left\langle \left\{ \hat a_\alpha^\dagger(E_1)\, \hat a_\alpha(E_1+\hbar\omega_1) - \left\langle \hat a_\alpha^\dagger(E_1)\, \hat a_\alpha(E_1+\hbar\omega_1)\right\rangle \right\} \right. \\ &\times \left. \left\{ \hat a_\beta^\dagger(E_2)\, \hat a_\beta(E_2+\hbar\omega_2) - \left\langle \hat a_\beta^\dagger(E_2)\, \hat a_\beta(E_2+\hbar\omega_2)\right\rangle \right\} \right\rangle . \end{aligned}$$

Taking into account that the average of the product of four operators is the sum of products of pair correlators we finally find,

$$J_{\alpha\beta}^{(in,in)}(E_{1,2},\omega_{1,2}) = \left\langle \hat a_\alpha^\dagger(E_1)\, \hat a_\beta(E_2+\hbar\omega_2)\right\rangle \left\langle \hat a_\alpha(E_1+\hbar\omega_1)\, \hat a_\beta^\dagger(E_2)\right\rangle .$$

Using Eq. (1.37), we calculate pair correlators,

$$\begin{aligned} \left\langle \hat a_\alpha^\dagger(E_1)\, \hat a_\beta(E_2+\hbar\omega_2)\right\rangle &= \delta_{\alpha\beta}\, \delta(E_1 - E_2 - \hbar\omega_2)\, f_\alpha(E_1)\,, \\ \left\langle \hat a_\alpha(E_1+\hbar\omega_1)\, \hat a_\beta^\dagger(E_2)\right\rangle &= \delta_{\alpha\beta}\, \delta(E_1 + \hbar\omega_1 - E_2)\, \{1 - f_\alpha(E_1+\hbar\omega_1)\}\,, \end{aligned}$$

and correspondingly,

$$\begin{aligned} J_{\alpha\beta}^{(in,in)}(E_{1,2},\omega_{1,2}) &= \delta_{\alpha\beta}\, \delta(E_1 - E_2 - \hbar\omega_2)\, \delta(E_1 + \hbar\omega_1 - E_2) \\ &\times f_\alpha(E_1)\, \{1 - f_\alpha(E_1+\hbar\omega_1)\}\,. \end{aligned}$$

In the same way we get,

$$J_{\beta\alpha}^{(in,in)}(E_{2,1},\omega_{2,1}) = \delta_{\alpha\beta}\,\delta(E_1+\hbar\omega_1-E_2)\,\delta(E_1-E_2-\hbar\omega_2) \times f_\alpha(E_1+\hbar\omega_1)\,\{1-f_\alpha(E_1)\}\,.$$

Substituting equations above into Eq. (2.44) and integrating over energy E_2, we represent this part of a current correlation function in the following way,

$$P_{\alpha\beta}^{(in,in)}(\omega_1,\omega_2) = 2\pi\,\delta(\omega_1+\omega_2)\,\mathcal{P}_{\alpha\beta}^{(in,in)}(\omega_1)\,,$$
$$\mathcal{P}_{\alpha\beta}^{(in,in)}(\omega_1) = \delta_{\alpha\beta}\,\frac{e^2}{h}\int\limits_0^\infty dE_1\,F_{\alpha\alpha}(E_1,E_1+\hbar\omega_1)\,. \tag{2.45}$$

Here we have introduced the following short notation,

$$F_{\alpha\beta}(E,E') = \frac{1}{2}\Big\{f_\alpha(E)\big[1-f_\beta(E')\big] + f_\beta(E')\big[1-f_\alpha(E)\big]\Big\}\,. \tag{2.46}$$

As it follows from Eq. (2.45), the currents flowing into the different leads, $\alpha\neq\beta$, to the scatterer are uncorrelated, $P_{\alpha\neq\beta}^{(in,in)}=0$. This is a consequence of our assumption that electrons at different reservoirs are uncorrelated.

2.2.2.2 Correlator for incoming and out-going currents

The part of a correlator dependent on an incoming current in the lead α and an out-going current in the lead β reads,

$$P_{\alpha\beta}^{(in,out)}(\omega_1,\omega_2) = -e^2 \iint\limits_0^\infty dE_1\, dE_2\, \frac{J_{\alpha\beta}^{(in,out)}(E_{1,2},\omega_{1,2}) + J_{\beta\alpha}^{(out,in)}(E_{2,1},\omega_{2,1})}{2}\,. \tag{2.47}$$

To calculate, for instance,

$$\begin{aligned} J_{\alpha\beta}^{(in,out)}(E_{1,2},\omega_{1,2}) &= \left\langle \left\{ \hat{a}_\alpha^\dagger(E_1)\,\hat{a}_\alpha(E_1+\hbar\omega_1) - \left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{a}_\alpha(E_1+\hbar\omega_1)\right\rangle \right\} \right. \\ &\times \left. \left\{ \hat{b}_\beta^\dagger(E_2)\,\hat{b}_\beta(E_2+\hbar\omega_2) - \left\langle \hat{b}_\beta^\dagger(E_2)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle \right\} \right\rangle \\ &= \left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle \left\langle \hat{a}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\right\rangle, \end{aligned}$$

we express b–operators in terms of a–operators, see Eq. (1.39),

$$\hat{b}_\beta^\dagger(E) = \sum_{\gamma=1}^{N_r} S_{\beta\gamma}^*(E)\hat{a}_\gamma^\dagger(E)\,, \quad \hat{b}_\beta(E) = \sum_{\gamma=1}^{N_r} S_{\beta\gamma}(E)\hat{a}_\gamma(E)\,,$$

and calculate pair correlators,

$$\left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle = \delta(E_1-E_2-\hbar\omega_2)\,S_{\beta\alpha}(E_2+\hbar\omega_2)\,f_\alpha(E_1)\,,$$

$$\left\langle \hat{a}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\right\rangle = \delta(E_1+\hbar\omega_1-E_2)\,S_{\beta\alpha}^*(E_2)\,\{1-f_\alpha(E_1+\hbar\omega_1)\}\,.$$

After that we find,

$$\begin{aligned} J_{\alpha\beta}^{(in,out)}(E_{1,2},\omega_{1,2}) &= \delta(E_1-E_2-\hbar\omega_2)\,\delta(E_1+\hbar\omega_1-E_2) \\ &\times\ S_{\beta\alpha}(E_2+\hbar\omega_2)\,S_{\beta\alpha}^*(E_2)\,f_\alpha(E_1)\,\{1-f_\alpha(E_1+\hbar\omega_1)\}\,. \end{aligned}$$

The similar calculations give us the second term in Eq. (2.47):

$$
\begin{aligned}
J_{\beta\alpha}^{(out,in)}(E_{2,1},\omega_{2,1}) &= \delta(E_1+\hbar\omega_1-E_2)\,\delta(E_1-E_2-\hbar\omega_2) \\
&\times\; S_{\beta\alpha}^{*}(E_2)\,S_{\beta\alpha}(E_2+\hbar\omega_2)\,f_\alpha(E_1+\hbar\omega_1)\,\{1-f_\alpha(E_1)\}\,.
\end{aligned}
$$

Using these equations in Eq. (2.47) and integrating over E_2, we calculate,

$$
\begin{aligned}
P_{\alpha\beta}^{(in,out)}(\omega_1,\omega_2) &= 2\pi\,\delta(\omega_1+\omega_2)\,\mathcal{P}_{\alpha\beta}^{(in,out)}(\omega_1)\,, \\
\mathcal{P}_{\alpha\beta}^{(in,out)}(\omega_1) &= -\frac{e^2}{h}\int\limits_0^\infty dE_1\,F_{\alpha\alpha}(E_1,E_1+\hbar\omega_1)\,S_{\beta\alpha}^{*}(E_1+\hbar\omega_1)\,S_{\beta\alpha}(E_1)\,.
\end{aligned}
\tag{2.48}
$$

This equation shows us that the current carrying by the electrons scattered into the lead β is correlated with a current carrying by the electrons incoming from the reservoir α. In fact these correlations are due to electrons scattered from the lead α into the lead β. That is indicated by the corresponding scattering matrix elements, $S_{\beta\alpha}$.

In the same way we calculate the third term in Eq. (2.43):

$$
\begin{aligned}
P_{\alpha\beta}^{(out,in)}(\omega_1,\omega_2) &= 2\pi\,\delta(\omega_1+\omega_2)\,\mathcal{P}_{\alpha\beta}^{(out,in)}(\omega_1)\,, \\
\mathcal{P}_{\alpha\beta}^{(out,in)}(\omega_1) &= -\frac{e^2}{h}\int\limits_0^\infty dE_1\,F_{\beta\beta}(E_1,E_1+\hbar\omega_1)\,S_{\alpha\beta}^{*}(E_1)\,S_{\alpha\beta}(E_1+\hbar\omega_1)\,.
\end{aligned}
\tag{2.49}
$$

This term is due to correlations between electrons coming from the reservoir β and electrons scattered in the reservoirs α.

2.2.2.3 Correlator for out-going currents

Finally we calculate the last term in Eq. (2.43):

$$
\begin{aligned}
P_{\alpha\beta}^{(out,out)}(\omega_1,\omega_2) &= \frac{e^2}{2}\iint\limits_0^\infty dE_1\,dE_2 \\
&\times \Big\{ \Big\langle \hat{b}_\alpha^\dagger(E_1)\,\hat{b}_\beta(E_2+\hbar\omega_2)\Big\rangle \Big\langle \hat{b}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\Big\rangle \\
&+ \Big\langle \hat{b}_\beta^\dagger(E_2)\,\hat{b}_\alpha(E_1+\hbar\omega_1)\Big\rangle \Big\langle \hat{b}_\beta(E_2+\hbar\omega_2)\,\hat{b}_\alpha^\dagger(E_1)\Big\rangle \Big\}.
\end{aligned} \tag{2.50}
$$

To calculate a pair correlator with $b-$operators we use Eqs. (1.39), (1.37) and obtain, for example,

$$
\begin{aligned}
\Big\langle \hat{b}_\alpha^\dagger(E_1)\,\hat{b}_\beta(E_2+\hbar\omega_2)\Big\rangle &= \delta(E_1-E_2-\hbar\omega_2) \\
&\times \sum_{\gamma=1}^{N_r} S_{\alpha\gamma}^{*}(E_1)\,S_{\beta\gamma}(E_2+\hbar\omega_2)\,f_\gamma(E_1)\,, \\
\Big\langle \hat{b}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\Big\rangle &= \delta(E_1+\hbar\omega_1-E_2) \\
&\times \sum_{\delta=1}^{N_r} S_{\alpha\delta}(E_1+\hbar\omega_1)\,S_{\beta\delta}^{*}(E_2)\,\{1-f_\delta(E_2)\}\,.
\end{aligned}
$$

Other pair correlators are calculated in the similar way. Then Eq. (2.50) results in the following:

$$
P_{\alpha\beta}^{(out,out)}(\omega_1,\omega_2) = 2\pi\,\delta(\omega_1+\omega_2)\,\mathcal{P}_{\alpha\beta}^{(out,out)}(\omega_1)\,, \tag{2.51}
$$

$$
\begin{aligned}
\mathcal{P}_{\alpha\beta}^{(out,out)}(\omega_1) &= \frac{e^2}{h}\int\limits_0^\infty dE_1 \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r} F_{\gamma\delta}(E_1,E_1+\hbar\omega_1)\,. \\
&\times S_{\alpha\gamma}^{*}(E_1)\,S_{\beta\gamma}(E_1)\,S_{\alpha\delta}(E_1+\hbar\omega_1)\,S_{\beta\delta}^{*}(E_1+\hbar\omega_1)\,.
\end{aligned}
$$

Note the correlator of scattered currents depends on the Fermi functions for all the reservoirs. In addition it depends not only on amplitudes of scattering between the leads α and β, where the currents are measured, but rather on all the possible scattering amplitudes. It emphasizes a non-locality inherent to phase-coherent systems.

Summing up Eqs. (2.45), (2.48), (2.49), and (2.51), we arrive at Eq. (2.33), where

$$\mathcal{P}_{\alpha\beta}(\omega) = \frac{e^2}{h}\int_0^\infty dE\Big\{F_{\alpha\alpha}(E,E+\hbar\omega)\left[\delta_{\alpha\beta} - S^*_{\beta\alpha}(E+\hbar\omega)\,S_{\beta\alpha}(E)\right]$$

$$- F_{\beta\beta}(E,E+\hbar\omega)\,S^*_{\alpha\beta}(E)\,S_{\alpha\beta}(E+\hbar\omega) \tag{2.52}$$

$$+\sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r} F_{\gamma\delta}(E,E+\hbar\omega)\,S^*_{\alpha\gamma}(E)\,S_{\beta\gamma}(E)\,S_{\alpha\delta}(E+\hbar\omega)\,S^*_{\beta\delta}(E+\hbar\omega)\Big\}.$$

The frequency dependence of a noise is due to internal and external factors. The internal factor is an energy dependence of the scattering amplitudes. The external factors, represented by the combination of the Fermi functions, $F_{\gamma\delta}(E,E+\hbar\omega)$, are chemical potentials and temperatures of reservoirs. The joint effect of internal and external factors is rather sample-specific. However in some simple cases the effect of bias and temperature can be analyzed.

2.2.3 Spectral noise power for energy independent scattering

Let the reservoirs have different potentials but the same temperature,

$$eV_{\alpha\beta} = \mu_\alpha - \mu_\beta\,; \quad T_\alpha = T_0\,, \ \ \forall\alpha\,. \tag{2.53}$$

We assume a bias and a temperature small compared to the Fermi energy,

$$|eV_{\alpha\beta}|\,,\ k_B T_0 \ll \mu_0\,. \tag{2.54}$$

Suppose also that the scattering matrix varies with energy only a little within the energy window of order k_BT_0, $|eV_{\alpha\beta}|$ near the Fermi energy μ_0. Then the scattering matrix elements in Eq. (2.52) can be calculated at the Fermi energy, $E \approx E + \hbar\omega = \mu_0$. The integration over energy becomes trivial,

$$\int_0^\infty dE\, F_{\alpha\beta}(E, E+\hbar\omega) = \frac{eV_{\alpha\beta}+\hbar\omega}{2}\,\mathrm{cth}\left(\frac{eV_{\alpha\beta}+\hbar\omega}{2k_BT_0}\right), \tag{2.55}$$

and we calculate the spectral noise power,

$$\begin{aligned}\mathcal{P}_{\alpha\beta}(\omega) = \frac{e^2}{h}\Bigg\{ & \frac{\hbar\omega}{2}\,\mathrm{cth}\left(\frac{\hbar\omega}{2k_BT_0}\right)\left[\delta_{\alpha\beta} - \left|S_{\beta\alpha}(\mu_0)\right|^2 - \left|S_{\alpha\beta}(\mu_0)\right|^2\right] \\ + & \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\frac{eV_{\gamma\delta}+\hbar\omega}{2}\,\mathrm{cth}\left(\frac{eV_{\gamma\delta}+\hbar\omega}{2k_BT_0}\right) S^*_{\alpha\gamma}(\mu_0)\,S_{\beta\gamma}(\mu_0)\,S_{\alpha\delta}(\mu_0)\,S^*_{\beta\delta}(\mu_0)\Bigg\}.\end{aligned} \tag{2.56}$$

Let us consider a particular case of $N_r = 2$. [23] Then we find,

$$\begin{aligned}\mathcal{P}_{11}(\omega) = \frac{e^2}{h}\Bigg\{ & \hbar\omega\,\mathrm{cth}\left(\frac{\hbar\omega}{2k_BT_0}\right) T_{12}^2 \\ + & R_{11}T_{12}\left[\frac{eV+\hbar\omega}{2}\,\mathrm{cth}\left(\frac{eV+\hbar\omega}{2k_BT_0}\right) + \frac{eV-\hbar\omega}{2}\,\mathrm{cth}\left(\frac{eV-\hbar\omega}{2k_BT_0}\right)\right]\Bigg\}.\end{aligned} \tag{2.57}$$

where $V = V_{12} = -V_{21}$, $T_{12} = |S_{12}(\mu_0)|^2$, $R_{11} = |S_{11}(\mu_0)|^2 = 1 - T_{12}$. Note in the two-terminal case the calculated quantity defines all other correlation functions: $\mathcal{P}_{12} = \mathcal{P}_{21} = -\mathcal{P}_{22} = -\mathcal{P}_{11}$.

The noise depends on a frequency ω at which the current is measured, on a bias V, and on a temperature T_0. If one of these factors exceeds other ones then we get,

$$\mathcal{P}_{11}(\omega) = \begin{cases} 2k_B T_0 G\,, & k_B T_0 \gg |eV|,\, \hbar\omega\,, \\ |eI|R_{11}\,, & |eV| \gg \hbar\omega,\, k_B T_0\,, \\ \frac{e^2}{2\pi}|\omega| T_{12}\,, & \hbar\omega \gg k_B T_0,\, |eV|\,, \end{cases} \tag{2.58}$$

where $G = (e^2/h)T_{12}$ is a conductance, $I = VG$ is a current through the sample. The first line represents a thermal noise which is linear in temperature. The coefficient 2 arises due to two reservoirs having the same temperature. If the temperatures are different then we should make a replacement, $2T_0 \to T_1 + T_2$. Taking into account Eq. (2.37) we see that this equation is exactly Eq. (2.3). The second line in Eq. (2.58) corresponds to a regime when the shot noise dominates. It reproduces Eq. (2.4). And, finally, the third line represents so called *a quantum noise* dependent on the measurement frequency ω. [24] Namely this last contribution is responsible for divergence of the mean square current fluctuations $\langle I_1^2 \rangle = P_{11}(t = 0)$, see Eq. (2.35).

As it follows from Eq. (2.58) the frequency dependence of a noise can be ignored if,

$$\hbar\omega \ll \max\left\{k_B T_0, |eV_{\alpha\beta}|\right\}, \quad \forall \alpha, \beta\,. \tag{2.59}$$

In this case the quantum noise becomes negligible and the main sources of current fluctuations are thermal and shot noises. At $T_0 \sim 10^{-2}$ K and/or $V \sim 10^{-6}$ V the quantum noise can be ignored up to the frequencies $\omega \sim 10^9$ Hz.

2.2.4 Zero frequency noise power

If the measurement is doing at enough small frequencies, Eq. (2.59), then the value of current fluctuations is defined by the noise power at zero frequency, $\omega = 0$, see Eq. (2.37). The quantity $\mathcal{P}_{\alpha\alpha}(0)$ is usually referred to as *the noise power*.

Let us represent a quantity $\mathcal{P}_{\alpha\beta}(0)$, Eq. (2.52), as the sum of two terms such that one of them vanishes at zero temperature, while another one vanishes in the absence of a current through the sample. To this end we write,

$$F_{\gamma\delta}(E,E) = \frac{1}{2}\left\{F_{\gamma\gamma}(E,E) + F_{\delta\delta}(E,E) + \left[f_\gamma(E) - f_\delta(E)\right]^2\right\}.$$

Then in Eq. (2.52) in the term with factor $F_{\gamma\gamma}(E,E)$ we sum up over δ and, taking into account Eq. (1.13), find,

$$\sum_{\gamma=1}^{N_r} F_{\gamma\gamma}\, S^*_{\alpha\gamma}\, S_{\beta\gamma} \sum_{\delta=1}^{N_r} S_{\alpha\delta}\, S^*_{\beta\delta} = \delta_{\alpha\beta} \sum_{\gamma=1}^{N_r} F_{\gamma\gamma}\, \left|S_{\alpha\gamma}\right|^2 .$$

The term with factor $F_{\delta\delta}(E,E)$ reads exactly the same. After then we write, [4]

$$\mathcal{P}_{\alpha\beta}(0) = \mathcal{P}^{(th)}_{\alpha\beta} + \mathcal{P}^{(sh)}_{\alpha\beta}\,, \tag{2.60}$$

where

$$\begin{aligned}\mathcal{P}^{(th)}_{\alpha\beta} &= \frac{e^2}{h}\int_0^\infty dE\left\{\delta_{\alpha\beta}\left[F_{\alpha\alpha}(E,E) + \sum_{\gamma=1}^{N_r} F_{\gamma\gamma}(E,E)\left|S_{\alpha\gamma}(E)\right|^2\right]\right.\\ &\left. - F_{\alpha\alpha}(E,E)\,\left|S_{\beta\alpha}(E)\right|^2 - F_{\beta\beta}(E,E)\,\left|S_{\alpha\beta}(E)\right|^2\right\},\end{aligned} \tag{2.61}$$

$$\mathcal{P}^{(sh)}_{\alpha\beta} = \frac{e^2}{h}\int_0^\infty dE \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r} \frac{\left[f_\gamma(E) - f_\delta(E)\right]^2}{2}\, S^*_{\alpha\gamma}(E)\, S_{\beta\gamma}(E)\, S_{\alpha\delta}(E)\, S^*_{\beta\delta}(E)\,. \tag{2.62}$$

The quantity $\mathcal{P}^{(th)}_{\alpha\alpha}$ can be referred to as *the thermal noise power*. This quantity vanishes at zero temperature, since at $T_\alpha = 0$ it is $F_{\alpha\alpha}(E,E) = 0,\ \forall\alpha$. While the quantity $\mathcal{P}^{(sh)}_{\alpha\alpha}$ can be called as *the shot noise power*. Since it vanishes in the absence of a current through the system. Remind that the current is driven by the Fermi function difference.

It should be noted that as $\mathcal{P}^{(th)}_{\alpha\beta}$ as $\mathcal{P}^{(sh)}_{\alpha\beta}$ depend on both the temperature and the bias voltage. That emphasizes the universal probabilistic nature of a noise. However there is an essential difference between the equilibrium (thermal) noise and the non-equilibrium (shot) noise. The thermal noise depends on the probabilities $|S_{\alpha\beta}|^2$ like the conductance $G_{\alpha\beta}$, Eq. (1.55), does. This is a consequence of the fluctuation-dissipation theorem, see e.g., Ref. [11]. While the shot noise depends on different combinations of the scattering matrix elements. That in general allows to extract additional information concerning the properties of a sample from the shot noise measurements. Further we consider general properties of the noise power.

2.2.4.1 Noise power conservation law

The sum of the zero-frequency current correlation function power over either incoming or outgoing indices is zero, [5]

$$\sum_{\alpha=1}^{N_r} \mathcal{P}_{\alpha\beta}(0) = \sum_{\beta=1}^{N_r} \mathcal{P}_{\alpha\beta}(0) = 0\,. \tag{2.63}$$

These conservation laws are quite analogous to the dc current conservation law, (1.48). They are due to particle number conservation at scattering (due to unitarity of the scattering matrix).

Remarkably the thermal noise and the shot noise are subject to these conservation laws separately. So using Eq. (1.51) we find for the thermal noise, Eq. (2.61),[1]

$$\sum_{\alpha=1}^{N_r} \mathcal{P}_{\alpha\beta}^{(th)} \sim \sum_{\alpha=1}^{N_r} \delta_{\alpha\beta}\left[F_{\alpha\alpha}(E,E) + \sum_{\gamma=1}^{N_r} F_{\gamma\gamma}(E,E)\left|S_{\alpha\gamma}(E)\right|^2\right]$$
$$- \sum_{\alpha=1}^{N_r} F_{\alpha\alpha}(E,E)\left|S_{\beta\alpha}(E)\right|^2 - F_{\beta\beta}(E,E)\sum_{\alpha=1}^{N_r}\left|S_{\alpha\beta}(E)\right|^2 =$$

[1] We drop an integration over energy since the conservation laws hold not only integrally but also separately for each energy

$$= F_{\beta\beta}(E,E) + \sum_{\gamma=1}^{N_r} F_{\gamma\gamma}(E,E) \left|S_{\beta\gamma}(E)\right|^2$$

$$- \sum_{\alpha=1}^{N_r} F_{\alpha\alpha}(E,E) \left|S_{\beta\alpha}(E)\right|^2 - F_{\beta\beta}(E,E) = 0\,.$$

In the same way using Eq. (1.46) we show that $\sum_{\beta=1}^{N_r} \mathcal{P}_{\alpha\beta}^{(th)}(0) = 0$.

In the case of a shot noise, Eq. (2.62), we use Eq. (1.12) and get,

$$\sum_{\alpha=1}^{N_r} \mathcal{P}_{\alpha\beta}^{(sh)} \sim \sum_{\gamma=1}^{N_r} \sum_{\delta=1}^{N_r} \frac{\left[f_\gamma(E) - f_\delta(E)\right]^2}{2} S_{\beta\gamma}(E)\, S_{\beta\delta}^*(E) \sum_{\alpha=1}^{N_r} S_{\alpha\gamma}^*(E)\, S_{\alpha\delta}(E)\,.$$

$$= \sum_{\gamma=1}^{N_r} \sum_{\delta=1}^{N_r} \frac{\left[f_\gamma(E) - f_\delta(E)\right]^2}{2} S_{\beta\gamma}(E)\, S_{\beta\delta}^*(E)\, \delta_{\gamma\delta} = 0\,.$$

Then with Eq. (1.13) we also prove, $\sum_{\beta=1}^{N_r} \mathcal{P}_{\alpha\beta}^{(sh)}(0) = 0$.

The conservation laws, Eq. (2.63), show that the auto-correlator and cross-correlators at zero frequency are not independent from each other. Some of them can be calculated if other were measured.

2.2.4.2 Sign rule for noise power

The auto-correlator is positive (or zero) while the cross-correlator is negative (or zero) [5],

$$\mathcal{P}_{\alpha\alpha}(0) \;\geq\; 0\,, \tag{2.64a}$$

$$\mathcal{P}_{\alpha\beta}(0) \;\leq\; 0\,,\; \alpha \neq \beta\,. \tag{2.64b}$$

The positiveness of $\mathcal{P}_{\alpha\alpha}(0)$ is clear, since this quantity is a mean square of a real quantity, Eq. (2.37). The negative sign of a cross-correlator is a consequence, first, of an indivisibility of electrons and, second, of the Pauli exclusion principle requiring (spinless) electrons with some energy pass one by one through the one-dimensional lead. Therefore, we can look at scattering of a single electron with given energy and forget about other electrons. Let us consider scattering of an electron flow moving to the sample in the lead γ. Electrons from this flow can be scatterer to any lead δ with probability $|S_{\delta\gamma}(E)|^2$. In particular some electrons will be scattered into the leads α and β. These electrons define the mean currents, $\langle I_\alpha^{(\gamma)}\rangle$ and $\langle I_\beta^{(\gamma)}\rangle$. On the other hand each particular electron can be scatterer to only one lead. It can be either lead α, or β, or any other lead δ. In any case the current pulse due to scattering of this particular electron arises only in one lead. Therefore, the product of instant currents in any two leads, for example in α and β, is zero, $I_\alpha^{(\gamma)} I_\beta^{(\gamma)} = 0$. Then we immediately conclude that the cross-correlator of currents in leads α and β due to single electrons coming with energy E from the reservoir γ is negative, $\mathcal{P}_{\alpha\beta}^{(\gamma)}(E) \sim \langle I_\alpha^{(\gamma)} I_\beta^{(\gamma)}\rangle - \langle I_\alpha^{(\gamma)}\rangle\langle I_\beta^{(\gamma)}\rangle \sim 0 - |S_{\alpha\gamma}(E)|^2 |S_{\beta\gamma}(E)|^2 \le 0$. In different reservoirs and at different energies electrons are statistically independent. Therefore, we can sum up correlation functions $\mathcal{P}_{\alpha\beta}^{(\gamma)}(E)$ over γ and integrate over E. Then we arrive at Eq. (2.64b).

Let us show that the thermal noise, Eq. (2.61), and the shot noise, Eq. (2.62), do satisfy the sign rule, Eqs. (2.64). We will omit an integration over energy which does not affect a sign of a current correlation function. First we consider a thermal noise. The auto-correlator gives,

$$
\begin{aligned}
\mathcal{P}_{\alpha\alpha}^{(th)} &\sim F_{\alpha\alpha}(E,E) + \sum_{\gamma=1}^{N_r} F_{\gamma\gamma}(E,E) \left|S_{\alpha\gamma}(E)\right|^2 - 2F_{\alpha\alpha}(E,E)\, |S_{\alpha\alpha}(E)|^2 = \\
&= F_{\alpha\alpha}(E,E) \left[1 - |S_{\alpha\alpha}(E)|^2\right] + \sum_{\gamma\ne\alpha=1}^{N_r} F_{\gamma\gamma}(E,E) \left|S_{\alpha\gamma}(E)\right|^2 \ge 0\,.
\end{aligned}
$$

Here we took into account $0 \le F_{\alpha\alpha}(E,E) \le 1$ and $|S_{\alpha\alpha}(E)|^2 \le 1$. For the cross-correlator, $\alpha \ne \beta$, we find a definitely negative expression, $P_{\alpha\ne\beta}^{(th)} \sim -|S_{\beta\alpha}|^2 f_\alpha [1 - f_\alpha] - |S_{\alpha\beta}|^2 f_\beta [1 - f_\beta] \le 0$.

Next we consider a shot noise. The auto-correlator is definitely positive, $P_{\alpha\alpha}^{(sh)} \sim \sum_{\gamma=1}^{N_r} \sum_{\delta=1}^{N_r} \frac{1}{2}\left(f_\gamma - f_\delta\right)^2 |S_{\alpha\gamma}|^2\, |S_{\alpha\delta}|^2 \ge 0$. To calculate a cross-correlator we use, $\left(f_\gamma - f_\delta\right)^2 = f_\gamma^2 + f_\delta^2 - 2 f_\gamma f_\delta$, use Eq. (1.13), and get,

$$\begin{aligned}
\mathcal{P}^{(sh)}_{\alpha\neq\beta} &\sim \frac{1}{2}\sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\left(f_\gamma^2 + f_\delta^2 - 2f_\gamma f_\delta\right) S^*_{\alpha\gamma} S_{\beta\gamma} S_{\alpha\delta} S^*_{\beta\delta} \\
&= \frac{1}{2}\sum_{\gamma=1}^{N_r} f_\gamma^2 S^*_{\alpha\gamma} S_{\beta\gamma}\sum_{\delta=1}^{N_r} S_{\alpha\delta} S^*_{\beta\delta} + \frac{1}{2}\sum_{\gamma=1}^{N_r} S^*_{\alpha\gamma} S_{\beta\gamma}\sum_{\delta=1}^{N_r} f_\delta^2\, S_{\alpha\delta} S^*_{\beta\delta} \\
&\quad -\sum_{\gamma=1}^{N_r} f_\gamma S^*_{\alpha\gamma} S_{\beta\gamma}\sum_{\delta=1}^{N_r} f_\delta S_{\alpha\delta} S^*_{\beta\delta} = -\left|\sum_{\gamma=1}^{N_r} f_\gamma S^*_{\alpha\gamma} S_{\beta\gamma}\right|^2 \leq 0\,.
\end{aligned}$$

In the second line we used $\sum_\delta S_{\alpha\delta} S^*_{\beta\delta} = \delta_{\alpha\beta} = 0$. Thus the sign rule for the current correlator power at zero frequency has proven.

To illustrate given above general properties we consider a simple example.

2.2.4.3 Scatterer with two leads

From Eq. (2.63) it follows that for $N_2 = 2$ the whole noise power matrix, $\hat{\mathcal{P}}(0)$, is defined by only a single element. This is true for the thermal noise and for the shot noise separately,

$$\begin{aligned}
\mathcal{P}^{(th)}_{11} &= \mathcal{P}^{(th)}_{22} = -\mathcal{P}^{(th)}_{12} = -\mathcal{P}^{(th)}_{21} \equiv \mathcal{P}^{(th)}\,, \\
\mathcal{P}^{(sh)}_{11} &= \mathcal{P}^{(sh)}_{22} = -\mathcal{P}^{(sh)}_{12} = -\mathcal{P}^{(sh)}_{21} \equiv \mathcal{P}^{(sh)}\,,
\end{aligned} \tag{2.65a}$$

where

$$\mathcal{P}^{(th)} = \frac{e^2 k_B}{h}\int_0^\infty dE\left(-T_1\frac{\partial f_1(E)}{\partial E} - T_2\frac{\partial f_2(E)}{\partial E}\right) T_{12}(E)\,, \tag{2.65b}$$

$$\mathcal{P}^{(sh)} = \frac{e^2}{h}\int_0^\infty dE\left[f_1(E) - f_2(E)\right]^2 T_{12}(E)\, R_{11}(E)\,. \tag{2.65c}$$

Here $T_{12}(E) = \left|S_{12}(E)\right|^2$, $R_{11}(E) = 1 - T_{12}(E)$ are, respectively, transmission and reflection probabilities for electrons with energy E. While transforming an expression for $\mathcal{P}^{(th)}$ we used the following identity for the Fermi distribution function,

$$F_{\alpha\alpha}(E,E) \equiv f_\alpha(E)[1 - f_\alpha(E)] = -k_B T_\alpha \frac{\partial f_\alpha(E)}{\partial E}. \tag{2.66}$$

From Eqs. (2.65) it follows that the character of a dependence of a transmission coefficient on energy is crucial for the dependence of a noise on both the temperature and the bias voltage. For instance, if the transmission coefficient $T_{12}(E)$ changes only a little within a relevant energy window (maximum of two, the reservoir temperature and the bias) then the thermal noise is linear in reservoir temperatures T_1, T_2 and it is independent of a bias: $\mathcal{P}^{(th)} = k_B (T_1 + T_2)G$, where $G = (e^2/h)T_{12}(\mu_0)$. In contrast, the shot noise, $\mathcal{P}^{(sh)}$, is a non-linear function of both the temperature and the bias. And only in the limit of a large bias, $|eV| \gg k_B T_1, k_B T_2$, the shot noise becomes merely proportional to a current, $I = VG$: $\mathcal{P}^{(sh)} = |eI| R_{11}(\mu_0)$.

2.2.5 Fano factor

The Fano factor, F, is a ratio of the shot noise to the dc current times the charge of carriers, see e.g., Ref. [20]:

$$F = \frac{P^{(sh)}}{|qI|}. \tag{2.67}$$

As it was shown by Schottky [21], for statistically independent carriers the Fano factor is unity. In the presence of correlations and/or interactions between carriers the Fano factor is generally different from unity.

In mesoscopics also one can introduce the Fano factor. However, as it follows from Eqs. (1.47) (for $N_r = 2$) and (2.65c), in general $F \neq 1$. Even in the simplest case, $T(E) = \text{const}$ and $eV \gg k_B T$, the Fano factor $F = 1 - T_{12} < 1$. At $T_{12} \to 0$ the quantity $F \approx 1$, therefore, one can say that in the case of a small conductance, $G/G_0 = T_{12} \ll 1$, the current is carried by the statistically independent particles. While with increasing conductance the factor Fano decreases, that is due to correlations between carriers. These correlations are consequence of the Pauli exclusion principle forcing electrons to pass a lead one by one.

Chapter 3

Non-stationary scattering theory

Applying a time-dependent bias or varying in time the properties of a sample we create conditions when the time-dependent currents flow through the system. Our aim is to consider how the non-stationary transport can be described within the scattering matrix formalism.

To calculate the scattering matrix elements, which are quantum-mechanical amplitudes, we need to solve the Schrödinger equation. Therefore, we first consider the methods of solution of the non-stationary Schrödinger equation and then analyze the properties of the scattering matrix of a non-stationary sample. We are interested in a particular case when the dependence on time is periodic.

3.1 Schrödinger equations with periodic in time potential

Let us consider the Schrödinger equation for the wave function Ψ of a particle with mass m in the case of a time-dependent Hamiltonian, $H(t,\vec{r})$,

$$\begin{aligned} i\hbar\,\frac{\partial \Psi(t,\vec{r})}{\partial t} &= H(t,\vec{r})\,\Psi(t,\vec{r})\,, \\ H(t,\vec{r}) &= H_0(\vec{r}) + V(t,\vec{r})\,. \end{aligned} \tag{3.1}$$

Here we split Hamiltonian into the two parts, a time-independent, $H_0(\vec{r})$, and dependent on time, $V(t,\vec{r})$. The corresponding boundary conditions are assumed to be stationary. We suppose that the solution to the stationary problem with Hamiltonian $H_0(\vec{r})$,

$$\begin{aligned} \Psi(t,\vec{r}) &= e^{-\frac{iEt}{\hbar}}\,\psi(\vec{r})\,, \\ H_0(\vec{r})\psi(\vec{r}) &= E\psi(\vec{r})\,. \end{aligned} \tag{3.2}$$

and with the same boundary conditions is known. That is, we found all the eigen-functions, $\psi_n(\vec{r})$, and eigen-energies,E_n,

$$H_0(\vec{r})\,\psi_n(\vec{r}) \;=\; E_n\psi_n(\vec{r})\,. \tag{3.3}$$

Note that it is $\Psi_n(t,\vec{r}) = e^{-\frac{iE_n t}{\hbar}}\psi_n(\vec{r})$. The index n (non necessary integer) numbers the states belonging to both discrete and continuous part of a spectrum.

We compare two method for solving of a non-stationary problem. The first method is the perturbation theory by P.A.M. Dirac [25], see, e.g., Ref. [10], which is applicable for a weak time-dependent potential with arbitrary dependence on time. The second one, based on the Floquet theorem, see, e.g., Ref. [26, 27], is applied for periodic in time potentials with arbitrary strength.

3.1.1 Perturbation theory

Let the time-dependent potential is small,

$$V(t,\vec{r}) \to 0\,, \tag{3.4}$$

and, therefore, can be considered as a perturbation which changes only a little the state of a quantum system with Hamiltonian $H_0(\vec{r})$.

We are looking for a solution to Eq. (3.1) as a series in stationary eigen-wavefunctions,

$$\Psi(t,\vec{r}) = \sum_n a_n(t)\,\Psi_n(t,\vec{r})\,. \tag{3.5}$$

Substituting Eq. (3.5) into Eq. (3.1) and using Eq. (3.3) we find,

$$i\hbar\sum_n \Psi_n(t,\vec{r})\,\frac{da_n(t)}{dt} = \sum_n a_n(t)\,V(t,\vec{r})\,\Psi_n(t,\vec{r})\,. \tag{3.6}$$

Further we multiply both parts of this equation with $\Psi_k^*(t,\vec{r})$ and integrate over space. Since the eigen-functions of the Hamiltonian are orthogonal,

$$\int d^3r\, \psi_k^*(\vec{r})\, \psi_n(\vec{r}) = \delta_{n,k}\,,$$

we arrive at the following equation for the coefficients a_k:

$$i\hbar \frac{da_k(t)}{dt} = \sum_n V_{kn}(t)\, a_n(t)\,, \tag{3.7}$$

where the perturbation matrix elements are:

$$V_{kn}(t) = \int d^3r\, \psi_k^*(\vec{r})\, V(t,\vec{r})\, \psi_n(\vec{r})\, e^{i\frac{E_k - E_n}{\hbar}t}\,. \tag{3.8}$$

To find the coefficients $a_n(t)$ we need to solve the system of an infinite number of differential equations of the first order, Eq. (3.7).

Up to now we did not use a fact that the perturbation is weak. Now we use it and solve the system of equations to the linear order in $V(t,\vec{r})$. To be more precise we consider the following problem:
The perturbation $V(t,\vec{r})$ is switched on at $t = 0$. We consider a particle which was in the state $\Psi_m(t,\vec{r})$ with energy E_m at $t \le 0$. We need to calculate its wave function $\Psi^{(m)}(t,\vec{r})$ at $t > 0$.
We will use an upper index (m) to show an initial state. So, we have a problem with following initial conditions,

$$\Psi^{(m)}(t=0,\vec{r}) = \Psi_m(t=0,\vec{r}) \Rightarrow \begin{cases} a_m^{(m)}(0) = 1\,, \\ a_n^{(m)}(0) = 0\,, \quad n \neq m\,, \end{cases}$$

where $a_n^{(m)}(t)$ are coefficients in Eq. (3.5) for the wave function of interest, $\Psi^{(m)}(t,\vec{r})$. After the perturbation is switched on the coefficients become functions of time, $a_n^{(m)}(t)$, which we look for as a series in powers of a small parameter $V(t,\vec{r})$. In the linear order we have,

$$
\begin{aligned}
a_m^{(m)}(t) &= 1 + a_m^{(m,1)}(t)\,, \\
a_n^{(m)}(t) &= 0 + a_n^{(m,1)}(t)\,, \quad n \neq m\,.
\end{aligned}
\tag{3.9}
$$

Substituting these equations into Eq. (3.7) and keeping only linear in V terms we find,

$$
i\hbar \frac{da_k^{(m,1)}(t)}{dt} = V_{km}(t)\,. \tag{3.10}
$$

This linear first order equation can be easily integrated out,

$$
a_k^{(m,1)}(t) = -\frac{i}{\hbar}\int_0^t dt'\, V_{km}(t')\,. \tag{3.11}
$$

Accordingly to the basic principles of the quantum mechanics the absolute value square, $|a_k^{(m)}(t)|^2$, defines a probability to observe a particle in the state $\Psi_k(t,\vec{r})$ with energy E_k at time t. Note at initial time $t = 0$ the particle was in the state with energy E_m. The change of particle's energy is due to the interaction with a time-dependent potential $V(t,\vec{r})$. The particle can either gain energy, $E_k > E_m$, or lose it, $E_k < E_m$.

Now we clarify when the potential can be treat as small, Eq. (3.4). Let us consider a uniform in space and periodic in time potential, $V(t,\vec{r}) = U(t)\,R(\vec{r})$, where

$$
U(t) = 2U\cos(\Omega_0 t)\,. \tag{3.12}
$$

Then we can solve Eq. (3.11),

$$
a_k^{(m,1)}(t) = -UR_{km}\left(\frac{e^{i(\omega_{km}-\Omega_0)t}-1}{\hbar(\omega_{km}-\Omega_0)} + \frac{e^{i(\omega_{km}+\Omega_0)t}-1}{\hbar(\omega_{km}+\Omega_0)}\right)\,, \tag{3.13}
$$

where $R_{km} = \int d^3 r \psi_k^*(\vec{r}) R(\vec{r}) \psi_n(\vec{r})$ and $\hbar\omega_{km} = E_k - E_m$. The perturbation theory is correct if the absolute value of $a_{k\neq m}^{(m)}(t)$ is small compared to a unity:

$$\frac{V_{km}}{\hbar(\omega_{km} \pm \Omega_0)} \sim \frac{UR_{km}}{\hbar(\omega_{km} \pm \Omega_0)} \ll 1\,. \tag{3.14}$$

In this case the particle with a large probability stays in its initial state and the effect of a time-dependent potential is really small as it was supposed.

If the perturbation frequency, Ω_0, is close to some difference, $\pm(E_{k_0} - E_m)/\hbar$, then the equation (3.14) can be easily violated and the perturbation theory fails. In such a case the time-dependent potential will cause a particle to pass over from the initial state $\Psi_m(t, \vec{r})$ to the state $\Psi_{k_0}(t, \vec{r})$ and back, since the coefficients $a_m^{(m)}(t)$ and $a_{k_0}^{(m)}(t)$ are of the same order.

Substituting Eqs. (3.13) and (3.9) into Eq. (3.5), written for the function $\Psi^{(m)}(t, \vec{r})$, we finally calculate,

$$\begin{aligned} \Psi^{(m)}(t, \vec{r}) &= e^{-i\frac{E_m}{\hbar}t} \sum_n \psi_n(\vec{r}) \\ &\times \left\{ \delta_{nm} - \frac{UR_{nm}\left(e^{-i\Omega_0 t} - e^{-i\omega_{nm}t}\right)}{\hbar(\omega_{nm} - \Omega_0)} - \frac{UR_{nm}\left(e^{i\Omega_0 t} - e^{-i\omega_{nm}t}\right)}{\hbar(\omega_{nm} + \Omega_0)} \right\}. \end{aligned} \tag{3.15}$$

Thus we found that the periodic perturbation results in additional terms in the expression for the wave function which correspond to initial energy shifted by $\pm\hbar\Omega_0$. Easy to understand that the spectral contents of the perturbation defines energies of additional side-bands of a wave function.

3.1.2 Floquet functions method

This method overcomes the restrictions put by Eq. (3.14) and allows to consider an arbitrary but periodic in time potential. The main idea is to use the Floquet theorem. Accordingly to this theorem the solution for the Schrödinger equation with periodic in time Hamiltonian,

$$H(t, \vec{r}) = H(t + \mathfrak{T}, \vec{r})\,, \tag{3.16}$$

can be written as follows,

$$\begin{aligned} \Psi(t,\vec{r}) &= e^{-i\frac{E}{\hbar}t}\phi(t,\vec{r})\,, \\ \phi(t,\vec{r}) &= \phi(t+\mathfrak{T},\vec{r})\,. \end{aligned} \tag{3.17}$$

To outline the proof of this theorem we consider the general solution $\Psi(t,\vec{r})$ to Eq. (3.1) with Hamiltonian, Eq. (3.16). Let us shift a time by one period, $t \to t+\mathfrak{T}$. Then the wave function $\Psi(t+\mathfrak{T},\vec{r})$ is also a solution to the same equation,

$$\begin{aligned} i\hbar\,\frac{\partial\Psi(t+\mathfrak{T},\vec{r})}{\partial t} &= H(t+\mathfrak{T},\vec{r})\,\Psi(t+\mathfrak{T},\vec{r}) \\ &= H(t,\vec{r})\,\Psi(t+\mathfrak{T},\vec{r})\,. \end{aligned}$$

Therefore, two general solutions have to be proportional each other,

$$\Psi(t+\mathfrak{T},\vec{r}) = C\,\Psi(t,\vec{r})\,. \tag{3.18}$$

Since the wave function is normalized,

$$\begin{aligned} \int d^3r|\Psi(t,\vec{r})|^2 &= 1\,, \\ \int d^3r|\Psi(t+\mathfrak{T},\vec{r})|^2 = |C|^2\int d^3r|\Psi(t,\vec{r})|^2 &= 1\,, \end{aligned}$$

we find for the constant C,

$$|C|^2 = 1 \quad\Rightarrow\quad C = e^{-i\alpha}\,. \tag{3.19}$$

The general expression for the function subject to Eq. (3.18) with coefficient given in Eq. (3.19) is the following,

$$\begin{aligned} \Psi(t,\vec{r}) &= e^{-i\frac{\alpha}{\mathfrak{T}}t}\phi(t,\vec{r})\,, \\ \phi(t,\vec{r}) &= \phi(t+\mathfrak{T},\vec{r})\,. \end{aligned} \tag{3.20}$$

Let us check that Eq. (3.18) holds,

$$\Psi(t+\mathfrak{T}) = e^{-i\frac{\alpha}{\mathfrak{T}}(t+\mathfrak{T})}\phi(t+\mathfrak{T}) = e^{-i\alpha}\left\{e^{-i\frac{\alpha}{\mathfrak{T}}t}\phi(t)\right\} = e^{-i\alpha}\Psi(t)\,.$$

Finally introducing $E = \hbar\alpha/\mathfrak{T}$ instead of α we see that Eq. (3.20) is reduced to Eq. (3.17). The Floquet theorem has proven.

Next we expand a periodic in time function $\phi(t,\vec{r})$ into the Fourier series,

$$\phi(t,\vec{r}) = \sum_{q=-\infty}^{\infty} e^{-iq\Omega_0 t}\psi_q(\vec{r})\,, \tag{3.21a}$$

$$\psi_q(\vec{r}) = \int_0^{\mathfrak{T}} \frac{dt}{\mathfrak{T}}\, e^{iq\Omega_0 t}\phi(t,\vec{r})\,, \tag{3.21b}$$

where $\Omega_0 = 2\pi/\mathfrak{T}$. Then the Floquet wave function, Eq. (3.17), becomes,

$$\Psi(t,\vec{r}) = e^{-i\frac{E}{\hbar}t}\sum_{q=-\infty}^{\infty} e^{-iq\Omega_0 t}\psi_q(\vec{r})\,. \tag{3.22}$$

In the case of a stationary Hamiltonian the solution corresponding to energy E has a factor $e^{-i\frac{E}{\hbar}t}$. Therefore, in the stationary case in Eq. (3.22) only the term with $q = 0$ survives. In the case of a time-dependent Hamiltonian the energy is a quantity which is not uniquely defined. For instance, if we change E in Eq. (3.22) by any number p of energy quanta $\hbar\Omega_0$, $E \to E + p\hbar\Omega_0$, then we arrive at the same wave function. To show it we need only to redefine functions $\psi_q(\vec{r})$ changing its indices, $q \to q + p$. Since the quantity E is defined up to energy quantum $\hbar\Omega_0$, then it is referred to as *the quasi-energy* or *the Floquet energy*. In each particular problem the quantity E is fixed just as it is convenient. For numerical calculations people often use, $0 \le E < \hbar\Omega_0$. On the other hand, exploring a problem how some stationary state evolves under the action of a periodic potential, it is convenient to choose E equal to energy of this initial stationary state. We will follow the latter way when we will consider scattering of electrons with fixed energy E onto the dynamic sample.

Comparing Eqs. (3.15) and (3.22) we conclude that the Floquet theorem predicts an existence of multi-photon processes when the energy changes by several quanta $\hbar\Omega_0$ in addition to single-photon processes taking place already in the case of a weak perturbation. The Floquet theorem gives an ansatz for the solution to the Schrödinger equation with periodic Hamiltonian. The unknown function $\psi_q(\vec{r})$ is a solution to some stationary problem. It should be noted in general case the functions $\psi_q(\vec{r})$ with different q are not independent. Therefore, the non-stationary problem is reduced to multi-channel stationary problem.

3.1.3 Uniform in space and oscillating in time potential

Let us consider a simple exactly solvable example to show that the solution of a period problem is really of a Floquet function type and at a weak perturbation only single-photon processes are allowed. So we consider the Schrödinger equation with a uniform potential, Eq. (3.12),

$$i\hbar\frac{\partial\Psi(t,\vec{r})}{\partial t} = \left\{H_0 + 2U\cos(\Omega_0 t)\right\}\Psi(t,\vec{r})\,. \tag{3.23}$$

The solution to this equation reads,

$$\Psi(t,\vec{r}) = e^{-i\left\{\frac{E}{\hbar}t+\frac{2U}{\hbar\Omega_0}\sin(\Omega_0 t)\right\}}\psi_E(\vec{r})\,, \tag{3.24}$$

where $\psi_E(\vec{r})$ is a solution to the following stationary equation,

$$H_0\,\psi_E(\vec{r}) = E\,\psi_E(\vec{r})\,. \tag{3.25}$$

Next we use the following Fourier series,

$$e^{-i\alpha\sin(\Omega_0 t)} = \sum_{q=-\infty}^{\infty} e^{-iq\Omega_0 t} J_q(\alpha)\,, \tag{3.26}$$

where J_q is the Bessel function of the first kind of the qth order, and rewrite Eq. (3.24) as follows,

$$\Psi(t,\vec{r}) = e^{-i\frac{E}{\hbar}t} \sum_{q=-\infty}^{\infty} e^{-iq\Omega_0 t} J_q\left(\frac{2U}{\hbar\Omega_0}\right)\psi_E(\vec{r})\,. \tag{3.27}$$

Comparing equation above with Eq. (3.22) we see that really the obtained solution is the Floquet function with $\psi_q(\vec{r}) = J_q(2U/\hbar\Omega_0)\psi_E(\vec{r})$.

Let us analyze Eq. (3.27) at small amplitude, $U/(\hbar\Omega_0) \ll 1$. To this end we expand the Bessel functions into the Tailor series in powers of a small parameter $\alpha = 2U/(\hbar\Omega_0)$,

$$J_0(\alpha) \approx 1 - \alpha^2/4\,, \quad J_{\pm 1}(\alpha) \approx \pm\,\alpha/2\,, \quad J_{\pm|n|} \sim \pm\,\alpha^{|n|}, \quad |n| > 1\,.$$

Then up to linear in U terms the solution Eq. (3.27) becomes,

$$\Psi(t,\vec{r}) \approx e^{-i\frac{E}{\hbar}t}\psi_E(\vec{r})\left\{1 + \frac{Ue^{-i\Omega_0 t}}{\hbar\Omega_0} - \frac{Ue^{i\Omega_0 t}}{\hbar\Omega_0}\right\}\,.$$

This equation is exactly Eq. (3.15) with $R_{nm} = \delta_{nm}$ and $\psi_m(\vec{r}) = \psi_E(\vec{r})$.

3.2 Floquet scattering matrix

The main difference of a dynamic scatterer compared to a stationary one is that it can change an energy of incident electrons. We are interested in a particular case when the parameters of a scatterer vary periodically in time. This variation can be caused by some external (classical) influence affecting the scattering properties of a sample. For instance, it can be an electric potential forming a barrier for propagating electrons.

We assume that the Hamiltonian describing an interaction of electrons with a scatterer depends periodically on time. Then the wave function of a scattered electron is of the Floquet function type, Eq. (3.22), having components corresponding to different energies. It is convenient to choose an energy E of an incident electron as the Floquet energy. Then the absolute value square of its qth side-band integrated over space defines a probability to absorb, $q > 0$, or emit, $q < 0$, an energy $|q|\hbar\Omega_0$ during scattering.

From the scattering theory point of view the fact that the scattering properties periodically vary in time results in scattering matrix dependent on two energies, incident and scattered. Such a scattering matrix is referred to as *the Floquet scattering matrix*, $\hat{S}_F$. The element $S_{F,\alpha\beta}(E_n, E)$ is a photon-assisted propagation amplitude times $\sqrt{k_n/k}$, where $k_n = \sqrt{2mE_n/\hbar^2}$. This amplitude describes a process when an electron with energy E incident from the lead β is scattered into the lead α and its energy is changed to $E_n = E + n\hbar\Omega_0$. [28] As in the stationary case we define scattering amplitudes as describing transitions between the states (carrying a unit flux) with fixed energy, which are eigen-wavefunctions for Hamiltonian in leads assumed to be stationary.

3.2.1 Floquet scattering matrix properties

3.2.1.1 Unitarity

Since the particle flow is conserved at scattering, the Floquet scattering matrix is unitary, [29]

$$\sum_n \sum_{\alpha=1}^{N_r} S^*_{F,\alpha\beta}(E_n\,,\,E_m)\,S_{F,\alpha\gamma}(E_n\,,\,E) = \delta_{m0}\,\delta_{\beta\gamma}\,, \tag{3.28a}$$

$$\sum_n \sum_{\beta=1}^{N_r} S_{F,\gamma\beta}(E_m\,,\,E_n)\,S^*_{F,\alpha\beta}(E\,,\,E_n) = \delta_{m0}\,\delta_{\alpha\gamma}\,. \tag{3.28b}$$

In the sum over n we keep only those terms which correspond to current-carrying states (with $E_n > 0$). Therefore, it is $n > -[E/\hbar\Omega_0]$, where $[X]$ stands for an integer part of X. In the case if

$$\epsilon = \frac{\hbar\Omega_0}{E} \ll 1 \tag{3.29}$$

the sum over n in Eq. (3.28) in fact runs from $-\infty$ to ∞. In what follows we assume this case.

Note the negative values, $E_n < 0$, correspond to the states localized on the scatterer. These states do not contribute to current. Strictly speaking the transitions between these localized states and current carrying states, $E > 0$, are also described by the Floquet scattering matrix elements. However in the steady state such transitions do not contribute to current. Therefore, they do not enter Eqs. (3.28). In below we use only a part of the Floquet scattering matrix corresponding to transitions between delocalized states and for shortness name it the Floquet scattering matrix.

3.2.1.2 Micro-reversibility

The invariance of the motion equations under the time reversal put some constraints onto the Floquet scattering matrix elements. As we considered earlier, see, Sec. 1.1.1.2, in the stationary case the Schrödinger equation remains invariant under $t \to -t$ if simultaneously to reverse a magnetic field direction and to replace the wave function by its complex conjugate. Note that the incoming and out-going scattering channels are interchanged.

In the case of a dynamical scattering the time reversal can change a time-dependent Hamiltonian. Let us assume that the Hamiltonian depends on N_p parameters $p_i(t)$, $i = 1, \dots, N_p$, which are all periodic in time,

$$p_i(t) = p_{i,0} + p_{i,1}\cos(\Omega_0 t + \varphi_i)\,. \tag{3.30}$$

Then under the time reversal, $t \to -t$, the Hamiltonian remains invariant if in addition we change the signs of all the phases, $\varphi_i \to -\varphi_i$, $\forall i$. Thus the micro-reversibility results in the following symmetry conditions, [30]

$$S_{F,\alpha\beta}(E, E_n; H, \{\varphi\}) = S_{F,\beta\alpha}(E_n, E; -H, \{-\varphi\}) \,, \tag{3.31}$$

where $\{\varphi\}$ is a set of phases φ_i.

3.3 Current operator

To calculate a current operator, Eq. (1.36), one needs to express the operators for scattered electrons, $\hat{b}_\alpha(E)$, in terms of operators for incident electrons, $\hat{a}_\alpha(E)$. These operators annihilate an electron in the state with definite energy. Taking into account that during scattering an electron can change its energy by several energy quanta $\hbar\Omega_0$, we arrive at the following generalization of Eq. (1.39) onto the case of periodic in time scattering, [28]

$$\hat{b}_\alpha(E) = \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} S_{F,\alpha\beta}(E, E_n)\, \hat{a}_\beta(E_n) \,, \tag{3.32a}$$

$$\hat{b}_\alpha^\dagger(E) = \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} S_{F,\alpha\beta}^*(E, E_n)\, \hat{a}_\beta^\dagger(E_n) \,. \tag{3.32b}$$

Note the summation over energy scattering channels is quite similar to a summation over orbital scattering channels. Given above equations together with unitarity conditions, Eqs. (3.28), guarantee anti-commutation relations for $b-$operators similar to ones for $a-$operators, (1.30).

It is natural to assume that the periodic in time varying of scattering properties results in periodic currents flowing in the system. [31] This guessing remains true even in the absence of both a bias voltage and a temperature difference. To analyze periodic currents it is convenient to go over to the frequency representation,

$$\hat{I}_\alpha(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \hat{I}_\alpha(\omega), \tag{3.33a}$$

$$\hat{I}_\alpha(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \hat{I}_\alpha(t). \tag{3.33b}$$

Using Eq. (1.36) we calculate,

$$\hat{I}_\alpha(\omega) = e \int_{0}^{\infty} dE \, \left\{ \hat{b}_\alpha^\dagger(E)\, \hat{b}_\alpha(E + \hbar\omega) - \hat{a}_\alpha^\dagger(E)\, \hat{a}_\alpha(E + \hbar\omega) \right\}, \tag{3.34}$$

where we used,

$$\int_{-\infty}^{\infty} dt\, e^{i\frac{E - E' + \hbar\omega}{\hbar} t} = 2\pi\hbar\, \delta\left(E - E' + \hbar\omega\right), \tag{3.35}$$

and,

$$\int_{0}^{\infty} dE'\, \delta\left(E - E' + \hbar\omega\right) X\left(E'\right) = X(E + \hbar\omega), \tag{3.36}$$

with $X = \hat{b}_\alpha(E'), \hat{a}_\alpha(E')$.

3.3.1 AC current

Substituting Eqs. (3.32) into Eq. (3.34) and averaging over the equilibrium state of reservoirs, we calculate a current spectrum, $I_\alpha(\omega) = \langle \hat{I}_\alpha(\omega) \rangle$, [32]

$$I_\alpha(\omega) = \sum_{l=-\infty}^{\infty} 2\pi\delta(\omega - l\Omega_0)\, I_{\alpha,l}, \tag{3.37a}$$

$$I_{\alpha,l} = \frac{e}{h}\int_0^\infty dE\left\{\sum_{\beta=1}^{N_r}\sum_{n=-\infty}^{\infty} S^*_{F,\alpha\beta}(E\ ,E_n)\,S_{F,\alpha\beta}(E_l,E_n)\,f_\beta(E_n) - \delta_{l0}\,f_\alpha(E)\right\}. \tag{3.37b}$$

Taking into account Eq. (3.28b) we rewrite $I_{l,\alpha}$ as follows,

$$I_{\alpha,l} = \frac{e}{h}\int_0^\infty dE\sum_{\beta=1}^{N_r}\sum_{n=-\infty}^{\infty} S^*_{F,\alpha\beta}(E_n,E)\,S_{F,\alpha\beta}(E_{l+n},E)\left\{f_\beta(E) - f_\alpha(E_n)\right\}, \tag{3.38}$$

where we additionally replaced $E_n \to E$ and $n \to -n$. The convenience of the last equation containing the difference of the Fermi functions becomes evident in the case of a slow variation of the scatterer parameters, $\Omega_0 \to 0$, when a current can be expanded in powers of Ω_0.

Substituting Eq. (3.37a) into Eq. (3.33a) we finally arrive at a time-dependent current,

$$I_\alpha(t) = \sum_{l=-\infty}^{\infty} e^{-il\Omega_0 t}\,I_{\alpha,l}\,, \tag{3.39}$$

which is really periodic in time, $I_\alpha(t) = I_\alpha(t + 2\pi/\Omega_0)$.

3.3.2 DC current

Of a special interest is a case when a current $I_\alpha(t)$ has a time-independent part. Emphasize, while an ac current is always generated by the dynamic scatterer, the dc current exists only under some specific conditions which we will discuss later on. Now we just give general expressions for a dc current, the term with $l = 0$ in Eq. (3.39).

Using $l = 0$ in Eq. (3.37b) we find,

$$I_{\alpha,0} = \frac{e}{h} \int_0^{\infty} dE \left\{ \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} \left| S_{F,\alpha\beta}(E, E_n) \right|^2 f_\beta(E_n) - f_\alpha(E) \right\}. \tag{3.40}$$

The dc current is subject to the conservation law, Eq. (1.48). To show it we transform expression above as follows. In the part with a factor $f_\beta(E_n)$ we shift $E \to E - n\hbar\Omega_0$ and $n \to -n$,[1] [28]

$$I_{\alpha,0} = \frac{e}{h} \int_0^{\infty} dE \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} \left\{ \left| S_{F,\alpha\beta}(E_n, E) \right|^2 f_\beta(E) - f_\alpha(E) \right\}. \tag{3.41}$$

Then using Eq. (3.28a) one can easily check that $\sum_{\alpha=0}^{N_r} I_{\alpha,0} = 0$.

Another expression for a dc current can be found if to substitute Eq.(3.28b) with $m = 0$ and $\alpha = \gamma$ into Eq. (3.41) as a unity in front of $f_\alpha(E)$ and to make a shift $E \to E - n\hbar\Omega_0$ and a substitution $n \to -n$: [28]

$$I_{\alpha,0} = \frac{e}{h} \int_0^{\infty} dE \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} \left| S_{F,\alpha\beta}(E_n, E) \right|^2 \left\{ f_\beta(E) - f_\alpha(E_n) \right\}. \tag{3.42}$$

From this equation it follows that (for $\hbar\Omega_0 \ll \mu$) only electrons with energy close to the Fermi energy contribute to current. Because only for such electrons the difference of the Fermi functions is noticeable, $f_\beta(E) - f_\alpha(E + n\hbar\Omega_0) \neq 0$. Note the energy window where the current flows is defined by the maximum of the following quantities, the energy quantum $\hbar\Omega_0$ dictated by the frequency of a drive, a possibly present bias $|eV_{\alpha\beta}|$, and a temperature, $k_B T_\alpha$.

And finally an intuitively clear expression for a current can be derived in the same way as Eq. (3.42) was derived from Eq. (3.41). We use Eq. (3.28a) instead of Eq. (3.28b) and replace $\alpha \to \beta$ and $\beta = \gamma \to \alpha$: [28]

[1] The limits of integration over energy are nor changed, because, as we already mentioned, only those elements of the Floquet scattering matrix contribute to current for which both $E > 0$ and $E_n > 0$

$$
\begin{aligned}
I_{\alpha,0} &= \frac{e}{h}\int_0^\infty dE \sum_{n=-\infty}^{\infty}\sum_{\beta=1}^{N_r} \\
&\left\{\left|S_{F,\alpha\beta}(E_n, E)\right|^2 f_\beta(E) - \left|S_{F,\beta\alpha}(E_n, E)\right|^2 f_\alpha(E)\right\}.
\end{aligned}
\tag{3.43}
$$

This equation represents a dc current in the lead α as a difference of two electron flows. First one is composed by the flows incident from various leads β and scattered with probability $\left|S_{F,\alpha\beta}(E_n, E)\right|^2$ into the lead α. And the second one is incident from the lead α and with probability $\left|S_{F,\beta\alpha}(E_n, E)\right|^2$ scattered into various leads β.

We emphasize all the equations (3.40) – (3.43) are equivalent. Which of them to use is dictated by the convenience reasons in each particular case.

3.4 Adiabatic approximation for the Floquet scattering matrix

To calculate the Floquet scattering matrix elements one needs to solve the non-stationary Schrödinger equation, that in general case is more complicated than to solve a stationary problem. In particular, the stationary scattering matrix $\hat{S}$ has $N_r \times N_r$ elements, while the Floquet scattering matrix $\hat{S}_F$, in addition depending on two energies, has much more elements, $N_r \times N_r \times (2n_{\max} + 1)^2$, where $n_{\max}$ is a maximum number of energy quanta $\hbar\Omega_0$ which an electron can absorb/emit interacting with a dynamic scatterer. Formally an electron can change its energy by $n \to \infty$ energy quanta $\hbar\Omega_0$. However in practice there is some number n_{max} such that the probability to absorb/emit $n_{max} + 1$ and more energy quanta is negligible within a given accuracy. For instance, if the amplitude δU of an oscillating potential is small compared to $\hbar\Omega_0$ then $n_{max} = 1$, that is only single-photon processes are relevant. In contrast if $\delta U \gg \hbar\Omega_0$ then $n_{max} \gg 1$.

In general the multi-photon processes become important if parameters of a scatterer vary slowly. Therefore, at $\Omega_0 \to 0$ we should calculate a huge number of scattering amplitudes that can be impossible in practice. On the other hand it is natural to expect that the scattering properties of a sample with parameters varying enough slowly should be close to scattering properties of a strictly stationary sample. Because at $\mathcal{T} = 2\pi/\Omega_0 \to \infty$ any finite time, spend by an electron within the scattering region, is always small compared to $\mathcal{T}$ and an electron should not feel that the scatterer is dynamic. However, as we show below, there is a principial difference between the properties of a dynamic scatterer and the properties of a stationary scatterer. [29, 30] For instance, a dynamic scatterer can generate a dc current in the absence of a bias applied to reservoirs.

3.4.1 Frozen scattering matrix

Let the stationary scattering matrix $\hat{S}$ depends on several parameters, $p_i \in \{p\}$, $i = 1, 2, \ldots, N_p$, which are varied periodically in time, Eq. (3.30). Then the matrix $\hat{S}$ becomes a periodic function of time, $\hat{S}(t, E) = \hat{S}(\{p(t)\}; E)$, $\hat{S}(t, E) = \hat{S}(t+\mathcal{T}, E)$. We stress the matrix $\hat{S}(t)$ does not describe scattering onto a dynamic scatterer. Its physical meaning is the following. Let us fix all the parameters at a time $t = t_0$ and will not change them any more. Then the matrix $\hat{S}(t_0, E)$ does describe scattering onto such a frozen scatterer. Treating a time t in this sense we can name a matrix $\hat{S}(t, E)$ as *the frozen scattering matrix*. Emphasize a variable t here is a parameter, relating to a given variation of the properties of a scatterer, rather than a true dynamical time entering the motion equation.

As we pointed out the frozen scattering matrix $\hat{S}(t, E)$ has not a direct relation to scattering onto a dynamic sample, since it depends on a single energy only. However at $\Omega_0 \to 0$ there exists some relation between the frozen and the Floquet scattering matrices. It becomes more clear if to expand $\hat{S}_F$ in powers of Ω_0,

$$\hat{S}_F = \sum_{q=0}^{\infty} (\hbar\Omega_0)^q \, \hat{S}_F^{(q)} \,. \qquad (3.44)$$

Below we relate the first and the second terms in this *adiabatic expansion* to the frozen scattering matrix.

3.4.2 Zeroth order approximation

To zeroth order, $q = 0$ in Eq. (3.44), all the terms proportional to Ω_0 (or its higher power) should be dropped. Within this accuracy an initial energy, E, and a final energy, $E_n = E + n\hbar\Omega_0$, are the same. Therefore, the term $\hat{S}_F^{(0)}$ depends, in fact, on only a single energy similar to the frozen scattering matrix. To establish a connection between these two matrices we take into account the following. The element $S_{F,\alpha\beta}(E_n, E)$ describe a scattering process when an electron energy is changed: $\Psi_{E_n,\alpha}^{(out)} \sim S_{F,\alpha\beta}(E_n, E)\,\Psi_{E,\beta}^{(in)}$, with $\Psi_{E,\beta}^{(in)} \sim e^{-iEt/\hbar}$ and $\Psi_{E_n,\alpha}^{(out)} \sim e^{-iE_nt/\hbar} = e^{-iEt/\hbar}e^{-in\Omega_0 t}$. On the other hand if to consider scattering onto the frozen scatterer, $\Psi_{E,\alpha}^{(out)} \sim S_{\alpha\beta}(t, E)\,\Psi_{E,\beta}^{(in)}$, and to use the following Fourier expansion,

$$\hat{S}(t, E) = \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t}\hat{S}_n(E)\,, \tag{3.45}$$

then one can see that the part of a wave function of a scattered electron proportional to $S_{\alpha\beta,n}$ has a time-dependent phase factor $e^{-iE_nt/\hbar}$, the same as that of due to $S_{F,\alpha\beta}(E_n, E)$. These simple arguments allow to conclude that to zeroth order in Ω_0 the Floquet scattering matrix elements are equal to the Fourier coefficients of the Frozen scattering matrix,

$$\hat{S}_F^{(0)}(E_n, E) = \hat{S}_n(E)\,, \tag{3.46a}$$

$$\hat{S}_F^{(0)}(E, E_n) = \hat{S}_{-n}(E)\,. \tag{3.46b}$$

To prove that this approximation does not violate unitarity we substitute equations above into Eq. (3.28). Then after the inverse Fourier transformation we find,

$$\hat{S}(t, E)\,\hat{S}^{\dagger}(t, E) = \hat{S}^{\dagger}(t, E)\,\hat{S}(t, E) = \hat{I}\,, \tag{3.47}$$

that is completely consistent with a unitarity condition, Eq. (1.10), for the stationary scattering matrix.

3.4.3 First order approximation

Up to terms of the first order in Ω_0 the initial energy, E, is different from the final energy, E_n. The simplest generalization of Eq. (3.46) could be the same relation but with frozen scattering matrix calculated at the middle energy, $(E + E_n)/2$. However it is easy to check that such a matrix is not unitary. To recover unitarity we need to introduce an additional term, $\hbar\Omega_0\hat{A}_n(E)$, where $\hat{A}_n(E)$ is a Fourier transform of some matrix $\hat{A}(t, E)$. Therefore, we arrive at the following ansatz for the first order in Ω_0 corrections to the frozen scattering matrix, the term with $q = 1$ in Eq. (3.44),

$$\hbar\Omega_0\hat{S}_F^{(1)}(E_n\,, E) = \frac{n\hbar\Omega_0}{2}\frac{\partial\hat{S}_n(E)}{\partial E} + \hbar\Omega_0\hat{A}_n(E)\,, \tag{3.48a}$$

$$\hbar\Omega_0\hat{S}_F^{(1)}(E\,, E_n) = \frac{n\hbar\Omega_0}{2}\frac{\partial\hat{S}_{-n}(E)}{\partial E} + \hbar\Omega_0\hat{A}_{-n}(E)\,. \tag{3.48b}$$

Notice the right hand side (RHS) of Eq. (3.48a) is calculated at the energy of an incident electron, while the RHS of Eq. (3.48b) is calculated at the energy of a scattered electron.

The equations (3.48) point out on the actual expansion parameter in Eq. (3.44). This, so called *an adiabaticity parameter*, is

$$\varpi = \frac{\hbar\Omega_0}{\delta E} \ll 1\,, \tag{3.49}$$

where δE is a characteristic energy scale over which the stationary scattering matrix changes significantly. For instance, if the energy, E, of an incident electron is close to the transmission resonance energy then δE is a width of a resonance. While if E is far from the resonance then δE is of the order of the distance between the resonances. In the case when the scatterer does not show a resonance transmission then, as a rule, δE is of order E. Emphasize, such a definition of adiabaticity is in general different from the one usually used in the Quantum mechanics and requiring a smallness of an energy quantum $\hbar\Omega_0$ compared to the difference between the energy levels.

The matrix $\hat{A}$ in Eqs. (3.48) can not be expressed in terms of the frozen scattering matrix $\hat{S}$. However the unitarity of the Floquet scattering matrix leads to some relation between these two matrix. [29] To find it we use

$$S_{F,\alpha\beta}(E_n, E) = S_{\alpha\beta,n}(E) + \frac{n\hbar\Omega_0}{2}\frac{\partial S_{\alpha\beta,n}}{\partial E} + \hbar\Omega_0 A_{\alpha\beta,n} + \mathcal{O}\left(\varpi^2\right), \tag{3.50}$$

in Eq. (3.28a) :

$$\sum_{n=-\infty}^{\infty}\sum_{\alpha=1}^{N_r}\left\{S^*_{\alpha\gamma,n-m}(E) + \frac{(n+m)\hbar\Omega_0}{2}\frac{\partial S^*_{\alpha\gamma,n-m}(E)}{\partial E} + \hbar\Omega_0\, A^*_{\alpha\gamma,n-m}(E)\right\}$$
$$\times\left\{S_{\alpha\beta,n}(E) + \frac{n\hbar\Omega_0}{2}\frac{\partial S_{\alpha\beta,n}(E)}{\partial E} + \hbar\Omega_0\, A_{\alpha\beta,n}(E)\right\} = \delta_{\beta\gamma}\delta_{m0}\,.$$

Taking into account that the matrix $\hat{S}(t, E)$ is unitary and omitting the terms of order Ω_0^2, we get

$$\sum_{n=-\infty}^{\infty}\sum_{\alpha=1}^{N_r}\left\{S_{\alpha\beta,n}\left(n - \frac{n-m}{2}\right)\frac{\partial S^*_{\alpha\gamma,n-m}}{\partial E} + \frac{n}{2}\frac{\partial S_{\alpha\beta,n}}{\partial E}S^*_{\alpha\gamma,n-m} + \right.$$
$$\left.\left[S_{\alpha\beta,n}A^*_{\alpha\gamma,n-m} + A_{\alpha\beta,n}S^*_{\alpha\gamma,n-m}\right]\right\} = 0\,.$$

Next we make the inverse Fourier transformation using the following properties,

$$n\,X_n = \frac{i}{\Omega_0}\left(\frac{\partial X}{\partial t}\right)_n, \qquad n\,X^*_n = -\frac{i}{\Omega_0}\left(\frac{\partial X^*}{\partial t}\right)_{-n},$$
$$X^*_n = (X^*)_{-n}, \qquad \sum_{n=-\infty}^{\infty}X_{-n}Y_{n-m} = (X\,Y)_{-m}\,. \tag{3.51}$$

and arrive at the following matrix equation,

$$\frac{i}{\Omega_0}\frac{\partial\hat{S}^{\dagger}}{\partial E}\frac{\partial\hat{S}}{\partial t}+\frac{i}{2\Omega_0}\left\{\frac{\partial^2\hat{S}^{\dagger}}{\partial t\partial E}\hat{S}+\hat{S}^{\dagger}\frac{\partial^2\hat{S}}{\partial t\partial E}\right\}+\hat{A}^{\dagger}\hat{S}+\hat{S}^{\dagger}\hat{A}=\hat{0}\,.$$

To simplify it we use the identity, $\partial^2(\hat{S}^{\dagger}\hat{S})/\partial t\partial E=\hat{0}$, following from Eq. (3.47), which can be rewritten as follows,

$$\frac{\partial^2\hat{S}^{\dagger}}{\partial t\partial E}\hat{S}+\hat{S}^{\dagger}\frac{\partial^2\hat{S}}{\partial t\partial E}=-\frac{\partial\hat{S}^{\dagger}}{\partial t}\frac{\partial\hat{S}}{\partial E}-\frac{\partial\hat{S}^{\dagger}}{\partial E}\frac{\partial\hat{S}}{\partial t}\,.$$

Then we arrive finally at the following equation (a consequence of the unitarity of scattering) for the matrix $\hat{A}$, [29]

$$\hbar\Omega_0\left[\hat{S}^{\dagger}(t,E)\,\hat{A}(t,E)+\hat{A}^{\dagger}(t,E)\,\hat{S}(t,E)\right]=\frac{1}{2}P\left\{\hat{S}^{\dagger}(t,E),\hat{S}(t,E)\right\}\,, \qquad (3.52)$$

where $P\{\hat{S}^{\dagger},\hat{S}\}$ is the Poisson bracket with respect to energy and time,

$$P\left\{\hat{S}^{\dagger},\,\hat{S}\right\}=i\hbar\left(\frac{\partial\hat{S}^{\dagger}}{\partial t}\frac{\partial\hat{S}}{\partial E}-\frac{\partial\hat{S}^{\dagger}}{\partial E}\frac{\partial\hat{S}}{\partial t}\right). \qquad (3.53)$$

It is a self-adjoint and traceless matrix,,

$$P\left\{\hat{S}^{\dagger},\,\hat{S}\right\}=\left(P\left\{\hat{S}^{\dagger},\,\hat{S}\right\}\right)^{\dagger}\,, \qquad (3.54)$$

$$\mathrm{Tr}\left[P\left\{\hat{S}^{\dagger},\,\hat{S}\right\}\right]\equiv\sum_{\alpha=1}^{N_r}P_{\alpha\alpha}\left\{\hat{S}^{\dagger},\,\hat{S}\right\}=0\,. \qquad (3.55)$$

To prove Eq. (3.54) we use $\left(\hat{X}^{\dagger}\hat{Y}\right)^{\dagger}=\hat{Y}^{\dagger}\hat{X}$. To prove Eq. (3.54) we use a unitarity, $\hat{S}^{\dagger}\hat{S} = \hat{S}\hat{S}^{\dagger} = \hat{I}$, its consequence, $\left(\partial\hat{S}^{\dagger}/\partial E\right)\hat{S} = -\hat{S}^{\dagger}\left(\partial S/\partial E\right)$, and a property of the trace, $\mathrm{Tr}\left[\hat{X}\hat{Y}\right] = \mathrm{Tr}\left[\hat{Y}\hat{X}\right]$:

$$\begin{aligned}\mathrm{Tr}\left[P\right] &= i\hbar\mathrm{Tr}\left[\frac{\partial\hat{S}^{\dagger}}{\partial t}\frac{\partial\hat{S}}{\partial E} - \frac{\partial\hat{S}^{\dagger}}{\partial E}\hat{S}\hat{S}^{\dagger}\frac{\partial\hat{S}}{\partial t}\right] = i\hbar\mathrm{Tr}\left[\frac{\partial\hat{S}^{\dagger}}{\partial t}\frac{\partial\hat{S}}{\partial E} - \hat{S}^{\dagger}\frac{\partial\hat{S}}{\partial E}\frac{\partial\hat{S}^{\dagger}}{\partial t}\hat{S}\right] \\ &= i\hbar\mathrm{Tr}\left[\frac{\partial\hat{S}^{\dagger}}{\partial t}\frac{\partial\hat{S}}{\partial E} - \frac{\partial\hat{S}^{\dagger}}{\partial t}\hat{S}\hat{S}^{\dagger}\frac{\partial\hat{S}}{\partial E}\right] = i\hbar\mathrm{Tr}\left[\frac{\partial\hat{S}^{\dagger}}{\partial t}\frac{\partial\hat{S}}{\partial E} - \frac{\partial\hat{S}^{\dagger}}{\partial t}\frac{\partial\hat{S}}{\partial E}\right] = 0\,.\end{aligned}$$

Note if we start from Eq. (3.28b) then we arrive at the following equation, [30]

$$\hbar\Omega_0\left[\hat{A}(t,E)\,\hat{S}^{\dagger}(t,E) + \hat{S}(t,E)\,\hat{A}^{\dagger}(t,E)\right] = \frac{1}{2}P\left\{\hat{S}(t,E),\hat{S}^{\dagger}(t,E)\right\}\,, \tag{3.56}$$

which is equivalent to Eq. (3.52). If we multiply Eq. (3.52) by $\hat{S}$ from the left and by $\hat{S}^{\dagger}$ from the right we arrive at Eq. (3.56).

The symmetry conditions for the Floquet scattering matrix, Eq. (3.31), result in some symmetry conditions for the matrix $\hat{A}(t,E)$. To derive them we proceed as follows. With parameters from Eq. (3.30) we have for the frozen scattering matrix, $\hat{S}(t,E;H,\{\varphi\}) = \hat{S}(-t,E;H,\{-\varphi\})$. Then from Eq. (3.52) we find, $\hat{A}(t,E;H,\{\varphi\}) = -\hat{A}(-t,E;H,\{-\varphi\})$. In terms of the Fourier coefficients these equations read, $\hat{S}_n(E;H,\{\varphi\}) = \hat{S}_{-n}(E;H,\{-\varphi\})$ and $\hat{A}_n(E;H,\{\varphi\}) = -\hat{A}_{-n}(E;H,\{-\varphi\})$. Finally substituting the sum of Eqs. (3.46) and (3.48) into Eq. (3.31) and taking into account given above relations between the Fourier coefficients we find the following symmetry condition, [30]

$$A_{\alpha\beta}(t,E;H,\{\varphi\}) = -A_{\beta\alpha}(t,E;-H,\{\varphi\})\,. \tag{3.57}$$

The analogous condition for the frozen scattering matrix follows from Eq. (1.29),

$$S_{\alpha\beta}(t,E;H,\{\varphi\}) = S_{\beta\alpha}(t,E;-H,\{\varphi\})\,. \tag{3.58}$$

Let us consider the case with $H = 0$. The non-diagonal elements of the matrix $\hat{A}$ change a sign under the reversal of incoming and out-going channels, that is in striking difference to the behavior of the non-diagonal elements of the frozen scattering matrix. Therefore, we name the matrix $\hat{A}$ as *the anomalous scattering matrix*. Such a sign reversal results in different probabilities for direct, $\alpha \to \beta$, and reverse, $\beta \to \alpha$, transmission through the dynamic scatterer. The diagonal elements of the anomalous scattering matrix are zero (in the absence of a magnetic field). Therefore, the reflection amplitudes up to terms of order Ω_0 are defined entirely by the frozen scattering matrix. This last circumstance justifies our representation for the elements of the Floquet scattering matrix in Eq. (3.48).

3.5 Beyond the adiabatic approximation

In some simple cases the Floquet scattering matrix can be calculated analytically. To this end it is convenient to turn to the mixed representation when the scattering matrix depends on energy and time.

3.5.1 Scattering matrix in mixed energy-time representation

Let us introduce the following scattering matrix, $\hat{S}_{in}(t,E)$ and $\hat{S}_{out}(E,t)$, in such a way that their Fourier coefficients are related to the Floquet scattering matrix elements as follows, [33]

$$\hat{S}_F(E_n, E) = \hat{S}_{in,n}(E) \equiv \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega_0 t} \hat{S}_{in}(t,E)\,, \tag{3.59a}$$

$$\hat{S}_F(E, E_n) = \hat{S}_{out,-n}(E) \equiv \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{-in\Omega_0 t} \hat{S}_{out}(E,t)\,. \tag{3.59b}$$

As we will see later on in examples, the elements of the matrix $\hat{S}_{in}(t,E)$ are scattering amplitudes for particles incident with energy E and leaving the scattering region at time t. The dual matrix $\hat{S}_{out}(E,t)$ composed of the scattering amplitudes for particles incident at time t and leaving the scatterer with energy E. Note this interpretation is consistent with the Heisenberg uncertainty principle. For instance, if the time when an electron leaves a scatterer is defined then its energy is not defined. In this case, in accordance with Eq. (3.59a), an electron energy can be one of $E_n = E + n\hbar\Omega_0$. Similarly, if an initial time is defined then the initial energy does not. This energy can differ from the energy E which an electron leaves the scatterer with. The probability, that an initial energy of an electron incident from the lead β and scattered into the lead α was $E_m = E + m\hbar\Omega_0$, is equal to $\left|S_{out,\alpha\beta,-m}(E)\right|^2$.

Substituting the definition for $\hat{S}_{in}$ into Eq. (3.28a) and the definition for $\hat{S}_{out}$ into Eq. (3.28b) and making the inverse transformation we get the following unitarity conditions, [30, 33]

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{im\Omega_0 t} \hat{S}^{\dagger}_{in}(t,E_m)\hat{S}_{in}(t,E) = \delta_{m,0}\hat{I}, \tag{3.60a}$$

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{im\Omega_0 t} \hat{S}_{out}(E_m,t)\hat{S}^{\dagger}_{out}(E,t) = \delta_{m,0}\hat{I}. \tag{3.60b}$$

To the zeroth order in the adiabaticity parameter, $\varpi \to 0$, as it follows from Eq. (3.46), the matrices $\hat{S}_{in}$ and $\hat{S}_{out}$ are the same and they are equal to the frozen scattering matrix. Already in the first order in ϖ these matrices become different. From Eq. (3.48) we can find, [30]

$$\hat{S}_{in}(t,E) = \hat{S}(t,E) + \frac{i\hbar}{2}\frac{\partial^2\hat{S}(t,E)}{\partial t\partial E} + \hbar\Omega_0\hat{A}(t,E) + \mathcal{O}\left(\varpi^2\right), \tag{3.61a}$$

$$\hat{S}_{out}(t,E) = \hat{S}(t,E) - \frac{i\hbar}{2}\frac{\partial^2\hat{S}(t,E)}{\partial t\partial E} + \hbar\Omega_0\hat{A}(t,E) + \mathcal{O}\left(\varpi^2\right), \tag{3.61b}$$

where $\mathcal{O}\left(\varpi^2\right)$ stands for the rest of order ϖ^2.

Despite their difference, the matrices $\hat{S}_{in}$ and $\hat{S}_{out}$ are related due to micro-reversibility. From Eqs. (3.31) and (3.59) it follows, [33]

$$\hat{S}_{in}(t, E; H, \{\varphi\}) = \hat{S}^{T}_{out}(E, -t; -H, \{-\varphi\}) . \tag{3.62}$$

Moreover, from Eq. (3.59) one can find,

$$\hat{S}_{in,\,n}(E) = \hat{S}_{out,\,n}(E_n) , \tag{3.63}$$

that in time representation reads,

$$\hat{S}_{in}(t, E) = \sum_{n=-\infty}^{\infty} \int_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}} e^{in\Omega_0(t'-t)} \hat{S}_{out}(E_n, t') , \tag{3.64a}$$

$$\hat{S}_{out}(E, t) = \sum_{n=-\infty}^{\infty} \int_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}} e^{-in\Omega_0(t'-t)} \hat{S}_{in}(t', E_n) . \tag{3.64b}$$

For the sake of completeness we give a current in terms of $\hat{S}_{in}$. To this end we use Eq. (3.59a) in Eq. (3.38) and then in Eq. (3.39) and finally calculate, [41]

$$\begin{aligned} I_\alpha(t) &= \frac{e}{h} \int_0^{\infty} dE \sum_{\beta=1}^{N_r} \sum_{n=-\infty}^{\infty} \left\{ f_\beta(E) - f_\alpha(E_n) \right\} \\ &\times \int_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}} e^{in\Omega_0(t-t')} S_{in,\alpha\beta}(t, E) S^{*}_{in,\alpha\beta}(t', E) . \end{aligned} \tag{3.65}$$

We transform this equation to exclude a reference to the periodicity of a driving potential. To this end we use the following correspondences,

$$
\begin{aligned}
n\Omega_0 &\to \omega\,, \\
\sum_{n=-\infty}^{\infty} &\to \frac{\mathcal{T}}{2\pi}\int_{-\infty}^{\infty} d\omega\,, \\
\int_0^{\mathcal{T}} dt'\, e^{in\Omega_0 t'} &\to \int_{-\infty}^{\infty} dt'\, e^{i\omega t'}\,.
\end{aligned}
\tag{3.66}
$$

which in fact means a passage from the discrete Fourier transformation to the continuous Fourier transformation. After that the current reads,

$$
\begin{aligned}
I_\alpha(t) = \frac{e}{h}\int dE\, \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \sum_{\beta=1}^{N_r}\left[f_\beta(E) - f_\alpha(E+\hbar\omega)\right] \\
\times \int_{-\infty}^{\infty} dt' e^{i\omega(t-t')} S_{in,\alpha\beta}(t,E) S^*_{in,\alpha\beta}(t',E)\,.
\end{aligned}
\tag{3.67}
$$

Thus we derived an expression which can be used to calculate a time-dependent current in terms of the scattering matrix elements in the case of driving with arbitrary (not necessarily periodic) dependence on time. In a particular case of a drive with period $\mathcal{T} = 2\pi/\Omega_0$ we have,

$$
\int_{-\infty}^{\infty} dt' e^{-i\omega t'} S^*_{in,\alpha\beta}(t',E) = \sum_{n=-\infty}^{\infty} 2\pi\,\delta(\omega - n\Omega_0) S^*_{in,\alpha\beta,n}(E)\,. \tag{3.68}
$$

The use of this equation transforms Eq. (3.67) into Eq. (3.65) as expected.

Further we consider several simple examples and calculate analytically the elements of the scattering matrix $\hat{S}_{in}$.

3.5.2 Point-like scattering potential

Let us consider a one-dimensional Schrödinger equation,

$$i\hbar\frac{\partial\Psi}{\partial t} = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(t,x)\right\}\Psi\,, \tag{3.69}$$

with point-like potential $V(t,x)$ whose strength oscillates in time,

$$V(t,x) = \delta(x)\,V(t)\,, \quad V(t) = V_0 + 2V_1\cos(\Omega_0 t + \varphi)\,. \tag{3.70}$$

Accordingly to the Floquet theorem the solution to Eq. (3.69) with periodic in time potential, Eq. (3.70) is of the following form,

$$\Psi(t,x) = e^{-i\frac{E}{\hbar}t}\sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t}\psi_n(x)\,, \tag{3.71}$$

where $\psi_n(x)$ is a general solution of the corresponding stationary problem. In all the places but $x = 0$ the potential is zero. Therefore, as a function $\psi_n(x \neq 0)$ we can take a general solution to the Schrödinger equation for a free particle,

$$\psi_n(x) = \begin{cases} a_n^{(-)}e^{ik_n x} + b_n^{(-)}e^{-ik_n x}\,, & x < 0\,, \\ a_n^{(+)}e^{ik_n x} + b_n^{(+)}e^{-ik_n x}\,, & x > 0\,, \end{cases} \tag{3.72}$$

with $k_n = \sqrt{2m(E + n\hbar\Omega_0)}/\hbar$.

To match the wave function on the left and on the right from $x = 0$ we use the following. At $x = 0$ the wave function should be continuous. To relate it's derivative we integrate out Eq. (3.69) over an infinitesimal vicinity of a point $x = 0$. We find that the derivative has a jump at this point. Therefore, we have the following boundary conditions,

$$\begin{aligned} \Psi(t, x=-0) &= \Psi(t, x=+0)\,, \\ \left.\frac{\partial \Psi(t,x)}{\partial x}\right|_{x=+0} - \left.\frac{\partial \Psi(t,x)}{\partial x}\right|_{x=-0} &= \frac{2m}{\hbar^2} V(t)\Psi(t, x=0)\,, \end{aligned} \tag{3.73}$$

which connect unknown coefficients of the wave function, Eq. (3.72) at $x > 0$ and at $x < 0$.

Now we formulate a proper scattering problem, which in particular includes the boundary conditions at $x \to \pm\infty$. The coefficients $a_n^{(-)}$ and $b_n^{(+)}$ in Eq. (3.72) correspond to incident waves, while the coefficients $a_n^{(+)}$ and $b_n^{(-)}$ correspond to out-going (scattered) waves. So we can write,

$$\psi_n(x) = \psi_n^{(in)}(x) + \psi_n^{(out)}(x)\,, \tag{3.74}$$

where

$$\psi_n^{(in)}(x) = \begin{cases} a_n^{(-)} e^{ik_n x}\,, & x < 0\,, \\ b_n^{(+)} e^{-ik_n x}\,, & x > 0\,, \end{cases} \tag{3.75a}$$

and

$$\psi_n^{(out)}(x) = \begin{cases} b_n^{(-)} e^{-ik_n x}\,, & x < 0\,, \\ a_n^{(+)} e^{ik_n x}\,, & x > 0\,, \end{cases} \tag{3.75b}$$

Correspondingly the wave function, Eq. (3.71) can be written as the sum, $\Psi(t,x) = \Psi^{(in)}(t,x) + \Psi^{(out)}(t,x)$. Note the coefficients $a_n^{(-)}$ and $b_n^{(+)}$ are defined by a given incident wave. In contrast the coefficients $a_n^{(+)}$ and $b_n^{(-)}$ should be calculated.

First we consider scattering of a wave with unit amplitude[2] corresponding to a particle with energy E incident from the left, Fig. 3.1,

$$\Psi_1^{(in)}(t,x) = e^{-i\frac{E}{\hbar}t} \begin{cases} e^{ikx}\,, & x < 0\,, \\ 0\,, & x > 0\,. \end{cases} \tag{3.76}$$

[2]This wave is not normalized on unite flux. Hence there is a factor $\sqrt{k_n/k} \equiv \sqrt{v_n/v}$ in Eq. (3.78).

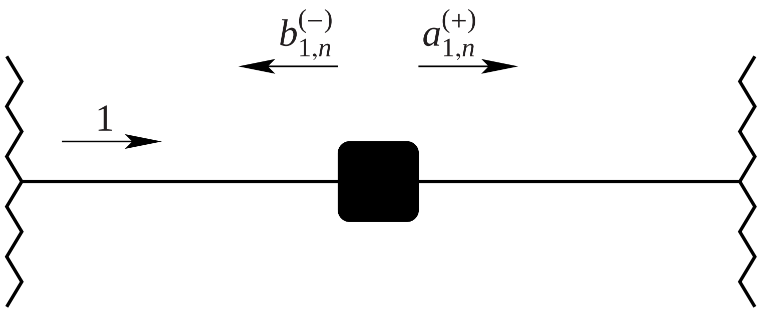

Figure 3.1: Scattering of a wave with unit amplitude onto the point-like potential barrier. Arrows and letters show the propagation direction and the amplitude of corresponding waves: 1 is an amplitude of an incoming wave, $b_{1,n}^{(-)}$ is an amplitude of a reflected wave, $a_{1,n}^{(+)}$ is an amplitude of a transmitted wave. Only a single (nth) component of the Floquet wave function for a scattered state is shown.

Comparing it with Eqs. (3.71) and (3.75a) we find, $a_{1,n}^{(-)} = \delta_{n0}$ and $b_{1,n}^{(+)} = 0$. To calculate the coefficients $a_{1,n}^{(+)}$, $b_{1,n}^{(-)}$ of the scattered wave $\Psi_1^{(out)}$ we use the boundary conditions, Eq. (3.73), and collect the coefficients having the same dependence on time, $\sim e^{-i\frac{E+n\hbar\Omega_0}{\hbar}t}$. As a result we arrive at the following set of linear equations, $n = 0, \pm 1, \pm 2, \dots$,

$$\begin{cases} \delta_{n0} + b_{1,n}^{(-)} = a_{1,n}^{(+)} , \\ \\ (k_n + ip_0)\, a_{1,n}^{(+)} = k\delta_{n0} - i\left(p_{+1}a_{1,n-1}^{(+)} + p_{-1}a_{1,n+1}^{(+)}\right) , \end{cases} \tag{3.77}$$

where $p_0 = mV_0/\hbar^2$ and $p_{\pm 1} = mV_1 e^{\mp i\varphi}/\hbar^2$ are the Fourier coefficients for $p\,(t) = mV\,(t)\,/\hbar^2$.

The coefficients $b_{1,n}^{(-)}/a_{1,n}^{(+)}$ define the corresponding Floquet scattering matrix elements for a point-like potential barrier, $\hat{S}_F^{(1)}(E_n, E)$, and, correspondingly, the elements of a matrix $\hat{S}_{in}^{(1)}(E)$,

$$S_{F,11}^{(1)}\,(E_n, E) = S_{in,11,n}^{(1)}\,(E) = \sqrt{\frac{k_n}{k}}\; b_{1,n}^{(-)} , \tag{3.78a}$$

$$S^{(1)}_{F,21}(E_n, E) = S^{(1)}_{in,21,n}(E) = \sqrt{\frac{k_n}{k}}\, a^{(+)}_{1,n}\,. \tag{3.78b}$$

Here the lower indices 1 and 2 correspond to left ($x \to -\infty$) and right ($x \to +\infty$) reservoirs, respectively. The square root $\sqrt{k_n/k}$ appeared because the absolute value square of the scattering matrix element is defined as a ratio of the current of scattered particles, $\sim k_n \left|\psi^{(out)}_n\right|^2$, to the current of incident particles, $\sim k \left|\psi^{(in)}_n\right|^2$.

Substituting Eq. (3.78) into Eq. (3.77) we find,

$$\begin{cases} \delta_{n0} + S^{(1)}_{in,11,n}(E) = S^{(1)}_{in,21,n}(E)\,, \\ (k_n + ip_0)\, S^{(1)}_{in,21,n}(E) = k\delta_{n0} \\ \qquad -ip_{+1}\sqrt{\frac{k_n}{k_{n-1}}}\, S^{(1)}_{in,21,n-1}(E) - ip_{-1}\sqrt{\frac{k_n}{k_{n+1}}}\, S^{(1)}_{in,21,n+1}(E)\,, \end{cases} \tag{3.79}$$

Let us solve this system of equations with accuracy to the first order in the parameter $\epsilon = \hbar\Omega_0/E$ introduced in Eq. (3.29). Notice in the problem under consideration the energy E only is a characteristic energy. Therefore, in this case the parameter ϵ coincides with the adiabaticity parameter, $\epsilon \sim \varpi$.

To the first order in ϵ we can approximate,

$$k_n = k + \frac{n\Omega_0}{v} + \mathcal{O}\left(\epsilon^2\right), \qquad \sqrt{\frac{k_n}{k_{n\mp1}}} = 1 \pm \frac{\Omega_0}{2vk} + \mathcal{O}\left(\epsilon^2\right)\,, \tag{3.80}$$

where $v = \hbar k/m$ is a velocity of an electron with energy E. Using these expansions in Eq. (3.79) and omitting terms of order ϵ^2 we find after the inverse Fourier transformation,

$$\begin{cases} 1 + S^{(1)}_{in,11}(t,E) = S^{(1)}_{in,21}(t,E) \ , \\ \{k + ip(t)\} S^{(1)}_{in,21}(t,E) = k - \dfrac{i}{v} \dfrac{\partial S^{(1)}_{in,21}(t,E)}{\partial t} + \dfrac{1}{2vk} \dfrac{dp(t)}{dt} S^{(1)}_{in,21}(t,E) \, , \end{cases} \tag{3.81}$$

Since these equations are derived to the first order in $\epsilon \sim \Omega_0$, we can solve them by iterations in those terms which have a time derivative. Omitting such terms we get a zero-order solution, i.e., the elements of the frozen scattering matrix,

$$S^{(1)}_{11}(t,E) = \frac{-ip(t)}{k + ip(t)} \, , \qquad S^{(1)}_{12}(t,E) = \frac{k}{k + ip(t)} \, . \tag{3.82}$$

Using this solution in Eq. (3.81) we calculate the elements $\hat{S}^{(1)}_{in}$ up to the first order in ϵ terms, [30]

$$\begin{aligned} S^{(1)}_{in,11}(t,E) &= \frac{-ip(t)}{k + ip(t)} - \frac{1}{2v} \frac{dp(t)}{dt} \frac{k - ip(t)}{\left[k + ip(t)\right]^3} \, , \\ S^{(1)}_{in,21}(t,E) &= \frac{k}{k + ip(t)} - \frac{1}{2v} \frac{dp(t)}{dt} \frac{k - ip(t)}{\left[k + ip(t)\right]^3} \, . \end{aligned} \tag{3.83}$$

With Eq. (3.82) we show that,

$$\frac{\partial^2 S^{(1)}_{11}(t,E)}{\partial t \partial E} = \frac{\partial^2 S^{(1)}_{21}(t,E)}{\partial t \partial E} = \frac{i}{\hbar v} \frac{dp(t)}{dt} \frac{k - ip(t)}{\left[k + ip(t)\right]^3} \, .$$

Therefore, Eq. (3.83) can be rewritten as follows,

$$\begin{aligned} S^{(1)}_{in,11}(t,E) &= S^{(1)}_{11}(t,E) + \frac{i\hbar}{2} \frac{\partial^2 S^{(1)}_{11}(t,E)}{\partial t \partial E} \, , \\ S^{(1)}_{in,21}(t,E) &= S^{(1)}_{21}(t,E) + \frac{i\hbar}{2} \frac{\partial^2 S^{(1)}_{21}(t,E)}{\partial t \partial E} \, . \end{aligned} \tag{3.84}$$

Solving the same problem but with a wave incident from the right,

$$\Psi_2^{(in)}(t,x) = e^{-i\frac{E}{\hbar}t}\begin{cases} 0\,, & x<0\,, \\ e^{-ikx}\,, & x>0\,, \end{cases} \tag{3.85}$$

(or just using the symmetry reasons), we calculate,

$$\begin{aligned} &S_{22}^{(1)}(t,E) = S_{11}^{(1)}(t,E)\,, \qquad S_{12}^{(1)}(t,E) = S_{21}^{(1)}(t,E)\,, \\ &S_{in,22}^{(1)}(t,E) = S_{in,11}^{(1)}(t,E)\,, \quad S_{in,12}^{(1)}(t,E) = S_{in,21}^{(1)}(t,E)\,. \end{aligned} \tag{3.86}$$

Thus using Eq. (3.84) we can write down the following relation between the scattering matrix $\hat{S}_{in}^{(1)}(t,E)$ and the frozen scattering matrix $\hat{S}\,(t,E)$:

$$\hat{S}_{in}^{(1)}(t,E) = \hat{S}^{(1)}(t,E) + \frac{i\hbar}{2}\frac{\partial^2\hat{S}^{(1)}(t,E)}{\partial t\partial E}\,, \tag{3.87}$$

with

$$\hat{S}^{(1)}(t,E) = \frac{1}{k+ip(t)}\begin{pmatrix} -ip(t) & k \\ k & -ip(t) \end{pmatrix}. \tag{3.88}$$

Remind the equation (3.87) is derived with accuracy of order ϵ which, in the case under consideration, is of the same order as the adiabaticity parameter ϖ. Comparing Eqs. (3.87) and (3.61a) we conclude that the anomalous scattering matrix is identically zero for a point-like scatterer,

$$\hat{A}^{(1)}(t,E) = 0\,. \tag{3.89}$$

Therefore, the dynamic point-like scatterer does not break a symmetry of scattering with respect to a spatial direction reversal inherent to stationary scattering. To break such a symmetry dynamically it is necessary a scatterer of a finite size which is able to keep an electron for a finite time [34, 35, 36, 37, 38] comparable with a period, $\mathcal{T} = 2\pi/\Omega_0$, of a drive.

In conclusion we give relations between the coefficients of a scattered wave and the elements of the Floquet scattering matrix in the case with incident waves from both the left and the right,

$$\Psi^{(in)}(t,x) = e^{-i\frac{E}{\hbar}t}\begin{cases} a_0^{(-)}e^{ikx}, & x<0\,, \\ b_0^{(+)}e^{-ikx}, & x>0\,. \end{cases} \tag{3.90}$$

Because of the superposition principle if the incident wave is $\Psi^{(in)} = a_0^{(-)}\Psi_1^{(in)} + b_0^{(+)}\Psi_2^{(in)}$ then the scatterer wave is $\Psi^{(out)} = a_0^{(-)}\Psi_1^{(out)} + b_0^{(+)}\Psi_2^{(out)}$. Using Eqs. (3.78) for the coefficients of $\Psi_1^{(out)}$ and the analogous relations between the coefficients of $\Psi_2^{(out)}$ and $S_{F,2j}^{(1)}(E_n,E)$, $j=1,2$ we find the coefficients of the scattered wave,

$$\Psi^{(out)}(t,x) = e^{-i\frac{E}{\hbar}t}\sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t}\begin{cases} b_n^{(-)}e^{-ik_n x}, & x<0\,, \\ a_n^{(+)}e^{ik_n x}, & x>0\,, \end{cases} \tag{3.91}$$

as follows,

$$b_n^{(-)} = \sqrt{\frac{k}{k_n}}\,S_{F,11}^{(1)}(E_n,E)\,a_0^{(-)} + \sqrt{\frac{k}{k_n}}\,S_{F,12}^{(1)}(E_n,E)\,b_0^{(+)}\,, \tag{3.92a}$$

$$a_n^{(+)} = \sqrt{\frac{k}{k_n}}\,S_{F,21}^{(1)}(E_n,E)\,a_0^{(-)} + \sqrt{\frac{k}{k_n}}\,S_{F,22}^{(1)}(E_n,E)\,b_0^{(+)}\,. \tag{3.92b}$$

Thus if the scattering matrix is known, then the solution to the boundary problem (3.73) with a wave function $\Psi(t,x) = \Psi^{(in)}(t,x) + \Psi^{(out)}(t,x)$, Eqs. (3.90) and (3.91), can be written down using the Floquet scattering matrix elements as it is given in Eq. (3.92). These equations can be written more compactly if to introduce a vector-column, $\hat{\Psi}_0^{(in)}$, for coefficients of an incident wave with energy E and a vector-column, $\hat{\Psi}_n^{(out)}$, for coefficients of a scattered wave with energy E_n,

$$\hat{\Psi}_0^{(in)} = \begin{pmatrix} a_0^{(-)} \\ b_0^{(+)} \end{pmatrix}, \qquad \hat{\Psi}_n^{(out)} = \begin{pmatrix} b_n^{(-)} \\ a_0^{(+)} \end{pmatrix}. \tag{3.93}$$

Then the equation (3.92) becomes,

$$\hat{\Psi}_n^{(out)} = \sqrt{\frac{k}{k_n}}\, \hat{S}_F\left(E_n, E\right) \hat{\Psi}_0^{(in)} . \tag{3.94}$$

In the case if the incident wave is also of the Floquet function type having side-bands with different energies E_m,

$$\Psi^{(in)}\left(t, x\right) = e^{-i\frac{E}{\hbar}t} \sum_{m=-\infty}^{\infty} e^{-im\Omega_0 t} \begin{cases} a_m^{(-)} e^{ik_m x}\,, & x < 0\,, \\ b_m^{(+)} e^{-ik_m x}\,, & x > 0\,, \end{cases} \tag{3.95}$$

we introduce corresponding vector-columns,

$$\hat{\Psi}_m^{(in)} = \begin{pmatrix} a_m^{(-)} \\ b_m^{(+)} \end{pmatrix} , \tag{3.96}$$

and, using the superposition principle, generalize Eq. (3.94) as follows,

$$\hat{\Psi}_n^{(out)} = \sum_{m=-\infty}^{\infty} \sqrt{\frac{k_m}{k_n}}\, \hat{S}_F\left(E_n, E_m\right) \hat{\Psi}_m^{(in)} . \tag{3.97}$$

This equation we need to consider a system comprising a set of point-like dynamic scatterers.

3.5.3 Double-barrier potential

Let the potential $V\left(t, x\right)$, in the Schrödinger equation (3.69), consists of two oscillating in time point-like potentials, $V_j\left(t\right)$, $j = L, R$, located at a distance d from each other and a uniform oscillating in time potential $U\left(t\right)$ between the

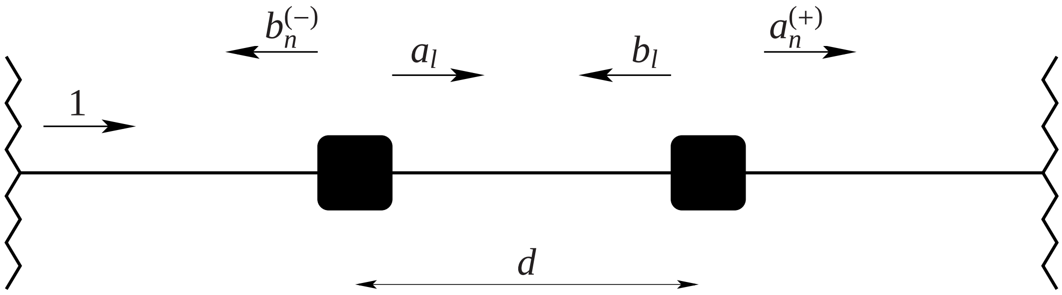

Figure 3.2: Two point-like potentials separated by a ballistic wire of length d. Arrows and letters indicate propagation direction and amplitude corresponding waves.

first two, Fig. 3.2,

$$\begin{aligned} V(t,x) &= V_L(t)\,\delta(x) + V_R(t)\,\delta(x-d) + U(t)\,\theta(x)\,\theta(d-x)\,, \\ V_j(t) &= V_{j,0} + 2V_{j,1}\cos\left(\Omega_0 t + \varphi_j\right), \qquad j = L, R\,, \\ U(t) &= 2U\cos\left(\Omega_0 t + \varphi_U\right)\,, \end{aligned} \tag{3.98}$$

where the Heaviside step function $\theta(x) = 1$ at $x > 0$ and $\theta(x) = 0$ at $x < 0$. Our aim is to calculate the Floquet scattering matrix $\hat{S}_F^{(2)}(E_n, E)$ for such a potential. [33]

To calculate the elements $S_{F,11}^{(2)}(E_n, E)$ and $S_{F,21}^{(2)}(E_n, E)$ we consider the scattering problem for a particle with energy E incident from the left. Its wave function is,

$$\Psi^{(in)}(t,x) = e^{-i\frac{E}{\hbar}t} \begin{cases} e^{ikx}\,, & x < 0\,, \\ 0\,, & x > 0\,. \end{cases} \tag{3.99}$$

The scattered wave is of the Floquet function type,

$$\Psi^{(out)}(t,x) = e^{-i\frac{E}{\hbar}t} \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} \begin{cases} b_n^{(-)} e^{-ik_n x}, & x < 0, \\ a_n^{(+)} e^{ik_n x}, & x > d, \end{cases} \tag{3.100}$$

where the coefficients $b_n^{(-)}$ and $a_n^{(+)}$ define the elements of the Floquet scattering matrix,

$$S_{F,11}^{(2)}(E_n, E) = S_{in,11,n}^{(2)}(E) = \sqrt{\frac{k_n}{k}}\, b_n^{(-)}, \tag{3.101a}$$

$$S_{F,21}^{(2)}(E_n, E) = S_{in,21,n}^{(2)}(E) = \sqrt{\frac{k_n}{k}}\, a_n^{(+)}\, e^{ik_n d}. \tag{3.101b}$$

Notice in the case of a finite-size structure the transmission amplitude includes a factor with corresponding propagation phase. In our case it is $e^{ik_n d}$.

The wave function inside the scattering region, $0 < x < d$, is also can be represented as the Floquet function, Eq. (3.71). To find the corresponding functions $\psi_n(x)$ we take into account the follows. In Sec. 3.1.3 we calculated the general solution to the Schrödinger equation with uniform oscillating potential, Eq. (3.24). In a one-dimensional case for the potential $U(t)$, Eq. (3.98), it reads as follows,

$$\Psi_E(t,x) = e^{-i\left\{\frac{E}{\hbar}t + \frac{2U}{\hbar\Omega_0}\sin(\Omega_0 t + \varphi_U)\right\}} \left(a_E\, e^{ikx} + b_E\, e^{-ikx}\right), \tag{3.102}$$

where a_E and b_E are constants (independent of t and x). This wave function corresponds to a particle with energy E and wave number $k = \sqrt{2mE}/\hbar$ in the region with a uniform oscillating in time potential $U(t)$. We use $\Psi_E(t,x)$ as a basis for calculating of a wave function at $0 < x < d$. In should be noted that interacting with a potential $V_L(t)$ an incident electron can change its initial energy E and, correspondingly, its initial wave number k. In such a case an electron enters a region with potential $U(t)$ having energy $E_l = E + l\hbar\Omega_0$ and wave number k_l. Therefore, the most general solution within the region $0 < x < d$ is the following,

$$\Psi^{(mid)}(t,x) = \sum_{l=-\infty}^{\infty} C_l \Psi_{E_l}(t,x) \; . \tag{3.103}$$

Next in Eq. (3.102) we expand a function,

$$\Upsilon(t) = e^{-i\frac{2U}{\hbar\Omega_0}\sin(\Omega_0 t + \varphi_U)} \, , \tag{3.104}$$

into the Fourier series, $\Upsilon(t) = \sum_{q=-\infty}^{\infty} e^{-iq\Omega_0 t}\,\Upsilon_q$, with

$$\Upsilon_q = J_q\left(\frac{2U}{\hbar\Omega_0}\right) e^{-iq\varphi_U} \, , \tag{3.105}$$

(J_q is the Bessel function of the first kind of the qth order). Then collecting together all the terms with the same dependence on time in Eq. (3.103) and introducing the following notation, $a_l = C_l\, a_{E_l}$ and $b_l = C_l\, b_{E_l}$, we finally get a required equation,

$$\begin{aligned} \Psi^{(mid)}(t,x) &= e^{-i\frac{E}{\hbar}t} \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t}\, \psi_n(x) \; , \\ \psi_n(x) &= \sum_{l=-\infty}^{\infty} \Upsilon_{n-l}\left(a_l\, e^{ik_l x} + b_l\, e^{-ik_l x}\right) \; , \quad 0 < x < d \, , \end{aligned} \tag{3.106}$$

which was suggested in Ref. [39, 40].

The sum of Eqs. (3.99), (3.100), and (3.106) determines an electron wave function,

$$\Psi(t,x) = \Psi^{(in)}(t,x) + \Psi^{(out)}(t,x) + \Psi^{(mid)}(t,x) \; , \tag{3.107}$$

at all the points but $x = 0$ and $x = d$. In these two points we should use the boundary conditions similar to ones given in Eq. (3.73):

$$\begin{aligned} \Psi(t, x=-0) &= \Psi(t, x=+0)\,, \\ \left.\frac{\partial \Psi(t,x)}{\partial x}\right|_{x=+0} - \left.\frac{\partial \Psi(t,x)}{\partial x}\right|_{x=-0} &= \frac{2m}{\hbar^2} V_L(t)\Psi(t, x=0)\,, \end{aligned} \tag{3.108}$$

$$\begin{aligned} \Psi(t, x=d-0) &= \Psi(t, x=d+0)\,, \\ \left.\frac{\partial \Psi(t,x)}{\partial x}\right|_{x=d+0} - \left.\frac{\partial \Psi(t,x)}{\partial x}\right|_{x=d-0} &= \frac{2m}{\hbar^2} V_R(t)\Psi(t, x=d)\,. \end{aligned} \tag{3.109}$$

Collecting terms having the same dependence on time we obtain an infinite system of equations for coefficients $b_n^{(-)}$, $a_n^{(+)}$, a_l and b_l.

The same system of equations can be derive in another way with the help of scattering matrices for constituting potentials. We designate as $\hat{L}_F$ the Floquet scattering matrix for a potential $V_L(t)$. Correspondingly, $\hat{R}_F$ is the Floquet scattering matrix for a potential $V_R(t)$. Further reasoning is quite analogous to what we used deriving Eq. (3.97) from the boundary conditions given in Eq. (3.73).

First we consider Eq. (3.108). Near $x = 0$ the wave function can be represented as follows, $\Psi(t,x) = \Psi_L^{(in)}(t,x) + \Psi_L^{(out)}(t,x)$, where $\Psi_L^{(in)}(t,x)$ corresponds to a wave incident to the barrier $V_L(t)$, while $\Psi_L^{(out)}(t,x)$ corresponds to a wave scattered by it. From Eqs. (3.99), (3.100), and (3.106) we find,

$$\Psi_L^{(in)}(t,x) = e^{-i\frac{E}{\hbar}t} \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} \begin{cases} \delta_{n0}\, e^{ikx}\,, & x<0\,, \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, b_l\, e^{-ik_l x}\,, & x>0\,, \end{cases} \tag{3.110}$$

$$\Psi_L^{(out)}(t,x) = e^{-i\frac{E}{\hbar}t} \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} \begin{cases} b_n^{(-)} e^{-ik_n x}\,, & x<0\,, \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, a_l\, e^{ik_l x}\,, & x>0\,, \end{cases} \tag{3.111}$$

Collecting all the wave function amplitudes corresponding to the same energy E_n into the vector-columns,

$$\hat{\Psi}_{Ln}^{(in)} = \begin{pmatrix} \delta_{n0} \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, b_l \end{pmatrix}, \qquad \hat{\Psi}_{Ln}^{(out)} = \begin{pmatrix} b_n^{(-)} \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, a_l \end{pmatrix}, \tag{3.112}$$

and using Eq. (3.97) we obtain the following matrix equation,

$$\begin{pmatrix} b_n^{(-)} \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, a_l \end{pmatrix} = \sum_{m=-\infty}^{\infty} \sqrt{\frac{k_m}{k_n}}\, \hat{L}_F\left(E_n, E_m\right) \begin{pmatrix} \delta_{m0} \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{m-l}\, b_l \end{pmatrix}, \tag{3.113}$$

which is completely equivalent to the boundary conditions given in Eq. (3.108) for the wave function given in Eq. (3.107).

The second pair of boundary conditions, Eq. (3.109), relate the coefficients of the wave function, Eq. (3.107), at $x = d$. Near this point the incident, $\Psi_R^{(in)}(t, x)$, and scattered, $\Psi_R^{(out)}(t, x)$, waves are,

$$\Psi_R^{(in)}(t, x) = e^{-i\frac{E}{\hbar}t} \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} \begin{cases} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, a_l\, e^{ik_l x}, & x < d\,, \\ 0\,, & x > d\,, \end{cases} \tag{3.114}$$

$$\Psi_R^{(out)}(t, x) = e^{-i\frac{E}{\hbar}t} \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} \begin{cases} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, b_l\, e^{-ik_l x}, & x < d\,, \\ a_n^{(+)} e^{ik_n x}\,, & x > d\,. \end{cases} \tag{3.115}$$

The corresponding vector-columns are,

$$\hat{\Psi}_{Rn}^{(in)} = \begin{pmatrix} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, a_l\, e^{ik_l d} \\ 0 \end{pmatrix}, \quad \hat{\Psi}_{Rn}^{(out)} = \begin{pmatrix} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, b_l\, e^{-ik_l d} \\ a_n^{(+)}\, e^{ik_n d} \end{pmatrix}. \tag{3.116}$$

Applying Eq. (3.97) to the right point-like potential, we get an equation,

$$\begin{pmatrix} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, b_l\, e^{-ik_l d} \\ a_n^{(+)}\, e^{ik_n d} \end{pmatrix} = \sum_{m=-\infty}^{\infty} \sqrt{\frac{k_m}{k_n}}\, \hat{R}_F\left(E_n, E_m\right) \begin{pmatrix} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{m-l}\, a_l\, e^{ik_l d} \\ 0 \end{pmatrix}, \tag{3.117}$$

which is equivalent to Eq. (3.109) for the wave function given in Eq. (3.107).

Let us solve the system of equations (3.113) and (3.117) with accuracy of the zeroth order in the parameter $\epsilon = \hbar\Omega_0/E \ll 1$, Eq. (3.29). Notice, in contrast to the case with a point-like scatterer, when the adiabaticity parameter ϖ coincides with a parameter ϵ, in the case of a finite-size scatterer, whose length d is much larger than the de-Broglie wave length, $\lambda_E = h/\sqrt{2mE}$, of an electron with energy E, the adiabaticity parameter is larger compared to ϵ: $\varpi \sim \epsilon d/\lambda_E \gg \epsilon$. This fact allows us to analyze both adiabatic, $\varpi \ll 1$, and non-adiabatic, $\varpi \gg 1$, regimes within the approach used.

So, to the zeroth order in ϵ we write,

$$\begin{aligned} \frac{k_m}{k_n} &= 1 + \mathcal{O}\left(\epsilon\right), \\ e^{\pm ik_l d} &= e^{\pm ikd}\, e^{\pm il\Omega_0\tau[1+\mathcal{O}(\epsilon)]}, \end{aligned} \tag{3.118}$$

where $\tau = L/v$ is a time of flight between the barriers for an electron with energy E. Further simplification is related to the following. As we showed earlier, the Floquet scattering matrix elements for a point-like barrier to the zeroth order in ϵ are the Fourier coefficients for the frozen scattering matrix, see, Eqs. (3.78) and (3.82). Designating the frozen scattering matrices for left and right barriers as $\hat{L}\left(t, E\right)$ and $\hat{R}\left(t, E\right)$, respectively, we have,

$$\hat{X}_F(E_n, E_m) = \hat{X}_{n-m}(E) + \mathcal{O}(\epsilon), \qquad X = L, R. \tag{3.119}$$

Then using Eqs. (3.118), (3.119), and (3.101) we can rewrite the system of equations (3.113) and (3.117) in the following way,

$$\begin{aligned}
\begin{pmatrix} S^{(2)}_{in,11,n}(E) \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, a_l \end{pmatrix} &= \sum_{m=-\infty}^{\infty} \hat{L}_{n-m}(E) \begin{pmatrix} \delta_{m0} \\ \sum\limits_{l=-\infty}^{\infty} \Upsilon_{m-l}\, b_l \end{pmatrix}, \\
\begin{pmatrix} e^{-ikd} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{n-l}\, b_l\, e^{-il\Omega_0\tau} \\ S^{(2)}_{in,21,n}(E) \end{pmatrix} &= \sum_{m=-\infty}^{\infty} \hat{R}_{n-m}(E) \begin{pmatrix} e^{ikd} \sum\limits_{l=-\infty}^{\infty} \Upsilon_{m-l}\, a_l\, e^{il\Omega_0\tau} \\ 0 \end{pmatrix}.
\end{aligned} \tag{3.120}$$

Next we use the following trick. We assume that the quantities a_l and b_l are the Fourier coefficients for some periodic in time functions $a(t) = a(t + \mathcal{T})$ and $b(t) = b(t + \mathcal{T})$. With these functions we can apply the inverse Fourier transformation to Eqs.(3.120) and calculate,

$$\begin{aligned}
\begin{pmatrix} S^{(2)}_{in,11}(t, E) \\ \Upsilon(t)\; a(t) \end{pmatrix} &= \hat{L}(t, E) \begin{pmatrix} 1 \\ \Upsilon(t)\; b(t) \end{pmatrix}, \\
\begin{pmatrix} e^{-ikd}\Upsilon(t)\; b(t+\tau) \\ S^{(2)}_{in,21}(t, E) \end{pmatrix} &= \hat{R}(t, E) \begin{pmatrix} e^{ikd}\Upsilon(t)\; a(t-\tau) \\ 0 \end{pmatrix},
\end{aligned} \tag{3.121}$$

where we took into account that the quantities $b_l\, e^{-il\Omega_0\tau}$ and $a_l\, e^{il\Omega_0\tau}$ are the Fourier coefficients for $b(t+\tau)$ and $a(t-\tau)$, respectively. It is easy to check. For instance,

$$\left[b(t+\tau)\right]_l = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}}\, e^{il\Omega_0 t}\, b(t+\tau) = \int_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}}\, e^{il\Omega_0 (t'-\tau)}\, b\left(t'\right) = b_l\, e^{-il\Omega_0\tau}.$$

Note the system of equations (3.121) contains only four equations, while initially we have an infinite system of equations, Eq. (3.120). The first and the fourth equations in (3.121) define the quantities of interest, $S^{(2)}_{in,11}(t,E)$ and $S^{(2)}_{in,21}(t,E)$, while the second and the third equations allow us to calculate $a(t)$ and $b(t)$. Substituting the third equation into the second one we get the following (for shortness we omit E),

$$a(t) = \Upsilon^{*}(t)\, L_{21}(t) + e^{i2kL} L_{22}(t)\, R_{11}(t-\tau)\, a(t-2\tau)\ . \qquad (3.122)$$

In addition here we used $\Upsilon^{-1}(t) = \Upsilon^{*}(t)$, because it is $|\Upsilon(t)|^2 = 1$ for the function $\Upsilon(t)$ introduced in Eq. (3.104). Since the absolute value of quantities entering Eq. (3.122) is less than unity, we can write down the solution for this equation as the following series,

$$\begin{aligned} a(t) &= \sum_{q=0}^{\infty} e^{i2qkd} \lambda^{(q)}(t)\, \Upsilon^{*}(t-2q\tau)\, L_{21}(t-2q\tau)\,, \\ \lambda^{(q>0)}(t) &= \prod_{j=0}^{q-1} L_{22}(t-2j\tau)\, R_{11}\left(t-[2j+1]\,\tau\right), \\ \lambda^{(0)}(t) &= 1\,. \end{aligned} \qquad (3.123)$$

This series can be found if to consider formally the second term on the right hand side of Eq. (3.122) as a perturbation and to sum up the terms in all the orders of the perturbation theory.

Using Eq. (3.123) in Eq. (3.121) we calculate $b(t)$ and then the Floquet scattering matrix elements, [33]

$$S^{(2)}_{in,\alpha 1}(t,E) = \sum_{q=0}^{\infty} e^{i2q_{\alpha 1}kd} \mathcal{S}^{(q)}_{\alpha 1}(t,E)\,, \quad \alpha = 1,2\,. \qquad (3.124)$$

where $2q_{\alpha 1} = 2q + 1 - \delta_{\alpha 1}$ and

$$\mathcal{S}_{\alpha 1}^{(q)}(t, E) = e^{-i\Phi_{q_{\alpha\beta}}} \sigma_{\alpha 1}^{(q)}(t, E) , \tag{3.125}$$

$$\Phi_{q_{\alpha 1}} = \frac{1}{\hbar} \int\limits_{t-2q_{\alpha 1}\tau}^{t} dt' U\left(t'\right) . \tag{3.126}$$

$$\sigma_{11}^{(0)}(t) = L_{11}(t) , \tag{3.127}$$
$$\sigma_{11}^{(q>0)}(t) = L_{12}(t) R_{11}(t-\tau) L_{21}(t-2q\tau) \lambda^{(q-1)}(t-2\tau) ,$$

$$\sigma_{21}^{(q)}(t) = R_{21}(t) L_{21}\left(t - \left[2q+1\right]\tau\right) \lambda^{(q)}(t-\tau) . \tag{3.128}$$

For shortness in Eqs. (3.127) and (3.128) we do not show an argument E. Note the time-dependent phase factor in Eq. (3.125) can be written as follows, $e^{-i\Phi_{q_{\alpha 1}}} = \Upsilon(t)\Upsilon^*(t - 2q_{\alpha 1}\tau)$.

Let us analyze Eq. (3.124). The scattering matrix element $\hat{S}_{in,\alpha 1}^{(2)}(t, E)$ is the sum of partial amplitudes, $e^{i2q_{\alpha 1}kd} \mathcal{S}_{in,\alpha 1}^{(q)}(t, E)$. Each such an amplitude corresponds to some path $\mathcal{L}_{\alpha 1}^{(q)}$ inside the scattering region. An electron with energy E enters the system through the lead 1, follows along this path undergoing $2q_{\alpha 1} - 1$ reflections, and leaves the system through the lead α at a time moment t. The trajectory $\mathcal{L}_{\alpha 1}^{(q)}$ consists of $2q_{\alpha 1}$ segments of length d. The partial scattering amplitude $e^{i2q_{\alpha 1}kd} \mathcal{S}_{in,\alpha 1}^{(q)}(t, E)$ is the product of some number of amplitudes $L_{\alpha\alpha}$ and $R_{\alpha\alpha}$, corresponding to an instant reflection from the point-like barriers, amplitudes $L_{\alpha\neq\beta}$ and $R_{\alpha\neq\beta}$, corresponding to an instant tunneling through the point-like barriers, and amplitudes $e^{i\left\{kd - \hbar^{-1}\int_{t_j-\tau}^{t_j} dt' U(t')\right\}}$, corresponding to a propagation (starting at time $t_j - \tau$ and lasting a time period $\tau = d/v$) between the two barriers in a uniform oscillating potential $U(t)$. The time moments, $t_j = t - j\tau$, at which the instantaneous reflection/transmission amplitudes are calculated, are counted backwards along the path $\mathcal{L}_{\alpha 1}^{(q)}$ in a descending order starting from the time moment t when the particle leaves the system through the left (for $\alpha = 1$) or right (for $\alpha = 2$) barrier.

Since $\mathcal{S}_{\alpha 1}^{(q)}(t, E)$ depends on scattering amplitudes calculated at different times, the scattering matrix $\hat{S}_{in}^{(2)}(t, E)$ is non-local in time, in contrast to the (local in time) frozen scattering matrix $\hat{S}\,(t, E)$. This non-locality arises as a consequence of a finite (minimal) time τ spent by an electron inside the scattering region. If the period $\mathcal{T}$ becomes as small as τ the system enters a non-adiabatic scattering regime. Therefore, a natural adiabaticity parameter for the system under consideration is the product, $\varpi_0 = \Omega_0 \tau/(2\pi)$.

To calculate remaining elements $S_{F,\alpha 2}^{(2)}(E_n, E)$, $\alpha = 1, 2$, and, correspondingly, $S_{in,\alpha 2,n}^{(2)}$, we have to consider scattering of an electron with energy E incident from the right. Then the corresponding elements of the scattering matrix $\hat{S}_{in}^{(2)}(t, E)$ are given by equations analogous to Eqs. (3.124) – (3.126) with

$$\sigma_{12}^{(q)} = L_{12}(t) R_{12}\left(t - \left[2q + 1\right]\tau\right) \rho^{(q)}(t - \tau)\,, \tag{3.129}$$

$$\begin{aligned} \sigma_{22}^{(0)} &= R_{22}(t)\,, \\ \sigma_{22}^{(q>0)} &= R_{21}(t) L_{22}(t - \tau) R_{12}(t - 2q\tau) \rho^{(q-1)}(t - 2\tau)\,. \end{aligned} \tag{3.130}$$

Here the quantity $\rho^{(q)}(t)$ is,

$$\begin{aligned} \rho^{(q>0)}(t) &= \prod_{j=0}^{q-1} R_{11}(t - 2j\tau) L_{22}\left(t - \left[2j + 1\right]\tau\right), \\ \rho^{(0)} &= 1\,. \end{aligned} \tag{3.131}$$

Thus, we have calculated the scattering matrix,

$$\hat{S}_{in}^{(2)}(t, E) = \sum_{q=0}^{\infty} e^{i2q_{\alpha 1}kd} \hat{\mathcal{S}}^{(q)}(t, E)\,, \tag{3.132}$$

allowing a description of the transport through the dynamic double-barrier as in adiabatic as in non-adiabatic regimes.

3.5.3.1 Adiabatic approximation

Let us consider the limit, $\varpi \to 0$, and calculate the anomalous scattering matrix, see, Eq. (3.48), for the double-barrier structure. We denote it as $\hat{A}^{(2)}(t, E)$. Remind this matrix is responsible for a chiral asymmetry of scattering at slow driving.

To the zeroth order in ϖ the matrix $\hat{S}_{in}^{(2)}(t, E)$ coincides with the frozen scattering matrix, which we denote as $\hat{S}^{(2)}(t, E)$ for the double-barrier under consideration. To calculate it we use Eq. (3.132) where we ignore a change of all the quantities during a time period τ. Then in Eqs. (3.123), (3.127) - (3.131) all the quantities are calculated at a time moment t, while the equation (3.126) (for $\beta = 1$) and an analogous one for $\beta = 2$, becomes,

$$\Phi_{q_{\alpha\beta}} \approx U(t)\tau\hbar^{-1}(2q + 1 - \delta_{\alpha\beta})\,.$$

As a result we get,

$$\begin{aligned} S_{\alpha\beta}^{(2)}(t, E) &= \sum_{q=0}^{\infty} \bar{S}_{\alpha\beta}^{(q)}(t, E)\,, \\ \bar{S}_{\alpha\beta}^{(q)}(t, E) &= e^{i(kd - U(t)\tau/\hbar)(2q + 1 - \delta_{\alpha\beta})}\bar{\sigma}_{\alpha\beta}^{(q)}(t, E)\,. \end{aligned} \tag{3.133}$$

Here the elements of a matrix $\hat{\bar{\sigma}}^{(q)}(t, E)$ are given in Eqs. (3.127) - (3.130) where we put $\tau = 0$.

To calculate the matrix $\hat{A}^{(2)}(t, E)$ we calculate $\hat{S}_{in}^{(2)}(t, E)$ in the first order in ϖ. To this end we expand the right hand side of Eq. (3.132) up to the linear in τ terms. Then we use Eq. (3.133) for the frozen scattering matrix and Eq. (3.61a) to extract the anomalous scattering matrix. Calculating the time and energy derivatives we take into account the following. The frozen matrix $\hat{S}^{(2)}$ depends on time via the potential $U(t)$ and the matrices $\hat{L}(t)$ and $\hat{R}(t)$. The energy dependence of $\hat{S}^{(2)}$, within the approximations used, Eqs. (3.118) and (3.119), is defined by the phase factor e^{2iqkd} only.[3] Then after the simple algebra we find,

[3]The energy dependence of the scattering matrices $\hat{L}$ and $\hat{R}$ results in corrections of order ϵ which we ignore.

$$\hbar\Omega A^{(2)}_{\alpha\beta}(t,E) = \sum_{q=0}^{\infty} \overline{S}^{(q)}_{\alpha\beta}(t,E)\mathcal{A}^{(q)}_{\alpha\beta}(t,\mu)\,, \tag{3.134a}$$

where

$$\mathcal{A}^{(q)}_{11} = \tau_0 q \frac{\partial}{\partial t}\ln\left(\frac{L_{12}}{L_{21}}\right)\,, \tag{3.134b}$$

$$\mathcal{A}^{(q)}_{21} = -\frac{\tau_0(2q+1)}{2}\frac{\partial}{\partial t}\ln\left(\frac{L_{21}}{R_{21}}\right) - \frac{\tau_0 q}{2}\frac{\partial}{\partial t}\ln\left(\frac{R_{11}}{L_{22}}\right)\,, \tag{3.134c}$$

$$\mathcal{A}^{(q)}_{12} = -\frac{\tau_0(2q+1)}{2}\frac{\partial}{\partial t}\ln\left(\frac{R_{12}}{L_{12}}\right) - \frac{\tau_0 q}{2}\frac{\partial}{\partial t}\ln\left(\frac{L_{22}}{R_{11}}\right)\,, \tag{3.134d}$$

$$\mathcal{A}^{(q)}_{22} = \tau_0 q \frac{\partial}{\partial t}\ln\left(\frac{R_{21}}{R_{12}}\right)\,. \tag{3.134e}$$

The equations above show that the anomalous scattering matrix $\hat{A}^{(2)}$ possesses symmetry properties with respect to interchange of lead indices which are different from those of the frozen scattering [30]. The symmetry of the $\hat{A}^{(2)}$ matrix depends on differences between the matrix elements of the $\hat{L}$ and $\hat{R}$ matrices. The main point is that the symmetry of the anomalous scattering matrix is fundamentally different from the frozen scattering matrix symmetry.

3.5.4 Unitarity and the sum over trajectories

One can expect that the scattering matrix elements for any structure comprising point-like scatterers connected via ballistic segments can be represented as the sum over trajectories similar to Eq. (3.132). On the other hand, as we saw, the use of the unitarity conditions allows us to simplify calculations. Therefore, it seems to be useful to formulate the unitarity conditions directly in terms of the partial scattering amplitudes, $\mathcal{S}^{(q)}_{\alpha\beta}(t,E)$, corresponding to the propagation of an electron along one of trajectories.

To this end we substitute Eq. (3.132) into Eq. (3.28b) and make the inverse Fourier transformation. Then we use an expansion given in Eq. (3.118) and get,

$$\begin{aligned} &\sum_{q=0}^{\infty} \hat{S}^{(q}(t,E)\hat{S}^{(q)\dagger}(t,E) + \\ &+\sum_{p=0}^{\infty}\sum_{s=1}^{\infty} e^{-2iskd}\hat{S}^{(p)}(t,E)\hat{S}^{(p+s)\dagger}(t+2\tau s,E) \\ &+\sum_{q=0}^{\infty}\sum_{s=1}^{\infty} e^{2iskd}\hat{S}^{(q+s)}(t,E)\hat{S}^{(q)\dagger}(t-2s\tau,E) \;=\; \hat{I}\,. \end{aligned} \tag{3.135}$$

This identity should hold at any energy E.

Note within the approximation used the quantities $\hat{S}^{(q)}$ should be kept as energy independent on the scale over which the phase kd changes by 2π. In such a case Eq. (3.135) can be considered as the Fourier expansion for the unit matrix $\hat{I}$ in the basis of plane waves, e^{2ilkd}, $l = 0, \pm 1, \pm 2, \dots$ Expanding the right hand side of Eq. (3.135) into this basis and calculating the corresponding Fourier coefficients we arrive at the following equations, [33]

$$\sum_{q=0}^{\infty} \hat{S}^{(q,\tau)}(t,E)\hat{S}^{(q,\tau)\dagger}(t,E) = \hat{I}\,, \tag{3.136a}$$

$$\sum_{p=0}^{\infty} \hat{S}^{(p,\tau)}(t,E)\hat{S}^{(p+s,\tau)\dagger}(t+2\tau s,E) = \hat{0}\,, \tag{3.136b}$$

$$\sum_{q=0}^{\infty} \hat{S}^{(q+s,\tau)}(t,E)\hat{S}^{(q,\tau)\dagger}(t-2\tau s,E) = \hat{0}\,, \tag{3.136c}$$

where $\hat{0}$ is a zero matrix.

We stress, compared to Eq. (3.60a) the equations given above are less general, since they rely essentially on the expansion (3.132), where the matrices $\hat{S}^{(q)}$ are energy independent over the scale of order $\hbar\Omega_0$.

3.5.5 Current and the sum over trajectories

Let us use Eq. (3.65) and calculate a current generated by the dynamic double-barrier structure connected to the reservoirs having the same potentials, $\mu_\alpha = \mu$, and temperatures, $T_\alpha = T$, hence $f_\alpha(E) = f_0(E)$, $\alpha = 1, 2$.

We substitute Eq. (3.132) into Eq. (3.65) and simplify it. To this end we assume that both the energy quantum $\hbar\Omega_0$ and the temperature are small compared to the Fermi energy,

$$\hbar\Omega_0,\, k_B T \ll \mu\,. \tag{3.137}$$

Then to integrate over energy in Eq. (3.65) we use the following expansion, $kd \approx k_\mu d + (E-\mu)/(\hbar\tau_\mu^{-1})$, where the lower index μ indicates that the corresponding quantity is evaluated at the Fermi energy. Within this accuracy we can treat the matrices $\hat{\mathbb{S}}^{(q)}$ as energy independent over the relevant energy window and evaluate them at $E = \mu$. The latter simplification is correct since the elements of scattering matrices $\hat{L}$ and $\hat{R}$ defining the elements of the matrix $\hat{\mathbb{S}}^{(q)}$ are changed significantly only if the energy $E \sim \mu$ changes by the quantity of order μ. Therefore, they can be kept as constant while integrating over energy over the window of the order of $\max(\hbar\Omega_0, k_B T) \ll \mu$.

Using introduced above simplifications we can integrate over energy in Eq. (3.65) and represent a time-dependent current, $I_\alpha(t)$, as the sum of diagonal, $I_\alpha^{(d)}(t)$, and non-diagonal, $I_\alpha^{(nd)}(t)$, contributions, [42, 33]

$$I_\alpha(t) = I_\alpha^{(d)}(t) + I_\alpha^{(nd)}(t)\,, \tag{3.138a}$$

The diagonal part comprises contributions of different dynamical scattering channels which can be labeled by the index q dependent on the number of reflections, (equals to $2q - \delta_{\alpha\beta}$ for $q > 0$) experienced by an electron propagating through the system, [33]

$$I_\alpha^{(d)}(t) = \sum_{q=0}^{\infty} J_\alpha^{(q)}(t)\,, \tag{3.138b}$$

where the contribution of the qth dynamical scattering channel is:

$$J_\alpha^{(q)}(t) = -i\,\frac{e}{2\pi}\left(\hat{\mathbb{S}}^{(q)}(t,\mu)\frac{\partial\hat{\mathbb{S}}^{(q)\dagger}(t,\mu)}{\partial t}\right)_{\alpha\alpha}. \tag{3.138c}$$

Notice the contribution $I_\alpha^{(d)}(t)$ is independent of the temperature.

The non-diagonal contribution to a current is the sum of temperature-dependent non-diagonal in dynamical scattering channels contributions, [33]

$$I_\alpha^{(nd)}(t) = \sum_{p=0}^{\infty}\sum_{\substack{q=0\\ q\neq p}}^{\infty} e^{i2(p-q)k_\mu d}\,\eta\left(\frac{[p-q]T}{T^*}\right) J_\alpha^{(p,q)}(t)\,, \tag{3.138d}$$

with

$$J_\alpha^{(p,q)}(t) = -i\frac{e}{2\pi}\left(\hat{\mathbb{S}}^{(p)}(t,\mu)\,\frac{\hat{\mathbb{S}}^{(q)\dagger}(t,\mu) - \hat{\mathbb{S}}^{(q)\dagger}(t-2\tau_\mu[p-q],\mu)}{2\tau_\mu[p-q]}\right)_{\alpha\alpha}. \tag{3.138e}$$

Here $\eta(x) = x/\sinh(x)$, where $x = |p-q|T/T^*$, and $k_B T^* = \hbar/(2\pi\tau_\mu)$.

The factor $\eta\left(|p-q|T/T^*\right)$ describes the effect of averaging over energies of incident electrons within the temperature widening of the edge of the Fermi distribution function. The time of flight, $\tau_\mu = d/v_\mu$, (for an electron with Fermi energy) between the barriers plays twofold role. On one hand, it separates adiabatic, $\mathcal{T} \gg \tau_\mu$, and non-adiabatic, $\mathcal{T} \leq \tau_\mu$, regimes. On the other hand, it defines the crossover temperature, T^*, separating low-temperature and high-temperature regimes. At low temperatures, $T \ll T^*$, the factor $\eta = 1$. While at relatively high temperatures, $T \gg T^*$, this factor is small, $\eta\left(|p-q|T/T^*\right) \approx 2|p-q|(T/T^*)\,e^{-|p-q|T/T^*}$. Therefore, at high temperatures the non-diagonal current, $I_\alpha^{(nd)}(t)$, is exponentially suppressed. Note the temperature effect we are discussing here is due to averaging over energy of incident electrons[4] and has nothing to do with inelastic (or other) processes destroying the phase coherence.

[4]The temperature T^* is known as the crossover temperature in the persistent current problem [18, 43], and it is appeared in the problem of stationary transport in ballistic mesoscopic structures with interference [44, 45, 46].

The unitarity conditions, Eq. (3.136), allows us to simplify $I_\alpha^{(nd)}(t)$ and to show that both the diagonal contribution and the non-diagonal contribution are real. So, taking a time derivative of Eq. (3.136a) we conclude that Eq. (3.138b) is real. Note each term $J_\alpha^{(q)}(t)$ in Eq. (3.138b) in general is not real, only their sum is necessarily real. Therefore, the interpretation of a quantity $J_\alpha^{(q)}(t)$ as a contribution of the qth dynamical scattering channel into the current $I_\alpha^{(d)}(t)$ is correct only in the case if $J_\alpha^{(q)}(t)$ is real.

To show that Eq. (3.138d) is real we first simplify it. From Eqs. (3.136b) and (3.136c) it follows that the product of scattering matrix elements corresponding to electrons leaving the scatterer at different times, t and $t-2\tau_\mu[p-q]$, drops out from Eq. (3.138d). Then the non-diagonal contribution is reduced to the following,

$$
\begin{aligned}
I_\alpha^{(nd)}(t) &= \frac{e}{2\pi\tau_\mu}\Im\sum_{s=1}^{\infty} e^{i2sk_\mu d}\,\frac{\eta\left(\frac{sT}{T^*}\right)}{s}\,\mathcal{C}_\alpha^{(s)}(t,\mu)\,, \\
\mathcal{C}_\alpha^{(s)}(t,\mu) &= \sum_{q=0}^{\infty}\left(\hat{\mathcal{S}}^{(q+s)}(t,\mu)\hat{\mathcal{S}}^{(q)\dagger}(t,\mu)\right)_{\alpha\alpha}\,.
\end{aligned}
\tag{3.139}
$$

Here the quantity $\mathcal{C}_\alpha^{(s)}$ is the sum of interference contributions from all the pairs of photon-assisted amplitudes corresponding to trajectories with the same length difference $2sd$. Note this length difference enters the phase factor $e^{i2sk_\mu d}$. All these amplitudes correspond to electrons leaving the scatterer at the time t when the current $I_\alpha^{(nd)}(t)$ is calculated.

The two parts, $I_\alpha^{(d)}(t)$ and $I_\alpha^{(nd)}(t)$, of the generated current result from different processes that lead to different temperature dependencies. The first part, $I_\alpha^{(d)}(t)$, is the sum of contribution, $J_{\alpha\beta}^{(q)}$, arising from different electron's paths $\mathcal{L}_{\alpha\beta}^{(q)}$ inside the system. These paths differ by incoming (β) and outgoing (α) leads, and by the index q counting the number of reflections inside the system. Therefore, one can consider the contribution $J_{\alpha\beta}^{(q)}$ as due to photon-assisted interference processes taking place within the same spatial path $\mathcal{L}_{\alpha\beta}^{(q)}$. Each such path can be characterized by a delay time $2q_{\alpha\beta}\tau$, i.e., the difference of times when an electron leaves and enters the system. If this time is not small compared with the driving period, $\mathcal{T}$, then the dynamical effects become important for an electron scattering off the system. Therefore, one can consider the path $\mathcal{L}_{\alpha\beta}^{(q)}$ as an effective *dynamical scattering channel*. Then we interpret $J_{\alpha}^{(q)}$ as arising due to intra-channel photon-assisted interference processes. Since all the quantum-mechanical amplitudes corresponding to such processes are multiplied by the same dynamical factor $e^{2iq_{\alpha\beta}kd}$, the corresponding probability is independent of energy. Consequently the energy integration becomes trivial.

In contrast, the second part, $I_{\alpha}^{(nd)}(t)$, due to interference between different paths (i.e., due to inter-channel interference) is defined as the sum of terms oscillating in energy. Consequently it vanishes at high temperatures.

From Eq. (3.138) it follows that with increasing temperature or driving frequency the different dynamical scattering channels contribute independently to the generated current, $I_{\alpha}(t) \approx I_{\alpha}^{(d)}(t)$. With regard to the temperature such conclusion is evident since $I_{\alpha}^{(d)}(t)$ is temperature-independent while $I_{\alpha}^{(nd)}(t)$ is exponentially suppressed at $T > T^*$. With regard to the frequency this follows from the observation that the ratio $I_{\alpha}^{(d)}(t)/I_{\alpha}^{(nd)}(t)$ behaves as $\Omega\tau_0$. Therefore, at $\Omega \to \infty$ the contribution $I_{\alpha}^{(d)}(t)$ dominates.

We emphasize that the current $I_{\alpha}^{(d)}(t)$ can not be considered as a classical part of a generated current $I_{\alpha}(t)$. This part is due to interference, therefore, it is of the quantum-mechanical nature. However it is due to interference taking pace within the same spatial trajectory, therefore this current is insensitive to energy averaging and, correspondingly, is temperature-independent.

Chapter 4

DC current generation

The current generated by the dynamical scatterer [31] has a dc component under some conditions [47]. In other words, the periodic in time excitation of a mesoscopic scatterer can result in an appearance of a dc current even in the absence of a bias between the reservoirs the scatterer is coupled to. This effect is called *the quantum pump effect*, and the dynamical mesoscopic scatterer generating a dc current is called *a quantum pump*. [47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68]

4.1 Steady particle flow

The existence of a dc current in the system means that there is a steady particle flow in the leads connecting a scatterer to the reservoirs. To characterize the intensity of such a flow in some direction [from the scatterer to the reservoir, the upper index (*in*), or back, the upper index (*out*)] it is conveniently to use *the distribution function* $f_{\alpha}^{(in/out)}(E)$, which defines how many particles with energy within the interval dE near E in unit time passes the cross-section of a lead α. The distribution function integrated over energy defines the total flow in some direction in the given lead. The dc current in the lead is defined as the difference of particle flows directed from the scatterer to the reservoir and back times an electron charge. The charge conservation requires the sum of dc currents flowing in all the leads is equal to zero.

4.1.1 Distribution function

Since we assume that the reservoirs are in equilibrium, then the electrons moving in leads from the reservoirs to the scatterer, the incident electrons, are described by the Fermi distribution function, $f_{\alpha}(E)$, where $\alpha = 1, \dots, N_r$ numbers the reservoirs. The distribution function $f_{\alpha}(E)$ depends on both the chemical potential, μ_{α}, and the temperature, T_{α}, of a corresponding reservoir. Below in this chapter we assume the chemical potentials and temperatures, hence the distribution functions, to be the same at all the reservoirs,

$$\begin{aligned} \mu_\alpha &= \mu_0\,, \quad T_\alpha = T_0\,, \quad \alpha = 1\,\dots, N_r\,, \\ f_\alpha(E) &= f_0(E)\,. \end{aligned} \tag{4.1}$$

In contrast, the electrons scattered by the dynamical sample are non-equilibrium. Therefore, they are characterized by the non-equilibrium distribution function. Let us show that the distribution function for scattered electrons is different from the Fermi distribution function.

The single-particle distribution function, $f_\alpha^{(out)}(E)$, for electrons scattered into the lead α and moving out of the scatterer, is defined as follows, [64]

$$\begin{aligned} \left\langle \hat{b}_\alpha^\dagger(E)\,\hat{b}_\beta\left(E'\right)\right\rangle &= \delta_{\alpha\beta}\delta\left(E - E'\right) f_\alpha^{(out)}(E)\,, \\ f_\alpha^{(out)}(E) &= \sum_{n=-\infty}^{\infty}\sum_{\beta=1}^{N_r}\left|S_{F,\alpha\beta}(E\,, E_n)\right|^2 f_\beta(E_n)\,. \end{aligned} \tag{4.2}$$

Using this definition we rewrite Eq. (3.40) for the dc current, $I_{\alpha,0}$, generated by the dynamical scatterer:

$$I_{\alpha,0} = \frac{e}{h}\int_0^\infty dE\left\{f_\alpha^{(out)}(E) - f_\alpha(E)\right\}. \tag{4.3}$$

From this equation it follows directly that the dc current exists in the case if the distribution function for scattered electrons is different from the one for incoming electrons.

In the case of a dynamical scatterer even if Eq. (4.1) is fulfilled the distribution function $f_\alpha^{(out)}(E)$, Eq. (4.2), differs from the Fermi distribution function,

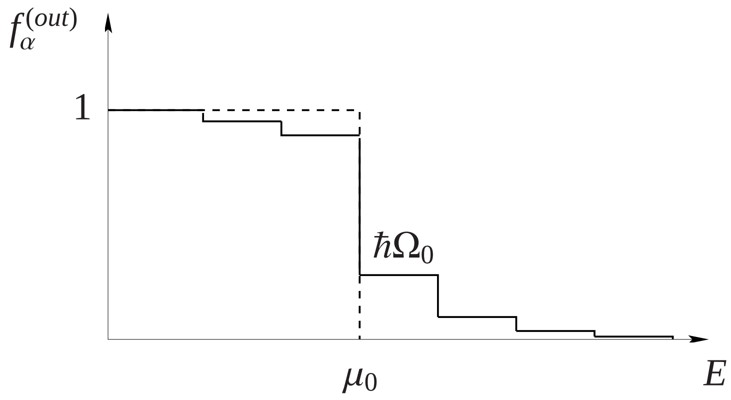

Figure 4.1: The non-equilibrium distribution function, $f_\alpha^{(out)}(E)$, for scattered electrons at zero temperature is shown schematically. The step width is $\hbar\Omega_0$. The zero-temperature Fermi function is shown by dashed line.

$f_0(E)$. Let us illustrate it in the case of zero temperatures, Fig. 4.1. In this case for each energy E the sum over n in Eq. (4.2) is restricted by those n for which $E_n \equiv E + n\hbar\Omega_0 \le \mu_0$. Therefore, we can write:

$$f_\alpha^{(out)}(E) = \sum_{n=-\infty}^{\left[\frac{\mu-E}{\hbar\Omega_0}\right]} \sum_{\beta=1}^{N_r} \left|S_{F,\alpha\beta}(E, E_n)\right|^2 = \begin{cases} <1, & E < \mu_0, \\ >0, & E > \mu_0, \end{cases} \tag{4.4}$$

where $[X]$ is an integer part of X. Given equation reaches unity only if the upper limit in the sum over n approaches infinity. This follows directly from the unitarity of the Floquet scattering matrix, see, Eq. (3.28b)

Note the distribution function $f_\alpha^{(out)}(E)$ is different from the equilibrium one only at energies near the Fermi level, $E \approx \mu_0$. For energies far from μ_0 the distribution function for scattered electrons is almost equilibrium:

$$f_\alpha^{(out)}(E) \approx \begin{cases} 1, & E \ll \mu_0, \\ 0, & E \gg \mu_0. \end{cases} \tag{4.5}$$

Therefore, we conclude: The dynamical scatterer runs an electron system out of equilibrium. This is, perhaps, the most prominent difference of the dynamical scatterer from the stationary one.

4.1.2 Adiabatic regime: Linear in pumping frequency current

Let us analyze a dc current in the limit of a small pumping frequency, see, Eq. (3.49). This is so called *the adiabatic regime* of a current generation. In this case it is convenient to use Eq. (3.42). With Eq. (4.1) we can write,

$$I_{\alpha,0} = \frac{e}{h}\int_0^\infty dE \sum_{n=-\infty}^{\infty}\left\{f_0(E) - f_0(E_n)\right\}\sum_{\beta=1}^{N_r}\left|S_{F,\alpha\beta}(E_n, E)\right|^2. \tag{4.6}$$

Expanding the difference of the Fermi functions up to linear in $\hbar\Omega_0$ terms and using the zero-order adiabatic approximation for the Floquet scattering matrix, see, Eq. (3.46a), we calculate:

$$I_{\alpha,0} = \frac{e\Omega_0}{2\pi}\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\sum_{\beta=1}^{N_r}\sum_{n=1}^{\infty} n\left\{\left|S_{\alpha\beta,n}(E)\right|^2 - \left|S_{\alpha\beta,-n}(E)\right|^2\right\}, \tag{4.7}$$

where the lower index n indicates the Fourier coefficient for the corresponding frozen scattering matrix element $S_{\alpha\beta}$.

As it follows from equation above the current $I_{\alpha,0}$ can be non-zero if the Fourier coefficients corresponding to the positive, $n > 0$ (emission), and negative, $n < 0$ (absorption), harmonics are different. After the inverse Fourier transformation the mentioned condition reads,

$$\hat{S}(t, E) \neq \hat{S}(-t, E). \tag{4.8}$$

Therefore, *the broken time-reversal symmetry of the frozen scattering matrix is a necessary condition for a dc current generation by the dynamical mesoscopic scatterer in the adiabatic regime.*

In fact we are speaking about a *dynamical* break of the time-reversal symmetry by the parameters of a scatterer, $p_i(t)$, varying under the action of external periodic in time perturbations. For instance, in the case of two parameters varying with the same frequency but shifted in phase,

$$\begin{aligned} p_1(t) &= p_{1,0} + p_{1,1}\cos(\Omega_0 t)\,, \\ p_2(t) &= p_{2,0} + p_{2,1}\cos(\Omega_0 t + \varphi)\,. \end{aligned} \tag{4.9}$$

the time-reversal symmetry is broken. To show it we note that in this case the time reversal, $t \to -t$, is equivalent to a phase reversal, $\varphi \to -\varphi$, which, at $\varphi \neq 0, 2\pi$, changes a parameter set for the frozen scattering matrix. As a result we arrive at Eq. (4.8).

Performing an inverse Fourier transformation in Eq. (4.7) we get a more compact expression for the adiabatic dc current: [47, 55, 66]

$$I_{\alpha,0} = -i\frac{e}{2\pi}\int_0^\infty dE\left(-\frac{\partial f_0(E)}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\hat{S}(E,t)\frac{\partial\hat{S}^\dagger(E,t)}{\partial t}\right)_{\alpha\alpha}. \tag{4.10}$$

To show that given above equation is real we use unitarity of the scattering matrix, $\hat{S}\hat{S}^\dagger = \hat{I}$. Where it follows from that the diagonal element $\left(\hat{S}\,d\hat{S}^\dagger\right)_{\alpha\alpha} = -\left(d\hat{S}\,\hat{S}^\dagger\right)_{\alpha\alpha}$ is imaginary, hence Eq. (4.10) is real.

Let us show that Eq. (4.10) conserves a charge. In the case of dc currents the charge conservation law (the continuity equation) reads as follows:

$$\sum_{\alpha=1}^{N_r} I_{\alpha,0} = 0\,. \tag{4.11}$$

Follow Ref. [66] we use the Birman-Krein formula (see, e.g., Ref. [36]),

$$d\ln\left(\det\hat{S}\right) = -\mathrm{Tr}\left(\hat{S}\,d\hat{S}^\dagger\right). \tag{4.12}$$

Summing up over α in Eq. (4.10) and using the identity Eq. (4.12), we find,

$$\sum_{\alpha\sim 1}^{N_r} I_{\alpha,0} \sim \int_0^{\mathcal{T}} dt\,\mathrm{Tr}\left(\hat{S}\,\frac{\partial \hat{S}^{\dagger}}{\partial t}\right) = -\int_0^{\mathcal{T}} dt\,\frac{d}{dt}\ln\left(\det \hat{S}\right)$$
$$= \ln\left(\det \hat{S}\,(0)\right) - \ln\left(\det \hat{S}\,(\mathcal{T})\right) = 0\,,$$

where in the last equality we have used the periodicity of the frozen scattering matrix.

In a particular case of a scatterer with two leads when the scattering matrix is given in Eq. (1.63), with phases γ, θ, ϕ and the reflection coefficient R all being periodic in time functions, the dc current generated, Eq. (4.10), is ($I_0 \equiv I_{1,0} = -I_{2,0}$):

$$I_0 = \frac{e}{4\pi}\int_0^{\infty} dE\left(-\frac{\partial f_0\,(E)}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left\{R(t)\,\frac{\partial\theta(t)}{\partial t} + T(t)\,\frac{\partial\phi(t)}{\partial t}\right\}. \tag{4.13}$$

As one can see, the current generated depends essentially on the phases of the scattering matrix elements. This fact emphasizes once more a quantum-mechanical nature of a current generated by the dynamical scatterer. Note that without a magnetic field it is $\phi \equiv 0$.

Notice the equation (4.10) defines a current at zero as well as at finite temperatures. From the formal point of view the expansion in powers of Ω_0 we used in Eq. (4.6) is valid only at $\hbar\Omega_0 \ll k_B T_0$. However one can show that Eq. (4.10) is valid in the opposite case, $\hbar\Omega_0 \gtrsim k_B T_0$, also. To this end we note that at zero temperature the integration over energy in each term in the sum over n in Eq. (4.6) is restricted by the interval of order $\sim |n|\hbar\Omega_0$ near the Fermi energy μ_0. At the same time the adiabatic approximation, Eq. (3.46a), is valid at the condition given in Eq. (3.49). This condition allows us to keep the frozen scattering matrix, $\hat{S}(t, E)$, as energy independent within the mentioned energy interval and to calculate it at $E = \mu_0$. The remaining integral over energy in

Eq. (4.6) becomes trivial. It gives $n\hbar\Omega_0$. As a result the first order in pumping frequency expression for a dc generated current reads ($\hbar\Omega_0 \gtrsim k_B T_0$):

$$I_{\alpha,0} = -i\frac{e}{2\pi}\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\hat{S}(t,\mu)\frac{\partial\hat{S}^\dagger(t,\mu)}{\partial t}\right)_{\alpha\alpha}. \tag{4.14}$$

The same equation can be obtained from Eq. (4.10) in the zero temperature limit (formally at $T_0 = 0$) when it is $-\partial f_0/\partial E = \delta(E-\mu)$.

The equation (4.14) admits an elegant geometrical formulation of the necessary condition for existence of a dc current generated in the adiabatic regime, see Ref. [47]. Let us consider a space of the frozen scattering matrix parameters p_i. Take a point, A(t), in this space with coordinates $p_i(t)$. During the period, $0 < t < \mathcal{T}$, the point A(t) follows a closed trajectory $\mathcal{L}$. We denote $\hat{S} \equiv \hat{S}(\{p_i(t)\},\mu)$, where $\{p_i(t)\}$ is a set of all the parameters, and rewrite Eq. (4.14) as follows: [55]

$$I_{\alpha,0} = -i\frac{e\Omega_0}{4\pi^2}\oint_{\mathcal{L}}\left(\hat{S}\,d\hat{S}^\dagger\right)_{\alpha\alpha}. \tag{4.15}$$

where the linear dependence on Ω_0 is explicit.

Further for the sake of simplicity we consider a case with only two parameters, $p_1(t)$ and $p_2(t)$, which vary with small amplitudes, $p_{i,1} \ll p_{i,0}$, $i = 1, 2$, see, Eq. (4.9). Then we can write,

$$d\hat{S}^\dagger = \frac{\partial\hat{S}^\dagger}{\partial p_1}dp_1 + \frac{\partial\hat{S}^\dagger}{\partial p_2}dp_2\,.$$

Using the Green theorem in Eq. (4.15),

$$\iint_{\mathcal{F}}\left\{\frac{\partial}{\partial p_1}\left(\hat{S}\,\frac{\partial\hat{S}^\dagger}{\partial p_2}\right) - \frac{\partial}{\partial p_2}\left(\hat{S}\,\frac{\partial\hat{S}^\dagger}{\partial p_1}\right)\right\}_{\alpha\alpha}dp_1dp_2 =$$
$$= \oint_{\mathcal{L}}\hat{S}\,\frac{\partial\hat{S}^\dagger}{\partial p_1}dp_1 + \hat{S}\,\frac{\partial\hat{S}^\dagger}{\partial p_2}dp_2\,,$$

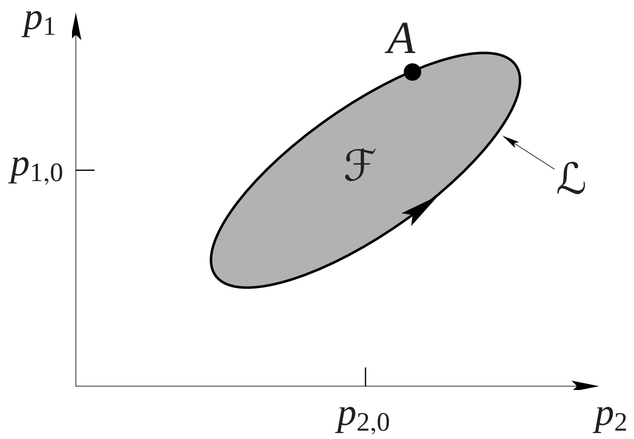

Figure 4.2: During one period the point $A(t)$ with coordinates $(p_1(t), p_2(t))$ follows a trajectory $\mathcal{L}$. $\mathcal{F}$ stands for a surface area. The arrow indicates a movement direction for $\varphi > 0$.

we finally arrive at the following [47]:

$$I_{\alpha,0} = \mathcal{F} \frac{e\Omega_0}{2\pi^2} \operatorname{Im} \left(\frac{\partial \hat{S}}{\partial p_1} \frac{\partial \hat{S}^\dagger}{\partial p_2} \bigg|_{p_i = p_{i,0}} \right)_{\alpha\alpha}, \tag{4.16}$$

where $\mathcal{F} = \pi\, p_{1,1} p_{2,1} \sin(\varphi)$ is an area of the surface (in the present case it is an ellipse) enclosed by the curve $\mathcal{L}$. The value of $\mathcal{F}$ is positive if the point A moves counterclockwise as it is shown in Fig. 4.2. In Eq. (4.16) we also took into account the following. If the parameters vary with small amplitudes then to the leading order we can keep the derivatives of the scattering matrix elements constant in the surface integral and calculate them at $p_i = p_{i,0}$.

So, *if the area $\mathcal{F}$ encircled by the representing point $A(t)$ in the parameter space of the scattering matrix during a period is non-zero, then in general case*[1] *the dc current generated in the adiabatic regime is non-zero.*

In the small amplitude limit the current is proportional to the area $\mathcal{F}$, that is the current is a quadratic form of the parameters amplitudes. Therefore,

[1] The current can be zero if the scattering matrix elements derivatives are zero. In addition in the large amplitude regime when the integrand is not constant and changes a sign, in some particular cases the current is zero even if the area is not zero. Then it is natural to speak about accidental current nullifying.

the pump effect is an essentially non-linear effect. Notice, as it follows from Eq. (4.16), the value and even the direction of a dc current can be changed simply by varying the phase difference φ between the parameters $p_1(t)$ and $p_2(t)$. It was shown experimentally in Ref. [48].

The equation (4.16) illustrates also an already mentioned relation between the existence of a dc current and the broken time-reversal symmetry. Such a relation is clearly seen from the following. Under the time reversal the direction of motion of a point A changes by its opposite. Therefore, the oriented surface $\mathcal{F}$ changes a sign.

In should be noted that there are frozen scattering matrix derivatives in Eq. (4.16). They do not connect directly to the driving. However at some particular values of parameters, $p_{i,0}$, these derivatives (or either of them) can vanish. That results in vanishing of a dc current. Therefore, the pump effect depends not only on parameters of a dynamical influence but also on the stationary characteristics of a scatterer. More precisely, the dc current arises only in the case of spatially asymmetric scatterer. To show it we use Eq. (3.43) which under conditions of Eq. (4.1) reads:

$$I_{\alpha,0} = \frac{e}{h}\int\limits_0^\infty dE\, f_0(E) \sum_{n=-\infty}^{\infty}\sum_{\beta=1}^{N_r}\left\{\left|S_{F,\alpha\beta}(E_n, E)\right|^2 - \left|S_{F,\beta\alpha}(E_n, E)\right|^2\right\}. \quad (4.17)$$

One can see, the dc current is non-zero if the photon-assisted probability for scattering from the lead β to the lead α is different from the probability for the scattering in the reversed direction.
So, *the necessary condition for the quantum pump effect is a spatial-inversion asymmetry of the scatterer.*

The use of Eq. (4.17) in the adiabatic regime, $\hbar\Omega_0 \ll \delta E$, allows us to represent a generated current as the sum of contributions due to electrons with different energies and to introduce a notion of *the spectral density of generated currents*, $dI_\alpha(t,E)/dE$, which we will need to analyze the quantum pump under external bias. While without a bias and at zero temperature, as it follows from Eq. (4.14), the current can be expressed in terms of quantities characterizing scattering of electrons with Fermi energy only.

Using Eq. (3.50) we find a square of the modulus of the Floquet scattering matrix element up to linear in Ω_0 terms:

$$\begin{aligned} \left|S_{F,\alpha\beta}(E_n,E)\right|^2 &\approx \left|S_{\alpha\beta,n}(E)\right|^2 + \frac{n\hbar\Omega_0}{2}\frac{\partial\left|S_{\alpha\beta,n}(E)\right|^2}{\partial E} \\ &+2\hbar\Omega_0 \mathrm{Re}\left[S^*_{\alpha\beta,n}(E)\,A_{\alpha\beta,n}(E)\right]. \end{aligned} \tag{4.18}$$

Also we use $\sum_n\sum_\beta\left|S_{F,\beta\alpha}(E_n,E)\right|^2 = 1$. Substituting these equations in Eq. (4.17), taking into account that $\sum_n\sum_\beta\left|S_{\alpha\beta,n}(E)\right|^2 = 1$, performing the inverse Fourier transformation, and using the identity (3.56), we finally calculate a dc current within the linear in pumping frequency, Ω_0, approximation, [66]

$$I_{\alpha,0} = \int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\int_0^\infty dE\, f_0(E)\,\frac{dI_\alpha(t,E)}{dE}, \tag{4.19}$$

where the spectral density, dI_α/dE, is related to the diagonal element of the following matrix Poisson brackets,

$$\frac{dI_\alpha(t,E)}{dE} = \frac{e}{h}P\left\{\hat{S},\hat{S}^\dagger\right\}_{\alpha\alpha} \equiv i\frac{e}{2\pi}\left(\frac{\partial\hat{S}}{\partial t}\frac{\partial\hat{S}^\dagger}{\partial E} - \frac{\partial\hat{S}}{\partial E}\frac{\partial\hat{S}^\dagger}{\partial t}\right)_{\alpha\alpha}. \tag{4.20}$$

This quantity is subject to the conservation law at each energy and at any time:

$$\sum_{\alpha=1}^{N_r}\frac{dI_\alpha}{dE} = \frac{e}{h}\sum_{\alpha=1}^{N_r}P\left\{\hat{S},\hat{S}^\dagger\right\}_{\alpha\alpha} = 0. \tag{4.21}$$

This equation is a direct consequence of the identity (3.55).

Stress both equations, (4.10) and (4.19), defines the same quantity, $I_{\alpha,0}$. The difference is a way of writing. Substituting Eq. (4.20) into Eq. (4.19) and integrating the first term by parts over the time t and both terms by parts over the energy E we arrive at Eq. (4.10).

Thus we see that the dynamical scatterer is in principle different from the stationary one even in the case of slowly varying parameters. The difference is in existence of currents with spectral density $dI_\alpha(t,E)/dE$ generated in the leads connecting a scatterer and the reservoirs.

4.1.3 Quadratic in pumping frequency current

If the phase difference φ of the pumping parameters, see, Eq. (4.9), is zero then the linear in frequency current, Eq. (4.16), vanishes. In particular such a current is absent if only one parameter of the scattering matrix varies in time. However even in this case the dynamical scatterer can generate a quadratic in pumping frequency dc current, $I_{\alpha,0} \sim \Omega_0^2$. The dc current proportional to Ω_0^n for $n > 1$ is usually called as *non-adiabatic*.

To calculate quadratic in Ω_0 dc current we substitute Eq. (4.18) into Eq. (4.6) and expand the difference of the Fermi functions up to terms proportional to Ω_0^2. After simple algebra we find a dc current,

$$I_{\alpha,0} = \frac{e}{2\pi}\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\,\mathrm{Im}\left\{\hat S\,\frac{\partial \hat S^\dagger}{\partial t} + 2\hbar\Omega_0\hat A\,\frac{\partial \hat S^\dagger}{\partial t}\right\}_{\alpha\alpha}. \tag{4.22}$$

If the frozen scattering matrix is time-reversal invariant, $\hat S(t) = \hat S(-t)$, then the first, linear in pumping frequency, term in the curly brackets in Eq. (4.22) does not contribute to current. In this case the quadratic in Ω_0 contribution is dominant,

$$I_{\alpha,0}^{(2)} = \frac{e\hbar\Omega_0}{\pi}\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\,\mathrm{Im}\left\{\hat A\,\frac{\partial \hat S^\dagger}{\partial t}\right\}_{\alpha\alpha}. \tag{4.23}$$

Earlier we showed that the linear in pumping frequency current is subject to the conservation law, Eq. (4.11). Since the current, $I_{\alpha,0}^{(2)}$ is also a dc current, it should satisfy the same conservation law. Therefore, we have,

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \, \mathrm{Im\,Tr} \left(\hat{A}(t,E) \, \frac{\partial \hat{S}^{\dagger}(t,E)}{\partial t} \right) = 0 \, . \tag{4.24}$$

This equation fulfilled at any energy E puts an additional constraint onto the anomalous scattering matrix $\hat{A}$.

4.2 Quantum pump effect

The dc current generation by the mesoscopic dynamical scatterer is due to asymmetric redistribution of (equal) electron flows incident to the scatterer from the reservoirs. It does not require any source (or drain) of a charge inside the scattering region. Before we outline a physical mechanism responsible for such an asymmetry, we give simple arguments to illustrate a possibility to generate a dc current without a bias.

4.2.1 Quasi-particle picture for a dc current generation

The appearance of a dc current can be clarified if to go over from the real particle picture to the quasi-particle picture. [64] The particle with energy above the Fermi level μ_0 we will call *a quasi-electron*, while an empty state with energy below μ_0 we will call *a hole*.

For the sake of simplicity we assume all the reservoirs being at zero temperature (and having the same chemical potentials). Then the quasi-particles are absent in equilibrium. Therefore, there is a zero quasi-particle flow incident to the scatterer. On the other hand the dynamical scatterer plays a role of a source of quasi-electron-hole pairs moving from the scatterer to the reservoirs. The quasi-electron-hole pair is created in the case when a (real) electron absorbs one, $n = 1$, or several, $n > 1$, energy quanta $\hbar\Omega_0$ interacting with a dynamical scatterer. During this process an electron empties the state with energy $E < \mu_0$ (a hole is created) and it occupies the state with energy $E_n = E + n\hbar\Omega_0 > \mu_0$ (a quasi-electron is created). We emphasize the created pair is charge neutral [2].

[2] The charge of a filled Fermi sea is treated as a reference point.

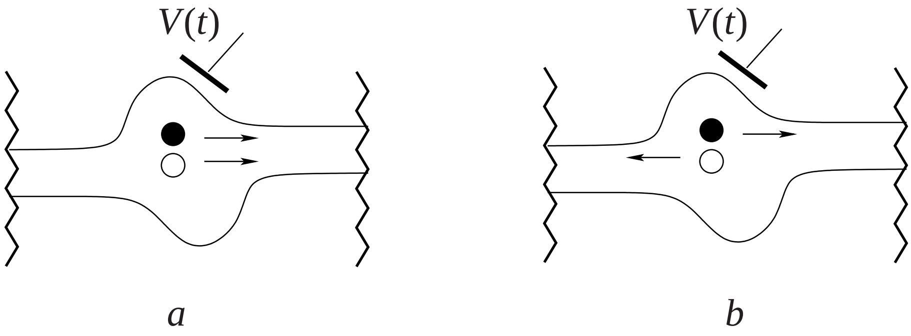

Figure 4.3: Under the action of a periodic in time potential, $V(t) = V(t + \mathcal{T})$ of the nearby metallic gate, an electron can absorb one or several energy quanta $\hbar\Omega_0$. As a result it jumps from the occupied level onto the non-occupied level, that can be viewed as a creation of a quasi-electron-hole pair. An electron (a dark circle) and a hole (a light circle) can leave the scattering region through the same lead (*a*) or through the different leads (*b*). In the latter case the current pulse is generated.

However if a quasi-electron and a hole leave the scattering region through the different leads, see, Fig. 4.3 (b), then the current pulse is generated between the corresponding reservoirs. The sum of currents in different leads is obviously zero. On the other hand, if a quasi-electron and a hole both are scattered into the same leads, then the current does not appear at all, see, Fig. 4.3 (a).

From this picture becomes clear, that the appearance of a dc current is a consequence of a broken symmetry between the quasi-electrons and holes. Otherwise the number of quasi-electrons and holes scattered to the same lead would be the same on average, hence the current averaged over a long time (a dc current) would not arise.

4.2.2 Interference mechanism of a dc current generation

As we already mentioned, within the real particle picture the appearance of a dc current is due to asymmetry in scattering of electrons from one lead to another and back, see Eq. (4.17). The physical mechanism leading to such an asymmetry is an interference of photon-assisted scattering amplitudes. [69]

To show it we consider a one-dimensional scatterer comprising two potentials, $V_1(t) = 2V\cos(\Omega_0 t + \varphi_1)$ and $V_2(t) = 2V\cos(\Omega_0 t + \varphi_2)$, oscillating with the same amplitude and located at a distance L from each other, Fig. 4.4. For simplicity we assume both potentials oscillate with small amplitude. Let an

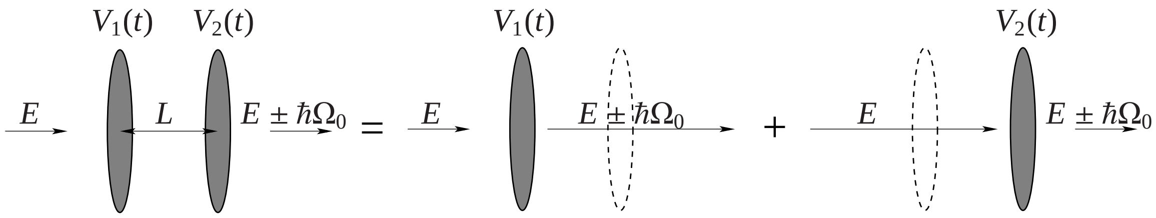

Figure 4.4: While propagating through the scatterer comprising two oscillating potentials an electron can absorb (or emit) an energy quantum $\hbar\Omega_0$ interacting either with a potential $V_1(t)$ or with a potential $V_2(t)$. Therefore, the photon-induced scattering amplitude is a sum of two terms.

electron with energy E falls upon the scatterer. Since in the case of a small amplitude of oscillations only the single-photon processes are relevant [38, 39, 40], there are three different outcomes:

(i) An electron does not interact with potentials, hence it does not change its energy. In this case an electron leaves the scattering region with energy $E^{(out)}$ equal to its initial one, $E^{(out)} = E$.

(ii) An electron absorbs one energy quantum: $E^{(out)} = E + \hbar\Omega_0$.

(iii) An electron emits one energy quantum: $E^{(out)} = E - \hbar\Omega_0$.

Since all these possibilities correspond to different final states, which differ in final energy $E^{(out)}$, the total transmission probability T is the sum of probabilities for mentioned above three processes,

$$T = T^{(0)}(E, E) + T^{(+)}(E + \hbar\Omega_0, E) + T^{(-)}(E - \hbar\Omega_0, E), \tag{4.25}$$

where the first argument is a final energy while the second one is an initial energy. The probability $T^{(0)}$, like the probability for scattering by the stationary scatterer, does not depend on the propagation direction. In contrast both probabilities $T^{(+)}$ and $T^{(-)}$ depend on it. Therefore, we concentrate on these last probabilities.

First we calculate $T^{(+)}$. Note there are two possibilities to pass through the scatterer and to absorb an energy quantum, see, Fig. 4.4. The first possibility is to absorb an energy quantum interacting with the potential $V_1(t)$. And the second possibility is to absorb an energy quantum interacting with $V_2(t)$. Since in both these cases the final state is the same, the corresponding amplitudes (not probabilities!) should be added up. Denoting corresponding amplitudes as $\mathcal{A}^{(j,+)}$, $j = 1, 2$, we calculate the probability,

$$T^{(+)} = \left|\mathcal{A}^{(1,+)} + \mathcal{A}^{(2,+)}\right|^2 . \tag{4.26}$$

Each of amplitudes $\mathcal{A}^{(j,+)}$ can be represented as the product of two terms, the amplitude, $\mathcal{A}^{(free)}(E) = e^{ikL}$, of a free propagation from one potential barrier to another one and the photon-assisted amplitude, $\mathcal{A}_j^{(+)}$, describing an absorption of an energy quantum $\hbar\Omega_0$ during an interaction with the potential V_j. The amplitude $\mathcal{A}_j^{(+)}$ is proportional to the Fourier coefficient for $V_j(t)$: $\mathcal{A}_j^{(+)} = \kappa V e^{-i\varphi_j}$, where κ is a proportionality constant.

We consider separately two cases. First, when an electron is incident from the side of the potential V_1 and, second, when an electron is incident form the side of the potential V_2. The corresponding probabilities we will label with the help of lower indices $\rightarrow$ and $\leftarrow$, respectively. Our aim is to show that

$$T_{\rightarrow}^{(+)} \neq T_{\leftarrow}^{(+)}. \tag{4.27}$$

Calculating $T_{\rightarrow}^{(+)}$ we take into account that an electron first meets the potential $V_1(t)$ and only then, after a distance L, it can reach the potential $V_2(t)$. Therefore, if an electron absorbs energy near V_1 then it propagates between the potential barriers with enhanced energy, $E_+ = E + \hbar\Omega$. The corresponding amplitude is: $\mathcal{A}_{\rightarrow}^{(1,+)} = \mathcal{A}_1^{(+)}\mathcal{A}^{(free)}(E_+)$. In contrast, if an electron absorbs energy near the potential $V_2(t)$ then it propagates between the barriers with initial energy E. The corresponding amplitude is: $\mathcal{A}_{\rightarrow}^{(2,+)} = \mathcal{A}^{(free)}(E)\mathcal{A}_2^{(+)}$. If $\hbar\Omega_0 \ll E$ we can expand the phase of an amplitude $\mathcal{A}^{(free)}(E_+)$ up to linear n Ω_0 terms: $k(E_+)L \approx kL + \Omega_0\tau$, where $k = k(E)$ and $\tau = Lm/(\hbar k)$ is a time of a free propagation between the barriers V_1 and V_2. After that we write,

$$\mathcal{A}_{\rightarrow}^{(1,+)} = \kappa V e^{-i\varphi_1} e^{i(kL + \Omega_0\tau)}, \quad \mathcal{A}_{\rightarrow}^{(2,+)} = e^{ikL}\kappa V e^{-i\varphi_2} . \tag{4.28}$$

Substituting these amplitudes into Eq. (4.26) we calculate,

$$T_{\rightarrow}^{(+)} = 2\kappa^2 V^2 \{1 + \cos(\varphi_1 - \varphi_2 - \Omega_0 \tau)\}. \tag{4.29}$$

Now we calculate the probability $T_{\leftarrow}^{(+)}$. Going from the right to the left an electron first meets V_2 and only then it meets V_1. By analogy with calculations presented above we find:

$$\mathcal{A}_{\leftarrow}^{(1,+)} = e^{ikL}\kappa V e^{-i\varphi_1}, \quad \mathcal{A}_{\leftarrow}^{(2,+)} = \kappa V e^{-i\varphi_2} e^{i(kL+\Omega_0\tau)}, \tag{4.30}$$

and correspondingly

$$T_{\leftarrow}^{(+)} = 2\kappa^2 V^2 \{1 + \cos(\varphi_1 - \varphi_2 + \Omega_0 \tau)\}. \tag{4.31}$$

Comparing Eqs. (4.29) and (4.31) we see that indeed the probability depends on the propagation direction, as it was announced in Eq. (4.27). The directional asymmetry of scattering can be characterized via the difference, $\Delta T^{(+)} = T_{\rightarrow}^{(+)} - T_{\leftarrow}^{(+)}$, which is equal to

$$\Delta T^{(+)} = 4\kappa^2 V^2 \sin(\Delta\varphi)\sin(\Omega_0\tau), \tag{4.32}$$

where $\Delta\varphi = \varphi_1 - \varphi_2$.

The probability of propagation with emission of the quantum energy $\hbar\Omega_0$ is characterized by the same asymmetry, $\Delta T^{(-)} = \Delta T^{(+)}$, for our simple model. Therefore, if the equal electron flows with intensity I_0 fall upon the scatterer from the both sides, then the asymmetric redistribution of scattered electrons results in a dc current, $I_{dc} = I_0\left(\Delta T^{(+)} + \Delta T^{(-)}\right) = 2I_0\Delta T^{(+)}$. This current depends on two phase factors. On one hand it depends on the difference of phase, $\Delta\varphi$, between the potentials $V_1(t)$ and $V_2(t)$. On the other hand the current depends on an additional contribution to the dynamical phase, $\Omega_0\tau = \Omega_0 L/v$ (where $v = \hbar k/m$ is an electron velocity), due to the energy change during scattering. The first factor breaks the time-reversal invariance allowing existence of a dc current in the system without a current in the stationary regime. While the second factor characterizes the system as spatially asymmetric (comprising two different potentials at a distance L). Interesting to note that in the case under consideration the spatial-inversion symmetry is broken only if $\varphi_1 \neq \varphi_2$, therefore, one can speak about the *dynamically* broken spatial symmetry. As it follows from Eq. (4.32), violation of only one of two symmetries, either spatial-inversion or time-reversal, is not enough for a dc current generation.

4.3 Single-parameter adiabatic current generation

Accordingly to Brouwer's arguments [47] [3] to generate a dc current in the adiabatic regime it is necessary to vary at least two parameters out of phase. The variation of a single parameter can result in at least quadratic in frequency dc current, see Sec. 4.1.3. This conclusion is confirmed by both the experiment [70, 71] and the theory [28, 62, 72, 73, 74, 75, 76, 77, 78]. However in Refs. [79, 80] it was shown theoretically, that at a slow rotation of a potential it is possible to generate a linear in rotation frequency dc current.[4] If the rotation angle is treated as a parameter then this is clearly an example of a single-parameter adiabatic dc current generation. It is natural to call this device as *a quantum Archimedes screw*. Below we give simple arguments showing that in the structures with a cyclic coordinate a single-parameter dc current generation is a rule rather than an exception.

Let the scattering matrix depends only on a single dependent on time parameter, $\hat{S}(t) = \hat{S}[p(t)]$. In the case when the system returns periodically to its initial state we have two possibilities: (i) The parameter p is a periodic function of time, $p(t) = p(t + \mathcal{T})$, or (ii) the parameter p is an angle, i.e., the scattering matrix depends periodically on p, see, e.g., Ref. [55], $S \sim e^{ip}$. In the latter case the parameter space can be rolled up into a cylinder (with $0 \le p < 2\pi$) and the parameter p can be a growing function of a time, for example, $p \sim t$.

If the parameter p is small, then the adiabatic time-dependent current $I_\alpha(t)$, Eq. (5.13), can be linearized,

$$I_\alpha(t) = e\, C_{\alpha\alpha}(0)\, \frac{\partial p}{\partial t}\,, \tag{4.33}$$

[3] See Fig. 4.2 and related discussion in the text.

[4] Note also that a uniformly translating potential can generate a dc current [81, 82]. At a slow translation the current is proportional to the speed. If to treat a spatial coordinate as a parameter then this is also an example of a single-parameter adiabatic dc current generation. Here the current results from the classical drag effect, i.e., the momentum transfer from the moving potential to the electron system is primary. In contrast in the quantum pump effect the energy transfer is primary.

where the constant

$$C_{\alpha\alpha}(p) = -\frac{i}{2\pi}\left(\hat{S}\,\frac{\partial\hat{S}^{\dagger}}{\partial p}\right)_{\alpha\alpha}$$

is calculated at $p = 0$. In the case (i) the current is periodic in time without a dc component. While in the case (ii) the current can have a dc component if $p \sim t$ and $C_{\alpha\alpha}(0) \neq 0$. Therefore, only a topologically non-trivial parameter space allows a single-parameter adiabatic pumping.

This conclusion remains valid at large p also, when C becomes function of p. In the case (i) we can expand $C(p)$ into the Tailor series in powers of p. Each term of this series results only in an ac current. In the case (ii) we expand $C(p)$ into the Fourier series. Again all the terms but the zero mode produce ac currents. In contrast, the zero mode results in a dc current (if $p \sim t$). Therefore, if the diagonal element α of a matrix $\hat{C} = \hat{S}\,\partial\hat{S}^{\dagger}/\partial p$ has a constant term (a zero mode) in the Fourier expansion in a cyclic coordinate p, then varying p with a constant speed, $p = \Omega_0 t$, one can generate a dc current $I_{\alpha} \sim \Omega_0$.

Chapter 5

AC current generation

In contrast to the dc currents, which exist only under the special conditions, the ac currents are generated as far as the properties of a scatterer changes periodically in time. As we will see below, there are several physical processes responsible for appearance of ac currents. First of all, it is a redistribution of incident electrons among the out-going channels, that is attributed to an intrinsic property of the dynamical scatterer to generate a current. The ac currents can arise also due to a possible periodical change of a charge localized onto the scatterer. And finally the potential difference between the electronic reservoirs can also lead to appearance of a current. Emphasize even the dc bias can result in ac currents since the conductance of a dynamical scatterer is changed in time.

5.1 Adiabatic ac current

Let us calculate the time-dependent current $I_\alpha(t)$, Eq. (3.39), flowing through the dynamical scatterer in the adiabatic regime, $\varpi = \hbar\Omega_0/\delta E \to 0$. To this end we transform Eq. (3.37b) for the Fourier harmonics of a current as follows. First, in the term having a factor $f_\beta(E_n)$ we make the following replacements: $E_n \to E$ and $n \to -n$. Then use an expansion (3.50) and calculate the product:

$$S^*_{F,\alpha\beta}(E_n, E)\, S_{F,\alpha\beta}(E_{l+n}, E) = S^*_{\alpha\beta,n} S_{\alpha\beta,l+n} + \hbar\Omega_0 \left\{ \frac{n}{2} \frac{\partial S^*_{\alpha\beta,n}}{\partial E} S_{\alpha\beta,l+n} \right.$$
$$\left. + \frac{(n+l)}{2} \frac{\partial S_{\alpha\beta,n+l}}{\partial E} S^*_{\alpha\beta,n} + \left(S^*_{\alpha\beta,n} A_{\alpha\beta,l+n} + A^*_{\alpha\beta,n} S_{\alpha\beta,n+l} \right) \right\} + \mathcal{O}\left(\varpi^2\right).$$

After that we sum up over n,

$$\sum_{n=-\infty}^{\infty} S^{*}_{F,\alpha\beta}(E_n, E)\, S_{F,\alpha\beta}(E_{l+n}, E) = \left(\left|S_{\alpha\beta}\right|^2\right)_l +$$

$$\frac{i\hbar}{2}\left(-\frac{\partial^2 S^{*}_{\alpha\beta}}{\partial t\partial E} S_{\alpha\beta} + \frac{\partial^2 S_{\alpha\beta}}{\partial t\partial E} S^{*}_{\alpha\beta}\right)_l + \hbar\Omega_0 \left(S^{*}_{\alpha\beta} A_{\alpha\beta} + A^{*}_{\alpha\beta} S_{\alpha\beta}\right)_l + \mathcal{O}\left(\varpi^2\right),$$

where on the right hand side (RHS) of the equation above the lower index l denotes a Fourier harmonics for the corresponding quantity. Then we get the following equation for the current in the linear in pumping frequency Ω_0 approximation as the sum of three terms,

$$I_\alpha(t) = I_\alpha^{(V)}(t) + I_\alpha^{(Q)}(t) + I_\alpha^{(gen)}(t). \tag{5.1}$$

The first term,

$$I_\alpha^{(V)}(t) = \frac{e}{h}\int_0^\infty dE \sum_{\beta=1}^{N_r} \left|S_{\alpha\beta}(t,E)\right|^2 \left\{f_\beta(E) - f_\alpha(E)\right\}, \tag{5.2}$$

is non-zero if the chemical potentials (and/or the temperatures) are different for different reservoirs. From the unitarity condition, Eq. (3.47), it follows that the quantity $I_\alpha^{(V)}(t)$ is subject to the conservation law,

$$\sum_{\alpha=1}^{N_r} I_\alpha^{(V)}(t) = 0, \tag{5.3}$$

the same as for a dc current, see, Eq. (1.48). This fact justify a separation of $I_\alpha^{(V)}(t)$ from the total current and allows us to relate this part to the potential (and/or temperature) difference between the reservoirs

The second term in Eq. (5.1),

$$I_\alpha^{(Q)}(t) = -e\frac{\partial}{\partial t}\int_0^\infty dE \sum_{\beta=1}^{N_r} f_\beta(E)\, \frac{dN_{\alpha\beta}(t,E)}{dE}, \tag{5.4}$$

is a part of a current due to the variation in time of a charge $Q(t)$ of a scatterer. In this equation we have introduced the frozen *partial density of states* (DOS),

$$\frac{dN_{\alpha\beta}(t.E)}{dE} = \frac{i}{4\pi}\left\{S_{\alpha\beta}(t,E)\frac{\partial S^{*}_{\alpha\beta}(t,E)}{\partial E} - \frac{\partial S_{\alpha\beta}(t,E)}{\partial E}S^{*}_{\alpha\beta}(t,E)\right\}, \tag{5.5}$$

which is expressed in terms of the elements of the frozen scattering matrix $\hat{S}(t,E)$ in the same way as the partial DOS of a stationary scatterer is expressed in terms of the stationary scattering matrix elements, see, Ref. [31].

Summing up currents $I^{(Q)}_{\alpha}$ in all the leads we arrive at the charge conservation law,

$$\sum_{\alpha=1}^{N_r} I^{(Q)}_{\alpha}(t) + \frac{\partial Q(t)}{\partial t} = 0, \tag{5.6}$$

where the charge localized on the scatterer is:

$$Q(t) = e\int_0^{\infty} dE \sum_{\alpha=1}^{N_r}\sum_{\beta=1}^{N_r} f_{\beta}(E)\frac{dN_{\alpha\beta}(E,t)}{dE}. \tag{5.7}$$

Strictly speaking the total current I_{α} should enter Eq. (5.6). However, as it follows from Eqs. (5.3) and (5.10) neither $I^{(V)}_{\alpha}(t)$ nor $I^{(gen)}_{\alpha}(t)$ do contribute to the equation under consideration. This allows us to interpret $I^{(Q)}_{\alpha}(t)$ as a current due to a variation of a scatterer charge.

We see as $I^{(V)}_{\alpha}$ as $I^{(Q)}_{\alpha}$ can be explained on the base of the characteristics (conductance and DOS) which are inherent to the stationary scatterer. In contrast the third contribution, a current generated by the dynamical scatterer in the lead α,

$$I^{(gen)}_{\alpha}(t) = \int_0^{\infty} dE \sum_{\beta=1}^{N_r} f_{\beta}(E)\frac{dI_{\alpha\beta}(t,E)}{dE}, \tag{5.8}$$

requires a quantity absent in the stationary case, [29]

$$\frac{dI_{\alpha\beta}}{dE} = \frac{e}{h}\left(2\hbar\Omega_0 \mathrm{Re}\left[S^{*}_{\alpha\beta}A_{\alpha\beta}\right] + \frac{1}{2}P\left\{S_{\alpha\beta}, S^{*}_{\alpha\beta}\right\}\right). \tag{5.9}$$

This is *a partial spectral current density* having a meaning of a flow generated by the dynamical scatterer from the reservoir β into the reservoir α.

The generated current, $I^{(gen)}_{\alpha}(t)$, is subject to the conservation law,

$$\sum_{\alpha=1}^{N_r} I^{(gen)}_{\alpha}(t) = 0, \tag{5.10}$$

which is directly follows from the property of the partial spectral current density,

$$\sum_{\alpha=1}^{N_r} \frac{dI_{\alpha\beta}(t,E)}{dE} = 0. \tag{5.11}$$

Above condition tells us that there is no any internal source of a charge (see, Sec. 4.2) : The scatterer takes a current $dI_{\alpha\beta}(E)/dE$ incoming from the lead β and pushes it into the lead α. The Fermi function $f_{\beta}(E)$ in Eq. (5.8) shows us how much this stream is populated.

To prove the identity (5.11) we use the diagonal element of the matrix expression (3.52),

$$4\hbar\Omega_0 \sum_{\alpha=1}^{N_r} \mathrm{Re}\left\{S^{*}_{\alpha\beta}A_{\alpha\beta}\right\} = P\left\{\hat{S}^{\dagger}, \hat{S}\right\}_{\beta\beta}, \tag{5.12}$$

and find,

$$\begin{aligned}\frac{2h}{e}\sum_{\alpha=1}^{N_r}\frac{dI_{\alpha\beta}}{dE} &= 4\hbar\Omega_0 \sum_{\alpha=1}^{N_r} \mathrm{Re}\left\{S^{*}_{\alpha\beta}A_{\alpha\beta}\right\} + \sum_{\alpha=1}^{N_r} P\left\{S_{\alpha\beta}, S^{*}_{\alpha\beta}\right\} \\ &= P\left\{\hat{S}^{\dagger}, \hat{S}\right\}_{\beta\beta} - P\left\{\hat{S}^{\dagger}, \hat{S}\right\}_{\beta\beta} = 0.\end{aligned}$$

If we sum up a quantity $dI_{\alpha\beta}/dE$ over all the incoming scattering channels (the index β) then we get the spectral current density generated into the lead α,

$$\frac{dI_\alpha}{dE} = \sum_{\beta=1}^{N_r} \frac{dI_{\alpha\beta}}{dE} = \frac{e}{2h}\left(4\hbar\Omega_0 \sum_{\beta=1}^{N_r} \mathrm{Re}\left\{S_{\alpha\beta}^{*} A_{\alpha\beta}\right\} + \sum_{\beta=1}^{N_r} P\left\{S_{\alpha\beta}, S_{\alpha\beta}^{*}\right\}\right)$$
$$= \frac{e}{2h}\left(P\left\{\hat{S}, \hat{S}^{\dagger}\right\}_{\alpha\alpha} + P\left\{\hat{S}, \hat{S}^{\dagger}\right\}_{\alpha\alpha}\right) = \frac{e}{h} P\left\{\hat{S}, \hat{S}^{\dagger}\right\}_{\alpha\alpha},$$

that coincides with Eq. (4.20).

The generated current $I_\alpha^{(gen)}(t)$ is essentially related to the anomalous scattering matrix, $\hat{A}(t,E)$, violating the symmetry of scattering with respect to a movement direction reversal, compare Eqs. (3.57) and (3.58). Note for the point-like scatterer it is $\hat{A} = \hat{0}$, see, Eq. (3.89), and also it is $P\left\{S_{\alpha\beta}, S_{\alpha\beta}^{*}\right\} = 0$, that is directly follows from Eq. (3.88), hence it is $I_\alpha^{(gen)} = 0$. Therefore, with no external bias (when it is $I_\alpha^{(V)} = 0$) the current of a dynamical point-like scatterer is only due to a variation of its charge, $I_\alpha(t) = I_\alpha^{(Q)}(t)$. For arbitrary dynamical scatterer having reservoirs with the same potentials and temperatures, $f_\alpha(E) = f_0(E)$, $\forall\alpha$, the current $I_\alpha(t) = I_\alpha^{(Q)}(t) + I_\alpha^{(gen)}(t)$ is

$$I_\alpha(t) = -\frac{ie}{2\pi}\int_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E}\right)\left(\hat{S}(t,E)\,\frac{\partial \hat{S}^{\dagger}(t,E)}{\partial t}\right)_{\alpha\alpha}, \tag{5.13}$$

which is nothing but a generalization of the Büttiker–Thomas–Prêtre formula [31].

5.2 External ac bias

Now we calculate a current flowing through the dynamical mesoscopic scatterer if the reservoirs are biased with periodic in time voltage $V_{\alpha\beta}(t) =$

$V_{\alpha\beta}(t+\mathcal{T}) \equiv V_\alpha(t) - V_\beta(t)$. This case is especial since the ac currents due to a bias, $V_{\alpha\beta}(t)$, do interfere with currents, $I_\alpha^{(gen)}(t)$, generated by the scatterer itself. As a result there arises an additional, so called *interference*, contribution to the current.

So, let the potentials applied to the reservoirs are varied with the same frequency as the parameters of a scatterer are,

$$V_\alpha(t) = V_\alpha \cos(\Omega_0 t + \phi_\alpha) , \quad \alpha = 1, \dots, N_r . \tag{5.14}$$

Due to the approach to phase-coherent transport phenomena of Refs. [83, 84] the periodic in time potential $V_\alpha(t)$ of an electron reservoir is treated as spatially uniform and it is accounted in the phase of a wave-function of electrons incident from the reservoir to the scatterer. At the same time the chemical potential μ_α entering the Fermi distribution function $f_\alpha(E)$ is constant and independent of $V_\alpha(t)$.

As we know, the Schrödinger equation with a spatially uniform potential $V_\alpha(t)$,

$$i\hbar \frac{\partial \Psi_\alpha}{\partial t} = H_{0,\alpha}\Psi_\alpha + eV_\alpha(t)\Psi_\alpha , \tag{5.15}$$

can be integrated out in time. Then the electron wave function can be written as follows, see Sec. 3.1.3,

$$\Psi_\alpha = \Psi_{0,\alpha}\, e^{-i\hbar^{-1}\int\limits_{-\infty}^{t} dt' eV_\alpha(t')} , \tag{5.16}$$

where $\Psi_{0,\alpha}$ is a solution to Eq. (5.15) with $V_\alpha(t) = 0$. Such a solution corresponding to energy E, is

$$\Psi_{0E,\alpha} = e^{-i\frac{E}{\hbar}t}\, \psi_{E,\alpha}(\vec{r}) . \tag{5.17}$$

With potential $V_\alpha(t)$, Eq. (5.14), the wave function, Eq. (5.16), corresponding to energy E, becomes ($eV_\alpha > 0$):

$$\Psi_{E,\alpha} = e^{-i\frac{E}{\hbar}t}\bar{\psi}_{E,\alpha}\left(\vec{r}\right)\sum_{n=-\infty}^{\infty} e^{-in\phi_\alpha} J_n\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) e^{-in\Omega_0 t}\,, \tag{5.18}$$

where we have used the following Fourier series,

$$e^{-iX\sin(\Omega_0 t+\phi_\alpha)} = \sum_{n=-\infty}^{\infty} J_n\left(X\right) e^{-in(\Omega_0 t+\phi_\alpha)}\,, \tag{5.19}$$

and have included the constant $C = e^{ieV_\alpha/(\hbar\Omega_0)\,\sin(\Omega_0 t'+\phi_\alpha)|_{t'=-\infty}}$ from Eq. (5.16) into the function $\bar{\psi}_{E,\alpha}\left(\vec{r}\right) = C\psi_{E,\alpha}\left(\vec{r}\right)$.

The wave function $\Psi_{E,\alpha}$ is of the Floquet function type, see Eqs. (3.22) and (3.27). Note the spatial part $\bar{\psi}_{E,\alpha}$ depends on the Floquet energy E but does not depend on the sub-band number n. Therefore, the Floquet wave function is normalized exactly as the stationary wave function $\psi_{E,\alpha}$ does:

$$\int d^3r\left|\Psi_{E,\alpha}\right|^2 = \int d^3r\left|\psi_{E,\alpha}\right|^2. \tag{5.20}$$

Indeed, using the following property of the Bessel functions,

$$\sum_{n=-\infty}^{\infty} J_n(X)\, J_{n+q}(X) = \delta_{q0}\,, \tag{5.21}$$

we find from Eq. (5.18),

$$\begin{aligned}\left|\Psi_{E,\alpha}\right|^2 &= \left|\psi_{E,\alpha}\right|^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} e^{-i(n-m)\phi_\alpha} e^{-i(n-m)\Omega_0 t} J_n\left(X\right) J_m\left(X\right) \\ &= \left|\psi_{E,\alpha}\right|^2 \sum_{q=-\infty}^{\infty} e^{iq\phi_\alpha} e^{iq\Omega_0 t} \sum_{n=-\infty}^{\infty} J_n\left(X\right) J_{n+q}\left(X\right) = \left|\psi_{E,\alpha}\right|^2.\end{aligned}$$

Here we denoted $X = eV_\alpha/(\hbar\Omega_0)$, introduced $q = m - n$, and took into account $|C|^2 = 1$.

The state with wave function $\Psi_{E,\alpha}$ can be occupied at most by one electron. While the measurement of an electron energy in the state $\Psi_{E,\alpha}$ can results in any of values $E_n = E + n\hbar\Omega_0$ with probability $J_n^2(eV_\alpha/\hbar\Omega_0)$. However the mean energy, $E[\Psi_{E,\alpha}]$, is equal to the energy E of the corresponding stationary state, $\Psi_{0E,\alpha}$:

$$E\left[\Psi_{E,\alpha}\right] = \sum_{n=-\infty}^{\infty} E_n J_n^2 = E\sum_{n=-\infty}^{\infty} J_n^2 + \hbar\Omega_0 \sum_{n=1}^{\infty} n\left(J_n^2 - J_{-n}^2\right) = E\,.$$

Therefore, the distribution function reflecting the occupation of the states $\Psi_{E,\alpha}$ is the Fermi distribution function dependent on the Floquet energy E.

5.2.1 Second quantization operators for incident and scattered electrons

Let us introduce the creation and annihilation operators, $\hat{a}'^{\dagger}_{\alpha}(E)$ and $\hat{a}'_{\alpha}(E)$, for electrons in the Floquet state $\Psi_{E,\alpha}$. They are anti-commuting, Eq. (1.30). The quantum-statistical average of the following product,

$$\left\langle \hat{a}'^{\dagger}_{\alpha}(E)\,\hat{a}'_{\beta}\left(E'\right)\right\rangle = \delta_{\alpha\beta}\,\delta\left(E - E'\right)\, f_\alpha(E)\,. \tag{5.22}$$

is expressed through the Fermi distribution function $f_\alpha(E)$ dependent on the Floquet energy E.

Strictly speaking we should consider scattering of the whole Floquet state, $\Psi_{E,\alpha}$, incident to the mesoscopic sample. However with Eq. (3.29) and if the amplitude of oscillating potential is small,

$$eV_\alpha \ll E\,, \tag{5.23}$$

the scattering of any sub-band of the Floquet state is independent of the scattering of other sub-bands. Therefore, following the approach of Ref. [84], we, as before, consider scattering of electrons in the states with fixed energy.

We suppose that the potential $V_\alpha(t)$ is present in the reservoir α but it is absent in the lead α connecting a reservoir and a scatterer. Then an electron in the lead is described by the wave function with fixed energy. For an incident electron in the lead α this wave function is: $\Psi_\alpha^{(in)} = e^{-iEt/\hbar}\psi_\alpha^{(in)}$, where $\psi_\alpha^{(in)}$ is given in Eq. (1.33). Notice there is a number of the Floquet states $\Psi_{E',\alpha}$, Eq. (5.18), having a sub-band with energy E in the reservoir α. For such states the Floquet energy E' should be different from E by the integer number of energy quanta $\hbar\Omega_0$. For instance, if $E' = E + n\hbar\Omega_0$ then the sub-band E'_{-n} has an energy E since it is

$$E'_{-n} = E' - n\hbar\Omega_0 = E + n\hbar\Omega_0 - n\hbar\Omega_0 = E\,.$$

All such Floquet states do contribute to the wave function, $\Psi_\alpha^{(in)}$, of an electron in a lead. Therefore, the operators $\hat{a}_\alpha^\dagger(E)/\hat{a}_\alpha(E)$ creating/annihilating an electron in the state $\Psi_\alpha^{(in)}$ in the lead α can be expressed in terms of the operators $\hat{a}_\alpha^\dagger(E_n)/\hat{a}_\alpha(E_m)$ creating/annihilating an electron in the reservoir α, as follows:

$$\begin{aligned}\hat{a}_\alpha(E) &= \sum_{m=-\infty}^{\infty} e^{-im\phi_\alpha} J_m\left(\frac{eV_\alpha}{\hbar\Omega_0}\right)\hat{a}'_\alpha(E - m\hbar\Omega_0)\,,\\ \hat{a}_\alpha^\dagger(E) &= \sum_{n=-\infty}^{\infty} e^{in\phi_\alpha} J_n\left(\frac{eV_\alpha}{\hbar\Omega_0}\right)\hat{a}'^\dagger_\alpha(E - n\hbar\Omega_0)\,.\end{aligned} \tag{5.24}$$

The spatial parts of the corresponding wave functions, Eq. (5.18), are assumed to be the same at the place where the reservoir is connected to the lead (an adiabatic connection condition). Therefore, they do not enter given above equations,

The operators $\hat{a}'$ are for electrons in a reservoir. They are anti-commuting by definition, see, Eq. (1.30). Let us show that the operators $\hat{a}$, Eq. (5.24), for electrons in the lead also are anti-commuting. Using Eq. (5.21) we calculate:

$$\begin{aligned}\left\{\hat{a}_\alpha^\dagger(E), \hat{a}_\beta(E')\right\} = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} e^{i\phi_\alpha n} e^{-i\phi_\beta m} J_n\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) J_m\left(\frac{eV_\beta}{\hbar\Omega_0}\right)\\ \times\left\{\hat{a}'^\dagger_\alpha(E - n\hbar\Omega_0), \hat{a}'_\beta(E' - m\hbar\Omega_0)\right\}\end{aligned}$$

$$= \delta_{\alpha\beta} \sum_{l=-\infty}^{\infty} e^{i\phi_\alpha l} \delta \left(E - E' - l\hbar\Omega_0\right) \sum_{n=-\infty}^{\infty} J_n \left(\frac{eV_\alpha}{\hbar\Omega_0}\right) J_{n-l} \left(\frac{eV_\alpha}{\hbar\Omega_0}\right)$$

$$= \delta_{\alpha\beta} \sum_{l=-\infty}^{\infty} e^{i\phi_\alpha l} \delta \left(E - E' - l\hbar\Omega_0\right) \delta_{l0} = \delta_{\alpha\beta} \delta \left(E - E'\right) ,$$

where we introduced $l = n - m$.

Next we calculate the distribution function, $\tilde{f}_\alpha (E) = \left\langle \hat{a}^\dagger_\alpha (E) \hat{a}_\alpha (E)\right\rangle$, for electrons in the lead α:

$$\tilde{f}_\alpha (E) = \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{eV_\alpha}{\hbar\Omega_0}\right) f_\alpha (E - n\hbar\Omega_0) . \tag{5.25}$$

This distribution function is non-equilibrium, that is due changing of conditions (the oscillating potential vanishes in the lead) with no relaxation processes present. Despite of the non-equilibrium state, the electrons in the lead incident to the scatterer carry a current $I_\alpha^{(in)}$ which is independent of the oscillating potential $V_\alpha(t)$. This current is time-independent and coincides with a current of equilibrium particles:

$$\begin{aligned} I_\alpha^{(in)} &= -\frac{e}{h} \int_0^\infty dE\, \tilde{f}_\alpha (E) = -\frac{e}{h} \int_0^\infty dE \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{eV_\alpha}{\hbar\Omega_0}\right) f_\alpha (E - n\hbar\Omega_0) \\ &= -\frac{e}{h} \int_0^\infty dE\, f_\alpha (E) \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{eV_\alpha}{\hbar\Omega_0}\right) = -\frac{e}{h} \int_0^\infty dE\, f_\alpha (E) . \end{aligned} \tag{5.26}$$

In the second line of this equation we made a shift $E \to E + n\hbar\Omega_0$ under the integral over energy. As always, we use a wide-band approximation, i.e., we assume that only electrons with energy $E \sim \mu$ are relevant for transport. Therefore, we can relax what is happening at $E \approx 0$, where, strictly speaking, the decomposition given in Eq. (5.24) fails.

As a next step we need to express the creation/annihilation operators, $\hat{b}_\alpha/\hat{b}^\dagger_\alpha$, for electrons scattered into the lead α in terms of operators, $\hat{a}'_\beta/\hat{a}'^\dagger_\beta$, for electrons in reservoirs. The relation between the operators $\hat{b}_\alpha$ for scattered electrons and the operators $\hat{a}_\beta$ for incident electrons is given in Eq. (3.32). Then using Eq. (5.24) we finally get:

$$\begin{aligned}
\hat{b}_\alpha(E) &= \sum_{\delta=1}^{N_r}\sum_{n'=-\infty}^{\infty}\sum_{p'=-\infty}^{\infty} S_{F,\alpha\delta}(E,E_{n'})\, e^{-i(n'+p')\phi_\delta} J_{n'+p'}\left(\frac{eV_\delta}{\hbar\Omega_0}\right)\hat{a}'_\delta\left(E_{-p'}\right), \\
\hat{b}^\dagger_\alpha(E) &= \sum_{\gamma=1}^{N_r}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} S^*_{F,\alpha\gamma}(E,E_n)\, e^{i(n+p)\phi_\gamma} J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right)\hat{a}'^\dagger_\gamma\left(E_{-p}\right).
\end{aligned} \tag{5.27}$$

These operators, as it should be for fermionic operators, are anti-commuting. To show it we write:

$$\begin{aligned}
\left\{\hat{b}^\dagger_\alpha(E), \hat{b}_\beta(E')\right\} &= \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty}\sum_{p'=-\infty}^{\infty} e^{i(n+p)\phi_\gamma} e^{-i(n'+p')\phi_\delta} \\
&\times J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{n'+p'}\left(\frac{eV_\delta}{\hbar\Omega_0}\right) S^*_{F,\alpha\gamma}(E,E_n)\, S_{F,\beta\delta}\left(E',E'_{n'}\right) \\
&\times \left\{\hat{a}'^\dagger_\gamma(E-p\hbar\Omega_0), \hat{a}'_\delta\left(E'-p'\hbar\Omega_0\right)\right\}.
\end{aligned}$$

Then we take into account that,

$$\left\{\hat{a}'^\dagger_\gamma(E-p\hbar\Omega_0), \hat{a}'^\dagger_\delta\left(E'-p'\hbar\Omega_0\right)\right\} = \delta_{\gamma\delta}\,\delta\left(E-E'+\left(p'-p\right)\hbar\Omega_0\right),$$

and proceed as follows. With the help of $\delta_{\gamma\delta}$ we sum up over δ. Because of the Dirac delta function we write $E' = E + (p'-p)\hbar\Omega_0 \equiv E_{p'-p}$ instead of E'.

Then we introduce $m = p' - p$ instead of p', $k = n - n' - m$ instead of n', and $q = n + p$instead of p. After that we calculate:

$$\left\{\hat{b}_{\alpha}^{\dagger}(E), \hat{b}_{\beta}\left(E'\right)\right\} = \sum_{\gamma=1}^{N_r} \sum_{m=-\infty}^{\infty} \delta\left(E - E' + m\hbar\Omega_0\right) \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} S_{F,\alpha\gamma}^{*}(E, E_n) \times e^{ik\phi_\gamma} S_{F,\beta\gamma}(E_m, E_{n-k}) \sum_{q=-\infty}^{\infty} J_q\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{q+k}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right).$$

Using Eq. (5.21) for Bessel functions we simplify given above equation as follows:

$$\left\{\hat{b}_{\alpha}^{\dagger}(E), \hat{b}_{\beta}\left(E'\right)\right\} = \sum_{\gamma=1}^{N_r} \sum_{m=-\infty}^{\infty} \delta\left(E - E' + m\hbar\Omega_0\right) \times \sum_{n=-\infty}^{\infty} S_{F,\alpha\gamma}^{*}(E, E_n) S_{F,\beta\gamma}(E_m, E_n).$$

Finally we take into account the unitarity of the Floquet scattering matrix, Eq. (3.28b), and find a required anti-commutation relation for operators of scattered electrons:

$$\left\{\hat{b}_{\alpha}^{\dagger}(E), \hat{b}_{\beta}\left(E'\right)\right\} = \delta\left(E - E'\right)\delta_{\alpha\beta}. \tag{5.28}$$

For the sake of completeness we give a distribution function, $f_{\alpha}^{(out)}(E) = \left\langle \hat{b}_{\alpha}^{\dagger}(E)\hat{b}_{\alpha}(E)\right\rangle$, for electrons scattered into the lead α: [29]

$$f_{\alpha}^{(out)}(E) = \sum_{\gamma=1}^{N_r} \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} S_{\alpha\gamma}^{*}(E, E_n) S_{\alpha\gamma}(E, E_{n'}) e^{i(n-n')\phi_\gamma} \times \sum_{p=-\infty}^{\infty} J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{n'+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) f_\gamma(E - p\hbar\Omega_0). \tag{5.29}$$

Note this equation is real. To show it one can calculate a complex conjugate quantity. Then after an irrelevant replacement, $n \leftrightarrow n'$, we arrive at the initial equation.

5.2.2 AC current

Substituting Eqs, (5.24) and (5.27) into Eq. (3.34) and taking into account Eq. (3.33a) we arrive at the current operator $\hat{I}_\alpha(t)$. Further, with Eq. (5.22) we average quantum-statistically over the equilibrium state of reservoirs and find the following equation for the time-dependent current, $I_\alpha(t) = \left\langle \hat{I}_\alpha(t) \right\rangle$:

$$I_\alpha(t) = \sum_{l=-\infty}^{\infty} e^{-il\Omega_0 t} I_{\alpha,l}, \tag{5.30a}$$

$$\begin{aligned} I_{\alpha,l} = \frac{e}{h}\int_0^\infty dE \Bigg\{ \sum_{\gamma=1}^{N_r} \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} e^{i(n-n'-l)\phi_\gamma} S_{\alpha\gamma}^*(E, E_n) S_{\alpha\gamma}(E_l, E_{n'+l}) \\ \times \sum_{p=-\infty}^{\infty} J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{n'+l+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) f_\gamma(E - p\hbar\Omega_0) - \delta_{l0} f_\alpha(E) \Bigg\}. \end{aligned} \tag{5.30b}$$

Let us transform this equation to have a difference of the Fermi functions. To this end we use Eqs. (3.28) and (5.21) and find the following expression for the Fourier harmonics of a current:

$$\begin{aligned} I_{\alpha,l} = \frac{e}{h}\int_0^\infty dE \sum_{\gamma=1}^{N_r} \sum_{p=-\infty}^{\infty} \left\{ f_\gamma(E - p\hbar\Omega_0) - f_\alpha(E) \right\} \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} e^{i(n-n'-l)\phi_\gamma} \\ \times S_{\alpha\gamma}^*(E, E_n) S_{\alpha\gamma}(E_l, E_{n'+l})\, J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{n'+l+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right). \end{aligned} \tag{5.31}$$

This equation is convenient to use in the adiabatic regime, when we can expand the Fermi function difference in powers of Ω_0.

5.2.3 DC current

A more compact equation can be obtained for a time-independent part of a current, $l = 0$. First of all we express the Floquet scattering matrix in terms of the scattering matrix $\hat{S}_{out}(E, t)$, see, Eq. (3.59b):

$$S_{\alpha\gamma}(E, E_{n'}) = S_{out,\alpha\gamma,-n'}(E), \quad S^*_{\alpha\gamma}(E, E_n) = S^*_{out,\alpha\gamma,-n}(E).$$

Then using a series (5.19) we express the Bessel functions in terms of the Fourier coefficients for some exponential function dependent on an oscillating potential, $V_\gamma(t)$:

$$J_{n'+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) = e^{i(n'+p)\phi_\gamma}\left(e^{-i\hbar^{-1}\int\limits_{-\infty}^{t} dt' eV_\gamma(t')}\right)_{n'+p}.$$

Note the lower limit in a time integral is irrelevant since it does not affect the value of the Fourier coefficient. Using equation above in Eq. (5.31) and summing up over n and n' with the help of the following property of the Fourier coefficients,

$$\sum_{n'=-\infty}^{\infty} A_{-n'} B_{p+n'} = (AB)_p \quad \sum_{n=-\infty}^{\infty} (A_{-n})^* (B^*)_{-p-n} = (A^* B^*)_{-p}, \tag{5.32}$$

we finally calculate the dc current in the lead α: [33]

$$I_{\alpha,0} = \frac{e}{h}\int\limits_0^{\infty} dE \sum_{\gamma=1}^{N_r}\sum_{p=-\infty}^{\infty}\left\{f_{\beta}(E-p\hbar\Omega_0)-f_{\alpha}(E)\right\} \times \left|\left(e^{-i\hbar^{-1}\int\limits_{-\infty}^{t} dt' eV_{\gamma}(t')} S_{out,\alpha\gamma}(E,t)\right)_p\right|^2 . \tag{5.33}$$

As we see, the reservoir oscillating potential can be taken into account as an additional phase factor in corresponding scattering matrix elements. Since, as it follows from Eq. (4.13), the phase of scattering matrix elements defines a generated current, we can guess that the presence of oscillating potentials at reservoirs modifies a generated current.

5.2.4 Adiabatic dc current

To clarify the effect of potentials $V_{\beta}(t)$ onto the dc current $I_{\alpha,0}$ we consider an adiabatic regime, $\varpi \ll 1$, and restrict ourselves by the terms linear in oscillating potentials,

$$|eV_{\beta}| \ll \hbar\Omega_0 \ll \delta E\,, \quad \forall\beta\,, \tag{5.34}$$

where δE is a characteristic energy introduced after Eq. (3.49). We assume also no bias conditions, Eq. (4.1).

Let us expand the difference of the Fermi functions in Eq. (5.33) in powers of pumping frequency:

$$f_0(E-p\hbar\Omega_0)-f_0(E) \approx \left(-\frac{\partial f_0}{\partial E}\right) p\hbar\Omega_0 + \frac{p^2(\hbar\Omega_0)^2}{2}\frac{\partial^2 f_0}{\partial E^2}\,. \tag{5.35}$$

Here we need to keep quadratic in Ω_0 terms. They are necessary since the phase factors dependent on $V_{\gamma}(t')$ results in a factor Ω_0^{-1}.

We substitute Eq. (5.35) into Eq. (5.33) and sum up over p. Then we take into account the adiabatic expansion, Eq. (3.61b), for the scattering matrix $\hat{S}_{out}$ and keep only terms linear in both $V_\gamma(t)$ and Ω_0. For short-notation we introduce $\Upsilon_\gamma(t) = \exp\left\{-\frac{i}{\hbar}\int\limits_{-\infty}^{t} dt' eV_\gamma(t')\right\}$. So, the linear in Ω_0 term in Eq. (5.35) results in the following:

$$\hbar \sum_{n=-\infty}^{\infty} \Omega_0 p \left| \left(\Upsilon_\gamma S_{out,\alpha\gamma}\right)_p \right|^2 = -i\hbar \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \Upsilon_\gamma S_{out,\alpha\gamma} \frac{\partial}{\partial t} \left(\Upsilon_\gamma^* S_{out,\alpha\gamma}^*\right)$$

$$= \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} eV_\gamma(t) \left\{ \left|S_{\alpha\gamma}\right|^2 - \frac{i\hbar}{2} \left(\frac{\partial^2 S_{\alpha\gamma}}{\partial t \partial E} S_{\alpha\gamma}^* - S_{\alpha\gamma} \frac{\partial^2 S_{\alpha\gamma}^*}{\partial t \partial E} \right) \right.$$

$$\left. + 2\hbar\Omega_0 \,\mathrm{Re}\left[S_{\alpha\gamma}^* A_{\alpha\gamma}\right] \right\} - i\hbar \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} S_{\alpha\gamma} \frac{\partial S_{\alpha\gamma}^*}{\partial t} + \mathcal{O}\left(\Omega_0^2\right).$$

While the quadratic in pumping frequency term in Eq. (5.35) is:

$$\sum_{n=-\infty}^{\infty} \Omega_0^2 p^2 \left| \left(\Upsilon_\gamma S_{out,\alpha\gamma}\right)_p \right|^2 = \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \frac{\partial}{\partial t} \left(\Upsilon_\gamma S_{out,\alpha\gamma}\right) \frac{\partial}{\partial t} \left(\Upsilon_\gamma^* S_{out,\alpha\gamma}^*\right) =$$

$$= \frac{i}{\hbar} \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} eV_\gamma(t) \left\{ \frac{\partial S_{\alpha\gamma}}{\partial t} S_{\alpha\gamma}^* - S_{\alpha\gamma} \frac{\partial S_{\alpha\gamma}^*}{\partial t} \right\} + \mathcal{O}\left(\Omega_0^2, V_\gamma^2\right).$$

The last equation enters the current, Eq. (5.33), with factor $\partial^2 f_0 / \partial E^2$. We integrate over energy by parts and calculate,

$$\int\limits_0^{\infty} dE \frac{\partial^2 f_0(E)}{\partial E^2} \left\{ \frac{\partial S_{\alpha\gamma}}{\partial t} S_{\alpha\gamma}^* - S_{\alpha\gamma} \frac{\partial S_{\alpha\gamma}^*}{\partial t} \right\} = \int\limits_0^{\infty} dE \left(-\frac{\partial f_0(E)}{\partial E} \right)$$

$$\times \left\{ \frac{\partial^2 S_{\alpha\gamma}}{\partial t \partial E} S_{\alpha\gamma}^* - S_{\alpha\gamma} \frac{\partial^2 S_{\alpha\gamma}^*}{\partial t \partial E} + \frac{\partial S_{\alpha\gamma}}{\partial t} \frac{\partial S_{\alpha\gamma}^*}{\partial E} - \frac{\partial S_{\alpha\gamma}}{\partial E} \frac{\partial S_{\alpha\gamma}^*}{\partial t} \right\},$$

where we used: $\partial f_0/\partial E|_{E=\infty} = 0$ and $\partial f_0/\partial E|_{E=0} = 0$. Note the latter is valid at $k_B T \ll \mu$.

With given above transformations we represent a dc current, $I_{\alpha,0}$, as the sum of three terms linear in both Ω_0 and V_γ: [29]

$$I_{\alpha,0} = I_{\alpha,0}^{(pump)} + I_{\alpha,0}^{(rect)} + I_{\alpha,0}^{(int)} . \tag{5.36a}$$

Here the current $I_{\alpha,0}^{(pump)}$, generated by the dynamical scatterer in the absence of an oscillating bias, is given in Eq. (4.10). The next terms,

$$I_{\alpha,0}^{(rect)} = \frac{e^2}{h} \int\limits_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E} \right) \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\gamma=1}^{N_r} V_\gamma(t) \left| S_{\alpha\gamma}(E,t) \right|^2 \tag{5.36b}$$

is a rectification current. It is due to rectifying of ac currents, produced by the time-dependent potentials $V_\gamma(t)$, onto the time-dependent conductance. The coexistence of rectified and generated currents was investigated theoretically [85, 58, 86, 87, 88] and experimentally [70, 71].

And, finally, the last term in Eq. (5.36a), an interference contribution,

$$\begin{aligned} I_{\alpha,0}^{(int)} = & \frac{e^2}{h} \int\limits_0^\infty dE \left(-\frac{\partial f_0}{\partial E} \right) \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\gamma=1}^{N_r} V_\gamma(t) \\ & \times \left(2\hbar\Omega_0 \mathrm{Re} \left[S_{\alpha\gamma}^* A_{\alpha\gamma} \right] + \frac{1}{2} P \left\{ S_{\alpha\gamma} S_{\alpha\gamma}^* \right\} \right) . \end{aligned} \tag{5.36c}$$

is due to a mutual influence (an interference) between the currents generated by the scatterer and the currents due to an ac bias. This part of a current shares features with both the generated current (it is proportional to Ω_0) and the rectified current (it is proportional to V_γ).

Physically the splitting of Eq. (5.36a) into three parts are justified by the fact that each part separately is subject to the conservation law, Eq. (4.11):

$$\sum_{\alpha=1}^{N_r} I_\alpha^{(x)} = 0\,, \quad x = pump,\, rect,\, int\,. \tag{5.37}$$

Let us analyze the conditions necessary for existence of each of the mentioned contributions. As we already showed, see, Eq. (4.8), the current $I_{\alpha,0}^{(pump)}$ is absent if the frozen scattering matrix is time-reversal invariant:

$$\hat{S}(t,E) = \hat{S}(-t,E)\,. \tag{5.38}$$

The rectified current, $I_{\alpha,0}^{(rect)}$, depends in fact on the potential difference, $\Delta V_{\gamma\alpha}(t) = V_\gamma(t) - V_\alpha(t)$, and it vanishes if the potentials of all the reservoirs are the same,

$$V_\gamma(t) = V(t)\,, \quad \forall\gamma\,. \tag{5.39}$$

To show it we use unitarity of the scattering matrix, see, Eq. (3.47), and find, $\sum_{\gamma=1}^{N_r} \left|S_{\alpha\gamma}(t,E)\right|^2 = 1$. Moreover, since the potentials are periodic we have, $\int_0^{\mathcal{T}} dt\, V_\alpha(t) = 0$. Using these two conditions we calculate:

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\gamma=1}^{N_r} V_\alpha(t) \left|S_{\alpha\gamma}(t,E)\right|^2 = 0\,.$$

And finally subtracting identity above from Eq. (5.36b), we find a required equation,

$$I_\alpha^{(rect)} = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\gamma=1}^{N_r} G_{\alpha\gamma}(t) \left\{V_\gamma(t) - V_\alpha(t)\right\}, \tag{5.40}$$

where the frozen conductance matrix elements,

$$G_{\alpha\gamma}(t) = G_0 \int_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E}\right) \left|S_{\alpha\gamma}(t,E)\right|^2. \tag{5.41}$$

are defined in the same way as in the stationary case, see, Eq. (1.54).

In contrast, the last contribution, $I_{\alpha,0}^{(int)}$, is present even if both Eqs. (5.38) and (5.39) are fulfilled, and neither pumped nor rectified currents do exist. To show it we first use Eq. (5.39) and rewrite Eq. (5.36c) as follows:

$$I_{\alpha,0}^{(int)} = \frac{e^2}{h}\int_0^{\infty} dE\left(-\frac{\partial f_0}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}V(t)\,P\left\{\hat{S}(t,E),\hat{S}^{\dagger}(t,E)\right\}_{\alpha\alpha}. \tag{5.42}$$

Here we have summed up over γ using the following identity,

$$4\hbar\Omega_0\sum_{\gamma=1}^{N_r}\mathrm{Re}\left\{S_{\alpha\gamma}^{*}A_{\alpha\gamma}\right\} = P\left\{\hat{S},\hat{S}^{\dagger}\right\}_{\alpha\alpha}. \tag{5.43}$$

To prove this equation we multiply a matrix equation (3.52) from the left by $\hat{S}$ and from the right by $\hat{S}^{\dagger}$ and take its diagonal element.

Under the conditions given in Eq. (5.38) the pumped current is zero, while the interference contribution, Eq. (5.42), can survive. The current $I_{\alpha,0}^{(int)}$, Eq. (5.42) is not zero if the potential $V(t)$ is shifted in phase with respect to varying in time parameters $p_i(t)$ of a scatterer. Therefore, to analyze the ability of the entire system, i.e., the scatterer plus reservoirs, to generate a dc current, $I_{\alpha,0} \neq 0$, it is necessary to take into account phases of all the time-dependent quantities, as parameters of a scatterer as possibly present time-dependent potentials at reservoirs.

Chapter 6

Noise of a dynamical scatterer

The current correlation function $P_{\alpha\beta}(t_1, t_2)$, is defined in Eqs. (2.30) and (2.39) in time and in frequency domains, respectively. Such defined correlator is called *a symmetrized correlator*. It satisfies the following symmetries,

$$P_{\alpha\beta}(t_1, t_2) = P_{\beta\alpha}(t_2, t_1)\,, \tag{6.1a}$$

$$P_{\alpha\beta}(\omega_1, \omega_2) = P_{\beta\alpha}(\omega_2, \omega_1)\,. \tag{6.1b}$$

which are a direct consequence of the fact that the currents, measured in leads α and β, enter symmetrically the correlator.

6.1 Noise spectral power

If currents are generated by the periodic dynamical scatterer, then the correlation function can be represented as follows (compare to Eq. (2.33) valid in the case of a stationary scatterer): [32]

$$P_{\alpha\beta}(\omega_1, \omega_2) = \sum_{l=-\infty}^{\infty} 2\pi\delta(\omega_1 + \omega_2 - l\Omega_0)\,\mathcal{P}_{\alpha\beta,l}(\omega_1, \omega_2)\,, \tag{6.2a}$$

where the spectral power $\mathcal{P}_{\alpha\beta,l}(\omega_1, \omega_2)$ is expressed in terms of the Floquet scattering matrix elements as follows:

$$\mathcal{P}_{\alpha\beta,l}(\omega_1, \omega_2) = \frac{e^2}{h}\int_0^{\infty} dE\,\Big\{\delta_{\alpha\beta}\,\delta_{l0}\,F_{\alpha\alpha}(E, E + \hbar\omega_1) \tag{6.2b}$$

$$- \sum_{n=-\infty}^{\infty} F_{\alpha\alpha}(E, E + \hbar\omega_1)\, S^{*}_{F,\beta\alpha}(E_n + \hbar\omega_1\,, E + \hbar\omega_1)\, S_{F,\beta\alpha}(E_{n+l}\,, E) -$$

$$- \sum_{n=-\infty}^{\infty} F_{\beta\beta}(E, E+\hbar\omega_2)\, S^{*}_{F,\alpha\beta}(E_n+\hbar\omega_2\,, E+\hbar\omega_2)\, S_{F,\alpha\beta}(E_{n+l}\,, E)$$

$$+ \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} F_{\gamma\delta}(E_{l+n}\,, E_m+\hbar\omega_1)\, S_{F,\beta\gamma}\left(E_{l+p}\,, E_{l+n}\right)$$

$$\times S^{*}_{F,\alpha\gamma}(E, E_{l+n})\, S_{F,\alpha\delta}(E+\hbar\omega_1, E_m+\hbar\omega_1)\, S^{*}_{F,\beta\delta}\left(E_p+\hbar\omega_1, E_m+\hbar\omega_1\right)\Big\}.$$

The quantity $F_{\alpha\beta}$ being a combination of the Fermi functions is defined in Eq. (2.46). To derive equation above we proceed in line with how we did in Sec. 2.2.2 but instead of Eq. (1.39) now we use Eq. (3.32) to relate the operators $\hat{b}_\alpha$ for scattered electrons to the operators $\hat{a}_\beta$ for incident electrons.

First of all we represent $P_{\alpha\beta}(\omega_1, \omega_2)$ as the sum of four quantities $P_{\alpha\beta}^{(i,j)}(\omega_1, \omega_2)$, $i, j = in, out$ accordingly to Eq. (2.43). For instance, $P_{\alpha\beta}^{(in,out)}(\omega_1, \omega_2)$ is a correlation function for a current of incident electrons in the lead α and a current of scattered electrons in the lead β. So, the spectral power reads:

$$\mathcal{P}_{\alpha\beta,l}(\omega_1, \omega_2) = \sum_{i,j=in,out} \mathcal{P}^{(i,j)}_{\alpha\beta,l}(\omega_1, \omega_2)\,, \tag{6.3}$$

Since incident electrons still did not interact with the scatterer then the part of a correlator related to incident currents is the same as in dynamical as in stationary cases. Therefore, for $P_{\alpha\beta}^{(in,in)}$ we can use Eq. (2.45) and write,

$$\mathcal{P}^{(in,in)}_{\alpha\beta,l}(\omega_1, \omega_2) = \delta_{\alpha\beta}\,\delta_{l0}\,\frac{e^2}{h}\int_0^\infty dE\, F_{\alpha\alpha}(E, E+\hbar\omega_1)\,. \tag{6.4}$$

Next we calculate $P_{\alpha\beta}^{(in,out)}$:

$$P^{(in,out)}_{\alpha\beta}(\omega_1, \omega_2) = e^2\int_0^\infty dE_1 \int_0^\infty dE_2 \Big\{$$

$$\left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{a}_\alpha(E_1+\hbar\omega_1)\right\rangle \left\langle \hat{b}_\beta^\dagger(E_2)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle$$
$$-\frac{1}{2}\left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{a}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle \tag{6.5}$$
$$-\frac{1}{2}\left\langle \hat{b}_\beta^\dagger(E_2)\,\hat{b}_\beta(E_2+\hbar\omega_2)\,\hat{a}_\alpha^\dagger(E_1)\,\hat{a}_\alpha(E_1+\hbar\omega_1)\right\rangle\Bigg\}.$$

Accordingly to the Wick's theorem (see, e.g. Ref. [19]) the mean of the product of four operators is the sum of products of two pair means. For instance:

$$\left\langle \hat{a}_\alpha^\dagger(E)\,\hat{a}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle =$$
$$\left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{a}_\alpha(E_1+\hbar\omega_1)\right\rangle \left\langle \hat{b}_\beta^\dagger(E_2)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle$$
$$+\left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle \left\langle \hat{a}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\right\rangle.$$

We can use the Wick's theorem since the operators $\hat{a}_\alpha$ correspond to particles in macroscopic reservoirs and the operators $\hat{b}_\beta$ are the linear combination of $\hat{a}_\alpha$. The first term on the right hand side (RHS) of an equation above does not contribute to the correlator, since it is compensated exactly by the corresponding product of currents [the first term on the RHS of Eq. (6.5)]. Therefore, only those pair means are relevant which comprise particle operators from both current operators $\hat{I}_\alpha^{(in)}$ and $\hat{I}_\beta^{(out)}$ simultaneously. To calculate such pair means we use Eq. (3.32). In particular we have:

$$\left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle = \sum_{\gamma=1}^{N_r}\sum_{m=-\infty}^{\infty} S_{F,\beta\gamma}(E_2+\hbar\omega_2, E_2+\hbar[\omega_2+m\Omega_0])$$
$$\times\left\langle \hat{a}_\alpha^\dagger(E_1)\,\hat{a}_\gamma(E_2+\hbar[\omega_2+m\Omega_0])\right\rangle$$

$$= \sum_{\gamma=1}^{N_r} \sum_{m=-\infty}^{\infty} S_{F,\beta\gamma}\left(E_2 + \hbar\omega_2, E_2 + \hbar\left[\omega_2 + m\Omega_0\right]\right)$$
$$\times \delta_{\alpha\gamma}\delta\left(E_1 - E_2 - \hbar\left[\omega_2 + m\Omega_0\right]\right) f_\alpha\left(E_1\right)$$

$$= \sum_{m=-\infty}^{\infty} S_{F,\beta\alpha}\left(E_2 + \hbar\omega_2, E_2 + \hbar\left[\omega_2 + m\Omega_0\right]\right)$$
$$\times \delta\left(E_1 - E_2 - \hbar\left[\omega_2 + m\Omega_0\right]\right) f_\alpha\left(E_1\right) .$$

By analogy we calculate all other pair means appeared in Eq. (6.5):

$$\left\langle \hat{a}_\alpha\left(E_1 + \hbar\omega_1\right) \hat{b}_\beta^\dagger\left(E_2\right) \right\rangle = \sum_{n=-\infty}^{\infty} S_{F,\beta\alpha}^*\left(E_2, E_2 + n\hbar\Omega_0\right)$$
$$\times \delta\left(E_1 + \hbar\omega_1 - E_2 - n\hbar\Omega_0\right) \left[1 - f_\alpha\left(E_1 + \hbar\omega_1\right)\right] ,$$

$$\left\langle \hat{b}_\beta^\dagger\left(E_2\right) \hat{a}_\alpha\left(E_1 + \hbar\omega_1\right) \right\rangle = \sum_{n=-\infty}^{\infty} S_{F,\beta\alpha}^*\left(E_2, E_2 + n\hbar\Omega_0\right)$$
$$\times \delta\left(E_1 + \hbar\omega_1 - E_2 - n\hbar\Omega_0\right) f_\alpha\left(E_1 + \hbar\omega_1\right) ,$$

$$\left\langle \hat{b}_\beta\left(E_2 + \hbar\omega_2\right) \hat{a}_\alpha^\dagger\left(E_1\right) \right\rangle = \sum_{m=-\infty}^{\infty} S_{F,\beta\alpha}\left(E_2 + \hbar\omega_2, E_2 + \hbar\left[\omega_2 + m\Omega_0\right]\right)$$
$$\times \delta\left(E_1 - E_2 - \hbar\left[\omega_2 + m\Omega_0\right]\right) \left[1 - f_\alpha\left(E_1\right)\right] .$$

Substituting these pair means into Eq. (6.5) we arrive at the sum of two terms. Then using the Dirac delta-function to integrate over energy, say over E_2, we get the following for each of these terms:

$$\int_0^\infty dE_2 \frac{1}{2} S_{F,\beta\alpha}\left(E_2 + \hbar\omega_2, E_{2,m} + \hbar\omega_2\right) \delta\left(E_1 - E_{2,m} - \hbar\omega_2\right)$$

$$\times f_\alpha(E_1)\, S^*_{F,\beta\alpha}\left(E_2, E_{2,n}\right) \delta\left(E_1 + \hbar\omega_1 - E_{2,n}\right) \left[1 - f_\alpha(E_1 + \hbar\omega_1)\right]$$

$$= \frac{1}{2\hbar} \delta\left(\omega_1 + \omega_2 + (m - n)\Omega_0\right) f_\alpha(E_1) \left[1 - f_\alpha(E_1 + \hbar\omega_1)\right]$$

$$\times S^*_{F,\beta\alpha}\left(E_{1,-n} + \hbar\omega_1, E_1 + \hbar\omega_1\right) S_{F,\beta\alpha}\left(E_{1,-m}, E_1\right),$$

$$\int_0^\infty dE_2 \frac{1}{2} S^*_{F,\beta\alpha}\left(E_2, E_{2,n}\right) \delta\left(E_1 + \hbar\omega_1 - E_{2,n}\right) f_\alpha(E_1 + \hbar\omega_1)$$

$$\times S_{F,\beta\alpha}\left(E_2 + \hbar\omega_2, E_{2,m} + \hbar\omega_2\right) \delta\left(E_1 - E_{2,m} - \hbar\omega_2\right) \left[1 - f_\alpha(E_1)\right]$$

$$= \frac{1}{2\hbar} \delta\left(\omega_1 + \omega_2 + (m - n)\Omega_0\right) f_\alpha(E_1 + \hbar\omega_1) \left[1 - f_\alpha(E_1)\right]$$

$$\times S^*_{F,\beta\alpha}\left(E_{1,-n} + \hbar\omega_1, E_1 + \hbar\omega_1\right) S_{F,\beta\alpha}\left(E_{1,-m}, E_1\right),$$

where $E_{i,k} = E_i + k\hbar\Omega_0$, $i = 1, 2$. Using equations above in Eq. (6.5), introducing $l = n - m$ instead of m, and replace $n \to -n$ and $E_1 \to E$, we finally find:

$$P^{(in,out)}_{\alpha\beta}(\omega_1, \omega_2) = \sum_{l=-\infty}^{\infty} 2\pi\delta\left(\omega_1 + \omega_2 - l\Omega_0\right) \mathcal{P}^{(in,out)}_{\alpha\beta,l}(\omega_1, \omega_2), \tag{6.6a}$$

with

$$\mathcal{P}^{(in,out)}_{\alpha\beta}(\omega_1, \omega_2) = -\frac{e^2}{h} \int_0^\infty dE \sum_{n=-\infty}^{\infty} F_{\alpha\alpha}(E, E + \hbar\omega_1) \times S^*_{F,\beta\alpha}(E_n + \hbar\omega_1, E + \hbar\omega_1)\, S_{F,\beta\alpha}(E_{n+l}, E). \tag{6.6b}$$

In the same way we find:

$$P_{\alpha\beta}^{(out,in)}(\omega_1,\omega_2) = e^2 \int\limits_0^\infty dE_1 \int\limits_0^\infty dE_2 \Big\{ \tag{6.7}$$

$$\left\langle \hat{b}_\alpha^\dagger(E_1)\,\hat{b}_\alpha(E_1+\hbar\omega_1)\right\rangle \left\langle \hat{a}_\beta^\dagger(E_2)\,\hat{a}_\beta(E_2+\hbar\omega_2)\right\rangle$$

$$-\frac{1}{2}\left\langle \hat{b}_\alpha^\dagger(E_1)\,\hat{b}_\alpha(E_1+\hbar\omega_1)\,\hat{a}_\beta^\dagger(E_2)\,\hat{a}_\beta(E_2+\hbar\omega_2)\right\rangle$$

$$-\frac{1}{2}\left\langle \hat{a}_\beta^\dagger(E_2)\,\hat{a}_\beta(E_2+\hbar\omega_2)\,\hat{b}_\alpha^\dagger(E_1)\,\hat{b}_\alpha(E_1+\hbar\omega_1)\right\rangle\Big\}.$$

Comparing it to Eq. (6.5) we see, that $P_{\alpha\beta}^{(out,in)}(\omega_1,\omega_2)$ can be calculated from Eq. (6.6) after the following replacements: $\alpha \leftrightarrow \beta$, $E_1 \leftrightarrow E_2$ and $\omega_1 \leftrightarrow \omega_2$. As a result we get for the spectral power (replace $E_2 \to E$):

$$\mathcal{P}_{\alpha\beta}^{(out,in)}(\omega_1,\omega_2) = -\frac{e^2}{h}\int\limits_0^\infty dE \sum_{n=-\infty}^{\infty} F_{\beta\beta}(E,E+\hbar\omega_2)$$
$$\times S_{F,\alpha\beta}^{*}(E_n+\hbar\omega_2, E+\hbar\omega_2)\, S_{F,\alpha\beta}(E_{n+l},E)\,. \tag{6.8}$$

Then we calculate the last contribution:

$$P_{\alpha\beta}^{(out,out)}(\omega_1,\omega_2) = \frac{e^2}{2}\int\limits_0^\infty dE_1 \int\limits_0^\infty dE_2 \Big\{$$

$$\left\langle \hat{b}_\alpha^\dagger(E_1)\,\hat{b}_\beta(E_2+\hbar\omega_2)\right\rangle \left\langle \hat{b}_\alpha(E_1+\hbar\omega_1)\,\hat{b}_\beta^\dagger(E_2)\right\rangle \tag{6.9}$$

$$+\left\langle \hat{b}_\beta^\dagger(E_2)\,\hat{b}_\alpha(E_1+\hbar\omega_1)\right\rangle \left\langle \hat{b}_\beta(E_2+\hbar\omega_2)\,\hat{b}_\alpha^\dagger(E_1)\right\rangle\Big\},$$

where we already expressed the mean for four operators in terms of pair means. The first of them is:

$$\left\langle \hat{b}^{\dagger}_{\alpha}(E_1)\,\hat{b}_{\beta}(E_2+\hbar\omega_2)\right\rangle = \sum_{\gamma=1}^{N_r}\sum_{r=-\infty}^{\infty}\sum_{\delta=1}^{N_r}\sum_{s=-\infty}^{\infty}\left\langle \hat{a}^{\dagger}_{\gamma}\left(E_{1,r}\right)\hat{a}_{\delta}\left(E_{2,s}+\hbar\omega_2\right)\right\rangle$$

$$\times S^{*}_{F,\alpha\gamma}\left(E_1,E_{1,r}\right)\,S_{F,\beta\delta}\left(E_2+\hbar\omega_2,E_{2,s}+\hbar\omega_2\right) = \sum_{\gamma=1}^{N_r}\sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} f_{\gamma}\left(E_{1,r}\right)$$

$$\times\delta\left(E_{1,r}-E_{2,s}-\hbar\omega_2\right)\,S^{*}_{F,\alpha\gamma}\left(E_1,E_{1,r}\right)\,S_{F,\beta\gamma}\left(E_2+\hbar\omega_2,E_{2,s}+\hbar\omega_2\right),$$

and, correspondingly, the second one equals to the following,

$$\left\langle \hat{b}_{\alpha}(E_1+\hbar\omega_1)\,\hat{b}^{\dagger}_{\beta}(E_2)\right\rangle = \sum_{\delta=1}^{N_r}\sum_{m=-\infty}^{\infty}\sum_{\gamma=1}^{N_r}\sum_{q=-\infty}^{\infty}\left\langle \hat{a}_{\delta}\left(E_{1,m}+\hbar\omega_1\right)\hat{a}^{\dagger}_{\gamma}\left(E_{2,q}\right)\right\rangle$$

$$\times S_{F,\alpha\delta}\left(E_1+\hbar\omega_1,E_{1,m}+\hbar\omega_1\right)\,S^{*}_{F,\beta\gamma}\left(E_2,E_{2,q}\right)$$

$$= \sum_{\delta=1}^{N_r}\sum_{m=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\left[1-f_{\delta}\left(E_{1,m}+\hbar\omega_1\right)\right]\delta\left(E_{1,m}+\hbar\omega_1-E_{2,q}\right)$$

$$\times S_{F,\alpha\delta}\left(E_1+\hbar\omega_1,E_{1,m}+\hbar\omega_1\right)\,S^{*}_{F,\beta\delta}\left(E_2,E_{2,q}\right).$$

Integrating the product of these means over E_2 we get:

$$\int_0^{\infty} dE_2\,\delta\left(E_{1,r}-E_{2,s}-\hbar\omega_2\right)\,\delta\left(E_{1,m}+\hbar\omega_1-E_{2,q}\right)\,f_{\gamma}\left(E_{1,r}\right)$$

$$\times\left[1-f_{\delta}\left(E_{1,m}+\hbar\omega\right)\right]\,S^{*}_{F,\alpha\delta}\left(E_1,E_{1,r}\right)\,S_{F,\beta\gamma}\left(E_2+\hbar\omega_2,E_{2,s}+\hbar\omega_2\right)$$

$$\times S_{F,\alpha\delta}\left(E_1+\hbar\omega_1,E_{1,m}+\hbar\omega_1\right)\,S^{*}_{F,\beta\delta}\left(E_2,E_{2,q}\right)$$

$$= \frac{1}{\hbar}\delta\left(\omega_1 + \omega_2 - \left[r + q - s - m\right]\Omega_0\right)\; f_\gamma\left(E_{1,r}\right)\left[1 - f_\delta\left(E_{1,m} + \hbar\omega_1\right)\right]$$

$$\times S^*_{F,\alpha\gamma}\left(E_1, E_{1,r}\right)\; S_{F,\beta\gamma}\left(E_{1,r-s}, E_{1,r}\right)$$

$$\times S_{F,\alpha\delta}\left(E_1 + \hbar\omega_1, E_{1,m} + \hbar\omega_1\right)\; S^*_{F,\beta\delta}\left(E_{1,m-q} + \hbar\omega_1, E_{1,m} + \hbar\omega_1\right)$$

$$= \frac{1}{\hbar}\delta\left(\omega_1 + \omega_2 - l\Omega_0\right)\; f_\gamma\left(E_{1,l+n}\right)\;\left[1 - f_\delta\left(E_{1,m} + \hbar\omega_1\right)\right]$$

$$\times S^*_{F,\alpha\gamma}\left(E_1, E_{1,l+n}\right)\; S_{F,\beta\gamma}\left(E_{1,l+p}, E_{1,l+n}\right)$$

$$\times S_{F,\alpha\delta}\left(E_1 + \hbar\omega_1, E_{1,m} + \hbar\omega_1\right)\; S^*_{F,\beta\delta}\left(E_{1,p} + \hbar\omega_1, E_{1,m} + \hbar\omega_1\right),$$

where at the end we introduced new indices: $p = m - q$ (instead of q), $n = s + m - q$ (instead of s), and $l = r - s + q - m$ (instead of r).

Comparing the first and the second terms on the RHS of Eq. (6.9) one can see that the later one results in the same expression as given above but with $f_\gamma\left(E_{l+n}\right)\left[1 - f_\delta\left(E_m + \hbar\omega\right)\right]$ being replaced by $f_\delta\left(E_m + \hbar\omega\right)\left[1 - f_\gamma\left(E_{l+n}\right)\right]$. Therefore, finally the equation (6.9) results in the following:

$$P^{(out,out)}_{\alpha\beta}\left(\omega_1, \omega_2\right) = \sum_{l=-\infty}^{\infty} 2\pi\delta\left(\omega_1 + \omega_2 - l\Omega_0\right)\mathcal{P}^{(out,out)}_{\alpha\beta,l}\left(\omega_1, \omega_2\right), \tag{6.10a}$$

where

$$\mathcal{P}^{(out,out)}_{\alpha\beta}\left(\omega_1, \omega_2\right) = \frac{e^2}{h}\int_0^\infty dE \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}$$

$$\times F_{\gamma\delta}\left(E_{l+n}, E_m + \hbar\omega_1\right) S^*_{F,\alpha\gamma}\left(E, E_{l+n}\right) S_{F,\beta\gamma}\left(E_{l+p}, E_{l+n}\right) \tag{6.10b}$$

$$\times S_{F,\alpha\delta}\left(E + \hbar\omega_1, E_m + \hbar\omega_1\right) S^*_{F,\beta\delta}\left(E_p + \hbar\omega_1, E_m + \hbar\omega_1\right).$$

Summing up Eqs. (6.4), (6.6b), (6.8), and (6.10b) we get announced result given in Eq. (6.2b).

6.2 Zero frequency noise spectral power

The quantity $\mathcal{P}_{\alpha\beta}(0) \equiv \mathcal{P}_{\alpha\beta,0}(0,0)$, referred to as the symmetrized *noise*, characterizes a mean square of current fluctuations (at $\alpha = \beta$) or a symmetrized current cross-correlator (at $\alpha \neq \beta$), averaged over long time period. It can be written as follows:

$$\mathcal{P}_{\alpha\beta}(0) = \frac{1}{2}\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \int_{-\infty}^{\infty} d\tau \left\langle \Delta\hat{I}_\alpha(t)\,\Delta\hat{I}_\beta(t+\tau) + \Delta\hat{I}_\beta(t+\tau)\,\Delta\hat{I}_\alpha(t)\right\rangle. \tag{6.11}$$

The noise expression in terms of the Floquet scattering matrix elements is given in Eq. (6.2b) at $l = 0$ and $\omega_1 = \omega_2 = 0$.

From Eq. (6.1b) it follows that the noise value does not change under the lead indices interchange,

$$\mathcal{P}_{\alpha\beta}(0) = \mathcal{P}_{\beta\alpha}(0)\,. \tag{6.12}$$

This is another reason why this quantity is called as a symmetrized noise.

Like its stationary counterpart the quantity $\mathcal{P}_{\alpha\beta}(0)$ can be represented as the sum of a thermal noise and a shot noise, see, Eq. (2.60). The thermal noise, $\mathcal{P}_{\alpha\beta}^{(th)}$, is due to fluctuations of quantum state occupations of electrons incident from the reservoirs with non-zero temperature. While the shot noise, $\mathcal{P}_{\alpha\beta}^{(sh)}$, is due to fluctuations of quantum state occupations of scattered electrons: If an electrons is scattered, say into the lead α, then in this contact the instant current is larger than the average current, while in other contacts, $\beta \neq \alpha$, the instant current is zero, i.e., it is smaller than the corresponding average current.

Let us calculate a noise when all the reservoirs have the same chemical potentials and temperatures,

$$\mu_\alpha = \mu\,, \quad T_\alpha = T\,. \tag{6.13}$$

Hence the distribution functions for electrons in reservoirs are the same,

$$f_\alpha(E) = f_0(E)\,. \tag{6.14}$$

Then from Eq. (6.2b) at $l = 0$, $\omega_1 = \omega_2 = 0$ it follows (see, also, Sec. 2.2.4) : $\mathcal{P}_{\alpha\beta}(0) = \mathcal{P}_{\alpha\beta}^{(th)} + \mathcal{P}_{\alpha\beta}^{(sh)}$, [89] where

$$\begin{aligned}\mathcal{P}_{\alpha\beta}^{(th)} &= \frac{e^2}{h}\int_0^\infty dE\, f_0(E)\left[1 - f_0(E)\right]\left\{\delta_{\alpha\beta}\left(1 + \sum_{n=-\infty}^{\infty}\sum_{\gamma=1}^{N_r}\left|S_{F,\alpha\gamma}(E_n, E)\right|^2\right)\right. \\ &\left. - \sum_{n=-\infty}^{\infty}\left(\left|S_{F,\alpha\beta}(E_n, E)\right|^2 + \left|S_{F,\beta\alpha}(E_n, E)\right|^2\right)\right\},\end{aligned} \tag{6.15}$$

$$\begin{aligned}\mathcal{P}_{\alpha\beta}^{(sh)} &= \frac{e^2}{h}\int_0^\infty dE \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\frac{\left[f_0(E_n) - f_0(E_m)\right]^2}{2} \\ &\times S_{F,\alpha\gamma}^{*}(E, E_n)\, S_{F,\alpha\delta}(E, E_m)\, S_{F,\beta\delta}^{*}\left(E_p, E_m\right) S_{F,\beta\gamma}\left(E_p, E_n\right).\end{aligned} \tag{6.16}$$

Clear that the thermal noise vanishes at zero temperature, since in that case it is $f_0(E)\left[1 - f_0(E)\right] = \theta(\mu - E)\,\theta(E - \mu) \equiv 0$. In contrast, the shot noise does exist at arbitrary temperature. However it vanishes in the equilibrium system, i.e., if the scatterer is stationary. In that case it is $\hat{S}_F(E_p, E) = \delta_{p0}\,\hat{S}(E)$, hence there are only terms with $n = 0$, $m = 0$, and $p = 0$ in Eq. (6.16). For these terms the difference of the Fermi functions is zero.

As we showed in Sec. (2.2.4.1) the unitarity of scattering results in conservation laws, Eq. (2.63), for the stationary noise. The noise due to a dynamical scatterer is also subject to the conservation laws. Moreover the thermal noise and the shot noise satisfy them separately:

$$\sum_{\beta=1}^{N_r}\mathcal{P}_{\alpha\beta}^{(th)} = 0\,, \qquad \sum_{\alpha=1}^{N_r}\mathcal{P}_{\alpha\beta}^{(th)} = 0\,, \tag{6.17a}$$

$$\sum_{\beta=1}^{N_r}\mathcal{P}_{\alpha\beta}^{(sh)} = 0\,, \qquad \sum_{\alpha=1}^{N_r}\mathcal{P}_{\alpha\beta}^{(sh)} = 0\,, \tag{6.17b}$$

that follows directly from Eqs. (6.15) and (6.16) if one uses in addition the unitarity condition, Eq. (3.28). Note to prove the second equality in Eq. (6.17b) it is necessary to make the following replacement in Eq. (6.16): $E \to E - p\hbar\Omega_0$, $n \to n - p$, and $m \to m - p$.

Let us analyze the sign of a zero-frequency noise power. The cross-correlator $\mathcal{P}_{\alpha\neq\beta}$ is negative in stationary case, see, Eq. (2.64b). It is negative in the dynamical case too:

$$\mathcal{P}_{\alpha\neq\beta}^{(th)} \le 0\,, \qquad \mathcal{P}_{\alpha\neq\beta}^{(sh)} \le 0\,. \tag{6.18}$$

For the thermal noise it follows directly from Eq. (6.15):

$$\begin{aligned}\mathcal{P}_{\alpha\neq\beta}^{(th)} &= -\frac{e^2}{h}\int_0^\infty dE\, f_0(E)\left[1 - f_0(E)\right] \\ &\times \sum_{n=-\infty}^{\infty}\left(\left|S_{F,\alpha\beta}(E_n, E)\right|^2 + \left|S_{F,\beta\alpha}(E_n, E)\right|^2\right) \le 0\,.\end{aligned}$$

To check this rule for the shot noise, let us rewrite Eq. (6.16) for $\alpha \neq \beta$ as follows:

$$\begin{aligned}\mathcal{P}_{\alpha\neq\beta}^{(sh)} &= -\frac{e^2}{h}\int_0^\infty dE \sum_{p=-\infty}^{\infty} \\ &\left|\sum_{n=-\infty}^{\infty}\sum_{\gamma=1}^{N_r} f_0(E_n)\, S_{F,\alpha\gamma}^{*}(E\,, E_n)\, S_{F,\beta\gamma}\left(E_p\,, E_n\right)\right|^2 \le 0\,.\end{aligned}$$

Here we took into account that the terms with squared Fermi functions vanish for $\alpha \neq \beta$ in Eq. (6.16). For instance, in the terms with $f_0^2(E_n)$ we can sum up over m and δ. Then using Eq. (3.28b) we find ($\alpha \neq \beta$):

$$\sum_{m=-\infty}^{\infty} \sum_{\delta=1}^{N_r} S_{F,\alpha\gamma}(E, E_m) S_{\beta\delta}^{*}(E_p, E_m) = \delta_{\alpha\beta} \delta_{p0} = 0 .$$

In the same way one can prove that the term with $f_0^2(E_m)$ is also zero.

The auto-correlator $\mathcal{P}_{\alpha\alpha}$ is a mean square of current fluctuations in the lead α, hence it is non-negative. From Eqs. (6.17) and (6.18) it follows,

$$\mathcal{P}_{\alpha\alpha}^{(th)} \geq 0 , \qquad \mathcal{P}_{\alpha\alpha}^{(sh)} \geq 0 . \tag{6.19}$$

The fact, that the thermal noise and the shot noise separately satisfy the sum rule, Eq. (6.17), and the sign rule, Eqs. (6.18) and (6.19), justifies splitting of a noise into these two parts.

Besides, the thermal noise and the shot noise depend differently on both the temperature T and the driving frequency Ω_0. Let us show it in the regime when the parameters of a scatterer vary slowly, $\Omega_0 \to 0$.

6.3 Noise in the adiabatic regime

The Floquet scattering matrix elements up to the linear in Ω_0 terms are given in Eq. (3.50). Remind that the adiabatic expansion implies that the frozen scattering matrix $\hat{S}$ changes only a little on the energy scale of order $\hbar\Omega_0$, see, Eq. (3.49).

6.3.1 Thermal noise

Substitute Eq. (3.50) into Eq. (6.15) and calculate the thermal noise up to terms linear in Ω_0: [89]

$$\mathcal{P}_{\alpha\beta}^{(th)} = \mathcal{P}_{\alpha\beta}^{(th,0)} + \mathcal{P}_{\alpha\beta}^{(th,\Omega_0)} , \tag{6.20a}$$

where

$$\mathcal{P}_{\alpha\beta}^{(th,0)} = k_B T \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \times \frac{e^2}{h} \left(2\delta_{\alpha\beta} - \left|S_{\alpha\beta}(t,E)\right|^2 - \left|S_{\beta\alpha}(t,E)\right|^2\right), \tag{6.20b}$$

$$\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)} = k_B T \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \times e \left(\delta_{\alpha\beta} \frac{dI_\alpha(t,E)}{dE} - \frac{dI_{\alpha\beta}(t,E)}{dE} - \frac{dI_{\beta\alpha}(t,E)}{dE}\right). \tag{6.20c}$$

As expected, the thermal noise is proportional to the temperature. Let us introduce an averaged over time frozen conductance matrix, see, (5.41),

$$\bar{\hat{G}} = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \hat{G}(t), \tag{6.21}$$

Then the quantity $\mathcal{P}_{\alpha\beta}^{(th,0)}$ depends on its elements

$$\mathcal{P}_{\alpha\beta}^{(th,0)} = k_B T \left(2\delta_{\alpha\beta} G_0 - \bar{G}_{\alpha\beta} - \bar{G}_{\beta\alpha}\right), \tag{6.22}$$

in the same way, as the equilibrium noise, the Nyquist-Johnson noise, Eq. (2.61), depends on elements of the conductance matrix, $\hat{G}$, Eq. (1.54), in the stationary case. Therefore, $\mathcal{P}_{\alpha\beta}^{(th,0)}$ can be called as *a quasi-equilibrium noise*. Comparing Eqs. (6.20b) and (2.61) it is necessary to use the identity Eq. (2.66) and the fact that Eq. (6.20b) was derived under conditions given in Eq. (6.14).

The second part of the thermal noise, Eq. (6.20c), indicates that the system is in fact non-equilibrium. The part $\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)}$ can be called as *a non-equilibrium thermal noise*, since, on one hand, it is proportional to the temperature (hence thermal), and, on the other hand, it depends on currents generated by the dynamical scatterer (hence non-equilibrium). Since the spectral current powers $dI_{\alpha\beta}(t,E)/dE$, Eq. (5.9), and $dI_{\alpha}(t,E)/dE$, Eq. (4.20), both are linear in pumping frequency Ω_0, then it is $\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)} \sim \Omega_0$.

6.3.2 Low-temperature shot noise

If the temperature is low enough,

$$k_B T \ll \hbar\Omega_0 \,, \tag{6.23}$$

then the thermal noise can be ignored. In this regime the main source of a noise is a dynamical scatterer generating a photon-assisted shot noise. Other source of the shot noise, the bias, is absent because of Eq. (6.13). The photon-assisted shot noise is a non-equilibrium noise. That follows (in the same way as in the stationary case with bias) from the fact that the noise is due to those of scattered electrons for which the distribution function $f_{\alpha}^{(out)}(E)$ is non-equilibrium, i.e., less than unity. As it follows from Eq. (4.4), see also, Eq. (4.5), the distribution function $f_{\alpha}^{(out)}(E)$ is non-equilibrium for energies different from the Fermi energy by the amount of order $\hbar\Omega_0$.

Let us calculate $\mathcal{P}_{\alpha\beta}^{(sh)}$, Eq. (6.16), in the lowest order in Ω_0. To this end it is enough to use the Floquet scattering matrix elements in zeroth order in Ω_0. For instance, with required accuracy we find from Eq. (3.50):

$$\hat{S}_F\left(E_m, E_p\right) = \hat{S}_{m-p}(E) + \mathcal{O}(\Omega_0) \,. \tag{6.24}$$

Remind in the lowest adiabatic approximation the frozen scattering matrix $\hat{S}$ should be treated as energy-independent over the scale of order $\hbar\Omega_0$. Therefore, under conditions given in Eq. (6.23) we can integrate over energy in Eq. (6.16) keeping the scattering matrix elements constant (for definiteness we will calculate them at $E = \mu$). Then the remaining integral over energy becomes trivial:

$$\int_0^\infty dE\,\{f_0(E_n) - f_0(E_m)\}^2 = \begin{cases} \hbar\Omega_0\,(m-n)\,, & m > n\,, \\ \hbar\Omega_0\,(n-m)\,, & m < n\,. \end{cases} \tag{6.25}$$

Using Eqs. (6.24) and (6.25) in Eq. (6.16) we find,

$$\begin{aligned} \mathcal{P}^{(sh)}_{\alpha\beta} &= \frac{e^2\Omega_0}{4\pi} \sum_{\gamma,\delta=1}^{N_r} \sum_{n,m,p=-\infty}^{\infty} \\ &\times |m-n|\, S^*_{\alpha\gamma,-n}(\mu) S_{\alpha\delta,-m}(\mu) S^*_{\beta\delta,p-m}(\mu) S_{\beta\gamma,p-n}(\mu)\,. \end{aligned} \tag{6.26}$$

So the photon-induced shot noise is linear in pumping frequency Ω_0 (see, also Ref. [66]). To simplify above equation we proceed as follows. For each fixed n we consider the sum over m and split it onto two parts, the sum over $m < n$ and the sum over $m > n$. Then we introduce a new index $q = m - n$ instead of m. After that we find for any quantity $X_{n,m}$ dependent on indices n and m the following:

$$\begin{aligned} \sum_{m=-\infty}^{\infty} |m-n|\, X_{m,n} &= \sum_{m=-\infty}^{n-1} (n-m)\, X_{m,n} + \sum_{m=n+1}^{\infty} (n-m)\, X_{m,n} \\ = \sum_{q=-\infty}^{-1} (-q)\, X_{q+n,n} &+ \sum_{q=1}^{\infty} q X_{q+n,n} = \sum_{q=1}^{\infty} q\left(X_{-q+n,n} + X_{q+n,n}\right). \end{aligned}$$

The term with $m = n$ is zero due to the factor $m - n = n - n \equiv 0$. Then the equation (6.26) is transformed into the following form,

$$\begin{aligned} \mathcal{P}^{(sh)}_{\alpha\beta} &= \frac{e^2\Omega_0}{4\pi} \sum_{q=1}^{\infty} \sum_{\gamma=1}^{N_r} \sum_{\delta=1}^{N_r} q \Big[\left\{S^*_{\alpha\gamma}(\mu)\, S_{\alpha\delta}(\mu)\right\}_{-q} \left\{S_{\beta\gamma}(\mu)\, S^*_{\beta\delta}(\mu)\right\}_q \\ &+ \left\{S^*_{\alpha\gamma}(\mu)\, S_{\alpha\delta}(\mu)\right\}_q \left\{S_{\beta\gamma}(\mu)\, S^*_{\beta\delta}(\mu)\right\}_{-q} \Big]. \end{aligned} \tag{6.27}$$

Going over from Eq. (6.26) to Eq. (6.27) we have summed up over n and p using the following identity valid for the Fourier coefficients of any periodic functions $A(t)$ and $B(t)$:

$$\sum_{n=-\infty}^{\infty} A_n \left(B_{n+q}\right)^* = \left(AB^*\right)_{-q} , \qquad \sum_{n=-\infty}^{\infty} A_{n+q} \left(B_n\right)^* = \left(AB^*\right)_q . \tag{6.28}$$

Easy to check that Eq. (6.27) satisfies the symmetry given in Eq. (6.12), $\mathcal{P}_{\alpha\beta}^{(sh)} = \mathcal{P}_{\beta\alpha}^{(sh)}$. To show it we need to use $\gamma \leftrightarrow \delta$ in an expression for $\mathcal{P}_{\beta\alpha}^{(sh)}$.

6.3.3 High-temperature shot noise

At higher temperatures,

$$k_B T \gg \hbar\Omega_0 , \tag{6.29}$$

the thermal noise dominates. In this regime the shot noise, Eq. (6.16), is only a small fraction of the total noise. However the thermal noise and the shot noise depend differently on both the pumping frequency Ω_0 and the temperature, that allows in principle to distinguish them.

With Eq. (6.29) we can expand the difference of Fermi functions in Eq. (6.16) in powers of Ω_0. Up to the first non-vanishing term we get:

$$f_0\left(E_n\right) - f_0\left(E_m\right) = \hbar\Omega_0 \frac{\partial f_0\left(E\right)}{\partial E} (n - m) .$$

Substituting this expansion and the adiabatic approximation for the Floquet scattering matrix, Eq. (6.24), into Eq. (6.16) we find the high-temperature shot noise ($k_B T \gg \hbar\Omega_0$) :

$$\mathcal{P}_{\alpha\beta}^{(sh)} = \frac{e^2}{4\pi}\hbar\Omega_0^2 \int_0^\infty dE \left(\frac{\partial f_0}{\partial E}\right)^2 \sum_{q=-\infty}^{\infty} q^2 \times \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r} \left\{S_{\alpha\gamma}^*(E)\, S_{\alpha\delta}(E)\right\}_q \left\{S_{\beta\gamma}(E)\, S_{\beta\delta}^*(E)\right\}_{-q} . \tag{6.30}$$

In this equation we keep the integration over energy since it is over the interval of order $k_B T \gg \hbar\Omega_0$ near the Fermi energy μ. While the use of an adiabatic approximation, Eq. (6.24), does not put any restrictions onto the energy dependence of the frozen scattering matrix, $\hat{S}$, over such an energy interval. The quadratic dependence of the high-temperature shot noise on the pumping frequency Ω_0 was shown in Ref. [90].

6.3.4 Shot noise within a wide temperature range

One can relax restrictions put by Eqs. (6.23) and (6.29) and calculate analytically a shot noise at arbitrary ratio of the temperature and the energy quantum $\hbar\Omega_0$ dictated by pumping. This is possible if the scattering matrix can be treated as an energy independent over the relevant energy interval:

$$\hbar\Omega_0,\ k_B T \ll \delta E\,. \tag{6.31}$$

Remind δE is an energy interval over which the scattering matrix changes significantly.

So, with Eq. (6.31) while calculating the shot noise, Eq. (6.16), in the adiabatic regime [when we use Eq. (6.24)] we can calculate the scattering matrix elements at $E = \mu$ only. Then the corresponding integral over energy is calculated analytically,

$$\int_0^\infty dE\, \{f_0(E_n) - f_0(E_m)\}^2 = (m-n)\,\hbar\Omega_0 \coth\left(\frac{(m-n)\,\hbar\Omega_0}{2k_B T}\right) - 2k_B T\,,$$

and finally we arrive at the following,

$$\mathcal{P}_{\alpha\beta}^{(sh)} = \frac{e^2}{h}\sum_{q=-\infty}^{\infty} F\left(q\hbar\Omega_0\,, k_BT\right) \times \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\left\{S_{\alpha\gamma}^{*}(\mu)\,S_{\alpha\delta}(\mu)\right\}_q\left\{S_{\beta\gamma}(\mu)\,S_{\beta\delta}^{*}(\mu)\right\}_{-q}, \tag{6.32a}$$

where

$$F\left(q\hbar\Omega_0\,, k_BT\right) = \frac{q\hbar\Omega_0}{2}\coth\left(\frac{q\hbar\Omega_0}{2k_BT}\right) - k_BT = \begin{cases} \frac{|q|\hbar\Omega_0}{2}, & k_BT \ll \hbar\Omega_0\,, \\ \frac{(q\hbar\Omega_0)^2}{12k_BT} & k_BT \gg \hbar\Omega_0\,. \end{cases} \tag{6.32b}$$

The obtained equation (6.32) reproduces both the equation (6.27) for the low-temperature shot noise, which is linear in Ω_0 and temperature-independent, and the equation (6.30) for the high-temperature shot noise, which is quadratic in pumping frequency and, under conditions of Eq. (6.31), inversally proportional to the temperature.

6.3.5 Noise as a function of a pumping frequency Ω_0

At zero temperature the dynamical scatterer generates only a shot noise, which is linear in Ω_0. With increasing temperature the thermal noise arises. It also depends on Ω_0. Therefore, the part $\delta\mathcal{P}_{\alpha\beta}^{(\Omega_0)}$ of the total high-temperature noise dependent on Ω_0 can be written as the sum of two contributions,

$$\delta\mathcal{P}_{\alpha\beta}^{(\Omega_0)} = \mathcal{P}_{\alpha\beta}^{(sh)} + \mathcal{P}_{\alpha\beta}^{(th,\Omega_0)}\,. \tag{6.33}$$

Let us compare these two terms. A non-equilibrium thermal noise, Eq. (6.20c), generated by the dynamical scatterer in the adiabatic regime ($\hbar\Omega_0 \ll \delta E$), is of the order

$$\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)} \sim k_B T \frac{\hbar\Omega_0}{\delta E}\,.$$

While a high-temperature shot noise, Eq. (6.32), can be estimated as follows:

$$\mathcal{P}_{\alpha\beta}^{(sh)} \sim \frac{(\hbar\Omega_0)^2}{k_B T}\,.$$

Their ratio is equal to

$$\frac{\mathcal{P}_{\alpha\beta}^{(sh)}}{\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)}} \sim \frac{\hbar\Omega_0\delta E}{(k_B T)^2}\,.$$

From this it is seen that at $k_B T \ll \sqrt{\hbar\Omega_0\delta E}$ the shot noise dominates. However at higher temperature namely a non-equilibrium thermal noise determines a dependence of the total noise on the pumping frequency Ω_0. Therefore, with increasing temperature one can expect the following: [89]

$$\delta\mathcal{P}_{\alpha\beta}^{(\Omega_0)} \sim \frac{e^2}{2h}\begin{cases} \hbar\Omega_0\,, & k_B T \ll \hbar\Omega_0\,, \\ \dfrac{(\hbar\Omega_0)^2}{6k_B T}\,, & \hbar\Omega_0 \ll k_B T \ll \sqrt{\hbar\Omega_0\delta E}\,, \\ \hbar\Omega_0\dfrac{k_B T}{\delta E}\,, & \sqrt{\hbar\Omega_0\delta E} \ll k_B T\,. \end{cases} \tag{6.34}$$

Stress the linear dependence on Ω_0 at low and high temperatures is due to different physical reasons. While at low temperatures it is due to the shot noise, at high temperatures it is due to the thermal noise.

Here we have presented the Floquet scattering theory for noise of quantum pumps. The same problem was also investigated within different approaches, the random matrix theory [91, 92, 93], the full counting statistics [94, 95, 96, 97, 98], and the Green function formalism [99, 100, 101, 102, 103]. Note also a prediction [94, 90, 95, 89, 102] that in the quantized emission regime[1] the noise vanishes. It seems that the experiment confirms it. [104]

[1] This is a regime when the integer number n of electrons is pumped during each period.

Chapter 7

Energetics of a dynamical scatterer

7.1 DC heat current

By analogy with a dc charge current, Eq. (4.3), we define a dc energy current I_α^E in the lead α as a difference between the energy flow $I_\alpha^{E(out)}$ carried by non-equilibrium electrons from the scatterer to the reservoir and an equilibrium energy flow $I_\alpha^{E(in)}$ from the reservoir to the scatterer:

$$I_\alpha^E = I_\alpha^{E(out)} - I_\alpha^{E(in)} . \tag{7.1}$$

Here the corresponding energy currents are defined as follows, [64]

$$I_\alpha^{E(in/out)} = \frac{1}{h} \int_0^\infty dE\, E\, f_\alpha^{(in/out)}(E) , \tag{7.2}$$

where $f_\alpha^{(out)}(E)$ is a distribution function for scattered electrons; $f_\alpha^{(in)}(E) \equiv f_\alpha(E)$ is an equilibrium distribution function for incident electrons. We are interested in a dc heat current I_α^Q which is the total energy current I_α^E reduced by the convective energy flow of electrons carrying a dc charge current I_α :

$$I_\alpha^Q = I_\alpha^E - \mu_\alpha \frac{I_\alpha}{e} . \tag{7.3}$$

The division of I_α^E into heat I_α^Q and convective $\mu_\alpha I_\alpha/e$ flows can be explained on the base of particle and energy balance for the reservoir α with fixed both the chemical potential μ_α and the temperature T_α (for macroscopic samples, see, e.g., [105]). If dc charge I_α and energy I_α^E currents enter the reservoir α, then its charge (the particle number) and its energy should change. At the same time the chemical potential and the temperature of a reservoir should be changed also. Let us analyze what should be done to maintain μ_α and T_α fixed. To keep μ_α fixed one needs to remove exceeding number of electrons with the rate

I_α/e they enter the reservoir. To this end the metallic contact, playing a role of reservoir for a mesoscopic sample, is connected to another much bigger conductor. This connection is located far away from the place where the mesoscopic sample is connected, so that all the injected non-equilibrium electrons become equilibrium. Therefore, all electrons which are removed (to keep μ_α fixed) are equilibrium and hence have an energy μ_α, see, e.g., [11]. It is clear that removing of electrons is accompanied by removing of energy with rate $\mu_\alpha I_\alpha/e$. Note this convective energy flow, $\mu_\alpha I_\alpha/e$, is removed at equilibrium, hence it can be reversibly given back to the reservoir.

Now analyze what to do to maintain the reservoir's temperature fixed. In general a taken away convective energy flow is not equal to the total energy flow I_α^E incoming to the reservoir α. To prevent heating of a reservoir it is necessary to remove additionally energy with rate I_α^Q, Eq. (7.3). Since, as a rule, the reservoir can not produce a work then only the way to take out of it energy, I_α^Q, (keeping the number of particles fixed) is to bring it into contact with some bath. Energy exchange between the reservoir and the bath is essentially irreversible. This is a reason to name as *a heat* the part of an energy flow denoted as I_α^Q. Emphasize, this part of an energy flow becomes a proper heat (i.e., it can change a temperature) only deep inside the reservoir, after thermalizing of non-equilibrium electrons. In the absence of a bath the reservoir's temperature will be changed under the action of a heat current I_α^Q, which, as we show, can be directed as to the reservoir as out of it: The dynamical scatterer can either heat some reservoir α or cool it, even when the temperatures of all the reservoirs were originally the same.

Expressing the distribution function $f_\alpha^{(out)}(E)$ for scattered electrons in terms of the Floquet scattering matrix elements and distribution functions $f_\beta(E)$ for incident electrons, Eq. (4.2), and using Eq. (3.40) for a dc charge current, $I_{\alpha,0}$, we finally get the following expression for a dc heat current I_α^Q, Eq. (7.3): [28]

$$I_\alpha^Q = \frac{1}{h}\int_0^\infty dE\,(E-\mu_\alpha)\sum_{n=-\infty}^{\infty}\sum_{\beta=1}^{N_r}\left|S_{F,\alpha\beta}(E,E_n)\right|^2\left\{f_\beta(E_n) - f_\alpha(E)\right\}. \tag{7.4}$$

Here as a factor at $f_\alpha(E)$ we used the following identity,

$$\sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} \left|S_{F,\alpha\beta}(E, E_n)\right|^2 = 1, \tag{7.5}$$

which is a consequence of the unitarity of the Floquet scattering matrix following from Eq. (3.28b) at $m = 0$ and $\gamma = \alpha$.

In addition we give also the following two equations for the dc heat current. The first one,

$$I_\alpha^Q = \frac{1}{h} \int_0^\infty dE \, (E_n - \mu_\alpha) \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} \left|S_{F,\alpha\beta}(E_n, E)\right|^2 \left\{ f_\beta(E) - f_\alpha(E_n) \right\}, \tag{7.6}$$

follows from Eq. (7.4) via the substitution $E \to E_n$ and $n \to -n$, And the second one,

$$I_\alpha^Q = \frac{1}{h} \int_0^\infty dE \left\{ \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} (E_n - \mu_\alpha) \left|S_{F,\alpha\beta}(E_n, E)\right|^2 f_\beta(E) - (E - \mu_\alpha) f_\alpha(E) \right\}. \tag{7.7}$$

is obtained via the same substitution but made only in the term with f_β, while in the term with f_α we used the identity (7.5).

Now we use the last equation to show the existence of two quite general effects due to the dynamical scatterer. For better clarity we assume that all the reservoirs have the same chemical potentials and the same temperatures:

$$\mu_\alpha = \mu_0, \quad T_\alpha = T_0, \quad f_\alpha(E) = f_0(E), \quad \alpha = 1 \ldots, N_r. \tag{7.8}$$

Therefore, all the possible energy/heat flows in the system are generated by the dynamical scatterer only. Presenting this problem we follow Ref. [106].

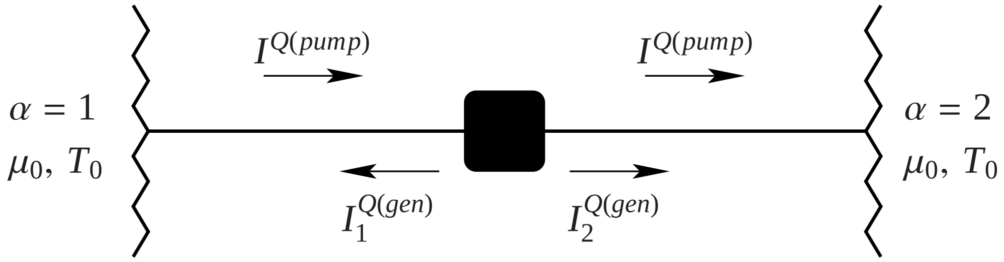

Figure 7.1: The heat flows caused by the dynamical scatterer with two contacts. $I_\alpha^{Q(gen)}$ is a generated heat flowing into the reservoir α; $I^{Q(pump)}$ is a pumped heat. The heat production rate is $I_{tot}^Q = I_1^{Q(gen)} + I_2^{Q(gen)}$.

7.1.1 Heat generation by the dynamical scatterer

The first of mentioned effects consists in the following:
The periodic in time variation of parameters of a mesoscopic scatterer is accompanied by pumping of an energy into an electron system. That after all leads to heating of electrons reservoirs. [90, 28, 107, 66]

In other words, functioning of the quantum pump is accompanied by a heat production as expected. To calculate the total heat production rate I_{tot}^Q we sum up the heat currents I_α^Q flowing into all leads. Using Eq. (7.7) under the conditions given in Eq. (7.8) we find

$$I_{tot}^Q \equiv \sum_{\alpha=1}^{N_r} I_\alpha^Q = \frac{\Omega_0}{2\pi} \int_0^\infty dE\, f_0(E) \sum_{n=-\infty}^{\infty} n \sum_{\alpha=1}^{N_r} \sum_{\beta=1}^{N_r} \left| S_{F,\alpha\beta}(E_n, E) \right|^2 . \tag{7.9}$$

Since the sum of dc heat currents is not zero, in contrast to the sum of dc change currents, see Eq. (4.11), we conclude that indeed the dynamical scatterer is a source of heat, Fig. 7.1. Taking into account a physical meaning of quantities entering Eq. (7.9) we can say that the quantity I_{tot}^Q is due to the energy absorbed by electrons scattered by the dynamical sample. The origin of this additional energy is driving external forces/fields causing a change of parameters of a scatterer.

7.1.2 Heat transfer between the reservoirs

The second effect is the following:
The dynamical scatterer plays a role of a heat pump producing a heat transfer between the electron reservoirs. [108, 109, 110, 111]

This effect is quite analogous to the considered earlier effect of a dc charge current generation. The dynamical scatterer can cause an appearance of heat flows, $I_\alpha^{Q(pump)}$, which are directed from the scatterer in some leads and to the scatterer in other leads, Fig. 7.1. At the same time the sum of these flows in all the leads is zero,

$$\sum_{\alpha=1}^{N_r} I_\alpha^{Q(pump)} = 0\,, \tag{7.10}$$

as in the case with a dc charge current, Eq. (4.11).

Above equation (7.10) means that the heat currents $I_\alpha^{Q(pump)}$ flow through the scatterer neither being accumulated nor disappearing, i.e., the dynamical scatterer is not a source (or a sink) for this part of heat flows. Its role consists only in providing conditions under which the heat currents flowing out of the reservoirs can be redistributed in such a way that the heat can be taken out of some reservoirs, $I_{\alpha_1}^{Q(pump)} < 0$, and be pushed into other reservoirs, $I_{\alpha_2}^{Q(pump)} > 0$. Note if some reservoir α_0 is at the zero temperature then in the lead connecting the scatterer to this reservoir the heat current is not negative, $I_{\alpha_0}^{Q(pump)} \ge 0$, since it is impossible to take heat out of such a reservoir.

To show the existence of a quantum heat pump effect we proceed as follows. Let us formally split the total heat production rate I_{tot}^Q into the parts, $I_\alpha^{Q(gen)}$, such that

$$I_{tot}^Q = \sum_{\alpha=1}^{N_r} I_\alpha^{Q(gen)}\,. \tag{7.11}$$

Comparing this equation with Eq. (7.9) we can write,

$$I_\alpha^{Q(gen)} = \frac{\Omega_0}{2\pi}\int_0^\infty dE\, f_0(E) \sum_{n=-\infty}^{\infty} n \sum_{\beta=1}^{N_r} \left|S_{F,\alpha\beta}(E_n, E)\right|^2\,. \tag{7.12}$$

One can interpret the quantity $I_\alpha^{Q(gen)}$ as a generated heat flowing into the reservoir α. Then comparing Eqs. (7.12) and Eq. (7.7) (at $f_\alpha = f_0\,, \forall\alpha$) we can see that $I_\alpha^{Q(gen)}$ is different from the heat current I_α^Q flowing into the lead α. The difference,

$$I_\alpha^{Q(pump)} = I_\alpha^Q - I_\alpha^{Q(gen)}\,, \tag{7.13}$$

is just a part of a heat current which is transferred between the reservoirs. This part is not related to the heat generated by the dynamical scatterer. Using Eqs. (4.7) and (4.10) into Eq. (4.11) we find,

$$I_\alpha^{Q(pump)} = \frac{1}{h}\int_0^\infty dE(E-\mu_0)\, f_0(E)\left\{\sum_{n=-\infty}^{\infty}\sum_{\beta=1}^{N_r}\left|S_{F,\alpha\beta}(E_n,E)\right|^2 - 1\right\}. \tag{7.14}$$

With the unitarity condition (3.28a) one can easily check that Eq. (7.14) satisfies the conservation law, Eq. (7.10).

Accordingly to Eq. (7.13) the heat flow I_α^Q in the lead α consists of two parts, Fig. 7.1. The first one, $I_\alpha^{Q(gen)}$, is a positive heat flow generated by the dynamical scatterer. The second one, $I_\alpha^{Q(pump)}$, is a transferred heat flow which can be either positive (the heat flow is directed to the reservoir α) or negative (the heat flow is directed from the reservoir α). Note if $I_\alpha^{Q(pump)} < 0$ and the transferred heat flow is larger than the generated heat flow in the same lead, $\left|I_\alpha^{Q(pump)}\right| > I_\alpha^{Q(gen)}$, then the reservoir α is cooled, since $I_\alpha^Q = I_\alpha^{Q(pump)} + I_\alpha^{Q(gen)} < 0$.

We have splitted a heat flow I_α^Q into the parts $I_\alpha^{Q(gen)}$ and $I_\alpha^{Q(pump)}$ to show that I_α^Q can be negative. Therefore, the electron reservoirs can not only be heated (that is intuitively expected since functioning of a device, in our case a quantum pump, is accompanied by the energy dissipation), but also can be cooled (that is a non-trivial effect). Strictly speaking we can rigorously calculate only a heat flow I_α^Q, Eq. (7.7), and the total generated heat rate I_{tot}^Q, Eq. (7.9). While the splitting presented in Eqs. (7.12) and (7.14) is not unique, since the equation (7.11) is not enough for unambiguous definition of the quantities $I_{\alpha}^{Q(gen)}$. In the next section we consider an adiabatic regime and give additional physical arguments supporting the splitting of the total heat flow I_{α}^{Q} into the sum of generated and transferred heat flows.

7.2 Heat flows in the adiabatic regime

At $\varpi \ll 1$, Eq. (3.49), up to the terms linear in frequency, Ω_0, of an external drive the Floquet scattering matrix elements are following [see Eqs. (3.44), (3.46a) and (3.48a)]:

$$\begin{aligned} \left|S_{F,\alpha\beta}(E_n,E)\right|^2 &= \left|S_{\alpha\beta,n}(E)\right|^2 + \frac{n\hbar\Omega_0}{2}\frac{\partial\left|S_{\alpha\beta,n}(E)\right|^2}{\partial E} \\ &\quad + 2\hbar\Omega_0 \mathrm{Re}\left[S^{*}_{\alpha\beta,n}(E)\,A_{\alpha\beta,n}(E)\right] + \mathcal{O}\left(\Omega_0^2\right). \end{aligned} \tag{7.15}$$

Substituting this expression into Eq. (7.6) we calculate a heat flow I_{α}^{Q} under the conditions given in Eq. (7.8) up to terms of order Ω_0^2. We consider separately finite temperature and zero temperature cases.

7.2.1 High temperatures

If it is,

$$k_B T_0 \gg \hbar\Omega_0 \tag{7.16}$$

then we can expand the difference of Fermi functions in Eq. (7.6) in powers of Ω_0:

$$f_0(E) - f_0(E_n) = \left(-\frac{\partial f_0}{\partial E}\right) n\hbar\Omega_0 - \frac{(n\hbar\Omega_0)^2}{2}\frac{\partial^2 f_0}{\partial E^2} + \mathcal{O}\left(\Omega_0^3\right). \tag{7.17}$$

Substituting above expansion into Eq. (7.6) and summing up over n, using the properties of the Fourier coefficients, we calculate ($k_B T_0 \gg \hbar\Omega_0$):

$$I_\alpha^Q = I_\alpha^{Q(gen)} + I_\alpha^{Q(pump)} + \mathcal{O}\left(\Omega_0^3\right) , \tag{7.18a}$$

where

$$I_\alpha^{Q(gen)} = \frac{\hbar}{4\pi} \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \left(\frac{\partial \hat{S}}{\partial t} \frac{\partial \hat{S}^\dagger}{\partial t}\right)_{\alpha\alpha} , \tag{7.18b}$$

and

$$I_\alpha^{Q(pump)} = \frac{1}{2\pi} \int_0^\infty dE\, (E - \mu_0) \left(-\frac{\partial f_0}{\partial E}\right) \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \mathrm{Im} \left\{ \left(\hat{S} + 2\hbar\Omega_0 \hat{A}\right) \frac{\partial \hat{S}^\dagger}{\partial t} \right\}_{\alpha\alpha} . \tag{7.18c}$$

The quantity $I_\alpha^{Q(pump)}$ satisfies the conservation law, Eq. (7.10). This follows from Eq. (4.11) with a dc charge current given in Eq. (4.22). Then at zero temperature we have an identity,

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \mathrm{Im\,Tr} \left\{ \left(\hat{S}(t,\mu_0) + 2\hbar\Omega_0 \hat{A}(t,\mu_0)\right) \frac{\partial \hat{S}^\dagger(t,\mu_0)}{\partial t} \right\} = 0 . \tag{7.19}$$

which should hold for any μ_0.

The separation of contributions given in Eq. (7.18a) can be justified by the following arguments.

1. The generated heat flow $I_\alpha^{Q(gen)}$ is definitely positive in all the leads, $\alpha = 1 \dots N_r$. This is exactly what is expected if the heat is generated by the dynamical scatterer and is dissipated into the reservoirs. To show its positiveness we rewrite Eq. (7.18b) in terms of the Fourier coefficients for the frozen scattering matrix elements:

$$I_{\alpha}^{Q(gen)} = \frac{\hbar\Omega_0^2}{4\pi} \int_0^{\infty} dE \left(-\frac{\partial f_0}{\partial E}\right) \sum_{n=-\infty}^{\infty} n^2 \sum_{\beta=1}^{N_r} \left|S_{\alpha\beta,n}(E)\right|^2. \tag{7.20}$$

It is obvious that $I_{\alpha}^{Q(gen)} > 0$. From above equation it follows that in the adiabatic regime the heat generated by the quantum pump is quadratic in pumping frequency Ω_0. [90]

2. The transferred heat flow $I_{\alpha}^{Q(pump)}$ vanishes at zero temperature, since it is impossible to take heat out of a reservoir with zero temperature in order to push it into other reservoir. This property follows from Eq. (7.18c) where at zero temperature it is $(E-\mu_0)\partial f_0/\partial E = 0$. From Eq. (7.18c) it follows that the transferred heat is linear in pumping frequency, $I_{\alpha}^{Q(pump)} \sim k_B T_0 \Omega_0$.

Note under the conditions given in Eq. (7.16) it is possible to realize a regime when the energy ($\sim k_B T_0 \Omega_0$) taken out of some reservoir is larger then its heating ($\sim \Omega_0^2$). Then such a reservoir will be cooled. To characterize a cooling efficiency let us introduce the coefficient K_{α} equal to a ratio of the dc heat current in the lead α and the total work produced by driving forces. Since the volume of the system remains constant, the mentioned work is equal to the total heat generated by the scatterer. Therefore, the coefficient K_{α} is,

$$K_{\alpha} = (-1)\frac{I_{\alpha}^{Q}}{I_{tot}^{Q}}, \tag{7.21}$$

where

$$I_{tot}^{Q} = \frac{\hbar}{4\pi} \int_0^{\infty} dE \left(-\frac{\partial f_0}{\partial E}\right) \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \mathrm{Tr}\left(\frac{\partial \hat{S}}{\partial t}\frac{\partial \hat{S}^{\dagger}}{\partial t}\right). \tag{7.22}$$

The positive/negative sign corresponds to cooling/heating of the reservoir α.

7.2.2 Low temperatures

In the case of ultra-low temperatures,

$$k_B T_0 \ll \hbar\Omega_0\,, \tag{7.23}$$

while integrating over energy in Eq. (7.6) we can relax an energy dependence of the Floquet scattering matrix elements and calculate them at $E = \mu_0$. Such a simplification is possible because of the following. The integration over energy in Eq. (7.6) is over the window of order $k_B T_0$ near the Fermi energy. The scattering matrix changes significantly if the energy changes by the value of order δE. Taking into account Eq. (7.23) we find that in the adiabatic regime, Eq. (3.49), it is $k_B T_0 \ll \delta E$. The last justifies the simplification used.

So, now the equation (7.6) reads:

$$\begin{aligned} I_\alpha^Q &= \frac{1}{h}\sum_{n=-\infty}^{\infty}\sum_{\beta=1}^{N_r}\left|S_{F,\alpha\beta}(\mu_0 + n\hbar\Omega_0, \mu_0)\right|^2 \int\limits_{\mu_0 - n\hbar\Omega_0}^{\mu_0} dE\,(E - \mu_0 + n\hbar\Omega_0) \\ &= \frac{\hbar\Omega_0^2}{4\pi}\sum_{n=-\infty}^{\infty} n^2 \sum_{\beta=1}^{N_r}\left|S_{F,\alpha\beta}(\mu_0 + n\hbar\Omega_0, \mu_0)\right|^2. \end{aligned}$$

Using Eq. (7.15) and making the inverse Fourier transformation we finally calculate ($k_B T_0 \ll \hbar\Omega_0$):

$$I_\alpha^Q = \frac{\hbar}{4\pi}\int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \left(\frac{\partial \hat{S}\,(t,\mu_0)}{\partial t}\frac{\partial \hat{S}^\dagger\,(t,\mu_0)}{\partial t}\right)_{\alpha\alpha} + \mathcal{O}\left(\Omega_0^3\right). \tag{7.24}$$

Comparing equation above with Eq. (7.18b) we conclude that at low temperatures, Eq. (7.23), the dynamical scatterer only heats the reservoirs. While the heat pump effect is absent, that is consistent with the conclusion made on the base of Eq. (7.18c) calculated at zero temperature.

Appendix A

Dynamical mesoscopic capacitor

The capacitor does not support a dc current. To model it one can consider a mesoscopic sample attached to only a single reservoir. We will call it *a mesoscopic capacitor* [112], since its capacitance depends not only on the geometry (as for a macroscopic capacitor) but also on the density of states (DOS) of electrons. Changing periodically the potential of a sample via a near gate or changing periodically the potential of a reservoir (or changing both potential simultaneously) one can generate an ac current flowing between the sample and the reservoir. Due to the gauge invariance the current depends on the potential difference rather than on each potential separately. Therefore, in what follows we consider the periodic potential applied to the sample and stationary reservoir. The reservoir is in equilibrium with the Fermi distribution function $f_0(E)$ with chemical potential μ_0 and temperature $k_B T_0$.

A.1 General theory for a single-channel scatterer

For the sake of simplicity we consider a lead connecting the sample to the reservoir to be one-dimensional. We ignore spin-flip processes, therefore, electrons can be teated as spinless. Then the sample can be viewed as a single-channel scatterer which has only one incoming and one outgoing orbital channels. In the stationary case the capacitor is characterized by the single scattering amplitude. If such a sample is driven by the periodic perturbation then the mentioned above scattering amplitude becomes a matrix in the energy space with elements $S_F(E_n, E)$, where $E_n = E + n\hbar\Omega_0$ with n integer. We call this matrix as *the Floquet scattering matrix*.

A.1.1 Scattering amplitudes

The scattering amplitudes $S_{in}(t, E)$ and $S_{out}(E, t)$ define elements of the Floquet scattering matrix as follows,

$$S_F(E + n\hbar\Omega_0,\ E) = S_{in,n}(E) \equiv \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega_0 t} S_{in}(t, E)\,, \tag{A.1}$$

$$S_F(E,\ E - n\hbar\Omega_0) = S_{out,n}(E) \equiv \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega_0 t} S_{out}(E, t)\,, \tag{A.2}$$

where $\mathcal{T} = 2\pi/\Omega_0$ is a period of a drive.

From the definition we get the following relation between in- and out-scattering amplitudes,

$$S_{in,\,n}(E) = S_{out,\,n}(E_n)\,. \tag{A.3}$$

In a time representation one can get,

$$\begin{aligned} S_{in}(t, E) &= \sum_{n=-\infty}^{\infty} \int_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}} e^{in\Omega_0(t'-t)} S_{out}(E_n, t')\,, \\ S_{out}(E, t) &= \sum_{n=-\infty}^{\infty} \int_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}} e^{in\Omega_0(t'-t)} S_{in}(t', E_{-n})\,. \end{aligned} \tag{A.4}$$

A.1.2 Unitarity conditions

The unitarity conditions read,

$$\sum_{n=-\infty}^{\infty} S_F^*(E_n, E_m) S_F(E_n, E) = \sum_{n=-\infty}^{\infty} S_F(E_m, E_n) S_F^*(E, E_n) = \delta_{m,0}\,. \tag{A.5}$$

Using Eqs. (A.1) and (A.2) we obtain from Eq. (A.5) the following relations between the amplitudes S_{in} and S_{out} [see Eq. (3.60)],

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{im\Omega_0 t} S^*_{in}(t, E_m) S_{in}(t, E) = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{im\Omega_0 t} S_{out}(E_m, t) S^*_{out}(E, t) = \delta_{m,0} . \tag{A.6}$$

Using the second equations in Eqs. (A.5) and (A.6) one can find,

$$\sum_{m=-\infty}^{\infty} e^{-im\Omega_0 t} \sum_{n=-\infty}^{\infty} S_F(E_m, E_n) S^*_F(E, E_n) = \\ = \sum_{m=-\infty}^{\infty} e^{-im\Omega_0 t} \left(S_{out}(E_m, t) S^*_{out}(E, t) \right)_m = \sum_{m=-\infty}^{\infty} e^{-im\Omega_0 t} \delta_{m,0} = 1 . \tag{A.7}$$

In fact we have proven the following useful identity being a direct consequence of the unitarity of scattering,

$$\sum_{n=-\infty}^{\infty} \int_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}} e^{-in\Omega_0 (t-t')} S_{in}(t, E_n) S^*_{in}(t', E_n) = 1 , \tag{A.8}$$

or equivalently,

$$\sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} S_{in}(t, E_n) S^*_{out,-n}(E) = 1 , \tag{A.9a}$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-im\Omega_0 t} S_{out, n+m}(E_m) S^*_{out, n}(E) = 1 , \tag{A.9b}$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-im\Omega_0 t} S_{in, n+m}(E_{-n}) S^*_{in, n}(E_{-n}) = 1 . \tag{A.9c}$$

A.1.3 Time-dependent current

The general expression for a time-dependent current [32] in the case of a periodically driven capacitor reads as follows:

$$I(t) = \frac{e}{h}\int dE \sum_{n=-\infty}^{\infty}\left\{ f_0(E) - f_0(E_n)\right\}\sum_{l=-\infty}^{\infty} e^{-il\Omega_0 t}\, S_F^*(E_n, E)\, S_F(E_{n+l}, E), \tag{A.10}$$

To simplify expression above we shift $E \to E_n$ in the part dependent on $f_0(E_n)$. Then from Eq. (A.7) we conclude that this part is reduced to $f_0(E)$. Using Eq. (A.1) in the remaining part of Eq. (A.10) we arrive at the following, [113]

$$I(t) = \frac{e}{h}\int dE\, f_0(E)\left\{\left|S_{in}(t,E)\right|^2 - 1\right\}. \tag{A.11}$$

Let us show that in the adiabatic regime this equation can be easily transformed into the following form,

$$I(t) = -\frac{ie}{2\pi}\int dE\left(-\frac{\partial f_0}{\partial E}\right) S(t,E)\,\frac{\partial S^*(t,E)}{\partial t}. \tag{A.12}$$

in accordance with a general theory developed in Ref. [31] (see, e.g., Ref. [69]). To this end in the adiabatic regime we use,

$$S_{in}(t,E) = S(t,E) + \frac{i\hbar}{2}\frac{\partial^2 S(t,E)}{\partial t\partial E}, \tag{A.13}$$

in the first order in Ω_0 approximation with S being the frozen scattering amplitude.[1] To calculate $|S_{in}(t,E)|^2$ we use $|S|^2 = 1$ and, correspondingly, $\partial^2|S|^2/(\partial t\partial E) = 0$. Also we use,

[1] See the first two terms on the RHS of Eq. (A.62) for a single-cavity capacitor but also Eq. (B.38) for a double-cavity capacitor. The appearance of an anomalous scattering amplitude A in the latter case does not affect Eq. (A.12).

$$\frac{\partial S^*}{\partial t}\frac{\partial S}{\partial E} = \frac{\partial S^*}{\partial t} S S^* \frac{\partial S}{\partial E} = S^* \frac{\partial S}{\partial t}\frac{\partial S^*}{\partial E} S = \frac{\partial S}{\partial t}\frac{\partial S^*}{\partial E},$$

and find (up to $\sim \Omega_0$ terms):

$$|S_{in}(t,E)|^2 \approx 1 - i\hbar \frac{\partial}{\partial E}\left(S\frac{\partial S^*}{\partial t}\right).$$

Substituting this equation into Eq. (A.11) and integrating over energy E by parts we arrive at Eq. (A.12).

A.1.4 The dissipation

The (dc) heat flowing out of the driven capacitor is,

$$I_E = \frac{1}{h}\int_0^\infty dE \sum_{n=-\infty}^{\infty} (E_n - \mu_0)\left[f_0(E) - f_0(E_n)\right]|S_F(E_n,E)|^2 . \qquad \text{(A.14)}$$

Using the unitarity of the Floquet scattering matrix, Eq. (A.5) with $m = 0$, and shifting the energy, $E_n \to E$, in the part with $f_0(E_n)$, we simplify,

$$\begin{aligned} I_E =& \frac{\Omega_0}{2\pi}\int_0^\infty dE\, f_0(E)\sum_{n=-\infty}^{\infty} n\,|S_F(E_n,E)|^2 \\ &+\frac{1}{h}\int_0^\infty dE\,(E-\mu_0)\, f_0(E)\sum_{n=-\infty}^{\infty}|S_F(E_n,E)|^2 \\ &-\frac{1}{h}\int_0^\infty dE\,(E-\mu_0)\, f_0(E)\sum_{n=-\infty}^{\infty}|S_F(E,E_{-n})|^2 \\ =& \frac{\Omega_0}{2\pi}\int_0^\infty dE\, f_0(E)\sum_{n=-\infty}^{\infty} n\,|S_F(E_n,E)|^2 . \end{aligned}$$

Then we introduce the scattering amplitude $S_{in,n}(E) = S_F(E_n, E)$, use the property of the Fourier coefficients $\Omega_0 n S^*_{in,n}(E) = -i\left\{\partial S^*_{in}(t,E)/\partial t\right\}_{-n}$, and finally obtain: [113]

$$I_E = -\frac{i}{2\pi}\int_0^\infty dE\, f_0(E)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\, S_{in}(t,E)\,\frac{\partial S^*_{in}(t,E)}{\partial t}\,. \tag{A.15}$$

Note Eqs. (A.11) and (A.15) are valid at arbitrary frequency and amplitude of the drive. The disadvantage only is that they involve an integration over all energies.

A.1.5 The dissipation versus squared current

Interesting to note that in the adiabatic regime ($\Omega_0 \to 0$) at zero temperature the heat production I_E for the capacitor can be related to the average square current $\langle I^2\rangle$. From Eq. (A.12) we find for zero temperature,

$$I(t) = -\frac{ie}{2\pi}\, S(t,\mu_0)\,\frac{\partial S^*(t,\mu_0)}{\partial t}\,. \tag{A.16}$$

Using $S\,dS^* = -dS\,S^*$ following from the unitarity of the frozen scattering amplitude, $|S(t,E)|^2 = 1$, we represent the square current as follows,

$$I^2(t) = -\frac{e^2}{4\pi^2}\, S\,\frac{\partial S^*}{\partial t}\, S\,\frac{\partial S^*}{\partial t} = \frac{e^2}{4\pi^2}\, S\, S^*\,\frac{\partial S}{\partial t}\,\frac{\partial S^*}{\partial t} = \frac{e^2}{4\pi^2}\left|\frac{\partial S}{\partial t}\right|^2,$$

and find for its average:

$$\langle I^2\rangle = \int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\, I^2(t) = \frac{e^2\Omega_0^2}{4\pi^2}\sum_{n=1}^{\infty} n^2\left\{|S_n|^2 + |S_{-n}|^2\right\}\,. \tag{A.17}$$

To calculate a heat current I_E in the adiabatic regime we use Eq. (A.14) with $f_0(E_n) \approx f_0(E) + n\hbar\Omega_0 \partial f_0(E)/\partial E + (n^2\hbar^2\Omega_0^2/2)\partial^2 f_0(E)/\partial E^2$. For $S_F(E_n, E) = S_{in,n}(E)$ we use Eq. (A.13) and find with accuracy up to Ω_0^2,

$$\begin{aligned} I_E &= \frac{\Omega_0}{2\pi}\int_0^\infty dE\,(E-\mu_0)\left(-\frac{\partial f_0(E)}{\partial E}\right)\sum_{n=-\infty}^{\infty} n\,|S_n(E)|^2 \\ &+\frac{\hbar\Omega_0^2}{4\pi}\int_0^\infty dE\left(-\frac{\partial f_0(E)}{\partial E}\right)\sum_{n=-\infty}^{\infty} n^2\,|S_n(E)|^2\,. \end{aligned}$$

The first term is identically zero for the capacitor. To show it we take into account that for a single channel capacitor the frozen scattering amplitude is of the following form,

$$S(t,E) = e^{i\phi(t,E)}\,. \tag{A.18}$$

Then we find,

$$\sum_{n=-\infty}^{\infty} in\Omega_0\,|S_n(E)|^2 = \int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\,S(t,E)\frac{\partial S^*(t,E)}{\partial t} = -i\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\frac{\partial\phi(t,E)}{\partial t} = 0\,.$$

Therefore, the heat flow generated by the dynamical capacitor is:

$$I_E = \frac{\hbar\Omega_0^2}{4\pi}\int_0^\infty dE\left(-\frac{\partial f_0(E)}{\partial E}\right)\sum_{n=1}^{\infty} n^2\left\{|S_n(E)|^2 + |S_{-n}(E)|^2\right\}\,. \tag{A.19}$$

Comparing Eqs. (A.17) and (A.19) we find the following at zero temperature: [113]

$$I_E = R_q \langle I^2 \rangle \,, \tag{A.20}$$

where $R_q = h/(2e^2)$ is the relaxation resistance for a single-channel scatterer and spinless electrons [112].

A.2 The chiral single-channel capacitor

Experiments demonstrate [114, 115] that a quantum capacitor in a 2D electron gas in the integer quantum Hall effect regime is a promising device to realize a sub-nanosecond, single- and few-electron, coherent quantum electronics. The quantum capacitor can be used as a single-particle emitter [115]. With such an emitter as an elementary block, several effects were predicted including shot-noise plateaus [116], two-particle emission and particle reabsorption [41], and a tunable two-particle Aharonov-Bohm effect [117].

A.2.1 Model and scattering amplitude

We consider a model [118, 114, 115, 119] consisting of a single circular edge state of circumference L (a cavity) coupled via a quantum point contact (QPC) to a linear edge state which in turn flows out of a reservoir of electrons with temperature T_0 and the Fermi energy μ_0, see Fig. A.1. A periodic in time potential $U(t) = U(t + \mathcal{T})$ induced nearby gate is applied uniformly over the cavity.

Using the method presented in Sec. 3.5.3 one can calculate the elements of the Floquet scattering matrix as follows:

$$S_F(E_n, E) = \int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega_0 t} S_{in}(t, E) \,, \tag{A.21a}$$

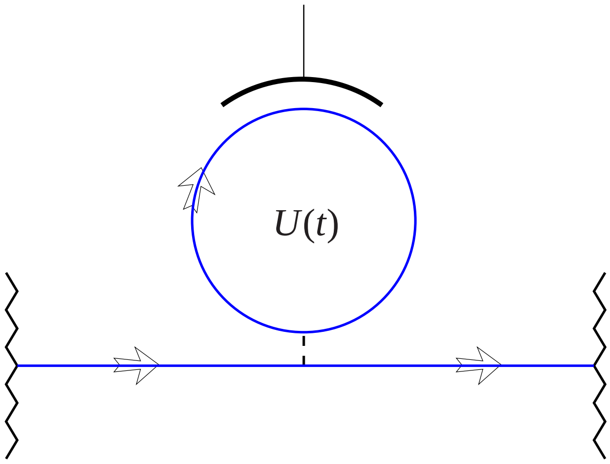

Figure A.1: The model of a single-cavity chiral quantum capacitor driven by the uniform potential $U(t)$ induced nearby metallic gate. The dotted line denotes the QPC. Arrows indicate the direction of movement of electrons.

$$S_{in}(t,E) = \sum_{q=0}^{\infty} e^{iqkL} S^{(q)}(t)\,, \tag{A.21b}$$

$$S^{(0)} = r\,, \quad S^{(q>0)}(t) = \overline{t}^{2}\, r^{q-1}\, e^{-i\Phi_q(t)}\,, \quad \Phi_q(t) = \frac{e}{\hbar}\int_{t-q\tau}^{t} dt' U(t')\,.$$

Here $r(E)/\overline{t}(E)$ is a reflection/transmission amplitude of a QPC connecting cavity to the linear edge state, $\tau = m_e L/(\hbar k)$ is a time necessary for electron with energy E to make one turn around a cavity of length L. The index q counts number of turns which electron makes in the cavity until escaping it. In above equation it is assumed that $\hbar\Omega_0 \ll E$ and the reflection/transmission amplitude of a QPC changes in energy over the scale $\delta E \sim E$ which is much larger than $\hbar\Omega_0$. Correspondingly we neglected the terms of order $\hbar\Omega_0/\delta E$ and smaller.

To calculate given above Floquet scattering matrix elements, $S_F(E_n, E)$, we consider scattering of a plane wave, $e^{-iEt/\hbar+ikx}$, with unit amplitude and with energy E onto an oscillating scatterer. We direct the axis x along the linear edge state and the axis y along the circular edge state of a cavity. We assume that the

QPC connects points $x = 0$ and $y = 0$. Then the wave function reads as follows:

$$\Psi(t,x) = \begin{cases} e^{-iEt/\hbar + ikx}, & x < 0\,, \\ e^{-iEt/\hbar} \sum\limits_{n=-\infty}^{\infty} \sqrt{\frac{k}{k_n}}\, S_F(E_n, E)\, e^{-in\Omega_0 t + ik_n x}, & x > 0\,, \end{cases} \tag{A.22a}$$

$$\Psi(t,y) = e^{-iEt/\hbar} \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} \sum_{l=-\infty}^{\infty} a_l \Upsilon_{n-l} e^{ik_l y}, \quad 0 < y < L\,, \tag{A.22b}$$

where Υ_p is a Fourier coefficient for $\Upsilon(t)$ dependent on a uniform periodic potential $U(t)$ of a cavity:

$$\Upsilon(t) = \exp\left(-\frac{ie}{\hbar} \int\limits_{-\infty}^{t} dt' U(t') \right). \tag{A.23}$$

In what follows we suppose,

$$\epsilon = \frac{\hbar \Omega_0}{E} \ll 1\,. \tag{A.24}$$

Then up to zeroth order in ϵ we have [for spatial coordinates $x, y \ll L/(\epsilon \Omega_0 \tau)$]:

$$\frac{k_n}{k} \approx 1, \quad e^{ik_n x} \approx e^{ikx} e^{in\Omega_0 x/v}, \tag{A.25}$$

where $v = \hbar k/m_e$ is an electron velocity.

We introduce the following periodic in time functions:

$$S_{in}(t,E) = \sum_{n=-\infty}^{\infty} e^{-in\Omega_0 t} S_F(E_n, E)\,, \quad a(t) = \sum_{l=-\infty}^{\infty} e^{-il\Omega_0 t} a_l\,. \tag{A.26}$$

With these functions one can perform inverse Fourier transformation in Eq. (A.22) and get,

$$\Psi(t,x) = \begin{cases} e^{-iEt/\hbar + ikx}, & x < 0, \\ S_{in}\left(t - \frac{x}{v}, E\right) e^{-iEt/\hbar + ikx}, & x > 0, \end{cases} \tag{A.27a}$$

$$\Psi(t,y) = a\left(t - \frac{y}{v}\right) \Upsilon(t)\, e^{-iEt/\hbar + iky}, \quad 0 < y < L. \tag{A.27b}$$

The amplitudes of a wave function at $x = 0$ and $y = 0$ are related to each other through the scattering matrix of a QPC. If its elements r and $\bar{t}$ can be kept as energy independent over the scale of order $\hbar\Omega_0$ then different terms in Eq. (A.22) have the same boundary conditions at $x = 0$ and $y = 0$. Therefore, one can use a wave function directly in the form of Eq. (A.27):

$$\begin{pmatrix} S_{in}(t,E) \\ a(t)\Upsilon(t) \end{pmatrix} = \begin{pmatrix} r(E) & \bar{t}(E) \\ \bar{t}(E) & r(E) \end{pmatrix} \begin{pmatrix} 1 \\ a(t-\tau)\Upsilon(t)\, e^{ikL} \end{pmatrix}. \tag{A.28}$$

The time of a single turn, $\tau = L/v$, was introduced after Eq. (A.21).

We solve the system of equations (A.28) by recursion. The equation for $a(t)$,

$$a(t)\Upsilon(t) = \bar{t} + r\, a(t-\tau)\, \Upsilon(t)\, e^{ikL},$$

has the following solution:

$$a(t) = \bar{t}\,\Upsilon^*(t) + \bar{t}\sum_{q=1}^{\infty} r^q\, e^{iqkL}\, \Upsilon^*(t - q\tau). \tag{A.29}$$

Substituting Eq. (A.29) into Eq. (A.28) we find:

$$S_{in}(t,E) = r + \bar{t}^2\, \Upsilon(t) \sum_{q=1}^{\infty} r^{q-1} e^{iqkL}\, \Upsilon^*(t - q\tau). \tag{A.30}$$

Then using Eq. (A.23) we arrive at Eq. (A.21b).

A.2.2 Unitarity

The Floquet scattering matrix is unitary. This puts the following constraint onto the scattering amplitude S_{in}: [30]

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} |S_{in}(t,E)|^2 = 1\,. \tag{A.31}$$

Let us show that Eq. (A.21b) satisfies this condition:

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} |S_{in}(t,E)|^2 = A + B\,,$$

$$A = R + \frac{T^2}{R}\sum_{q=1}^{\infty} R^q = R + T = 1\,,$$

$$\begin{aligned}
B &= \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} 2\,\Re\left\{ -T\sum_{q=1}^{\infty} r^q\, e^{i\{qkL-\Phi_q(t)\}} \right. \\
&\quad \left. + \frac{T^2}{R}\sum_{m=1}^{\infty} R^m \sum_{q=1}^{\infty} r^q\, e^{iqkL}\, e^{i\left\{\Phi_m(t)-\Phi_{m+q}(t)\right\}} \right\} \\
&= 2\,T\,\Re\sum_{q=1}^{\infty} r^q\, e^{iqkL} \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \left\{ \frac{T}{R}\sum_{m=1}^{\infty} R^m\, e^{-i\Phi_q(t-m\tau)} - e^{-i\Phi_q(t)} \right\} = 0\,.
\end{aligned}$$

Here $T = |t|^2$ and $R = |r|^2$ are transmission and reflection probabilities, respectively. In the last line of equation above we use, first, $\Phi_{m+q}(t) - \Phi_m(t) = \Phi_q(t - m\tau)$. Then using a periodicity of $\Phi_q(t)$ in time we make a shift $t - m\tau \to t$ in this term under the integration for a time period $\mathcal{T}$. After that one can sum up over m and get zero.

Note in the stationary case, $\Upsilon(t) = 1$, the elements of the Floquet scattering matrix become $S_F(E_n, E) = \delta_{n,0}\, S(E)$, where the stationary scattering amplitude is:

$$S(E) = r + \frac{\tilde{t}^2\, e^{ikL}}{1 - r\, e^{ikL}}\,. \tag{A.33a}$$

This quantity can be presented in the following form:

$$S(E) = -\, e^{ikL}\, \frac{r - R\, e^{-ikL}}{(r - R\, e^{-ikL})^*}\,, \tag{A.33b}$$

which is manifestly unitary.

A.2.3 Gauge invariance

Now we show that the model we use is gage-invariant, i.e., we get the same current either applying a periodic potential $U(t)$ at the reservoir or applying a potential $-U(t)$ at the cavity.

We consider the stationary cavity but suppose that the periodic potential $U(t) = U(t + 2\pi/\Omega_0)$ is applied at the reservoir. In this case the state of an electron in the reservoir is the Floquet state, see Eqs. (3.27) and (5.18). Let the operator $\hat{a}'^{\dagger}(E)$ creates an electron in the reservoir in the Floquet state,

$$\Psi_E(t, \vec{r}) = e^{i\vec{k}\vec{r}}\, e^{-i\frac{E}{\hbar}t} \sum_{n=-\infty}^{\infty} \Upsilon_n\, e^{-in\Omega t}\,, \tag{A.34}$$

where Υ_n is the Fourier coefficient for $\Upsilon(t)$ defined in Eq. (A.23). If $U(t) = U\cos(\Omega t)$ then $\Upsilon_n = J_n(eU/\hbar\Omega)$, where J_n is the Bessel function of the first kind of the nth order. The operators $\hat{a}'^{\dagger}(E)$ and $\hat{a}'(E)$ describe equilibrium fermions,

$$\langle \hat{a}'^{\dagger}(E), \hat{a}'(E') \rangle = \delta(E - E')\, f_0(E)\,. \tag{A.35}$$

We assume also that there is no potential within the lead connecting the cavity and the reservoir. Therefore, the wave function for electrons in the lead is a plane wave, $\psi_E(t,x) = e^{ikx - i\frac{E}{\hbar}t}$. Note that the wave number k for $\psi_E(t,x)$ and the wave vector $\vec{k}$ for $\Psi_E(t,\vec{r})$ in the reservoir depends on energy differently. While in the lead $k = \sqrt{2m_e E/\hbar^2}$ depends on a total energy E of an electron, in the reservoir k depends on the Floquet energy, E, and it is independent of an additional side-band energy $E_n - E = n\hbar\Omega$.

Let the operator $\hat{a}^\dagger(E)$ creates an electron within the lead. Then matching the wave functions with the same total energy, see Eq. (5.24), one can write,

$$\hat{a}(E) = \sum_{n=-\infty}^{\infty} \Upsilon_n \, \hat{a}'(E_{-n}) \,. \tag{A.36}$$

Note that we ignore the reflection due to the wave number changing. The corresponding reflection coefficient is as small as $(\hbar\Omega/\mu_0)^2 \ll 1$. We usually ignore such small quantities.

After scattering by the stationary cavity an electron acquires the scattering amplitude $S(E)$. Therefore, the operator $\hat{b}(E)$ annihilating the scattered electron with energy E is:

$$\hat{b}(E) = S(E)\hat{a}(E) = \sum_{n=-\infty}^{\infty} S(E)\Upsilon_n \hat{a}'(E_{-n}) \,. \tag{A.37}$$

Now we calculate the current $I(t)$, flowing in the lead,

$$I(t) = \frac{e}{h} \iint_0^\infty dE dE' \, e^{i\frac{E-E'}{\hbar}t} \left\{ \langle \hat{b}^\dagger(E)\hat{b}(E')\rangle - \langle \hat{a}^\dagger(E)\hat{a}(E')\rangle \right\} . \tag{A.38}$$

The lth harmonic of this current reads,

$$I_l = \frac{e}{h} \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{il\Omega_0 t} \iint_0^\infty dE dE' \, e^{i\frac{E-E'}{\hbar}t} \left\{ \langle \hat{b}^\dagger(E)\hat{b}(E')\rangle - \langle \hat{a}^\dagger(E)\hat{a}(E')\rangle \right\} . \tag{A.39}$$

Using Eqs.(A.35) - (A.37) and making the shift $E \to E + n\hbar\Omega$ we finally calculate,

$$I_l = \frac{e}{h} \int_0^\infty dE\, f_0(E) \sum_{n=-\infty}^{\infty} \Upsilon_n^\star \Upsilon_{n+l} \left\{ S^\star(E_n)\, S(E_{n+l}) - 1 \right\}. \tag{A.40}$$

To simplify above equation we introduce a time-dependent function,

$$S(t, E) = \sum_{n=-\infty}^{\infty} S(E_n)\, \Upsilon_n\, e^{-in\Omega t}, \tag{A.41}$$

and take into account that $\sum_{n=-\infty}^{\infty} \Upsilon_n^\star \Upsilon_{n+l} = \delta_{l,0}$. Then after the inverse Fourier transformation we get from Eq.(A.40):

$$I(t) = \frac{e}{h} \int dE\, f_0(E) \left\{ \left| S(t, E) \right|^2 - 1 \right\}. \tag{A.42}$$

This equation defines the same current as Eq. (A.11) in the case when the potential $-U(t)$ is applied to the cavity. To check it we need to show that the function $S(t, E)$ differs from the function $S_{in}(t, E)$, Eq.(A.21b), first, by the phase factor (in fact by the factor $\Upsilon(t)$) which is irrelevant for the current, and, second, by the replacement $U \to -U$. To this end we substitute

$$S(E) = \sum_{q=0}^{\infty} e^{iqkL} S^{(q)}, \quad S^{(0)} = r, \quad S^{(q>0)}(t) = \tilde{t}^2\, r^{q-1},$$

into Eq.(A.41) and calculate,

$$S(t, E) = \sum_{q=0}^{\infty} e^{iqkL} S'^{(q)}(t), \tag{A.43}$$

$$S'^{(0)}(t) = \Upsilon(t)\, r,$$

$$
\begin{aligned}
S'^{(q>0)}(t) &= t^2 r^{q-1} \sum_n e^{iqn\Omega\tau} \Upsilon_n e^{-in\Omega t} \\
&= \bar{t}^2 r^{q-1} \Upsilon(t-q\tau) = \Upsilon(t) \bar{t}^2 r^{q-1} e^{-i\tilde{\Phi}_q(t)} , \\
\tilde{\Phi}_q(t) &= \frac{e}{\hbar} \int\limits_{t-q\tau}^{t} dt' \Big(-U(t') \Big) ,
\end{aligned}
$$

Here we used, $k(E_n) \approx k(E) + n\Omega/v$, and $L/v = \tau \equiv h/\Delta$. Comparing Eqs.(A.21b) and (A.43) we see that,

$$
S\left(U(t), E\right) = \Upsilon(t) S_{in}\left(-U(t), E\right) . \tag{A.44}
$$

One can understand above equation as follows. The particle leaving a reservoir at time t has a phase $\Upsilon(t)$ induced by the oscillating potential. However to calculate a current we need to count particles leaving the cavity at the time t. If the particle leaving the cavity at time t spent in cavity q turns then it leaved the reservoir at time $t-q\tau$. Such a particle has a time-dependent phase $\Upsilon(t-q\tau)$. The common for all the amplitudes phase is irrelevant for the measurable quantities. Therefore, one can take out the largest time-dependent phase $\Upsilon(t)$. After such an artificial transformation the time-dependent phases become $\Upsilon^{\star}(t)\Upsilon(t-q\tau)$. This is exactly the phase which the particle spending q turns in the cavity would feel if the potential $-U(t)$ would be applied at the cavity instead to be applied at the reservoir.

A.2.4 Time-dependent current

Replacing in Eq. (A.10) the Floquet scattering matrix elements by the elements of $S_{in}(t,E)$ we obtain [compare to Eq. (3.65)]:

$$
I(t) = \frac{e}{h} \int\limits_0^{\infty} dE \sum_{n=-\infty}^{\infty} \{ f_0(E) - f_0(E_n) \} \int\limits_0^{\mathcal{T}} \frac{dt'}{\mathcal{T}} e^{in\Omega_0(t-t')} S_{in}^{*}(t',E) S_{in}(t,E) . \tag{A.45}
$$

Then substituting Eqs. (A.21b) into Eq. (A.45) and integrating over energy E we find the current as a sum of two terms $I^{(d)}$ and $I^{(nd)}$: [119]

$$I(t) = I^{(d)}(t) + I^{(nd)}(t)\,, \tag{A.46a}$$

$$I^{(d)}(t) = \frac{e^2}{h} T^2 \sum_{q=1}^{\infty} R^{q-1} \left\{U(t) - U(t - q\tau)\right\}\,, \tag{A.46b}$$

$$I^{(nd)} = \frac{eT^2}{\pi\tau} \Im \left\{ \sum_{s=1}^{\infty} \frac{\eta\left(s\frac{T_0}{T^*}\right)\left\{r\, e^{ik_F L}\right\}^s}{s} \sum_{q=1}^{\infty} R^{q-1} \left(e^{-i\Phi_s(t-q\tau)} - e^{-i\Phi_s(t)}\right) \right\}. \tag{A.46c}$$

Here $R = 1 - T$ is a reflection probability of a QPC, $k_F = \sqrt{2m_e\mu_0}/\hbar$, $k_B T^* = \hbar/(\pi\tau) = \Delta/(2\pi^2)$. In Eq. (A.46) the time τ is calculated for electrons with Fermi energy, $E = \mu_0$. Such an approximation is valid in zeroth order in $k_B T_0/\mu_0 \to 0$. The function $\eta(x) = x/\sinh(x)$ has appeared after an integration over energy:

$$\eta\left(2\pi^2 s \frac{k_B T_0}{\Delta}\right) = i\,\frac{2\pi s}{\Delta} \int_0^{\infty} dE\, f_0(E)\, e^{i2\pi \frac{E-\mu_0}{\Delta} s}. \tag{A.47}$$

Note that when we integrated over energy a term with $f_0(E_n)$ in Eq. (A.45) we made a shift $E_n \to E$ and expanded exponential factors in accordance with Eq. (A.25):

$$e^{iqk_n L} \approx e^{iqkL}\, e^{in\Omega_0\tau}\,,$$

where $k_n = k(E_n)$. The double sum appeared after substituting Eq. (A.21) into Eq. (A.45) we presented as follows:

$$\sum_{q=1}^{\infty}\sum_{p=1}^{\infty} A_q B_p = \sum_{q=1}^{\infty} A_q B_q + \sum_{q=1}^{\infty}\sum_{s=1}^{\infty} A_q B_{q+s} + \sum_{p=1}^{\infty}\sum_{s=1}^{\infty} A_{p+s} B_p\,.$$

In Eq. (A.46) we assume that the energy scale δE over which the reflection/transmission amplitude of a QPC changes significantly is much larger than the temperature, $\delta E \gg k_B T_0$, and take r and $\bar{t}$ at $E = \mu_0$. This is correct if $k_B T_0 \ll \mu_0$ since for a QPC $\delta E \sim E$ and only electrons with energies $E \sim \mu_0$ are relevant for transport.

The contribution $I^{(d)}$, which we will name as diagonal, arises due to interference of photon-assisted amplitudes corresponding to the spatial paths with the same length [the same index q in Eq. (A.21b)] which electron follows to propagating through the system. This contribution is temperature independent. Since we neglect inelastic processes the temperature can not be too high. The non-diagonal part, $I^{(nd)}$, is due to interference of photon-assisted amplitudes corresponding to different number of turns, $q_1 \neq q_2$. This part is suppressed by the temperature (at $T_0 \gtrsim T^*$) since it is a sum of contributions which oscillates strongly with an electron energy. Therefore, at high temperatures, $T_0 \gg T^*$, the only linear in cavity's potential part is present, $I(t) \approx I^{(d)}(t)$. While at $T_0 \ll T^*$ both parts, $I^{(d)}(t)$ and $I^{(nd)}(t)$, do contribute and the current is a non-linear function of $U(t)$.

The current $I(t)$ depends on a driving frequency Ω_0 periodically with period $\delta\Omega_0 = 2\pi/\tau$. The corresponding periodicity is governed by the time τ of a single turn around the cavity. If $\tau = n\mathcal{T}$, hence $\Omega_0 = n\delta\Omega_0$, then the oscillating potential $U(t) = U(t + \mathcal{T})$ does not change the phase of electrons contributing to the current. Such electrons enter the cavity, make several q turns, and escape the cavity. Therefore, they visit a cavity for a finite time period $\delta t = q\tau = qn\mathcal{T}$. These electrons see an effectively stationary cavity since the time-dependent phase is zero, $\Phi_q(t) = 0$. In such a case the current does not arise, $I(t) = 0$. At frequencies different from these particular values the phase accumulated by an electron within a cavity becomes dependent on time. Consequently, in accordance with the Friedel sum rule [12], the charge accumulated within a cavity becomes dependent on time that, in turn, causes an appearance of a time dependent current, $I(t) \neq 0$.

A.2.5 High-temperature current

Since at $T_0 \gg T^*$ the current, $I(t) \approx I^{(d)}(t)$, is linear in potential, we can introduce a frequency dependent response function (conductance),

$$G_l^{(d)} = \frac{I_l^{(d)}}{U_l}\,, \tag{A.48}$$

where U_l and I_l are Fourier coefficients for potential and current respectively:

$$U(t) = \sum_{l=-\infty}^{\infty} U_l\, e^{-il\Omega_0 t}\,, \quad I^{(d)}(t) = \sum_{l=-\infty}^{\infty} I_l^{(d)} e^{-il\Omega_0 t}\,, \tag{A.49}$$

Taking into account that

$$U(t - q\tau) = \sum_{l=-\infty}^{\infty} U_l\, e^{il\Omega_0 q\tau}\, e^{-il\Omega_0 t}\,,$$

we calculate from Eq. (A.46b):

$$G_l^{(d)} = \frac{e^2}{h} T \frac{1 - e^{il\Omega_0\tau}}{1 - R\, e^{il\Omega_0\tau}}\,. \tag{A.50}$$

The ac conductance $G_l^{(d)}$ shows a strong non-linear dependence on the frequency Ω_0 of a drive. The frequency affects both the magnitude and the phase of a response function. In particular, at $0 < l\Omega_0\tau \mod 2\pi < \pi$ the response is capacitive-like. While at $\pi < l\Omega_0\tau \mod 2\pi < 2\pi$ it is inductive-like. It is interesting that at $l\Omega_0\tau \mod 2\pi = \pi$ the response is purely ohmic, $G_l^{(d)} = 1/R_q$ (for R_q see below).

In general to model a mesoscopic system under consideration via an equivalent electric circuit one needs to use some frequency-dependent element. However at small frequencies, $\Omega_0\tau \ll 1$, one can model it as a capacitance $C_q^{(d)}$, a resistance $R_q^{(d)}$, and an inductance $L_q^{(d)}$ connected in series, Fig. A.2. The conductance of such a circuit equals to

$$\frac{1}{G(\omega)} = R_q + i\left\{\frac{1}{\omega C_q} - \omega L_q\right\}. \tag{A.51}$$

Comparing it to Eq. (A.50) at

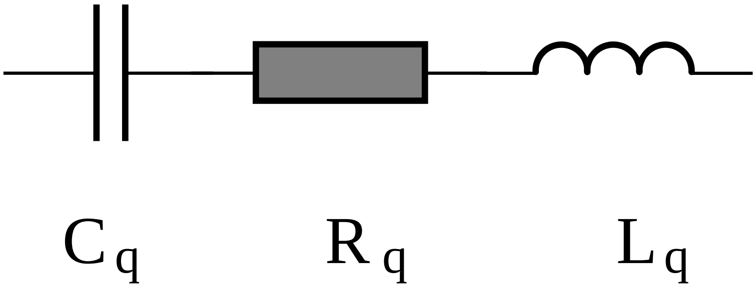

Figure A.2: An equivalent electrical circuit to model a low-frequency response of a quantum capacitor.

$$\omega = l\Omega_0 \ll \frac{T}{\tau}\,, \tag{A.52}$$

we find:

$$C_q^{(d)} = \frac{e^2}{\Delta}\,, \quad R_q^{(d)} = \frac{h}{e^2}\left(\frac{1}{T} - \frac{1}{2}\right), \quad L_q^{(d)} = \frac{h^2}{12e^2\Delta}\,, \tag{A.53}$$

where $\Delta = h/\tau \ll \mu_0$ is a level spacing in the isolated cavity. The upper index (d) indicates a high-temperature regime.

At lower temperatures, $T_0 \lesssim T^*$, both parts, $I^{(d)}$ and $I^{(nd)}$, contribute to a current. The current $I^{(nd)}$, Eq. (A.46c), is a non-linear function of both the magnitude and the frequency of a driving potential $U(t)$.

A.2.6 Linear response regime

At small amplitude of an oscillating potential,

$$eU_l \ll l\hbar\Omega_0\,, \tag{A.54}$$

one can simplify the expression for $I^{(nd)}$. At zero temperature we use Eq. (A.46c) and find a compact expression valid at arbitrary frequency. On the other hand, at small driving frequencies, $\Omega_0\tau \ll 1$, it is more convenient to expand directly in the expression for the scattering matrix, Eqs. (A.21), and then to calculate the current $I(t)$, Eq. (A.45). In such a way one can obtain a simple expression allowing us to analyze a temperature dependence of a current.

A.2.6.1 Zero-temperature linear response current

We expand exponents depending on Φ_s in Eq. (A.46c) up to linear in U_l terms. Then, taking into account that at zero temperature $\eta(0) = 1$, one can sum up over s and q. After that we can calculate $G_l^{(nd)} = I_l^{(nd)}/U_l$.

We use the following transformation taking into account that $U_l = U_{-l}^*$, since $U(t)$ is real:

$$
\begin{aligned}
I(t) &= \Im \sum_{l=-\infty}^{\infty} U_l\, G'_l\, e^{-il\Omega_0 t} \\
&= \sum_{l=-\infty}^{\infty} \frac{U_l\, G'_l\, e^{-il\Omega_0 t} - U_l^*\, G'^*_l\, e^{il\Omega_0 t}}{2i} = \sum_{l=-\infty}^{\infty} I_l\, e^{-il\Omega_0 t}\,, \\
I_l &= U_l\, G_l\,, \quad G_l = \frac{G'_l - G'^*_{-l}}{2i}\,.
\end{aligned}
$$

Then the total zero-temperature ac conductance $G_l = G_l^{(d)} + G_l^{(nd)}$ is found to be,

$$
G_l = G_l^{(d)} \left\{ 1 + \frac{i}{l\Omega_0 \tau} \ln \left(\frac{1 + R\, e^{2il\Omega_0 \tau} - 2\sqrt{R}\, e^{il\Omega_0 \tau} \cos(\chi_F)}{1 + R - 2\sqrt{R}\, \cos(\chi_F)} \right) \right\}. \tag{A.55}
$$

Here we use the following notation: $r = \sqrt{R}\, e^{i\chi_r}$, $\chi_F = k_F L + \chi_r - 2\pi e U_0/\Delta$, where $|eU_0| \ll \mu_0$ is an average value of an oscillating potential. We stress that Eq. (A.55) is valid for small amplitude but arbitrary frequency of an oscillating potential.

To get the parameters of a low-frequency equivalent circuit (at low temperatures we denote them as C_q, R_q, and L_q) we evaluate Eq. (A.55) in the limit of

$l\Omega_0\tau \to 0$ and obtain after the comparison with Eq. (A.51):

$$C_q = \frac{e^2}{\Delta}\frac{T}{1+R-2\sqrt{R}\,\cos(\chi_F)} \equiv e^2\nu(\mu_0)\,, \quad R_q = \frac{h}{2e^2},$$
$$L_q = \frac{h^2\nu(\mu_0)}{12e^2}\left\{1 + \frac{8R - 2(1+R)\sqrt{R}\,\cos(\chi_F) - 4R\,\cos^2(\chi_F)}{T^2}\right\}. \tag{A.56}$$

Here $\nu(E) = i/(2\pi)S(E)\partial S^*/\partial E$ is the DOS of a stationary cavity coupled to a linear edge state [for $S(E)$ see Eq. (A.33)].

A.2.6.2 Low-frequency linear response current

At small frequency,

$$\Omega_0\tau \ll 1\,, \tag{A.57}$$

the Floquet scattering matrix, Eq. (A.21), can be expressed in terms of a stationary scattering matrix $S(E)$, Eq. (A.33), calculated at $k(U_0) = \sqrt{2m_eE}/\hbar - 2\pi eU_0/(L\Delta)$. To this end we expand $U'(t') = U(t') - U_0$, entering equation for $\Phi_q(t)$ in Eq. (A.21b), in powers of $t' - t$,

$$U'(t') \approx U'(t) + \left(t' - t\right)\frac{dU'(t)}{dt} + \frac{(t'-t)^2}{2}\frac{d^2U'(t)}{dt^2}\,, \tag{A.58}$$

and integrate over t'. Then expanding corresponding exponents we calculate up to linear in $U'(t)$ and quadratic in Ω_0 terms:

$$S_{in}(t,E) \approx S(U_0,E) - eU'(t)\frac{\partial S(U_0,E)}{\partial E} - \frac{i\hbar}{2}\frac{edU'(t)}{dt}\frac{\partial^2 S(U_0,E)}{\partial E^2}$$
$$+\frac{\hbar^2}{6}\frac{ed^2U'(t)}{dt^2}\frac{\partial^3 S(U_0,E)}{\partial E^3}\,. \tag{A.59}$$

Note that first three terms in the right-hand side of equation above can be found from adiabatic expansion Eq. (A.13) if one expands the frozen scattering matrix up to linear in $U'(t)$ terms,

$$S(t,E) \equiv S\,(U(t),E) \approx S\,(U_0,E) + U'(t)\,\frac{\partial S(U_0,E)}{\partial U_0}\,,$$

discard $\sim\ \Omega_0^2$ terms, and take into account that in our model $\partial S/\partial U_0 = -e\,\partial S/\partial E$.

Substituting Eq. (A.59) into Eq. (A.45), where in addition we expand,

$$f_0(E) - f_0(E_n) \approx -\frac{\partial f_0}{\partial E}\,n\hbar\Omega_0 - \frac{\partial^2 f_0}{\partial E^2}\,\frac{(n\hbar\Omega_0)^2}{2}\,, \tag{A.60}$$

we find a low-frequency conductance:

$$\begin{aligned} G_l &= \int\limits_0^\infty dE\left(-\frac{\partial f_0(E)}{\partial E}\right) G_l(E)\,, \\ G_l(E) &= -ie^2 l\Omega_0\,\nu(E) + e^2 h\,\frac{(l\Omega_0)^2}{2}\,\nu^2(E) \\ &\quad -ie^2h^2\,\frac{(l\Omega_0)^3}{6}\left\{\frac{1}{8\pi^2}\,\frac{\partial^2\nu(E)}{\partial E^2} - \nu^3(E)\right\}. \end{aligned} \tag{A.61}$$

At zero temperature Eq. (A.61) leads to parameters of an equivalent electric circuit given in Eq. (A.56). It is less evident but still true that at high temperatures ($T_0 \gg T^*$) from Eq. (A.61) one can find parameters given in Eq. (A.53).

A.2.7 Non-linear low-frequency regime

In the limit of low frequencies, Eq. (A.57), one can go beyond the linear response regime, Eq. (A.54). Substituting Eq. (A.58) into Eq. (A.21b) and expanding up to terms of order Ω_0^2 we calculate the scattering matrix as follows:

$$
\begin{aligned}
S_{in}(t,E) &= S(t,E) + \frac{i\hbar}{2}\frac{\partial^2 S(t,E)}{\partial t \partial E} + \frac{\hbar^2}{6}\frac{ed^2U(t)}{dt^2}\frac{\partial^3 S(t,E)}{\partial E^3} \\
&\quad - \frac{\hbar^2}{8}\left(\frac{edU(t)}{dt}\right)^2 \frac{\partial^4 S(t,E)}{\partial E^4} + \mathcal{O}\left\{(\Omega_0\tau)^3\right\}.
\end{aligned}
\qquad (A.62)
$$

Remind that the frozen scattering matrix is: $S(t,E) = S\left(U(t),E\right)$. To calculate it one can use Eq. (A.33) and replace $kL \to kL - 2\pi eU(t)/\Delta$. Note Eq. (A.59) is nonlinear in U_0 but linear in $U'(t) = U(t) - U_0$. In contrast Eq. (A.62) is nonlinear in a full time-dependent potential $U(t)$.

Substituting Eqs. (A.62) and (A.60) into Eq. (A.45) we calculate a low-frequency current as follows: [119]

$$
I(t) = \int_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E}\right)\left\{\mathcal{J}^{(1)}(t,E) + \mathcal{J}^{(2)}(t,E) + \mathcal{J}^{(3)}(t,E)\right\}, \qquad (A.63a)
$$

$$
\mathcal{J}^{(1)}(t,E) = e^2\, \nu(t,E)\frac{dU(t)}{dt}, \qquad (A.63b)
$$

$$
\mathcal{J}^{(2)}(t,E) = -\frac{e^2 h}{2}\frac{\partial}{\partial t}\left\{\nu^2(t,E)\frac{dU(t)}{dt}\right\}, \qquad (A.63c)
$$

$$
\begin{aligned}
\mathcal{J}^{(3)}(t,E) &= -\frac{e^2h^2}{6}\frac{\partial^2}{\partial t^2}\left\{\left(\frac{1}{8\pi^2}\frac{\partial^2\nu(t,E)}{\partial E^2} - \nu^3(t,E)\right)\frac{dU(t)}{dt}\right\} \\
&\quad - \frac{e^3h^2}{96\pi^2}\frac{\partial}{\partial t}\left\{\frac{\partial^3\nu(t,E)}{\partial E^3}\left(\frac{dU(t)}{dt}\right)^2\right\},
\end{aligned}
\qquad (A.63d)
$$

where the frozen DOS is:

$$\nu(t,E) = \frac{i}{2\pi} S(t,E) \frac{\partial S^*(t,E)}{\partial E} = \frac{1}{\Delta} \left\{ 1 + 2\Re \sum_{q=1}^{\infty} r^q e^{iq[kL - 2\pi e U(t)/\Delta]} \right\}. \quad \text{(A.64)}$$

Note in Eq. (A.63) we used $\partial \nu/\partial t = -e(dU/dt)(\partial \nu/\partial E)$, since the DOS depends on time via an oscillating uniform potential only.

To illustrate the physical meaning of Eq. (A.63) it is instructive to rewrite this equation. Integrating over energy by parts one can represent it in the form of the continuity equation for a charge current,

$$I(t) + \frac{\partial Q(t)}{\partial t} = 0\,, \quad \text{(A.65a)}$$

$$Q(t) = e \int_0^{\infty} dE\, f_0(E)\, \nu_{dyn}(t,E)\,, \quad \text{(A.65b)}$$

$$\begin{aligned} \nu_{dyn}(t,E) &= \nu(t,E) - \frac{h}{2} \frac{\partial \nu^2(t,E)}{\partial t} + \frac{h^2}{6} \frac{\partial^2 \nu^3(t,E)}{\partial t^2} \\ &\quad - \frac{h^2}{96\pi^2} \frac{\partial^2}{\partial E^2} \left\{ 2 \frac{\partial^2 \nu(t,E)}{\partial t^2} - \frac{\partial^2 \nu(t,E)}{\partial E^2} \left(\frac{dU}{dt} \right)^2 \right\}. \end{aligned} \quad \text{(A.65c)}$$

Here $Q(t)$ is a charge accumulated on a mesoscopic capacitor, $\nu_{dyn}(t,E)$ can be called as *a dynamical density of states*.

The dynamical DOS takes into account a retardation effect, i.e., a finiteness of a time spend by an electron inside a capacitor. As a result the charge $Q(t)$ accumulated on a capacitor depends on the frequency of a drive. At small driving frequencies, $\Omega_0 \to 0$, such a dependence (up to terms of order Ω_0^2) can be accounted by introducing an effective resistance R_q connected in series with a capacitance C_q. In the linear response regime these quantities are constant parameters, see Eq.(A.56) for low- and Eq. (A.53) for high-temperature regimes. In the non-linear regime these parameters become dependent on a driving potential, i.e., the capacitor is characterized by a non-linear dependence of a charge

Q on the potential drop U_C and the resistor has a non-linear current-voltage (I-V) characteristic. In such a case it is more convenient to introduce the differential parameters, the differential capacitance, $C_\partial(U_C) = \partial Q(U_C)/\partial U_C$, and the differential resistance, $R_\partial(V) = \partial V/\partial I(V)$. In terms of these quantities the current $I(t)$ flowing into an equivalent electrical circuit subject to the potential $U(t) = U_C + V$ reads as follows (at $\Omega_0 \to 0$): [119]

$$I(t) = C_\partial \frac{dU}{dt} - R_\partial C_\partial \frac{\partial}{\partial t}\left(C_\partial \frac{dU}{dt}\right) . \tag{A.66}$$

Comparing Eqs. (A.63) and (A.66) we find:

$$C_\partial(t) = e^2 \int\limits_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E}\right) \nu(t,E) , \tag{A.67a}$$

$$R_\partial(t) = \frac{h}{2e^2} \frac{\int\limits_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{\partial}{\partial t}\left(\nu^2(t,E) \frac{dU}{dt}\right)}{\int\limits_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \nu(t,E) \int\limits_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{\partial}{\partial t}\left(\nu(t,E) \frac{dU}{dt}\right)} . \tag{A.67b}$$

We conclude, in the non-linear low-frequency regime the DOS defines an intrinsic capacitance of a mesoscopic sample (which is coupled in series with a geometrical one if any). That is in accordance with Ref. [112] where the linear response regime was considered. The difference consists in the following: In the non-linear regime the DOS is related to the differential capacitance while in the linear response regime the DOS is related to an ordinary capacitance. Another difference we found concerns the effective resistance. In the linear regime for our system it has a universal value at zero temperature, $R_q = h/(2e^2)$, see, Ref. [112] and Eq. (A.56). While in the non-linear regime R_∂ becomes dependent on the sample's properties (on the DOS) and the potential $U(t)$.

Note the third contribution in Eq. (A.63), $\mathcal{J}^{(3)}$, defines a differential inductance $L_\partial(t) = \partial\Phi/\partial I$ (where Φ is a magnetic flux). The corresponding equation can be calculated straightforwardly. We do not show it because it is lengthy.

A.2.8 Transient current caused by a step potential

Let the potential changes abruptly at some time moment t_0:

$$U(t) = \begin{cases} 0\,, & t < t_0\,, \\ U_0\,, & t > t_0\,. \end{cases} \tag{A.68}$$

Strictly speaking we suppose that the potential U jumps from zero to U_0 for some time interval $\delta t \gg \hbar\mu_0^{-1}$. The last inequality allows us to use the scattering matrix, Eq. (A.21), valid if all the relevant energy scales much smaller than the Fermi energy μ_0. On the other hand δt should be small enough compared with intrinsic time scales (τ, RC-time, etc) to speak about abrupt change.

Using Eq. (A.21) in Eq. (A.45) we represent a current as a sum of two contributions [see Eq. (3.138)]:

$$I(t) = I^{(d)}(t) + I^{(nd)}(t)\,, \tag{A.69a}$$

$$I^{(d)}(t) = -i\,\frac{e}{2\pi}\sum_{q=0}^{\infty} S^{(q)}(t)\,\frac{\partial S^{(q)*}(t)}{\partial t}\,, \tag{A.69b}$$

$$I^{(nd)}(t) = \frac{e}{\pi\tau}\Im\sum_{s=1}^{\infty}\frac{\eta\left(s\,\frac{T}{T^*}\right)e^{isk_FL}}{s}C_s(t)\,,\quad C_s(t) = \sum_{q=0}^{\infty} S^{(q+s)}(t)S^{(q)*}(t)\,. \tag{A.69c}$$

These equations are equivalent to equations (A.46).

Note originally Eq. (A.45) for a time-dependent current was derived within the Floquet scattering theory. However it can be cast into the form which does not appeal to periodicity of a drive, see general Eq. (3.67) and particular Eqs. (A.46) and (A.69) as examples. Then one can use this equation to calculate aperiodic current also. Therefore, we use Eq. (A.69) to analyze a transient current caused by the potential Eq. (A.68).

A.2.8.1 High-temperature current

At high temperatures, $T_0 \gg T^*$, only the diagonal current $I^{(d)}(t)$ survives. For the potential $U(t)$, Eq. (A.69), this current is (for $t \geq t_0$): [119]

$$I^{(d)}(t) = \frac{e^2 U_0}{\Delta} \frac{T}{\tau} R^{N(t)}, \tag{A.70}$$

where $N(t) = [t/\tau]$ is an integer part of the ratio t/τ.

As we see at high temperatures the current, $I(t) = I^{(d)}(t)$, decays in time in a step-like manner: It is constant over the time interval τ and it exponentially decreases with increase time. Over the time scale larger than τ one can write $I(t) \sim I_0 e^{-(t-t_0)/\tau_D}$, where $I_0 = e^2 U_0 T/h$ and

$$\tau_D = \frac{\tau}{\ln\left(\frac{1}{R}\right)}, \tag{A.71}$$

is a decay time. At small transparency of a QPC, $T \to 0$, the decay time is $\tau_D \approx \tau/T$.

A.2.8.2 Low-temperature current

At lower temperatures, $T_0 \lesssim T^*$, the current $I(t) = I^{(d)}(t) + I^{(nd)}(t)$ still decays in time but in addition it shows fast oscillations with a period $h/(eU_0)$. To calculate $I^{(nd)}(t)$ we need to know $C_s(t)$, Eq. (A.69). We substitute Eq. (A.68) into Eq. (A.21b) and find,

$$S^{(q)} = \begin{cases} r, & q = 0, \\ t^2 r^{q-1} \times \begin{cases} e^{-i\frac{eU_0}{\hbar}\tau q}, & 1 \leq q \leq N, \\ e^{-i\frac{eU_0}{\hbar}t}, & N+1 \leq q. \end{cases} \end{cases} \tag{A.72}$$

Then we calculate,

$$S^{(s)}S^{(0)*} = -Tr^s \times \begin{cases} e^{-i\frac{eU_0}{\hbar}\tau s}\,, & 1 \le s \le N\,,\ \ 1 \le N\,, \\ e^{-i\frac{eU_0}{\hbar}t}\,, & N+1 \le s\,,\ \ \forall N\,, \end{cases} \tag{A.73}$$

and

$$\begin{aligned} S^{(q+s)}S^{(q)*} &= T^2 R^{q-1} r^s \\ &\times \begin{cases} e^{-i\frac{eU_0}{\hbar}\tau s}\,, & 1 \le q \le N-s\,,\ 1 \le s \le N-1\,,\ 2 \le N\,, \\ e^{i\frac{eU_0}{\hbar}(\tau q - t)}\,, & \begin{cases} N-s+1 \le q \le N\,,\ s \le N\,,\ 1 \le N\,, \\ 1 \le q \le N\,,\ N+1 \le s\,,\ 1 \le N\,, \end{cases} \\ 1\,, & N+1 \le q\,,\ \forall s\,,\ \forall N\,. \end{cases} \end{aligned} \tag{A.74}$$

Finally we find,

$$\begin{aligned} C_s &= \gamma_N(t)\, T\, r^s R^N \left\{ 1 - \frac{\theta(N-s)}{\left(R\, e^{i\frac{eU_0}{\hbar}\tau}\right)^s} \right\} - \chi(t)\theta(s-N-1)\,, \\ \gamma_N(t) &= 1 - T\, e^{-i\frac{eU_0}{\hbar}t} \frac{e^{i\frac{eU_0}{\hbar}\tau(N+1)}}{1 - R\, e^{i\frac{eU_0}{\hbar}\tau}}\,, \quad \chi(t) = e^{-i\frac{eU_0}{\hbar}t} \frac{1 - e^{i\frac{eU_0}{\hbar}\tau}}{1 - R\, e^{i\frac{eU_0}{\hbar}\tau}}\,. \end{aligned} \tag{A.75}$$

Then we calculate from Eq. (A.69c):

$$I^{(nd)}(t) = \frac{eT}{\pi\tau} \int\limits_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \Im\left\{ J^{(1)}(t,E) + J^{(2)}(t,E) + J^{(3)}(t,E) \right\}\,, \tag{A.76a}$$

$$J^{(1)}(t,E) = -R^N \ln\left(1 - re^{ikL}\right)\left\{1 - Te^{-i2\pi\frac{eU_0}{\Delta}\frac{t}{\tau}}\frac{e^{i2\pi\frac{eU_0}{\Delta}(N+1)}}{1 - Re^{i2\pi\frac{eU_0}{\Delta}}}\right\}, \tag{A.76b}$$

$$J^{(2)}(t,E) = -R^N \sum_{s=1}^{N\geq 1} \frac{e^{is\left(kL - 2\pi\frac{eU_0}{\Delta}\right)}}{s}\frac{r^s}{R^s}\left\{1 - Te^{-i2\pi\frac{eU_0}{\Delta}\frac{t}{\tau}}\frac{e^{i2\pi\frac{eU_0}{\Delta}(N+1)}}{1 - Re^{i2\pi\frac{eU_0}{\Delta}}}\right\}, \tag{A.76c}$$

$$J^{(3)}(t,E) = (-1)\sum_{s=N+1}^{\infty}\frac{e^{iskL}\,r^s}{s}\,e^{-i2\pi\frac{eU_0}{\Delta}\frac{t}{\tau}}\frac{1 - e^{i2\pi\frac{eU_0}{\Delta}}}{1 - Re^{-i2\pi\frac{eU_0}{\Delta}}}. \tag{A.76d}$$

Note, the equations above differ from Eq. (A.69c) in the following. Using Eq. (A.47) we reintroduced an integration over energy E and then used the following identity:

$$\int_0^{\infty} dE\, f_0(E)\, e^{\frac{i2\pi s}{\Delta}(E-\mu_0)} = \frac{\Delta}{i2\pi s}\int_0^{\infty} dE\left(-\frac{\partial f_0}{\partial E}\right) e^{\frac{i2\pi s}{\Delta}(E-\mu_0)}. \tag{A.77}$$

The current $I^{(nd)}(t)$, Eq. (A.76), can be greatly simplified if $eU_0 = n\Delta$:

$$J^{(1)}(t,E) = -R^N \ln\left(1 - re^{ikL}\right)\left\{1 - e^{-i2\pi\frac{eU_0}{\Delta}\frac{t}{\tau}}\right\}, \tag{A.78a}$$

$$J^{(2)}(t,E) = (-1)R^N \sum_{s=1}^{N\geq 1}\frac{e^{iskL}}{s}\frac{r^s}{R^s}\left\{1 - e^{-i2\pi\frac{eU_0}{\Delta}\frac{t}{\tau}}\right\}, \tag{A.78b}$$

$$J^{(3)}(t,E) = 0. \tag{A.78c}$$

If in addition the temperature is low [such that only $E = \mu_0$ is relevant in Eq. (A.76a)] and the Fermi level lies exactly in the middle between the cavity's levels, $r^s e^{isk_F L} = (-1)^s R^{s/2}$, then the total current, $I(t) = I^{(d)}(t) + I^{(nd)}(t)$, reads as follows,

$$I(t) = \frac{Q}{\tau} T R^{N(t)} \left\{ 1 + \frac{\sin\left(2\pi n \frac{t}{\tau}\right)}{\pi n} \zeta(t) \right\},$$

$$\zeta(t) = -\ln\left(1 + \sqrt{R}\right) - T_0 (N-1) \sum_{s=1}^{N} \frac{(-1)^s}{s R^{\frac{s}{2}}}, \tag{A.79a}$$

$t < \tau$: (A.79b)

$$I(t) = \frac{Q}{\tau} T \left\{ 1 - \frac{\sin\left(2\pi n \frac{t}{\tau}\right)}{\pi n} \ln\left(1 + \sqrt{R}\right) \right\},$$

$t \gg \tau$: (A.79c)

$$I(t) \approx \frac{Q}{\tau} T R^{N} \left\{ 1 - \frac{\sin\left(2\pi n \frac{t}{\tau}\right)}{\pi n} \ln \sqrt{R} \right\}.$$

A.2.8.3 Emitted charge

Let us calculate a charge,

$$Q = \int_0^\infty dt \left\{ I^{d}(t) + I^{(nd)}(t) \right\}, \tag{A.80}$$

emitted from the cavity under the action of the potential $U(t)$, Eq. (A.68). To this end we integrate a current $I(t)$ over a time interval of duration τ (over which N is constant) and then sum over N from zero to infinity,

$$Q = \sum_{N=0}^{\infty} \int_{N\tau}^{(N+1)\tau} dt \left\{ I^{(d)}(t) + I^{(nd)}(t) \right\}. \tag{A.81}$$

After the simple algebra we calculate,

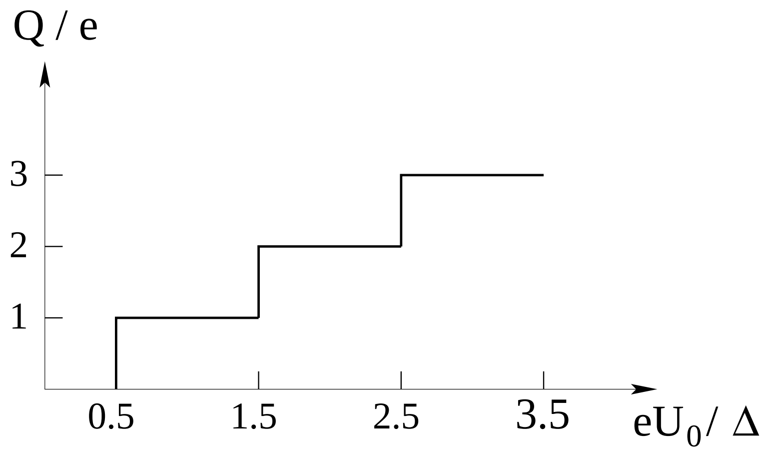

Figure A.3: The dependence of an emitted charge Q on the potential step height U_0 at zero temperature. The Fermi level is centered in the middle of cavity's levels.

$$Q = \frac{e^2 U_0}{\Delta} + \frac{e}{\pi} \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E} \right) \Im \ln \left(\frac{1 - r e^{i\left(kL - 2\pi \frac{eU_0}{\Delta}\right)}}{1 - r e^{ikL}} \right) . \tag{A.82}$$

At $T_0 \gg T^*$ above equation gives: $Q = e^2 U_0 / \Delta$.

At lower temperatures we consider the limit $T \to 0$ when the density of states can be approximated as a sum of delta-function peaks centered at eigenenergies E_n of an isolated cavity. At $\mu_0 \gg \Delta$ the spectrum near the Fermi energy is equidistant, $E_n = E_0 + n\Delta$. Then we use in Eq. (A.82),

$$\frac{1}{\pi} \Im \ln \left(1 - e^{i 2\pi \frac{E - E_0}{\Delta}} \right) = -\frac{1}{2} + \left\{ \left\{ \frac{E - E_0}{\Delta} \right\} \right\} ,$$

where $\{\{X\}\}$ is a fractional part of X, and find:

$$Q = \frac{e^2 U_0}{\Delta} + e \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E} \right) \left(\left\{ \left\{ \frac{E - E_0 - eU_0}{\Delta} \right\} \right\} - \left\{ \left\{ \frac{E - E_0}{\Delta} \right\} \right\} \right) . \tag{A.83}$$

From this equation it follows that at zero temperature the emitted (absorbed) charge is quantized, Fig. A.3. For instance, if μ_0 is centered exactly in the middle of two cavity's levels, then we get:

$$Q = e\left[\left[\frac{1}{2} + \frac{eU_0}{\Delta}\right]\right], \quad k_BT_0 = 0\,, \quad T \to 0\,, \tag{A.84}$$

where $[[X]]$ is an integer part of X.

At finite but small temperatures, $k_BT_0 \ll \Delta$, the deviation δQ from this quantized value is:

$$\delta Q = \mathrm{sgn}(1-2v_0)\,\frac{1 - e^{(|1-2v_0|-1)\frac{\Delta}{k_BT_0}}}{e^{|1-2v_0|\frac{\Delta}{2k_BT_0}} + 1}\,, \tag{A.85}$$

where $v_0 = \{\{eU_0/\Delta\}\}$ lies within the following interval: $0 \le v_0 < 1$, $\mathrm{sgn}(X) = +1$ for $X > 0$ and -1 for $X < 0$. The function $\delta Q(v_0)$ has the following asymptotics:

$$\delta Q(v_0) = \begin{cases} v_0\,\frac{2\Delta}{k_BT_0}\,e^{-\frac{\Delta}{2k_BT_0}}\,, & v_0 \to 0\,, \\ \pm\,\frac{1}{2}\,, & v_0 = \frac{1}{2} \mp 0, \\ -(1-v_0)\,\frac{2\Delta}{k_BT_0}\,e^{-\frac{\Delta}{2k_BT_0}}\,, & v_0 \to 1\,. \end{cases} \tag{A.86}$$

The violation of a charge quantization is exponentially small at low temperatures unless we are at the transition point from one plateau to another.

Next we consider how the quantization of an emitted charge is affected by the finiteness of QPC's transmission coefficient. For the scattering amplitude $S(E)$, Eq. (A.33), at $T \ll 1$ the density of states can be approximated by the sum of Breit-Wigner resonances [120] with width $\Gamma = T\Delta/(4\pi) \ll \Delta$,

$$\nu(E) = \frac{1}{\pi}\sum_n \frac{\Gamma}{(E - E_n)^2 + \Gamma^2}\,. \tag{A.87}$$

Then at zero temperature, $k_B T_0 = 0$, the deviation δQ from Eq. (A.84) reads:

$$\delta Q = -\theta(2\nu_0 - 1) + \frac{1}{2}\left\{1 - \frac{\arctan\frac{(1-2\nu_0)\,2\pi}{T}}{\arctan\frac{2\pi}{T}}\right\}, \tag{A.88}$$

where $\theta(X)$ is the Heaviside theta-function equal to zero for $X < 0$ and unity for $X > 0$. The asymptotics for $\delta Q(\nu_0)$ are following:

$$\delta Q(\nu_0) = \begin{cases} \nu_0 \frac{T}{\pi^2}\,, & \nu_0 \to 0\,, \\ \pm\frac{1}{2}\,, & \nu_0 = \frac{1}{2} \mp 0\,, \\ -(1-\nu_0)\frac{T}{\pi^2}\,, & \nu_0 \to 1\,. \end{cases} \tag{A.89}$$

In contrast to the temperature, the effect of a finite QPC transmission is more crucial, since δQ is linear in transmission T.

Appendix B

Mesoscopic capacitor as a particle emitter

In the below we consider the model introduces in Sec. A.2.1 in the large amplitude ($\sim \Delta$) adiabatic regime at low temperatures. We are particularly interested in the case of a cavity with small transparency, $T \to 0$. In this case electrons and holes emitted by the cavity are well separated in time and we can treat them as separate particles in quite intuitive manner. On the other hand, as we show, they are subject to the Pauli exclusion principle and are able to interfere. Therefore, they are quantum particles.

B.1 Quantized emission regime

First we show that at zero temperature the current generated by the capacitor slowly driven by the large-amplitude periodic potential, $U(t) = U(t + \mathcal{T})$, consists of a series of positive and negative pulses corresponding to the emission of electrons and holes. When we speak about an electron emitted by the cavity we mean the following. With increasing the potential energy $eU(t)$ the position of quantum levels in the cavity changes. One of the occupied levels can rises above the Fermi level and an electron occupying this level leaves the cavity. Therefore, the stream of electrons in the linear edge state (which the capacitor is connected to) is increased by one: The electron is emitted. In contrast, when $eU(t)$ decreases, some empty level can sink below the Fermi level. Then one electron enters the cavity leaving a hole in the stream of electrons in the linear edge state: The hole is emitted. Therefore, the quantum capacitor can serve as a single particle source (SPS). Since after the period $\mathcal{T}$ the charge on the capacitor returns to its initial value, such an SPS emits the same number electrons and holes, i.e., it is a source of quantized ac currents. [115].

Let the capacitor, see Fig. A.1, is driven by the potential

$$U(t) = U_0 + U_1 \cos(\Omega_0 t + \varphi) \, . \tag{B.1}$$

In the adiabatic regime to describe a capacitor it is enough to know its frozen scattering amplitude. In our model $S(t,E)$ is given in Eq. (A.33b) where we need to replace $kL \to kL - 2\pi eU(t)/\Delta$. If in addition the temperature is zero,

$$k_B T_0 = 0\,, \tag{B.2}$$

then we need the scattering amplitude at $E = \mu_0$ only. We rewrite $S(t) = S(t,\mu_0)$ as follows,

$$S(t) = e^{i\theta_r}\frac{\sqrt{1-T} - e^{i\phi(t)}}{1 - \sqrt{1-T}e^{i\phi(t)}}\,, \tag{B.3}$$

where θ_r is a phase of the reflection amplitude of a QPC connecting the SPS to the linear edge state, $r = \sqrt{R}\,e^{i\theta_r}$, $\phi(t) = \phi(\mu_0) - 2\pi eU(t)/\Delta$ is a phase accumulated by an electron with energy $E = \mu_0$ during one trip along the cavity, $\phi(\mu_0) = \theta_r + k_F L$.

To proceed analytically we assume that the amplitude U_1 of an oscillating potential is chosen in such a way that during a period only one level of the SPS crosses the Fermi level. The time of crossing t_0 is defined by the condition $\phi(t_0) = 0 \mod 2\pi$. Introducing the deviation of a phase from its resonance value, $\delta\phi(t) = \phi(t) - \phi(t_0)$, we obtain the scattering amplitude in the limit

$$T \to 0\,, \tag{B.4}$$

as follows:

$$S(t) = -e^{i\theta_r}\frac{T + 2i\delta\phi(t)}{T - 2i\delta\phi(t)} + \mathcal{O}(T^2)\,. \tag{B.5}$$

We keep only terms in the leading order in T.

There are two time moments when resonance conditions occur (two times of crossing). First time is when the level rises above the Fermi level and the second one is when the level sinks below the Fermi level. We denote these times as $t_0^{(-)}$ and $t_0^{(+)}$, respectively. At a time $t_0^{(-)}$ one electron is emitted by the cavity, while at a time $t_0^{(+)}$ one electron enters the cavity, a hole is emitted.

We suppose that the constant part of the potential U_0 accounts for a detuning of the nearest electron level E_n in the SPS from the Fermi level. Then the resonance times can be found from the following equation:

$$E_n + eU\left(t_0^{(\mp)}\right) = \mu_0 \quad \Rightarrow \quad U_0 + U_1 \cos\left(\Omega_0 t_0^{(\mp)} + \varphi\right) = 0\,. \tag{B.6}$$

For $|eU_0| < \Delta/2$ and $|eU_0| < |eU_1| < \Delta - |eU_0|$ we find the resonance times,

$$t_0^{(\mp)} = \mp t_0^{(0)} - \frac{\varphi}{\Omega_0}\,, \qquad t_0^{(0)} = \frac{1}{\Omega_0}\arccos\left(-\frac{U_0}{U_1}\right). \tag{B.7}$$

The deviation from the resonance time, $\delta t^{(\mp)} = t - t_0^{(\mp)}$, can be related to a deviation from the resonance phase, $\delta\phi^{(\mp)} = \mp M\Omega_0\delta t^{(\mp)}$, where $\mp M = d\phi/dt|_{t=t_0^{(\mp)}}/\Omega_0 = \mp 2\pi|e|\Delta^{-1}\sqrt{U_1^2 - U_0^2}$. With these definitions we can rewrite Eq. (B.5) as follows:

$$S(t) = e^{i\theta_r}\begin{cases} \dfrac{t - t_0^{(+)} - i\Gamma_\tau}{t - t_0^{(+)} + i\Gamma_\tau}\,, & \left|t - t_0^{(+)}\right| \lesssim \Gamma_\tau\,, \\[2ex] \dfrac{t - t_0^{(-)} + i\Gamma_\tau}{t - t_0^{(-)} - i\Gamma_\tau}\,, & \left|t - t_0^{(-)}\right| \lesssim \Gamma_\tau\,, \\[2ex] 1\,, & \left|t - t_0^{(\mp)}\right| \gg \Gamma_\tau\,. \end{cases} \tag{B.8}$$

Here Γ_τ is (a half of) a time during which the level rises above or sink below the Fermi level:

$$\Omega_0\Gamma_\tau = \frac{T\Delta}{4\pi\left|eU_1\sin\left(\Omega_0 t_0^{(0)} + \varphi\right)\right|} = \frac{T\Delta}{4\pi|e|\sqrt{U_1^2 - U_0^2}}\,. \tag{B.9}$$

The equation (B.8) assumes that the overlap between the resonances is small,

$$\left| t_0^{(+)} - t_0^{(-)} \right| \gg \Gamma_\tau \,. \tag{B.10}$$

Substituting Eq. (B.8) into Eq. (A.12) we find an adiabatic current at zero temperature (for $0 < t < \mathcal{T}$):

$$I(t) = \frac{e}{\pi} \left\{ \frac{\Gamma_\tau}{\left(t - t_0^{(-)}\right)^2 + \Gamma_\tau^2} - \frac{\Gamma_\tau}{\left(t - t_0^{(+)}\right)^2 + \Gamma_\tau^2} \right\} . \tag{B.11}$$

This current consists of two pulses of the Lorentzian shape with half-width Γ_τ corresponding to an emission of an electron and a hole. Integrating over time easy to check that the first pulse carries a charge e while the second pulse carries a charge $-e$.

In this regime the frozen density of states, Eq. (A.64) reads:

$$\nu(t, \mu_0) = \frac{4}{\Delta T} \left\{ \frac{\Gamma_\tau^2}{\left(t - t_0^{(-)}\right)^2 + \Gamma_\tau^2} + \frac{\Gamma_\tau^2}{\left(t - t_0^{(+)}\right)^2 + \Gamma_\tau^2} \right\} , \tag{B.12}$$

With this equation one can estimate the adiabaticity condition, i.e., the condition under which the current $I^{(2)} \sim \Omega_0^2$ is small compared to a linear in Ω_0 current $I^{(1)}$, see Eq. (A.63). We use $\nu \sim 1/(T\Delta)$. In the linear response regime we have, $I^2 \sim e^2 h \nu^2 \, d^2 U/dt^2$, and correspondingly find:

$$\varpi_{lin} \sim \frac{I^{(2)}}{I^{(1)}} \sim h\, \nu\, \Omega_0 \sim \frac{\tau \Omega_0}{T} \ll 1 \,. \tag{B.13a}$$

While in the non-linear regime to leading order in $\Omega_0 \Gamma_\tau$ we can write: $I^{(2)} \sim e^2 h \nu \, (\partial \nu / \partial t) \, (dU/dt)$. Then using $\partial \nu / \partial t \sim 1/(\Gamma_\tau T \Delta)$ we calculate:

$$\varpi_{n/lin} \sim \frac{I^{(2)}}{I^{(1)}} \sim \frac{h}{\Gamma_\tau T \Delta} \sim \frac{\tau \Omega_0}{T^2} \ll 1 \,. \tag{B.13b}$$

Comparing Eqs. (B.13a) and (B.13b) we conclude that in the quantized emission regime the adiabaticity condition is more restrictive compared to the linear response regime. For instance, if Eq. (B.13a) can be rewrittent as $\tau_D \ll \mathcal{T}$, then Eq. (B.13b) can be rewritten as $\tau_D \ll \Gamma_\tau$.

We calculate also the heat production rate I_E in the quantized emission regime. For the current $I(t)$, Eq. (B.11), we find to leading order in $\Omega_0\Gamma_\tau \ll 1$,

$$\mathcal{T}\left\langle I^2\right\rangle = \frac{e^2}{\pi}\frac{1}{\Gamma_\tau}, \tag{B.14}$$

and substituting it into Eq. (A.20) we finally find, [113]

$$I_E = \frac{\hbar}{\Gamma_\tau}\frac{1}{\mathcal{T}}. \tag{B.15}$$

This heat flow is due to additional (over the μ_0) energy $\hbar/(2\Gamma_\tau)$ carried by each particle (electron or hole) emitted during the period $\mathcal{T}$.

B.2 Shot noise quantization

Let us show that the quantized ac current generated by the SPS results in a quantized shot noise [121, 116, 122] in a geometry of Fig. B.1.

We calculate the zero-frequency symmetrized correlation function power $\mathcal{P}_{12}$ for currents $I_1(t)$ and $I_2(t)$ flowing into the contacts 1 and 2. At zero temperature it reads [see Eq. (6.27)],

$$\mathcal{P}_{12} = \frac{e^2\Omega}{4\pi}\sum_{q=-\infty}^{\infty}|q|\sum_{\gamma,\delta=1}^{2}\left\{\tilde{S}_{0,1\gamma}\tilde{S}^*_{0,1\delta}\right\}^*_q\left\{\tilde{S}_{0,2\gamma}\tilde{S}^*_{0,2\delta}\right\}_q. \tag{B.16}$$

The frozen scattering matrix $\hat{\tilde{S}}_0(t)$ for the entire system is:

$$\hat{\tilde{S}}_0(t) = \begin{pmatrix} e^{ik_FL_{11}}S(t)r_C & e^{ik_FL_{12}}t_C \\ e^{ik_FL_{21}}S(t)t_C & e^{ik_FL_{22}}r_C \end{pmatrix}, \tag{B.17}$$

where $L_{\gamma\delta}$ is a length of a path along the linear edge states from the contact δ to

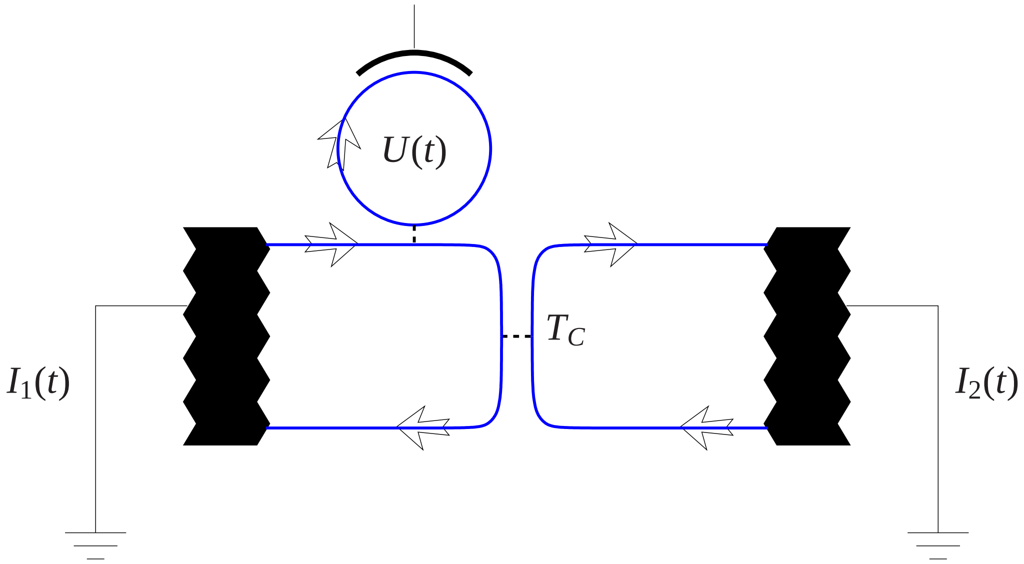

Figure B.1: The quantum capacitor is connected to the linear edge state which in turn is connected via the central QPC with transmission T_C to another linear edge state. The arrows indicate the direction of motion. The potential $U(t)$ induced by the back-gate acting onto the capacitor generates an ac current $I(t)$ which is splitted at the central QPC into the currents $I_1(t)$ and $I_2(t)$ flowing into the leads.

the contact γ, r_C/t_C is a reflection/transmission amplitude of the central QPC, $S(t)$ is a scattering amplitude of the capacitor. Remind that at zero temperature we need all quantities only at $E = \mu_0$. After the simple algebra we find,

$$\mathcal{P}_{12} = -\mathcal{P}_0 \sum_{q=1}^{\infty} q \left\{ \left|S_q\right|^2 + \left|S_{-q}\right|^2 \right\}, \tag{B.18}$$

where

$$\mathcal{P}_0 = e^2 R_C T_C \frac{\Omega_0}{2\pi}. \tag{B.19}$$

To calculate the shot noise we need the Fourier coefficients,

$$S_q = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{iq\Omega_0 t} S(t), \tag{B.20}$$

which in the limit

$$\Gamma_\tau \ll \mathcal{T}\,, \tag{B.21}$$

can be calculated as follows. The function $S(t)$, Eq. (B.8), is almost constant and changes only in a tiny ($\sim \Gamma_\tau$) vicinity of times $t_0^{(\mp)}$. Since only integrating over those small intervals plays a role in Eq. (B.20), we can formally extend integral, $\int_0^{\mathcal{T}} \to \int_{-\infty}^{\infty}$, and evaluate it closing the contour in the complex t-plane, $\int_{-\infty}^{\infty} \to \oint$, in the upper, Im$t > 0$, for $q > 0$ or in the lower, Im$t < 0$, for $q < 0$ semi-plane. The corresponding contour integral is evaluated using the Cauchy integral,

$$\frac{1}{2\pi i}\oint dt \sum_{j=1}^{N_p} \frac{f_j(t)}{(t-t_{pj})^{n_j+1}} = \sum_{j=1}^{N_p} \frac{1}{n_j!}\,\frac{d^{n_j} f_j}{dt^{n_j}}\Big|_{t=t_{pj}}\,, \tag{B.22}$$

where t_{pj} is a pole of the n_jth order, N_p is a number of poles which lie inside the integration contour.

The function $S(t)$, Eq. (B.8), has poles $t_p^{(-)} = t_0^{(-)} + i\Gamma_\tau$ and $t_p^{(+)} = t_0^{(+)} - i\Gamma_\tau$ in the upper and lower semi-planes of the complex variable t, respectively. Simple evaluation gives:

$$S_q = -2\Omega_0\Gamma_\tau\, e^{-|q|\Omega_0\Gamma_\tau}\, e^{i\theta_r} \begin{cases} e^{iq\Omega_0 t_0^{(-)}}\,, & q > 0\,, \\ e^{iq\Omega_0 t_0^{(+)}}\,, & q < 0\,. \end{cases} \tag{B.23}$$

Substituting above equation into Eq. (B.18) and evaluating the following sum to leading order in the small parameter $\epsilon = \Omega_0\Gamma_\tau$,

$$\sum_{q=1}^{\infty} q\left|S_q\right|^2 = 1 + \mathcal{O}\left(\epsilon^2\right), \tag{B.24}$$

we finally find the noise,

$$\mathcal{P}_{12} = -2\,\mathcal{P}_0\,, \tag{B.25}$$

which is independent of both the parameters of the SPS and the parameters of a driving potential.

If the amplitude U_1 of an oscillating potential $U(t)$, Eq. (B.1) is larger, for instance if n electrons and n holes are emitted during a period, then the noise is n times larger [1], $\mathcal{P}_{12} = -2n\mathcal{P}_0$, see the upper solid (black) line in Fig. B.5 in Sec. B.4.

Remarkably the noise produced by the SPS is quantized. The increment $\mathcal{P}_0$, Eq. (B.25), depends on the frequency Ω_0 of the oscillating voltage and on the transparency T_C of the central QPC. Therefore the quantization is not universal.

B.2.1 Probability interpretation for the shot noise

The noise $\mathcal{P}_{12}$, Eq. (B.25), can be understood as the shot noise due to one electron and one hole emitted by the source during the period $\mathcal{T} = 2\pi/\Omega_0$. The shot noise originates from the fact that in each particular event the indivisible particle has either to be reflected from or transmitted through the central QPC [20]. Since an electron and a hole are emitted at different times they are uncorrelated and contribute to the noise independently. Since the electron-hole symmetry is not violated in our system they contribute to noise equally, leading to a factor 2 in Eq. (B.25). Further for definiteness we consider an electron contribution,

$$\mathcal{P}_{12}^{(e)} = -\mathcal{P}_0 = -e^2 R_C T_C \frac{\Omega_0}{2\pi} . \tag{B.26}$$

The hole contribution can be considered similarly.

To interpret $\mathcal{P}_{12}^{(e)}$ we introduce the following probabilities which are evaluated by averaging over many periods. First, we introduce a single-particle probability $\mathcal{N}_\alpha$ having a meaning of a probability to detect an electron at the reservoir $\alpha = 1, 2$ during a period. Taking into account that the SPS emits only one electron during a period, we calculate for the circuit under consideration,

[1] The authors of the Ref. [121] considered the Lorentzian current pulses generated by carefully shaped external voltage pulses across two-terminal conductors and showed the the shot noise is proportional to the number of excitations. The operator algebra describing these excitations is also derived.

Fig. B.1:

$$\mathcal{N}_1 = R_C\,, \quad \mathcal{N}_2 = T_C\,. \tag{B.27}$$

Second, we introduce a two-particle probability $\mathcal{N}_{\alpha\beta}$ which means a probability to detect two particles at different contacts during a period. Since in our case there is only one electron emitted during a period, we have,

$$\mathcal{N}_{12} = 0\,. \tag{B.28}$$

And, finally, we introduce the particle-particle correlation function,

$$\delta\mathcal{N}_{12} = \mathcal{N}_{12} - \mathcal{N}_1\mathcal{N}_2\,. \tag{B.29}$$

From Eqs.(B.27) - (B.29) we find:

$$\delta\mathcal{N}_{12} = -R_C T_C\,. \tag{B.30}$$

Comparing above equation and Eq. (B.26) we find the following relation between the noise power and the particle correlator:

$$\mathcal{P}_{12} = \frac{e^2\Omega_0}{2\pi}\,\delta\mathcal{N}_{12}\,. \tag{B.31}$$

The equations (B.29) and (B.31) show how the current cross-correlator $\mathcal{P}_{12}$ relates to the two-particle detection probability $\mathcal{N}_{12}$. We will show that this relation holds also for circuits with several SPSs when $\mathcal{N}_{12} \neq 0$.

B.3 Two-particle source

Two cavities placed in series, Fig. B.2, and driven by the potentials $U_L(t)$ and $U_R(t)$ with the same period $\mathcal{T}$ can serve as a two-particle source. Depending on the phase difference between the potentials $U_L(t)$ and $U_R(t)$ such a double-cavity capacitor can emit electron and hole pairs, or electron-hole pairs, or emit single particles, electrons and holes. [41]

B.3.1 Scattering amplitude

If the cavities placed at a small distance, $L_{LR} \approx 0$, of each other, then the Floquet scattering amplitude of a capacitor reads,

$$S_F^{(2)}(E_n, E) = \sum_{m=-\infty}^{\infty} S_{R,F}(E_n, E_m) S_{L,F}(E_m, E) \,. \tag{B.32}$$

Here $S_{j,F}(E_n, E)$ is the Floquet scattering amplitude for a single cavity, $j = L, R$. Then introducing the amplitude $S_{in}^{(2)}(t, E)$ whose Fourier coefficients define the elements of the Floquet scattering matrix of a double-cavity capacitor,

$$S_F^{(2)}(E_n, E) = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega_0 t} S_{in}^{(2)}(t, E) \,, \tag{B.33}$$

and using Eq. (A.21) for a single-cavity scattering amplitude, we find:

$$S_{in}^{(2)}(t, E) = \sum_{p=0}^{\infty} e^{ipkL_R} S_R^{(p)}(t) \sum_{r=0}^{\infty} e^{irkL_L} S_L^{(r)}(t - p\tau) \,, \tag{B.34}$$

where L_j is a length of the cavity $j = L, R$.

B.3.2 Adiabatic approximation

In the limit of a slow excitation, $\Omega_0 \to 0$, we can approximate the single-cavity Floquet scattering matrix as follows:

$$S_{j,F}(E_n, E) = S_{j,n}(E) + \frac{\hbar\Omega_0 n}{2} \frac{\partial S_{j,n}(E)}{\partial E} + \mathcal{O}\left(\Omega_0^2\right) \,, \tag{B.35}$$

where $S_{j,n}(E)$ is the nth Fourier coefficient for the frozen scattering matrix, $S_j(t, E)$, of a single cavity. For the double-cavity capacitor we have to write:

$$S_F^{(2)}(E_n, E) = S_n^{(2)}(E) + \frac{\hbar\Omega_0 n}{2} \frac{\partial S_n^{(2)}(E)}{\partial E} + \hbar\Omega_0 A_n(E) + \mathcal{O}\left(\Omega_0^2\right) \,, \tag{B.36}$$

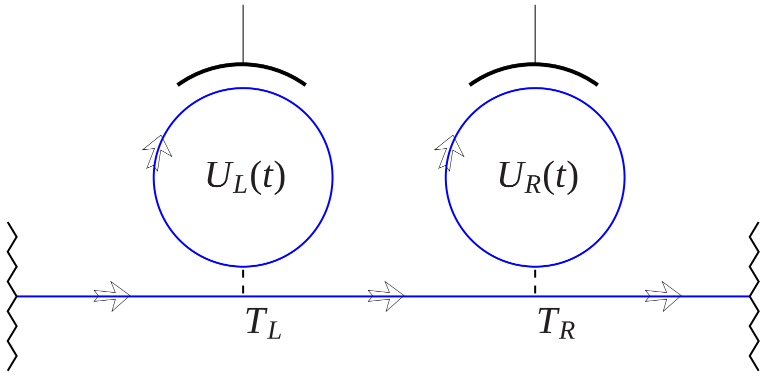

Figure B.2: The model of a double-cavity chiral quantum capacitor. The periodic potentials $U_L(t) = U_L(t + \mathcal{T})$ and $U_R(t) = U_R(t + \mathcal{T})$ act uniformly onto the corresponding single cavities connected via the QPSs with transmission T_L and T_R to the same linear edge state. Arrows indicate the direction of movement of electrons.

where $S_n^{(2)}$ is a Fourier coefficient for the frozen scattering matrix of a double-cavity system,

$$S^{(2)}(t,E) = S_R(t,E)S_L(t,E)\,. \tag{B.37}$$

Correspondingly, the inverse Fourier transform gives:

$$S_{in}^{(2)}(t,E) = S^{(2)}(t,E) + \frac{i\hbar}{2}\frac{\partial^2 S^{(2)}(t,E)}{\partial t\partial E} + \hbar\Omega_0 A(t,E)\,. \tag{B.38}$$

To find the anomalous scattering amplitude $A(t,E)$ for a double-cavity system, we substitute Eq. (B.35) into Eq. (B.32). Then after the inverse Fourier transformation we find:

$$\begin{aligned} S_{in}^{(2)}(t,E) &= S_R(t,E)\,S_L(t,E) + i\hbar\frac{\partial S_L}{\partial t}\frac{\partial S_R}{\partial E} \\ &+ \frac{i\hbar}{2}\left\{S_L\frac{\partial^2 S_R}{\partial t\partial E} + S_R\frac{\partial^2 S_L}{\partial t\partial E}\right\}\,. \end{aligned} \tag{B.39}$$

Comparing Eqs. (B.38) and (B.39) we finally get:

$$\hbar\Omega_0 A(t,E) = \frac{i\hbar}{2}\left\{\frac{\partial S_L}{\partial t}\frac{\partial S_R}{\partial E} - \frac{\partial S_L}{\partial E}\frac{\partial S_R}{\partial t}\right\}. \tag{B.40}$$

B.3.2.1 Time-dependent current

In lines with calculations presented in Sec. 4.1.3 we calculate the time-dependent current generated by the double-cavity capacitor up to Ω_0^2 terms:

$$\begin{aligned} I^{(2)}(t) \;=\; & \frac{e}{2\pi}\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\left\{\Im\left(S^{(2)}\frac{\partial S^{(2)*}}{\partial t}\right) + 2\hbar\Omega_0\Im\left(A\frac{\partial S^{(2)*}}{\partial t}\right)\right. \\ & \left. + \frac{\partial}{\partial t}\left(\frac{\hbar}{2}\frac{\partial S^{(2)}}{\partial E}\frac{\partial S^{(2)*}}{\partial t} - i\hbar\Omega_0 S^{(2)}A^*\right)\right\}. \end{aligned} \tag{B.41}$$

Using Eqs. (B.37) and (B.40) we find:

$$I^{(2)}(t) \;=\; e^2\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\left\{J^{(2,1)}(t,E) + J^{(2,2)}(t,E)\right\}. \tag{B.42a}$$

$$J^{(2,1)}(t,E) = \nu_L(t,E)\frac{dU_L(t)}{dt} + \nu_R(t,E)\frac{dU_R(t)}{dt}, \tag{B.42b}$$

$$J^{(2,2)}(t,E) = -\frac{h}{2}\frac{\partial}{\partial t}\left\{\nu_L^2\frac{dU_L}{dt} + \nu_R^2\frac{dU_R}{dt} + 2\nu_L\nu_R\frac{dU_L}{dt}\right\}. \tag{B.42c}$$

Here $\nu_j(t,E)$ is the frozen DOS of the cavity $j = L, R$.

B.3.3 Mean square current

To recognize a regime when both cavities emit particles simultaneously we calculate the mean square current: [41]

$$\langle I^2 \rangle = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \left(I^{(2)}(t) \right)^2 . \tag{B.43}$$

To leading order in Ω_0 we should keep only $J^{(2,1)}$ in Eq. (B.42). Alternatively one can express an adiabatic current directly in terms of the Fourier coefficients $S_{0,q}^{(2)}$ for the double-cavity frozen scattering amplitude, Eq. (B.37). Then, by analogy with Eq. (A.17), we get at zero temperature:

$$\langle I^2 \rangle = \frac{e^2 \Omega_0^2}{4\pi^2} \sum_{q=1}^{\infty} q^2 \left\{ \left| S_{0,q}^{(2)} \right|^2 + \left| S_{0,-q}^{(2)} \right|^2 \right\} . \tag{B.44}$$

To calculate the Fourier coefficients,

$$S_q^{(2)} = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{iq\Omega_0 t} S_L(t) S_R(t) , \tag{B.45}$$

we proceed similarly to how we calculated Eq. (B.23). For amplitudes $S_j(t)$ we use Eq. (B.8) with lower indeces $_L$ and $_R$ indicating cavity-specific quantities θ_{rj}, $\Gamma_{\tau j}$ and $t_{0j}^{(\mp)}$, $j = L, R$. We assume that each cavity emits only one electron and one hole during a period. Then the functions $S_j(t)$ for $0 < t < \mathcal{T}$ have one pole, $t_{pj}^{(-)} = t_{0j}^{(-)} + i\Gamma_{\tau j}$, in the upper and one pole, $t_{pj}^{(+)} = t_{0j}^{(+)} - i\Gamma_{\tau j}$, in the lower semi-plane of a complex variable t. Therefore, we calculate:

$$S_q^{(2)} = \begin{cases} S_R\left(t_{pL}^{(-)}\right) S_{L,q} + S_L\left(t_{pR}^{(-)}\right) S_{R,q} , & q > 0 , \\ S_R\left(t_{pL}^{(+)}\right) S_{L,q} + S_L\left(t_{pR}^{(+)}\right) S_{R,q} , & q < 0 , \end{cases} \tag{B.46}$$

where, $S_{j,q}$ are given in Eq. (B.23) with θ_r, Γ_τ and $t_0^{(\mp)}$ being replaced by θ_{rj}, $\Gamma_{\tau j}$ and $t_{0j}^{(\mp)}$, respectively. The squared Fourier coefficient reads:

$$\begin{aligned}\left|S_q^{(2)}\right|^2 &= \left|S_R\left(t_{pL}^{(\chi)}\right)\right|^2\left|S_{L,q}\right|^2 + \left|S_L\left(t_{pR}^{(\chi)}\right)\right|^2\left|S_{R,q}\right|^2 + \xi_q^{(\chi)}, \\ \xi_q^{(\chi)} &= 2\Re\left\{S_R\left(t_{pL}^{(\chi)}\right) S_{L,q} S_L^*\left(t_{pR}^{(\chi)}\right) S_{R,q}^*\right\},\end{aligned} \tag{B.47}$$

where $\chi = -$ for $q > 0$ and $\chi = +$ for $q < 0$.

To proceed further we need to define more precisely whether two cavities emit particles at close or at different times. To this end we introduce the difference of times,

$$\Delta t_{L,R}^{(\chi,\chi')} = t_{0L}^{(\chi)} - t_{0R}^{(\chi')}, \tag{B.48}$$

where $\chi = \mp$ and $\chi' = \mp$ depending on particles (an electron or a hole) of interest, and compare $\Delta t_{L,R}^{(\chi,\chi')}$ to the duration of current pulses $\Gamma_{\tau L}, \Gamma_{\tau R}$.

B.3.3.1 Emission of separate particles

First, we assume that all the particles are emitted at different times,

$$\left|\Delta t_{L,R}^{(\chi,\chi')}\right| \gg \Gamma_{\tau L}, \Gamma_{\tau R}. \tag{B.49}$$

In this case,

$$S_j\left(t_{0\bar{j}}^{(\chi)}\right) = e^{i\theta_{rj}}, \tag{B.50}$$

where $j \neq \bar{j}$, and from Eq. (B.47) we find:

$$\begin{aligned}\left|S_{0,q}^{(2)}\right|^2 &= 4\Omega_0^2\left\{\Gamma_{\tau L}^2 e^{-2|q|\Omega_0\Gamma_{\tau L}} + \Gamma_{\tau R}^2 e^{-2|q|\Omega_0\Gamma_{\tau R}}\right\} + \xi_q^{(\chi)}, \\ \xi_q^{(\chi)} &= 8\,\Omega_0^2\,\Gamma_{\tau L}\Gamma_{\tau R}\, e^{-|q|\Omega_0(\Gamma_{\tau L}+\Gamma_{\tau R})}\cos\left(q\Omega_0\Delta t_{L,R}^{(\chi,\chi)}\right).\end{aligned} \tag{B.51}$$

Next we need to sum up over q in Eq. (B.44).

It is convenient to introduce the following quantities:

$$A_{1,j} = \sum_{q=1}^{\infty} e^{-2q\Omega_0\Gamma_{\tau j}} = \frac{e^{-2\Omega_0\Gamma_{\tau j}}}{1-e^{-2\Omega_0\Gamma_{\tau j}}} = \frac{1}{2\Omega_0\Gamma_{\tau j}} + \mathcal{O}(1)\,, \tag{B.52}$$

$$\begin{aligned} A_2 &= \sum_{q=1}^{\infty} e^{-q\Omega_0\Gamma_{\tau\Sigma}} \cos(q\Omega_0\Delta t) = \frac{(-1)}{2}\frac{\cos(\Omega_0\Delta t) - e^{-\Omega_0\Gamma_{\tau\Sigma}}}{\cos(\Omega_0\Delta t) - \cosh(\Omega_0\Gamma_{\tau\Sigma})} \\ &\overset{\Delta t \gg \Gamma_\tau}{=} -\frac{1}{2} + \mathcal{O}(\Omega_0\Gamma_{\tau\Sigma})\,, \end{aligned} \tag{B.53}$$

$$\begin{aligned} A_3 &= \sum_{q=1}^{\infty} e^{-q\Omega_0\Gamma_{\tau\Sigma}} \sin(q\Omega_0\Delta t) = \frac{(-1)}{2}\frac{\sin(\Omega_0\Delta t)}{\cos(\Omega_0\Delta t) - \cosh(\Omega_0\Gamma_{\tau\Sigma})} \\ &\overset{\Delta t \gg \Gamma_\tau}{=} \frac{1}{2}\mathrm{ctg}\left(\frac{\Omega_0\Delta t}{2}\right) + \mathcal{O}(\Omega_0\Gamma_{\tau\Sigma})\,. \end{aligned} \tag{B.54}$$

where $\Gamma_{\tau\Sigma} = \Gamma_{\tau L} + \Gamma_{\tau R}$ and $\Delta t = \Delta t_{L,R}^{(\chi,\chi)}$. Then we see, to leading order in $\Omega_0\Gamma_{\tau j} \ll 1$ the term with $\xi_q^{(\chi)}$ does not contribute to the sum over q unless $\Delta t_{L,R}^{(\chi,\chi)} \lesssim \Gamma_{\tau j}$.

Substituting Eq. (B.51) into Eq, (B.44) and using the following sum,

$$\sum_{q=1}^{\infty} q^2 e^{-2q\Omega_0\Gamma_{\tau j}} = \frac{1}{4\Omega_0^2}\frac{\partial^2 A_{1,j}}{\partial\Gamma_{\tau j}^2} \approx \frac{1}{4\Omega_0^3\Gamma_{\tau j}^3}\,,$$

we find,

$$\mathcal{T}\langle I^2\rangle = \frac{e^2}{\pi}\left(\frac{1}{\Gamma_{\tau L}} + \frac{1}{\Gamma_{\tau R}}\right)\,. \tag{B.55}$$

Comparing above equation with a single-cavity result, Eq. (B.14), we conclude:

If all particles are emitted at different times then both cavities contribute additively to $\langle I^2\rangle$. Note, because of Eq. (A.20) the same is correct with respect to a generated heat flow.

B.3.3.2 Particle reabsorption regime

Let one cavity of the capacitor emits an electron (a hole) at the time when another cavity emits a hole (an electron). We expect that the source comprising both cavities does not generate a current, since the particle emitted by the first cavity is absorbed by the second cavity. The subsequent calculations of both the quantity $\left\langle I^2\right\rangle$ and the shot noise (in Sec. B.3.4.2) support such an expectation.

So, we suppose that,

$$\begin{aligned} \left|\Delta t_{L,R}^{(+,-)}\right|, \left|\Delta t_{L,R}^{(-,+)}\right| &\lesssim \Gamma_{\tau L}, \Gamma_{\tau R}, \\ \left|\Delta t_{L,R}^{(-,-)}\right|, \left|\Delta t_{L,R}^{(+,+)}\right| &\gg \Gamma_{\tau L}, \Gamma_{\tau R}. \end{aligned} \tag{B.56}$$

In this case $\xi_q^{(\chi)}$ in Eq. (B.47) still does not contribute, since it depends on $\Delta t_{1,2}^{(\chi,\chi)}$ which is large. Other quantities, we need to calculate Eq. (B.47), are the following:

$$\begin{aligned} S_L\left(t_{pR}^{(-)}\right) &= e^{i\theta_{rL}} \frac{\Delta t_{L,R}^{(+,-)} + i(\Gamma_{\tau L} - \Gamma_{\tau R})}{\Delta t_{L,R}^{(+,-)} - i(\Gamma_{\tau L} + \Gamma_{\tau R})}, \quad & S_L\left(t_{pR}^{(+)}\right) &= e^{i\theta_{rL}} \frac{\Delta t_{L,R}^{(-,+)} - i(\Gamma_{\tau L} - \Gamma_{\tau R})}{\Delta t_{L,R}^{(-,+)} + i(\Gamma_{\tau L} + \Gamma_{\tau R})}, \\ S_R\left(t_{pL}^{(-)}\right) &= e^{i\theta_{rR}} \frac{\Delta t_{L,R}^{(-,+)} + i(\Gamma_{\tau L} - \Gamma_{\tau R})}{\Delta t_{L,R}^{(-,+)} + i(\Gamma_{\tau L} + \Gamma_{\tau R})}, \quad & S_R\left(t_{pL}^{(+)}\right) &= e^{i\theta_{rR}} \frac{\Delta t_{L,R}^{(+,-)} - i(\Gamma_{\tau L} - \Gamma_{\tau R})}{\Delta t_{L,R}^{(+,-)} - i(\Gamma_{\tau L} + \Gamma_{\tau R})}. \end{aligned}$$

After squaring we find,

$$\begin{aligned} \left|S_L\left(t_{pR}^{(-)}\right)\right|^2 = \left|S_R\left(t_{pL}^{(+)}\right)\right|^2 = \gamma\left(\Delta t_{L,R}^{(+,-)}\right), \\ \left|S_L\left(t_{pR}^{(+)}\right)\right|^2 = \left|S_R\left(t_{pL}^{(-)}\right)\right|^2 = \gamma\left(\Delta t_{L,R}^{(-,+)}\right), \end{aligned}$$

where

$$\gamma(\Delta t) = \frac{(\Delta t)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2}{(\Delta t)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2}\,. \tag{B.57}$$

Remarkably $\gamma(\Delta t)$ is independent of q, i.e., when an electron and a hole are emitted at close times then all the photon-assisted probabilities are reduced by the same factor. Therefore, we can immediately write instead of Eq. (B.55) the following equation,

$$\mathcal{T}\langle I^2\rangle = \frac{e^2}{2\pi}\left(\frac{1}{\Gamma_{\tau L}} + \frac{1}{\Gamma_{\tau R}}\right)\left\{\gamma\left(\Delta t_{L,R}^{(-,+)}\right) + \gamma\left(\Delta t_{L,R}^{(+,-)}\right)\right\}. \tag{B.58}$$

For the identical cavities, $\Gamma_{\tau L} = \Gamma_{\tau R}$, emitting in synchronism, $\Delta t_{L,R}^{(-,+)} = \Delta t_{L,R}^{(+,-)} = 0$, the mean square current vanishes. Therefore, one can say that in this case the second (R) cavity re-absorbs all the particles emitted by the first (L) cavity.

B.3.3.3 Two-particle emission regime

Next we consider the cases when the two particles of the same kind are emitted near simultaneously. Due to the Pauli exclusion principle it is impossible to have two (spinless) electrons (or two holes) in the same state. Therefore, the second emitted particle should have energy larger than the first one. To be more precise, the electron pair (or the hole pair) has energy larger then the sum of energies of two separately emitted electrons (holes). Therefore, the heat flow I_E should be enhanced and, because of Eq. (A.20), the mean square current also should be enhanced [compared to Eq. (B.55)].

We assume:

$$\begin{aligned} \left|\Delta t_{L,R}^{(-,-)}\right|, \left|\Delta t_{L,R}^{(+,+)}\right| &\lesssim \Gamma_{\tau L}, \Gamma_{\tau R}, \\ \left|\Delta t_{L,R}^{(-,+)}\right|, \left|\Delta t_{L,R}^{(+,-)}\right| &\gg \Gamma_{\tau L}, \Gamma_{\tau R}. \end{aligned} \tag{B.59}$$

In this case the two poles of $S^{(2)}(t) = S_L(t)S_R(t)$ as a function of a complex time t in the upper (and/or in the lower) semi-plane become close to each other, that affects calculations significantly. Now the quantities A_2, Eq. (B.53), and A_3, Eq. (B.54), become of order $A_{1,j}$, Eq. (B.52). Therefore, we should keep $\xi_q^{(\chi)}$ in Eq. (B.47). From Eq. (B.59) it follows that $\Omega_0 \Delta t_{L,R}^{(\chi,\chi)} \ll 1$ and we find:

$$A_2 = \frac{\Omega_0(\Gamma_{\tau L} + \Gamma_{\tau R})}{\left(\Omega_0 \Delta t_{L,R}^{(\chi,\chi)}\right)^2 + \Omega_0^2(\Gamma_{\tau L} + \Gamma_{\tau R})^2} + \mathcal{O}(1)\,, \qquad \text{(B.60a)}$$

$$A_3 = \frac{\Omega_0 \Delta t_{L,R}^{(\alpha,\alpha)}}{\left(\Omega_0 \Delta t_{L,R}^{(\chi,\chi)}\right)^2 + \Omega_0^2\,(\Gamma_{\tau L} + \Gamma_{\tau R})^2} + \mathcal{O}(1)\,. \qquad \text{(B.60b)}$$

Also we will use the following quantities,

$$S_L\left(t_{pR}^{(-)}\right) = e^{i\theta_{rL}}\,\frac{\Delta t_{L,R}^{(-,-)} - i(\Gamma_{\tau L}+\Gamma_{\tau R})}{\Delta t_{L,R}^{(-,-)} + i(\Gamma_{\tau L}-\Gamma_{\tau R})}\,, \quad S_L\left(t_{pR}^{(+)}\right) = e^{i\theta_{rL}}\,\frac{\Delta t_{L,R}^{(+,+)} + i(\Gamma_{\tau L}+\Gamma_{\tau R})}{\Delta t_{L,R}^{(+,+)} - i(\Gamma_{\tau L}-\Gamma_{\tau R})}\,,$$

$$S_R\left(t_{pL}^{(-)}\right) = e^{i\theta_{rR}}\,\frac{\Delta t_{L,R}^{(-,-)} + i(\Gamma_{\tau L}+\Gamma_{\tau R})}{\Delta t_{L,R}^{(-,-)} + i(\Gamma_{\tau L}-\Gamma_{\tau R})}\,, \quad S_R\left(t_{pL}^{(+)}\right) = e^{i\theta_{rR}}\,\frac{\Delta t_{L,R}^{(+,+)} - i(\Gamma_{\tau L}+\Gamma_{\tau R})}{\Delta t_{L,R}^{(+,+)} - i(\Gamma_{\tau L}-\Gamma_{\tau R})}\,,$$

$$S_R\left(t_{pL}^{(-)}\right) S_L^*\left(t_{pR}^{(-)}\right) = e^{i(\theta_{rR}-\theta_{rL})}\,\frac{\left(\Delta t_{L,R}^{(-,-)}\right)^2 - (\Gamma_{\tau L}+\Gamma_{\tau R})^2 + 2i\Delta t_{L,R}^{(-,-)}(\Gamma_{\tau L}+\Gamma_{\tau R})}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2}\,,$$

$$S_R\left(t_{pL}^{(+)}\right) S_L^*\left(t_{pR}^{(+)}\right) = e^{i(\theta_{rR}-\theta_{rL})}\,\frac{\left(\Delta t_{L,R}^{(+,+)}\right)^2 - (\Gamma_{\tau L}+\Gamma_{\tau R})^2 - 2i\Delta t_{L,R}^{(+,+)}(\Gamma_{\tau L}+\Gamma_{\tau R})}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2}\,,$$

and some squares,

$$\left|S_L\left(t_{pR}^{(-)}\right)\right|^2 = \left|S_R\left(t_{pL}^{(-)}\right)\right|^2 = \frac{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2}\,, \qquad \text{(B.61)}$$

$$\left|S_L\left(t_{pR}^{(+)}\right)\right|^2 = \left|S_R\left(t_{pL}^{(+)}\right)\right|^2 = \frac{\left(\Delta t_{L,R}^{(+,+)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2}{\left(\Delta t_{L,R}^{(+,+)}\right)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2}\,. \qquad \text{(B.62)}$$

In Eq. (B.44) we need to calculate the following sum,

$$\sum_{q=1}^{\infty} q^2 \left|S_q^{(2)}\right|^2 = \Phi_1 + \Phi_2 \,, \tag{B.63a}$$

where

$$\Phi_1 = \left|S_L\left(t_{pR}^{(-)}\right)\right|^2 \sum_{q=1}^{\infty} q^2 \left\{\left|S_{L,q}\right|^2 + \left|S_{R,q}\right|^2\right\} = \left|S_L\left(t_{pR}^{(-)}\right)\right|^2$$

$$\times \sum_{j=L,R} \Gamma_{\tau j}^2 \frac{\partial^2 A_{1,j}}{\partial \Gamma_{\tau j}^2} = \frac{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2} \frac{1}{\Omega_0} \left\{\frac{1}{\Gamma_{\tau L}} + \frac{1}{\Gamma_{\tau R}}\right\}, \tag{B.63b}$$

and

$$\Phi_2 = \sum_{q=1}^{\infty} q^2 \xi_q^{(-)} = 2\sum_{q=1}^{\infty} q^2 \Re\left\{S_R\left(t_{pL}^{(-)}\right) S_{L,q} S_L^*\left(t_{pR}^{(-)}\right) S_{R,q}^*\right\}$$

$$= \frac{8\Omega_0^2 \Gamma_{\tau L}\Gamma_{\tau R}}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2} \left\{\frac{\left(\Delta t_{L,R}^{(-,-)}\right)^2 - (\Gamma_{\tau L} + \Gamma_{\tau R})^2}{\Omega_0^2} \frac{\partial^2 A_2}{\partial \Gamma_{\tau L}^2} - \frac{2\Delta t_{L,R}^{(-,-)}(\Gamma_{\tau L} + \Gamma_{\tau R})}{\Omega_0^2} \frac{\partial^2 A_3}{\partial \Gamma_{\tau L}^2}\right\}, \tag{B.63c}$$

From Eqs. (B.60) we calculate,

$$\frac{\partial A_2}{\partial \Gamma_{\tau L}} = \frac{\left(\Delta t_{L,R}^{(\chi,\chi)}\right)^2 - (\Gamma_{\tau L} + \Gamma_{\tau R})^2}{\Omega_0 \left(\left(\Delta t_{L,R}^{(\chi,\chi)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2\right)^2}, \qquad \frac{\partial A_3}{\partial \Gamma_{\tau L}} = \frac{-2\Delta t_{L,R}^{(\chi,\chi)}(\Gamma_{\tau L} + \Gamma_{\tau R})}{\Omega_0 \left(\left(\Delta t_{L,R}^{(\chi,\chi)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2\right)^2},$$

and

$$\frac{\partial^2 A_2}{\partial \Gamma_{\tau L}^2} = \frac{-2(\Gamma_{\tau L} + \Gamma_{\tau R})\left(3\left(\Delta t_{L,R}^{(\chi,\chi)}\right)^2 - \Gamma_{\tau \Sigma}^2\right)}{\Omega_0 \left(\left(\Delta t_{L,R}^{(\chi,\chi)}\right)^2 + \Gamma_{\tau \Sigma}^2\right)^3}, \qquad \frac{\partial^2 A_3}{\partial \Gamma_{\tau L}^2} = \frac{-2\Delta t_{L,R}^{(\chi,\chi)}\left(\left(\Delta t_{L,R}^{(\chi,\chi)}\right)^2 - 3\Gamma_{\tau \Sigma}^2\right)}{\Omega_0 \left(\left(\Delta t_{L,R}^{(\chi,\chi)}\right)^2 + \Gamma_{\tau \Sigma}^2\right)^3},$$

Using above equations in Eq. (B.63c), we find,

$$\Phi_2 = \frac{8\Gamma_{\tau L}\Gamma_{\tau R}\Pi}{\Omega_0\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2\right)\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2\right)^3},$$

with

$$\Pi = -2\Gamma_{\tau\Sigma}\left\{\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 - \Gamma_{\tau\Sigma}^2\right)\left(3\left(\Delta t_{L,R}^{(-,-)}\right)^2 - \Gamma_{\tau\Sigma}^2\right)\right.$$
$$\left. -2\left(\Delta t_{L,R}^{(-,-)}\right)^2\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 - 3\Gamma_{\tau\Sigma}^2\right)\right\} = -2\Gamma_{\tau\Sigma}\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 + \Gamma_{\tau\Sigma}^2\right)^2.$$

After the simplification it becomes:

$$\Phi_2 = \frac{-16\Gamma_{\tau L}\Gamma_{\tau R}(\Gamma_{\tau L} + \Gamma_{\tau R})}{\Omega_0\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2\right)\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2\right)}. \tag{B.64}$$

Next, substituting Eqs. (B.63b) and (B.64) into Eq. (B.63a) we get:

$$\sum_{q=1}^{\infty} q^2 \left|S_{,q}^{(2)}\right|^2 = \frac{(\Gamma_{\tau L}+\Gamma_{\tau R})\left\{\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2+(\Gamma_{\tau L}+\Gamma_{\tau R})^2\right)^2-16\Gamma_{\tau L}^2\Gamma_{\tau R}^2\right\}}{\Omega_0\Gamma_{\tau L}\Gamma_{\tau R}\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2+(\Gamma_{\tau L}-\Gamma_{\tau R})^2\right)\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2+(\Gamma_{\tau L}+\Gamma_{\tau R})^2\right)}$$

$$= \frac{\Gamma_{\tau\Sigma}\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2+\Gamma_{\tau\Sigma}^2+4\Gamma_{\tau L}\Gamma_{\tau R}\right)}{\Omega_0\Gamma_{\tau L}\Gamma_{\tau R}\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2+\Gamma_{\tau\Sigma}^2\right)} = \frac{1}{\Omega_0}\left(\frac{1}{\Gamma_{\tau L}} + \frac{1}{\Gamma_{\tau R}}\right)\left\{2 - \gamma\left(\Delta t_{L,R}^{(-,-)}\right)\right\},$$

where $\gamma(\Delta t)$ is defined in Eq. (B.57). The sum $\sum_{q=1}^{\infty} q^2 \left|S_{-q}^{(2)}\right|^2$ gives the same result but with $\Delta t_{1,2}^{(-,-)}$ being replaced by $\Delta t_{1,2}^{(+,+)}$. Finally, from Eq. (B.44) we have the mean square current,

$$\mathcal{T}\langle I^2\rangle = \frac{e^2}{2\pi}\left(\frac{1}{\Gamma_{\tau L}}+\frac{1}{\Gamma_{\tau R}}\right)\left\{4-\gamma\left(\Delta t_{L,R}^{(-,-)}\right)-\gamma\left(\Delta t_{L,R}^{(+,+)}\right)\right\}. \tag{B.65}$$

For the identical cavities, $\Gamma_{\tau L} = \Gamma_{\tau R}$, emitting electrons and holes in synchronism, $\Delta t_{L,R}^{(-,-)} = \Delta t_{L,R}^{(+,+)} = 0$, the mean square current, hence the heat production rate, is as twice as larger compared to the one in the regime of separately emitted particles, Eq. (B.55).

Combining Eq. (B.58) with Eq. (B.65) we obtain an equation describing all the considered regimes: [41]

$$\begin{aligned}\mathcal{T}\langle I^2\rangle &= \frac{e^2}{2\pi}\left(\frac{1}{\Gamma_{\tau L}}+\frac{1}{\Gamma_{\tau R}}\right)\left\{2+\gamma\left(\Delta t_{L,R}^{(-,+)}\right)+\gamma\left(\Delta t_{L,R}^{(+,-)}\right)\right.\\ &\left. -\gamma\left(\Delta t_{L,R}^{(-,-)}\right)-\gamma\left(\Delta t_{L,R}^{(+,+)}\right)\right\}.\end{aligned} \tag{B.66}$$

Note that this equation is in the leading order in $\Omega_0\Gamma_{\tau j} \ll 1$. The higher order corrections arise from the current $J^{(2,2)}$ in Eq. (B.42) and from approximations we made evaluating the Fourier coefficients, Eq. (B.20).

B.3.4 Shot noise of a two-particle source

Let the double-cavity capacitor is connected to a linear edge state which in turn is connected to another linear edge state via a central QPC with transmission T_C, Fig. B.3. Our aim is to investigate how the shot noise, arising when emitted particles (electrons and holes) are scattered at the central QPC, depends on the regime of emission of the double-cavity capacitor.

By analogy with the single-cavity capacitor case, Eq. (B.18), we can write:

$$\mathcal{P}_{12} = -\mathcal{P}_0\sum_{q=1}^{\infty} q\left\{\left|S_q^{(2)}\right|^2+\left|S_{-q}^{(2)}\right|^2\right\}, \tag{B.67a}$$

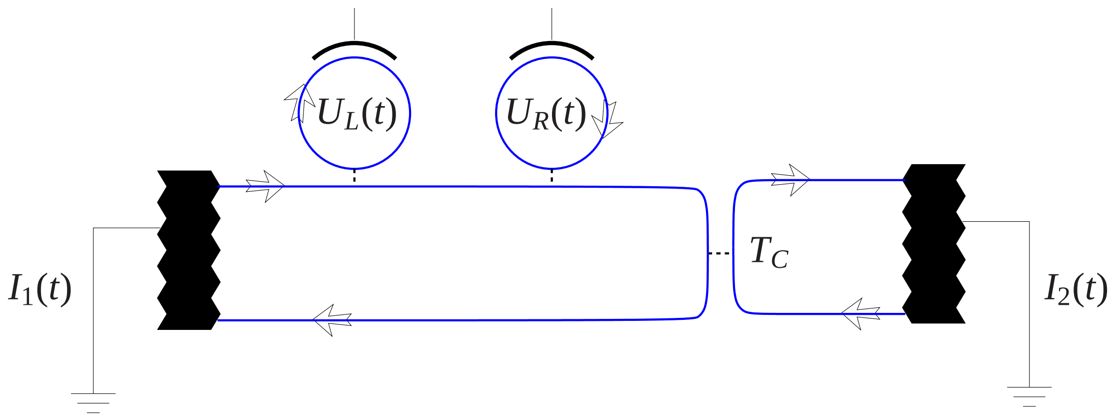

Figure B.3: The double-cavity quantum capacitor is connected to the linear edge state which in turn is connected via the central QPC with transmission T_C to another linear edge state. The arrows indicate the direction of motion. The potentials $U_L(t)$ and $U_R(t)$ induced by the back-gates acting on the corresponding cavities generate an ac current $I(t)$ which is splitted at the central QPC into the currents $I_1(t)$ and $I_2(t)$ flowing into the leads.

and taking into account that $S^{(2)} = S_L S_R$:

$$\mathcal{P}_{12} = -\mathcal{P}_0 \sum_{q=1}^{\infty} q \left\{ \left|(S_L S_R)_q\right|^2 + \left|(S_L S_R)_{-q}\right|^2 \right\} . \tag{B.67b}$$

To evaluate this cross-correlator for the different emission regimes we proceed similarly to what we did in Sec. B.3.3.

B.3.4.1 Emission of separate particles

If all the particles are emitted at different times, Eq. (B.49), then calculating the sum over q we can neglect the term $\xi_q^{(\chi)}$ in Eq. (B.51). Then using the following sum (to the leading order in $\Omega_0 \Gamma_{\tau j} \ll 1$),

$$\sum_{q=1}^{\infty} q e^{-2q\Omega_0\Gamma_{\tau j}} = \frac{-1}{2\Omega_0} \frac{\partial A_{1,j}}{\partial \Gamma_{\tau j}} \approx \frac{1}{4\Omega_0^2 \Gamma_{\tau j}^2} ,$$

(see Eq. (B.52) for $A_{1,j}$) we calculate,

$$\mathcal{P}_{12} = -4\,\mathcal{P}_0\,, \tag{B.68}$$

This noise is due to four particles (two electrons and two holes) emitted by both cavities during the period $\mathcal{T} = 2\pi/\Omega_0$.

B.3.4.2 Particle reabsorption regime

Under conditions given in Eq. (B.56) all the photon-assisted probabilities are reduced by the same factor, see Eq. (B.57). Therefore, we can immediately write instead of Eq. (B.68) the following equation,

$$\mathcal{P}_{12} = -2\,\mathcal{P}_0 \left\{ \gamma\left(\Delta t_{L,R}^{(-,+)}\right) + \gamma\left(\Delta t_{L,R}^{(+,-)}\right) \right\} . \tag{B.69}$$

If electrons and holes are emitted at close times, then both the noise $\mathcal{P}_{12}$, Eq. (B.69), and the mean square current $\langle I^2 \rangle$, Eq. (B.58), are suppressed. On the other hand their ratio remains the same as in the regime of emission of separate particles. It tells us that in the reabsorption regime the rarely emitted (not absorbed) electrons and holes remain uncorrelated.

B.3.4.3 Two-particle emission regime

We will show that the shot noise is not sensitive whether two electrons (two holes) are emitted at close times, Eq. (B.59), or not. This means that two particles are scattered at the central QPC independently alike they are emitted at different times. Therefore, despite the fact that the energy of two electrons (two holes) emitted simultaneously is enhanced compared to the sum of energies of two separately emitted electrons (holes), they remain uncorrelated rather than constitute a pair.

To calculate Eq. (B.67) we use Eqs. (B.47), (B.60), and (B.61) and evaluate the following sum,

$$\sum_{q=1}^{\infty} q \left|S_q^{(2)}\right|^2 = F_1 + F_2 \,, \tag{B.70a}$$

where

$$F_1 = \left|S_L\left(t_{pR}^{(-)}\right)\right|^2 \sum_{q=1}^{\infty} q \left\{\left|S_{L,q}\right|^2 + \left|S_{R,q}\right|^2\right\} = -2\Omega_0 \left|S_L\left(t_{pR}^{(-)}\right)\right|^2 \tag{B.70b}$$

$$\times \sum_{j=L,R} \Gamma_{\tau j}^2 \frac{\partial A_{1,j}}{\partial \Gamma_{\tau j}} = 2\frac{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L} - \Gamma_{\tau R})^2} \,,$$

and

$$F_2 = \sum_{q=1}^{\infty} q \xi_q^{(-)} = 8\Omega_0^2 \Gamma_{\tau L}\Gamma_{\tau R}$$
$$\times \Re\left\{S_R\left(t_{pL}^{(-)}\right) S_L^*\left(t_{pR}^{(-)}\right) e^{i(\theta_{rL}-\theta_{rR})} \sum_{q=1}^{\infty} q e^{-q\Omega_0\Gamma_{\tau\Sigma}} e^{iq\Omega_0\Delta t_{L,R}^{(-,-)}}\right\}. \tag{B.70c}$$

Using the product $S_R\left(t_{pL}^{(-)}\right) S_L^*\left(t_{pR}^{(-)}\right)$ given just before Eq. (B.61) we write:

$$F_2 = \frac{8\Omega_0^2\Gamma_{\tau L}\Gamma_{\tau R}}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2}\left\{-2\Delta t_{L,R}^{(-,-)}\Gamma_{\tau\Sigma}\sum_{q=1}^{\infty} q e^{-q\Omega_0\Gamma_{\tau\Sigma}} \sin\left(q\Omega_0\Delta t_{L,R}^{(-,-)}\right)\right.$$
$$\left. + \left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 - \Gamma_{\tau\Sigma}^2\right)\sum_{q=1}^{\infty} q e^{-q\Omega_0\Gamma_{\tau\Sigma}} \cos\left(q\Omega_0\Delta t_{L,R}^{(-,-)}\right)\right\},$$

and rewrite it, using Eqs. (B.53) and (B.54):

$$F_2 = \frac{8\Omega_0^2\Gamma_{\tau L}\Gamma_{\tau R}}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2}\left\{\frac{2\Delta t_{L,R}^{(-,-)}\Gamma_{\tau\Sigma}}{\Omega_0}\frac{\partial A_3}{\partial \Gamma_{\tau L}} - \frac{\left(\Delta t_{L,R}^{(-,-)}\right)^2 - \Gamma_{\tau\Sigma}^2}{\Omega_0}\frac{\partial A_2}{\partial \Gamma_{\tau L}}\right\}.$$

In the regime under consideration the derivatives $\partial A_2/\partial\Gamma_{\tau L}$ and $\partial A_3/\partial\Gamma_{\tau L}$ are given just below Eq. (B.63). Then we find:

$$F_2 = \frac{(-8\Gamma_{\tau L}\Gamma_{\tau R})\left\{\left(2\Delta t_{L,R}^{(-,-)}\Gamma_{\tau\Sigma}\right)^2 + \left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 - \Gamma_{\tau\Sigma}^2\right)^2\right\}}{\left\{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2\right\}\left(\left(\Delta t_{L,R}^{(-,-)}\right)^2 + \Gamma_{\tau\Sigma}^2\right)^2} = \frac{-8\Gamma_{\tau L}\Gamma_{\tau R}}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2}\,.$$

Substituting above equation and Eq. (B.70b) into Eq. (B.70a) we get,

$$\sum_{q=1}^{\infty} q\left|S_q^{(2)}\right|^2 = \frac{2\left(\Delta t_{L,R}^{(-,-)}\right)^2 + 2(\Gamma_{\tau L}+\Gamma_{\tau R})^2 - 8\Gamma_{\tau L}\Gamma_{\tau R}}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2} = 2\,\frac{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2}{\left(\Delta t_{L,R}^{(-,-)}\right)^2 + (\Gamma_{\tau L}-\Gamma_{\tau R})^2} = 2\,.$$

The same result is for negative harmonics, $\sum_{q=1}^{\infty} q\left|S_{-q}^{(2)}\right|^2 = 2$.

Thus the noise power, Eq. (B.67), is $\mathcal{P}_{LR} = -4\mathcal{P}_0$. This is the same as in the regime when the particles are emitted at different times, Eq. (B.68). Therefore, the noise is not sensitive to whether two electrons (two holes) are emitted simultaneously or not. Note the equation (B.69) is applicable for all considered regimes.

B.4 Mesoscopic electron collider

Consider the circuit presented in Fig. B.4 where the two quantum capacitors (two SPSs) are placed in arms located at the different sides of the central QPC. The particles emitted by the different SPSs are uncorrelated, hence they contribute to noise independently. However if the two SPSs emit electrons (holes) simultaneously, then these particles become correlated after scattering at the central QPC. The correlations arise due to the Pauli exclusion principle: Two electrons (holes) can not be scattered to the same edge state, instead they are necessarily scattered to different edge states and arrive at different contacts. Therefore, in this regime the system comprising two SPSs and the central

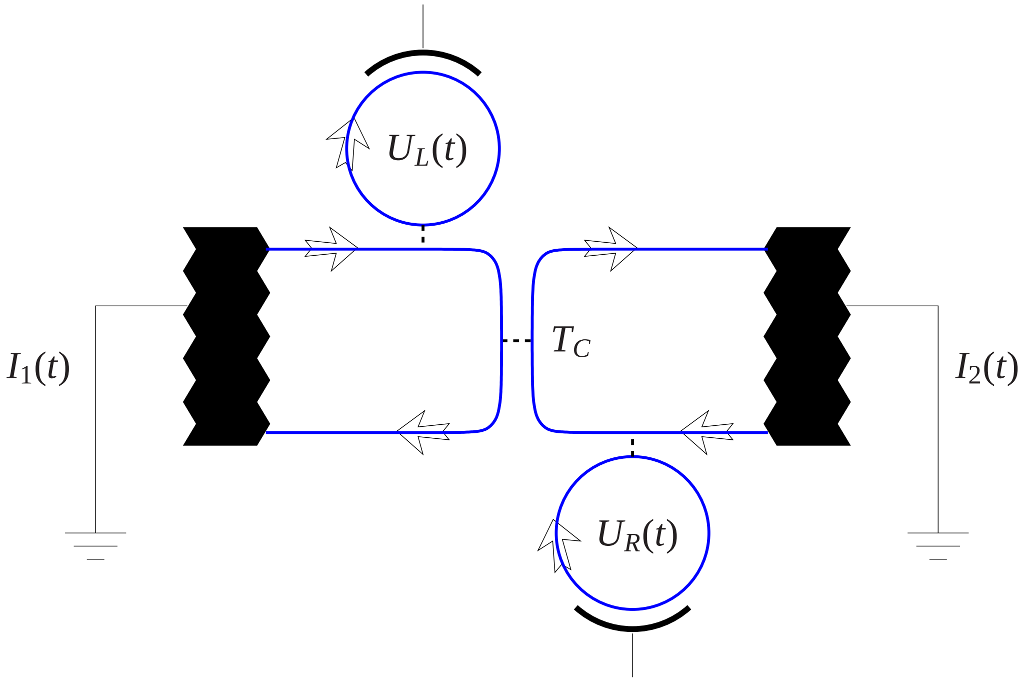

Figure B.4: Two quantum capacitors are connected to linear edge states which in turn are connected via the central QPC with transmission T_C. The arrows indicate the direction of motion. The potentials $U_L(t)$ and $U_R(t)$ induced by back-gates acting on the corresponding capacitors generate ac currents $I_1(t)$ and $I_2(t)$ at leads.

QPC serves as a two-particle source emitting only particles in pairs whose constituents are directed to different contacts.

By mere changing the phase difference between the potential driving two cavities,[2] one can switch the statistics of particles emitted during a period from classical to quantum (fermionic).

B.4.1 Shot noise suppression

The elements of the frozen scattering matrix $\hat{\hat{S}}_0(t)$ for the circuit under study, Fig. B.4, are:

[2]This phase difference controls a difference of emission times of particles exiting the cavities

$$\hat{\tilde{S}}_0(t) = \begin{pmatrix} e^{ik_F L_{11}} S_L(t) r_C & e^{ik_F L_{12}} S_R(t) t_C \\ e^{ik_F L_{21}} S_L(t) t_C & e^{ik_F L_{22}} S_R(t) r_C \end{pmatrix}, \tag{B.71}$$

where $S_j(t)$ is a frozen scattering amplitude of the capacitor $j = L, R$. Other quantities are the same as in Eq. (B.17). With this scattering matrix from Eq. (B.16) we calculate:

$$\mathcal{P}_{12} = -\mathcal{P}_0 \sum_{q=1}^{\infty} q \left\{ \left|(S_L^* S_R)_q\right|^2 + \left|(S_L^* S_R)_{-q}\right|^2 \right\}. \tag{B.72}$$

Comparing Eq. (B.72) with Eq. (B.67b) we see that the difference is only a replacement $S_L \to S_L^*$. From Eq. (B.8) we conclude that the complex conjugate scattering amplitude corresponds to emission of a hole (an electron) if a bare scattering amplitude corresponds to emission of an electron (a hole). Therefore, one can use the results of Sec. B.3.4 if one to replace $\Delta t_{L,R}^{(-,-)} \to \Delta t_{L,R}^{(+,-)}$, etc.

If two capacitors emit particles at different times or they emit an electron and a hole at close times, then the noise,

$$\mathcal{P}_{12} = -4\mathcal{P}_0, \tag{B.73}$$

is due to independent contributions of four uncorrelated particles emitted during a period by both capacitors. Note the possible collision of an electron and a hole at the central QPC does not affect the shot noise, since an electron and a hole have different energies (above and below[3] the Fermi energy, respectively) and are not subject to the Pauli exclusion principle which could lead to appearance of correlations crucial for a noise.

In contrast, if two electrons (two holes) are emitted at close times, $\Delta t_{L,R}^{(-,-)} = t_{0L}^{(-)} - t_{0R}^{(-)} \lesssim \Gamma_{\tau L}, \Gamma_{\tau R}$ ($\Delta t_{L,R}^{(+,+)} = t_{0L}^{(+)} - t_{0R}^{(+)} \lesssim \Gamma_{\tau L}, \Gamma_{\tau R}$) then the noise is suppressed: [116]

$$\mathcal{P}_{12} = \mathcal{P}_{12}^{(e)} + \mathcal{P}_{12}^{(h)}, \tag{B.74a}$$

[3] There is no contradiction with the fact that a hole carries a positive heat, see Eq. (B.15). Since heat is defined as an extra energy obtained by the reservoir with fixed chemical potential. To maintain it fixed we need to add one electron with energy μ_0 after a hole will enter the reservoir.

where electron and hole contributions, $\mathcal{P}_{12}^{(e)}$ and $\mathcal{P}_{12}^{(h)}$, are:

$$\mathcal{P}_{12}^{(e)} = -2\mathcal{P}_0 \gamma\left(\Delta t_{L,R}^{(-,-)}\right) = -2\mathcal{P}_0 \left\{1 - \frac{4\Gamma_{\tau L}\Gamma_{\tau R}}{\left(t_{0L}^{(-)} - t_{0R}^{(-)}\right)^2 + \Gamma_{\tau\Sigma}^2}\right\}, \tag{B.74b}$$

$$\mathcal{P}_{12}^{(h)} = -2\mathcal{P}_0 \gamma\left(\Delta t_{L,R}^{(+,+)}\right) = -2\mathcal{P}_0 \left\{1 - \frac{4\Gamma_{\tau L}\Gamma_{\tau R}}{\left(t_{0L}^{(+)} - t_{0R}^{(+)}\right)^2 + \Gamma_{\tau\Sigma}^2}\right\}. \tag{B.74c}$$

We give the noise as a sum of electron and hole parts since they contribute independently.

When each of the time differences $\Delta t_{L,R}^{(\chi,\chi)}$ is larger than the sum of half-widths of current pulses, then the two sources contribute to a shot noise independently, Fig. B.5, lower solid (green) line. In this case Eq. (B.74) leads to Eq. (B.73). In contrast if there is some overlap in time between the particle wave packets arriving at the central QPC, $\Delta t_{L.R}^{(-,-)} \sim \Gamma_{\tau L} + \Gamma_{\tau R}$ ($\Delta t_{L.R}^{(+,+)} \sim \Gamma_{\tau L} + \Gamma_{\tau R}$), then the correlations between electrons (holes) arise and the noise decreases. In the case of the full overlap, $\Delta t_{L,R}^{(\chi,\chi)} = 0$ and $\Gamma_{\tau L} = \Gamma_{\tau R}$, the noise is suppressed down to zero:

$$\mathcal{P}_{12}^{(e)} = 0\,, \quad \text{if} \quad t_{0L}^{(-)} = t_{0R}^{(-)}\,, \tag{B.75a}$$

$$\mathcal{P}_{12}^{(h)} = 0\,, \quad \text{if} \quad t_{0L}^{(+)} = t_{0R}^{(+)}\,. \tag{B.75b}$$

In Fig. B.5 the dashed (red) line shows a noise generated by the two identical sources as a function of the amplitude $U_{L,1}$ of a potential acting onto the capacitor L. If $U_{L,1} \neq U_{R,1}$ then the times when particles are emitted by the different sources are different. In this case both sources contribute to noise independently. However if $eU_{L,1}$ approaches $eU_{R,1} = 0.5\Delta_R$ then the time differences $\Delta t_{L,R}^{(\chi,\chi)} \to 0$ that results in a suppression of a shot noise.

In contrast to the case considered in Sec. B.3.4.2, where the noise decreases together with a current, here the noise vanishes while the currents remain non-zero, $I_1(t) \neq 0$, $I_2(t) \neq 0$. Taking into account the conservation law for the

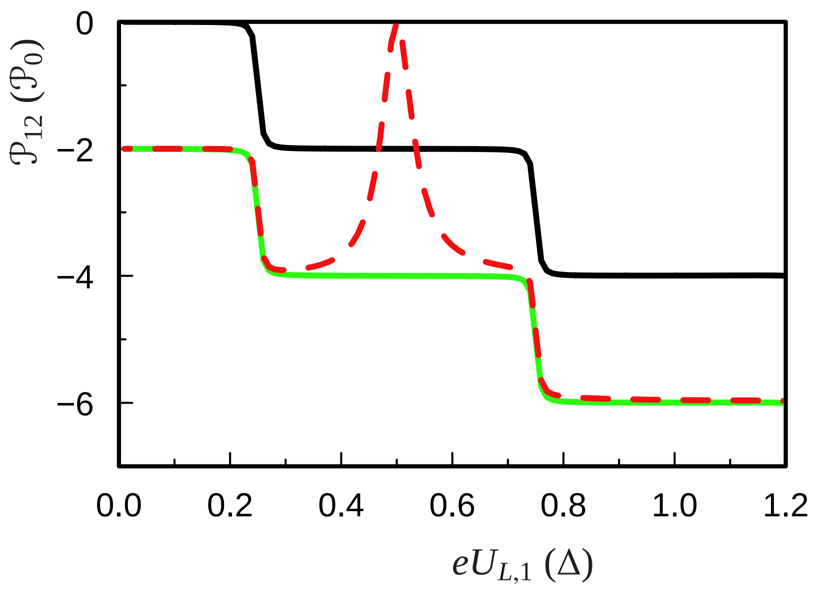

Figure B.5: The noise $\mathcal{P}_{12}$, Eq. (B.72), as a function of the amplitude $U_{L,1}$ of a potential $U_L(t) = U_{L,0} + U_{L,1}\cos(\Omega_0 t + \varphi_L)$ acting upon the left capacitor, see Fig. B.4. Upper solid (black) line: The right capacitor is stationary. Lower solid (green) line: The right capacitor is driven by the potential $U_R(t) = U_{R,0} + U_{R,1}\cos(\Omega_0 t + \varphi_R)$ which is out of phase, $\varphi_R = \pi$, and have an amplitude $eU_{R,1} = 0.5\Delta_R$. Dashed (red) line: The right capacitor is driven by the in phase potential, $\varphi_R = 0$, with amplitude $eU_{R,1} = 0.5\Delta_R$. Other parameters are: $eU_{L,0} = eU_{R,0} = 0.25\Delta_R$ ($\Delta_L = \Delta_R$), $\varphi_L = 0$, $T_L = T_R = 0.1$.

zero-frequency noise power, $\sum_{\beta=1,2}\mathcal{P}_{\alpha\beta} = 0$, we derive from Eqs. (B.75) that $\mathcal{P}_{11}^{(x)} = \mathcal{P}_{22}^{(x)} = 0$, where $x = e, h$. In other words, there are regular electron (hole) flows entering the contacts $\alpha = 1, 2$. This regularity is due to the following. First, the electrons (holes) are regularly emitted by the sources. And, second, due to the Pauli exclusion principle, each two electrons (holes) incident upon the QPC will be scattered into different contacts.

While electrons (holes) emitted by the different SPSs are statistically independent, after the collision at the central QPC electrons (holes) become correlated (i.e., indistinguishable in the quantum-statistical sense) since they lose their origin: It is impossible to indicate which SPS emitted an electron (hole) arrived at the given contact. Thus the disappearance of a shot noise [116] indicates an appearance of the Fermi correlations between electrons (holes) after colliding at the QPC. This effect looks similar to the Hong, Ou, and Mandel

[123] effect in optics. However as we show for electrons the probability to detect particles at the different contacts peaks while for photons it shows a dip [123].

B.4.2 Particle probability analysis

For definiteness we will concentrate on electrons. Remind that during the period $\mathcal{T} = 2\pi/\Omega_0$ each SPS emits one electron at $t_{0L}^{(-)}$ and $t_{0R}^{(-)}$, respectively for L and R sources. The single-particle probability $\mathcal{N}_\alpha$, i.e., the probability to detect an electron at the contact $\alpha = 1, 2$ during a period, is independent of the time difference $\Delta t_{L,R}^{(-,-)} = t_{0L}^{(-)} - t_{0R}^{(-)}$. In contrast the two-particle probability $\mathcal{N}_{12}$, i.e., the probability to detect electrons at both contacts during a period depends crucially on this time difference. Moreover at $\Delta t_{L,R}^{(-,-)} = 0$ the two-particle probability $\mathcal{N}_{12}$ becomes *a joint detection probability* introduced by Glauber [124], which means a probability to detect two particles at two contacts simultaneously, i.e., on the time-scale $\Gamma_{\tau j} \ll \mathcal{T}$.

B.4.2.1 Single-particle probabilities

At $\Delta t_{L,R}^{(-,-)} \gg \Gamma_\tau = \Gamma_{\tau L} = \Gamma_{\tau R}$ the particles emitted by the different sources remain distinguishable and we can write,

$$\mathcal{N}_1 = \mathcal{N}_1^{(L)} + \mathcal{N}_1^{(R)}, \tag{B.76a}$$

$$\mathcal{N}_2 = \mathcal{N}_2^{(L)} + \mathcal{N}_2^{(R)}, \tag{B.76b}$$

where the upper indices (L) and (R) stand for the origin of an electron. The single particle probability can be calculated as the square of a single-particle amplitude for the particle emitted by some SPS to arrive at the given contact, $\mathcal{N}_\alpha^{(j)} = \left|\mathcal{A}_{\alpha j}\right|^2$, with

$$\begin{aligned} \mathcal{A}_{1L} &= e^{ik_F L_{1L}}\, r_C\,, \quad \mathcal{A}_{1R} = e^{ik_F L_{1R}}\, t_C\,, \\ \mathcal{A}_{2L} &= e^{ik_F L_{2L}}\, t_C\,, \quad \mathcal{A}_{2R} = e^{ik_F L_{2R}}\, r_C\,. \end{aligned} \tag{B.77}$$

where $L_{\alpha j} = L_{\alpha C} + L_{Cj}$ is the distance from the source $j = L, R$ though the quantum point contact C to the contact $\alpha = 1, 2$ along the linear edge state, see Fig. B.4 and compare to Eq. (B.71). Then we find,

$$\mathcal{N}_1^{(L)} = R_C\,, \quad \mathcal{N}_1^{(R)} = T_C\,, \tag{B.78a}$$

$$\mathcal{N}_2^{(L)} = T_C\,, \quad \mathcal{N}_2^{(R)} = R_C\,, \tag{B.78b}$$

and

$$\mathcal{N}_1 = \mathcal{N}_2 = 1\,. \tag{B.79}$$

For $\Delta t_{L,R}^{(-,-)} = 0$ we can not distinguish the SPS an electron came from. However apparently one electron should be detected at each contact. Therefore, Eq. (B.79) remains valid.

B.4.2.2 Two-particle probability for classical regime

At $\Delta t_{L,R}^{(-,-)} \gg \Gamma_\tau = \Gamma_{\tau L} = \Gamma_{\tau R}$ the electrons emitted by the different SPSs remain uncorrelated. Therefore, we can write,

$$\mathcal{N}_{12}^{(LR)} = \mathcal{N}_1^{(L)}\mathcal{N}_2^{(R)}\,, \quad \mathcal{N}_{12}^{(RL)} = \mathcal{N}_1^{(R)}\mathcal{N}_2^{(L)}\,. \tag{B.80}$$

For $N_\alpha^{(j)}$ see Eqs. (B.78). Taking into account that the two-electron probability can be represented as follows,

$$\mathcal{N}_{12} = \mathcal{N}_{12}^{(L)} + \mathcal{N}_{12}^{(LR)} + \mathcal{N}_{12}^{(RL)} + \mathcal{N}_{12}^{(R)}\,, \tag{B.81}$$

and that a single electron can not be detected at two distant places,

$$\mathcal{N}_{12}^{(L)} = \mathcal{N}_{12}^{(R)} = 0\,, \tag{B.82}$$

we find,

$$\mathcal{N}_{12} = \mathcal{N}_{12}^{(LR)} + \mathcal{N}_{12}^{(RL)} = R_C^2 + T_C^2\,. \tag{B.83}$$

Note $\mathcal{N}_{12} < 1$ since with probability $R_C T_C$ the two electrons can reach the same (either 1 or 2) contact.

Using Eq. (B.79) we find, $\delta\mathcal{N}_{12} = \mathcal{N}_{12} - \mathcal{N}_1\mathcal{N}_2 = -2R_C T_C$, that is, by virtue of Eq. (B.31), consistent with a shot noise due to electrons, $\mathcal{P}_{12}^{(e)} = -2\mathcal{P}_0$, see Eq. (B.73) for the total noise. Alternatively we can proceed as follows. Since in this regime the electrons emitted by the different SPSs are statistically independent, the particle correlation function, $\delta\mathcal{N}_{12} = \mathcal{N}_{12} - \mathcal{N}_1\mathcal{N}_2$, can be represented as the sum,

$$\delta\mathcal{N}_{12} = \delta\mathcal{N}_{12}^{(L)} + \delta\mathcal{N}_{12}^{(R)}, \tag{B.84}$$

where the single-particle correlation functions are,

$$\delta\mathcal{N}_{12}^{(L)} = -\mathcal{N}_1^{(L)}\mathcal{N}_2^{(L)}, \quad \delta\mathcal{N}_{12}^{(R)} = -\mathcal{N}_1^{(R)}\mathcal{N}_2^{(R)}. \tag{B.85}$$

Using Eqs. (B.78) we find again, $\delta\mathcal{N}_{12} = -2R_C T_C$.

B.4.2.3 Two-particle probability for quantum regime

If $\Delta t_{L,R}^{(-,-)} = 0$ then the electrons collide at the central QPC and become correlated, i.e. they acquire fermionic statistics. Therefore, we can not use Eq. (B.80). Strictly speaking, we even can not introduce the upper indices, since we can not indicate the origin of an electron arriving at the given contact. In this regime we can still use Eq. (B.79). Since there are no events with two electrons arriving at the same contact, we find:

$$\mathcal{N}_{12} = 1. \tag{B.86}$$

This quantum result is independent of the parameters of the central QPC in contrast to its classical counterpart, Eq. (B.83). Using Eqs. (B.79) and (B.86) we calculate the particle correlation function: $\delta\mathcal{N}_{12} = 0$. This is consistent with a zero noise result, Eq. (B.75a), if one uses Eq. (B.31) relating a shot noise and a particle correlation function.

The result given in Eq. (B.86) can also be calculated as a two-particle probability, $\mathcal{N}_{12} = \left|\mathcal{A}^{(2)}\right|^2$. Note due to colliding at the central QPC electrons become indistinguishable. Then scattering of two electrons, describing by the following two two-particle amplitudes $\mathcal{A}_a^{(2)} = \mathcal{A}_{1L}\mathcal{A}_{2R}$ and $\mathcal{A}_b^{(2)} = -\mathcal{A}_{1R}\mathcal{A}_{2L}$[4], result in the same final state. Therefore, these amplitudes should be added up, $\mathcal{A}^{(2)} = \mathcal{A}_a^{(2)} + \mathcal{A}_b^{(2)}$, and the two-particle amplitude can be written as the Slater determinant,

$$\mathcal{A}^{(2)} = \det \begin{vmatrix} \mathcal{A}_{1L} & \mathcal{A}_{1R} \\ \mathcal{A}_{2L} & \mathcal{A}_{2R} \end{vmatrix}. \tag{B.87}$$

Using single-particle amplitudes given in Eq. (B.77) and taking into account that $L_{1L} + L_{2R} = L_{1R} + L_{2L}$ (due to crossing of the trajectories at the central QPC) and $r_C t_C^* = -r_C^* t_C$ (due to unitarity) we arrive at Eq. (B.86).

Comparing Eqs. (B.86) and (B.79) one can see that $\mathcal{N}_{12} = \mathcal{N}_1\mathcal{N}_2$. This equation seems to tell us that the arrival of electrons at one contacts is not correlated with the arrival of electrons at another contact. However we found that electrons arrive at contacts in pairs, i.e., electrons are strongly correlated. This seeming inconsistency is due to a special value of single-particle probabilities, $\mathcal{N}_j = 1$. In the next section we consider a circuit with $\mathcal{N}_j < 1$ when the single particles as well as the pairs of correlated (due to colliding at the central QPC) particles do contribute to noise. We show that colliding particles are positively correlated.

B.5 Noisy mesoscopic electron collider

In Fig. B.6 we show a set-up where the regular flows, emitted by the two quantum capacitors S_L and S_R, become fluctuating (noisy) after passing the quantum point contacts L and R, respectively. There are events with two, one, or zero particles entering the central part of the circuit (CPC) and contributing to the cross-correlator $\mathcal{P}_{12}$ of the currents $I_1(t)$ and $I_2(t)$ flowing into the contacts 1 and 2, respectively. Under the conditions when the electrons (holes) emitted by the different SPSs can collide at the quantum point contact C, there

[4]The sign minus is due to fermionic statistics: Two electrons are interchanged in incoming scattering channels

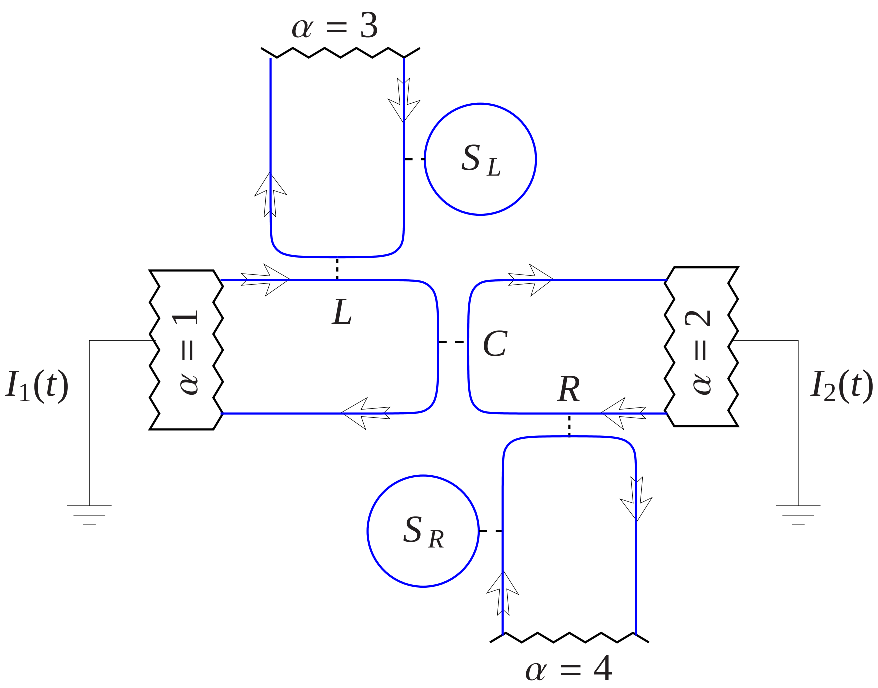

Figure B.6: The two noisy flows originated from the quantum point contacts L and R can collide at the quantum point contact C.

are different contributions into $\mathcal{P}_{12}$.[5] Namely, there are single- and two-particle contributions. In the case if the two particles enter the CPC they become correlated (after colliding at the contact C) and cause a two-particle contribution to the noise. While if only one particle (either from the contact L or from the contact R) enters the CPC it causes a negative single-particle contribution, see Eq. (B.26). In the case $T_L = T_R$ the cross-correlator is zero, $\mathcal{P}_{12} = 0$. Therefore, the two-particle contribution is positive: After colliding at the quantum point contact C the two electrons (holes) become positively correlated.

B.5.1 Current cross-correlator suppression

The elements of the frozen scattering matrix $\hat{\hat{S}}_0(t)$ for the circuit, Fig. B.6, we need to calculate $\mathcal{P}_{12}$ are the following:

[5] If the electrons (holes) do not collide at the contact C then there are only negative single-particle contributions into the noise

$$\tilde{S}_{0,11}(t) = e^{ik_F L_{11}} r_L r_C\,, \qquad \tilde{S}_{0,12}(t) = e^{ik_F L_{12}} r_R t_C\,,$$
$$\tilde{S}_{0,13}(t) = e^{ik_F L_{13}} S_L(t) t_L r_C\,, \quad \tilde{S}_{0,14}(t) = e^{ik_F L_{14}} S_R(t) t_R t_C\,, \tag{B.88a}$$

$$\tilde{S}_{0,21}(t) = e^{ik_F L_{21}} r_L t_C\,, \qquad \tilde{S}_{0,22}(t) = e^{ik_F L_{22}} r_R r_C\,,$$
$$\tilde{S}_{0,23}(t) = e^{ik_F L_{23}} S_L(t) t_L t_C\,, \quad \tilde{S}_{0,24}(t) = e^{ik_F L_{24}} S_R(t) t_R r_C\,, \tag{B.88b}$$

where the lower indices L, R, and C at the reflection and transmission coefficients denote the corresponding quantum point contacts. Using these elements in Eq. (B.16) we calculate by analogy with Eqs. (B.74):

$$\mathcal{P}_{12} = \mathcal{P}_{12}^{(e,1)} + \mathcal{P}_{12}^{(e,2)} + \mathcal{P}_{12}^{(h,1)} + \mathcal{P}_{12}^{(h,2)}\,, \tag{B.89a}$$

where the single particle,

$$\mathcal{P}_{12}^{(e,1)} = \mathcal{P}_{12}^{(h,1)} = -\mathcal{P}_0\left(T_L^2 + T_R^2\right)\,, \tag{B.89b}$$

and the two-particle,

$$\mathcal{P}_{12}^{(e,2)} = 2\mathcal{P}_0 T_L T_R \frac{4\Gamma_{\tau L}\Gamma_{\tau R}}{\left(t_{0L}^{(-)} - t_{0R}^{(-)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2}\,, \tag{B.89c}$$

$$\mathcal{P}_{12}^{(h,2)} = 2\mathcal{P}_0 T_L T_R \frac{4\Gamma_{\tau L}\Gamma_{\tau R}}{\left(t_{0L}^{(+)} - t_{0R}^{(+)}\right)^2 + (\Gamma_{\tau L} + \Gamma_{\tau R})^2}\,, \tag{B.89d}$$

contributions to the current cross-correlator are introduced. The cross-correlator $\mathcal{P}_{12}$ is given in Fig. B.7. The equation (B.89a) describes a shot noise at the second plateau ($\mathcal{P}_{12} \sim -\mathcal{P}_0$) of this plot.

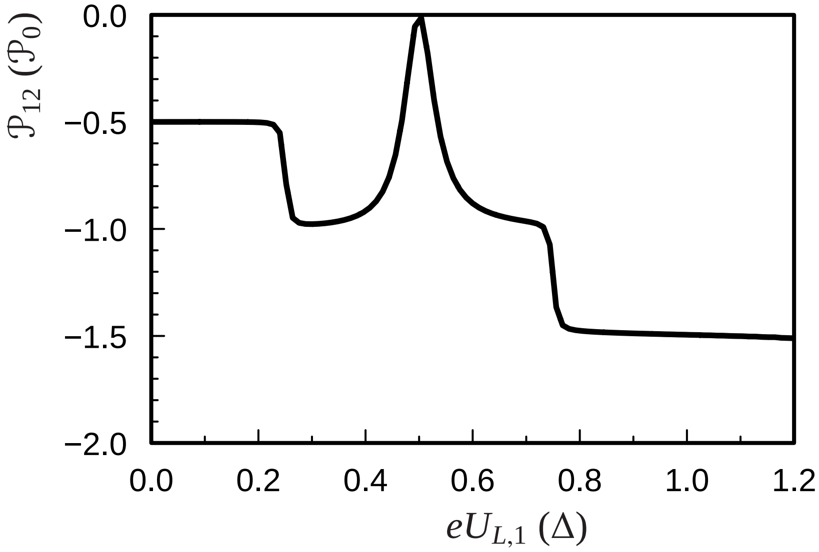

Figure B.7: The current cross-correlator $\mathcal{P}_{12}$ as a function of the amplitude $U_{L,1}$ of a potential $U_L(t) = U_{L,0} + U_{L,1}\cos(\Omega_0 t + \varphi_L)$ driving the left SPS. The parameters of SPSs are the same as in Fig. B.5 but $\varphi_L = \varphi_R$. Other parameters are: $T_L = T_R = 0.5$.

In the classical regime, $\Delta t_{L,R}^{(\chi,\chi)} \gg \Gamma_{\tau L}, \Gamma_{\tau R}$, when the emitted particles remain statistically independent, the two-particle contribution is not present, $\mathcal{P}_{12}^{(x,2)} \sim \mathcal{O}\left(\Gamma_{\tau j}/\Delta t_{L,R}^{(\chi,\chi)}\right)^2 \approx 0$, where $x = e, h$, and the cross-correlator,

$$\mathcal{P}_{12} = -2\mathcal{P}_0\left\{T_L^2 + T_R^2\right\}, \tag{B.90}$$

is due to a contribution of single particles only. In this classical regime the measurements at contacts 1 and 2 can give the following outcomes (during one period separately for electrons and holes): (i) Two particles can be detected at the same contact, (ii) two particle can be detected at different contacts, (iii) one particle can be detected at either of contacts, and (iv) no particle can be detected at all.

While if the electrons (holes) can collide at the central QPC, $\Delta t_{L,R}^{(\chi,\chi)} = 0$, the cross-correlator is suppressed compared to Eq. (B.90). Assuming $\Gamma_{\tau L} = \Gamma_{\tau R} \equiv \Gamma_\tau$ we find from Eq. (B.89a),

$$\mathcal{P}_{12} = -2\mathcal{P}_0 \left(T_R - T_L\right)^2 . \tag{B.91}$$

The current cross-correlator becomes zero in the symmetric case, $T_L = T_R$. We should stress that the current flowing into either of contacts is noisy, $\left\langle \delta I_\alpha^2 \right\rangle > 0$, $\alpha = 1, 2$, despite the fact that the current cross-correlator is suppressed. This suppression is due to a positive two-particle contribution compensating a negative single-particle one. This regime differs from the classical one considered above in two points: (i) There are no events with two electrons (holes) detected at the same contact, (ii) if two electrons (holes) are detected at different contacts they are detected simultaneously.

B.5.2 Particle probability analysis

As before, we concentrate on electrons. The holes can be considered in the same way.

B.5.2.1 Single-particle probabilities

The single-particle probabilities are insensitive to whether electrons emitted by different sources can collide at the central quantum point contact C or not. Therefore, we assume $\Delta t_{L,R}^{(-,-)} \gg \Gamma_{\tau L}, \Gamma_{\tau R}$ and use Eqs. (B.76) with the following single-particle amplitudes:

$$\begin{aligned} \mathcal{A}_{1L} &= e^{ik_F L_{1L}}\, t_L\, r_C\,, \quad & \mathcal{A}_{1R} &= e^{ik_F L_{1R}}\, t_R\, t_C\,, \\ \mathcal{A}_{2L} &= e^{ik_F L_{2L}}\, t_L\, t_C\,, \quad & \mathcal{A}_{2R} &= e^{ik_F L_{2R}}\, t_R\, r_C\,. \end{aligned} \tag{B.92}$$

Then we find,

$$\mathcal{N}_1^{(L)} = T_L R_C\,, \quad \mathcal{N}_1^{(R)} = T_R T_C\,, \tag{B.93a}$$

$$\mathcal{N}_2^{(L)} = T_L T_C\,, \quad \mathcal{N}_2^{(R)} = T_R R_C\,, \tag{B.93b}$$

and

$$\begin{aligned} \mathcal{N}_1 &= T_L + T_C\,(T_R - T_L)\,, \\ \mathcal{N}_2 &= T_R - T_C\,(T_R - T_L)\,. \end{aligned} \tag{B.94}$$

Apparently it should be $\mathcal{N}_1 + \mathcal{N}_2 = T_L + T_R$: How many electrons enter the CPC so many electrons reach contacts 1 and 2.

B.5.2.2 Two-particle probability for classical regime

We can use results of Sec. B.4.2.2. Substituting Eqs. (B.93) into Eq. (B.80) and then into Eq. (B.83) we arrive at the following:

$$\mathcal{N}_{12} = T_L T_R \left(R_C^2 + T_C^2\right). \tag{B.95}$$

This equation differs from Eq. (B.83) by the factor $T_R T_L$ which is a probability for two particles to enter the CPC and to contribute to $\mathcal{N}_{12}$.

Calculating $\delta\mathcal{N}_{12} = \mathcal{N}_{12} - \mathcal{N}_1\mathcal{N}_2$ with Eqs. (B.94) and (B.95) we find,

$$\delta\mathcal{N}_{12} = -R_C T_C \left(T_L^2 + T_R^2\right), \tag{B.96}$$

that is consistent with a single-electron contribution to the cross-correlator, Eq. (B.89b) by virtue of Eq. (B.31).

Alternatively Eq. (B.96) can be represent as Eq. (B.84) with

$$\delta\mathcal{N}_{12}^{(L)} = -R_C T_C T_L^2\,, \quad \delta\mathcal{N}_{12}^{(R)} = -R_C T_C T_R^2\,. \tag{B.97}$$

B.5.2.3 Two-particle probability for quantum regime

If $\Delta t_{L,R}^{(-,-)} = 0$ then those electrons emitted by the SPSs which reach the contacts 1 and 2 are correlated. Therefore, instead of Eq. (B.84) we should write:

$$\delta\mathcal{N}_{12} = \delta\mathcal{N}_{12}^{(L)} + \delta\mathcal{N}_{12}^{(R)} + \delta\mathcal{N}_{12}^{(\widehat{LR})}, \tag{B.98}$$

where the single-particle cross-correlation functions are given in Eq. (B.97) and the two-particle correlation function is:

$$\delta\mathcal{N}_{12}^{(\widehat{LR})} = \mathcal{N}_{12} - \mathcal{N}_1^{(L)}\mathcal{N}_2^{(R)} - \mathcal{N}_1^{(R)}\mathcal{N}_2^{(L)}. \tag{B.99}$$

The two-particle probability $\mathcal{N}_{12}$ can be calculated as follows, $\mathcal{N}_{12} = \left|\mathcal{A}^{(2)}\right|^2$, with a two-particle amplitude (in the case of indistinguishable electrons) being the Slater determinant, Eq. (B.87). Using the single-particle amplitudes given in Eq. (B.92) we calculate,

$$\mathcal{N}_{12} = T_L T_R. \tag{B.100}$$

Note this equation is independent of the parameters of the central QPC, that can be used as an indication of a quantum regime. Stress in the quantum regime the two-particle probability becomes the Glauber joint detection probability [124].

The equation (B.100) can be understood as follows: If and only if the two electrons enter the CPC (one electron from L and one electron from R) then they necessarily collide at the central QPC and reach different contacts. Therefore, the probability to detect one electron at the contact 1 and one electron at the contact 2 is equal to a probability for two electrons to enter the CPC.

Using Eqs. (B.100) and (B.93) we calculate the two-particle cross-correlation function, Eq. (B.99),

$$\delta\mathcal{N}_{12}^{(\widehat{LR})} = 2T_L T_R R_C T_C. \tag{B.101}$$

which, by virtue of Eq. (B.31), is consistent with a two-particle contribution to the cross-correlation function, Eq. (B.89c), at $\Gamma_{\tau L} = \Gamma_{\tau R}$ and $t_{0L}^{(-)} = t_{0R}^{(-)}$.

With Eqs. (B.97) and (B.101) we calculate the total particle cross-correlation function, Eq. (B.98):

$$\delta\mathcal{N}_{12} = -R_C T_C (T_R - T_L)^2 , \tag{B.102}$$

which is consistent with an electron contribution to the current cross-correlation function $\mathcal{P}_{12}^{(e)} = \mathcal{P}_{12}/2 = -\mathcal{P}_0 (T_R - T_L)^2$, see Eq. (B.91) for the total current cross-correlation function.

B.6 Two-particle interference effect

Let us consider a circuit with two interferometers, Fig. B.8, and show that the particles emitted by the SPSs can show even such a subtle effect as a two-particle interference effect. [117]

In contrast to previous sections, now we consider a non-adiabatic regime: We take into account the time necessary for electrons (holes) to propagate along circuit's branches, while the process of emission by the SPS is treated adiabatically. If the difference of times of a propagation along the different arms, U and D, of an interferometer is larger than $\Gamma_{\tau j}$, then the single-particle interference is suppressed and the currents flowing into contacts are insensitive to the magnetic flux though the interferometer. However if the parameters of a circuit are adjusted in such a way that the particles emitted by S_L and S_R can collide at the outputs $L1$ and $R2$, then the current cross-correlation function becomes sensitive to magnetic fluxes Φ_L and Φ_R of both interferometers. This effect is a manifestation of a two-particle interference taking place in the system.

B.6.1 Model and definitions

The circuit, Fig. B.8, has four contacts and, therefore, it is described by the 4×4 scattering matrix $\hat{S}_{in}(t, E)$ defining the corresponding elements of the Floquet scattering matrix, $S_{F,\alpha\beta}(E_n, E) = S_{in,\alpha\beta,n}(E)$, $\alpha, \beta = 1, 2, 3, 4$. All the contacts are in equilibrium and have the same Fermi distribution function, $f_i(E) = f_0(E), \forall i$, with chemical potential μ_0 and temperature T_0. Each source S_L and S_R emits one electron and one hole during a period.

We are interested in a zero-frequency cross-correlation function $\mathcal{P}_{12}$ for currents I_1 and I_2. At zero temperature, $k_B T_0 = 0$, it reads [see Eq. (6.16)],

$$\mathcal{P}_{12} = \frac{e^2}{2h} \sum_{q=-\infty}^{\infty} \text{sign}(q) \int_{\mu_0 - q\hbar\Omega_0}^{\mu_0} dE \sum_{n.m=-\infty}^{\infty} \sum_{\gamma,\delta=1}^{4} \times S_{F,1\gamma}(E_n, E) S_{F,1\delta}^{*}(E_n, E_q) S_{F,2\delta}(E_m.E_q) S_{F,2\gamma}^{*}(E_m, E)\,, \tag{B.103}$$

where $E_n = E + n\hbar\Omega_0$. Using $\hat{S}_{in}$ and summing up over n and m we find,

$$\mathcal{P}_{12} = \frac{e^2}{2h} \sum_{q=-\infty}^{\infty} \text{sign}(q) \int_{\mu_0 - q\hbar\Omega_0}^{\mu_0} dE \sum_{\gamma,\delta=1}^{4} \times \left\{S_{in,1\gamma}(E) S_{in,1\delta}^{*}(E_q)\right\}_q \left\{S_{in,2\gamma}(E) S_{in,2\delta}^{*}(E_q)\right\}_q^{*}\,. \tag{B.104}$$

In the circuit under consideration there are no paths from the contact 4 to the contact 1 and from the contact 3 to the contact 2. Therefore, the relevant indices are $\gamma, \delta = 1, 2$. Thus the final expression for the current cross-correlator becomes:

$$\mathcal{P}_{12} = \frac{e^2}{2h} \sum_{q=-\infty}^{\infty} \text{sign}(q) \int_{\mu_0 - q\hbar\Omega_0}^{\mu_0} dE \left\{A_q + B_q + C_q + D_q\right\}\,, \tag{B.105a}$$

where

$$A_q = \left\{S_{in,11}(E) S_{in,11}^{*}(E_q)\right\}_q \left\{S_{in,21}(E) S_{in,21}^{*}(E_q)\right\}_q^{*}\,, \tag{B.105b}$$

$$B_q = \left\{S_{in,11}(E) S_{in,12}^{*}(E_q)\right\}_q \left\{S_{in,21}(E) S_{in,22}^{*}(E_q)\right\}_q^{*}\,, \tag{B.105c}$$

$$C_q = \left\{S_{in,12}(E) S_{in,11}^{*}(E_q)\right\}_q \left\{S_{in,22}(E) S_{in,21}^{*}(E_q)\right\}_q^{*}\,, \tag{B.105d}$$

$$D_q = \left\{S_{in,12}(E) S_{in,12}^{*}(E_q)\right\}_q \left\{S_{in,22}(E) S_{in,22}^{*}(E_q)\right\}_q^{*}\,. \tag{B.105e}$$

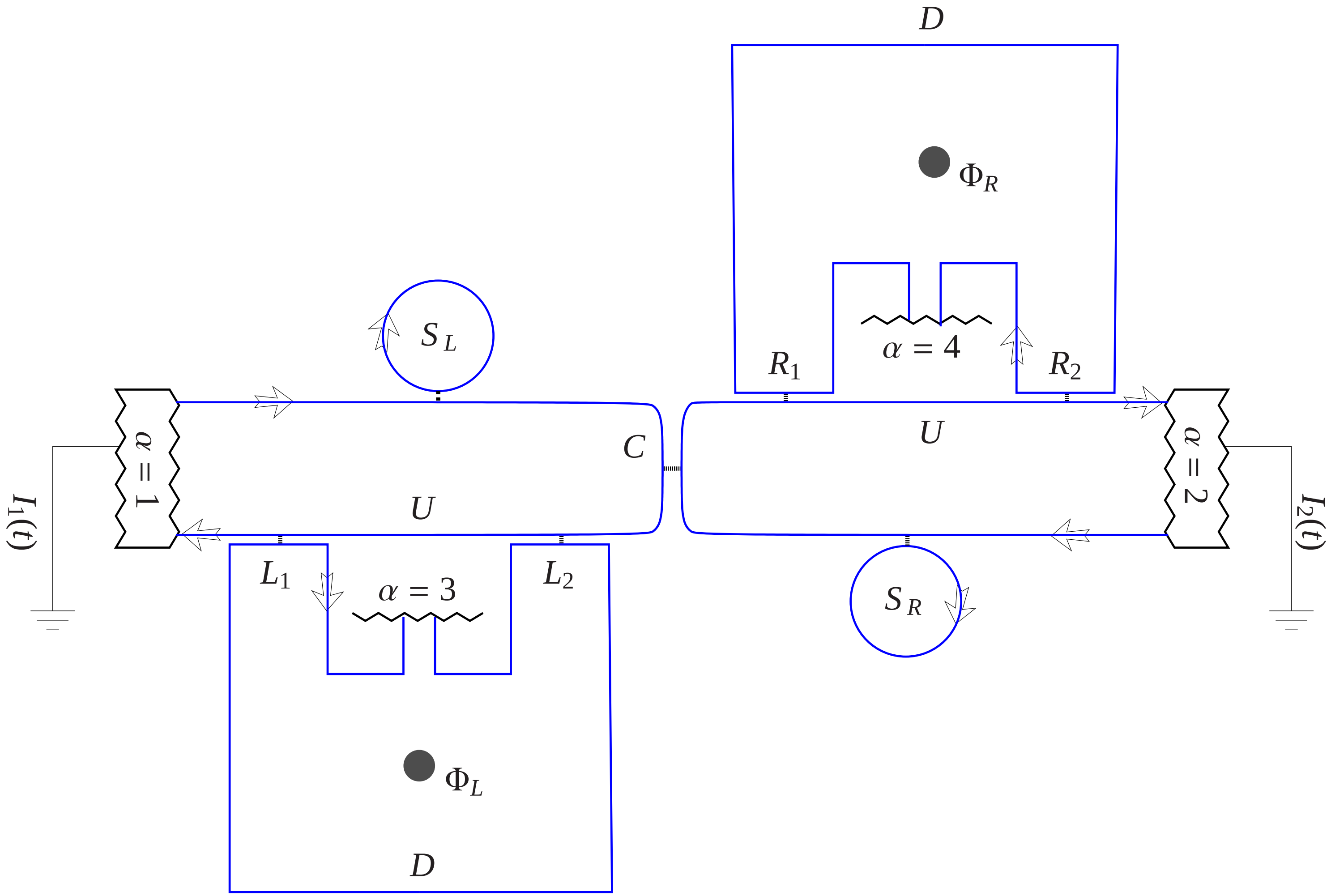

Figure B.8: The circuit comprising two single particle sources S_L and S_R and two Mach-Zehnder interferometers with magnetic fluxes Φ_L and Φ_R, respectively.

Note because of integration over energy in Eq. (B.105a) only the quantities with $q \neq 0$ are relevant. Shifting $E_q \to E$ (under the integration over energy) and replacing $q \to -q$ (under the sum over q) one can show that the contributions due to A_q and D_q are real, while the contributions due to C_q and B_q are complex conjugate each other.

Manipulating with Fourier coefficients we will use the following relations:

$$\begin{aligned} \{X(t)\}_q \, e^{iq\Omega_0\tau} &= \{X(t-\tau)\}_q \,, \\ \{X(t)\}_q \, \{Y(t)\}_q^* &= \{X(t-\tau)\}_q \, \{Y(t-\tau)\}_q^* \,. \end{aligned} \tag{B.106}$$

Before presenting expressions for the scattering matrix elements we introduce some definitions. We assume that kinematic phase $\varphi_{\mathcal{L}}(E)$ acquired by an electron with energy E along the trajectory $\mathcal{L}$ of a length $L_{\mathcal{L}}$ is linear in energy,

$$\varphi_{\mathcal{L}}(E) = \varphi_{\mathcal{L}} + (E - \mu_0)\, \tau_{\mathcal{L}}/\hbar \,, \tag{B.107}$$

where $\varphi_{\mathcal{L}} = k_F L_{\mathcal{L}}$ and $\tau_{\mathcal{L}}$ is an independent of energy time spent by an electron within the trajectory $\mathcal{L}$. We will label each trajectory by the three-letter lower index, where the first letter is a number of a destination contact, the second letter indicates the branch of a corresponding MZI, and the third letter indicates the source of electrons. For instance, the label $\mathcal{L} = 2UL$ indicates a trajectory starting at the left single-particle source S_L, passing across the upper branch of the (right) MZI, and finishing at the contact 2. We will name MZI's branch as upper (the lower index U) or down (the lower index D) if an electron going through this branch encircles the magnetic flux counter-clockwise or clockwise, respectively, see Fig. B.8.

It is convenient to introduce an interferometer imbalance time $\Delta\tau_j$, $j = L, R$ and a time delay $\Delta\tau_{LR}$,

$$\Delta\tau_L = \tau_{1Uj} - \tau_{1Dj} \,, \quad \Delta\tau_R = \tau_{2Uj} - \tau_{2Dj} \,, \quad \Delta\tau_{LR} = \tau_{\alpha YL} - \tau_{\alpha YR} \,, \tag{B.108}$$

where $\alpha = 1, 2$, $Y = U, D$, and $j = L, R$. The quantity $\Delta\tau_{LR}$ characterizes the asymmetry in position of the sources S_L and S_R with respect to the central QPC. With these definitions some differences which we need below read as follows,

$$\begin{aligned} \tau_{1UL} - \tau_{1DR} &= \Delta\tau_L + \Delta\tau_{LR}\,, \quad \tau_{1UR} - \tau_{1DL} = \Delta\tau_L - \Delta\tau_{LR}\,, \\ \tau_{2UL} - \tau_{2DR} &= \Delta\tau_R + \Delta\tau_{LR}\,, \quad \tau_{2UR} - \tau_{2DL} = \Delta\tau_R - \Delta\tau_{LR}\,. \end{aligned} \tag{B.109}$$

The magnetic flux, Φ_j, $j = L, R$, through the corresponding MZI we present as the sum of fluxes associated with upper and lower branches,

$$\Phi_j = \Phi_{jU} + \Phi_{jD}\,, \tag{B.110}$$

Each MZI has two quantum point contacts which we will label by $j1$ and $j2$, $j = L, R$. Without loss of generality we choose the scattering matrices for these QPCs as

$$\hat{S}_{j\alpha} = \begin{pmatrix} \sqrt{R_{j\alpha}} & i\sqrt{T_{j\alpha}} \\ i\sqrt{T_{j\alpha}} & \sqrt{R_{j\alpha}} \end{pmatrix}. \tag{B.111}$$

Here $\alpha = 1, 2$. For the central quantum point contact C connecting two branches of the circuit we use a scattering matrix of the same form but with index $j\alpha$ being replaced by the index C.

We assume also that the dwell time τ_j for electrons in each single-particle source, $j = L$ for S_L and $j = R$ for S_R, is short compared to the period of a drive [see Eq. (A.57)],

$$\Omega_0 \tau_j \ll 1\,. \tag{B.112}$$

Therefore, for the left and right single-particle sources we can use the frozen scattering amplitudes, which we denote as $S_L(t, E)$ and $S_R(t, E)$, respectively. In particular, within this approximation one can use $S_j(t, E) \approx S_j(t, E_n)$. In what follows we use $S_j(t, E) \approx S_j(t, \mu_0) \equiv S_j(t)$. The amplitudes $S_j(t)$ are given in Eq. (B.8) with θ_r, Γ_τ, and $t_0^{(\mp)}$ being replaced by θ_{rj}, $\Gamma_{\tau j}$, and $t_{0j}^{(\mp)}$, respectively.

We are interested in the regime when the interferometer imbalance times are large compared to the duration of wave packets but small compared to the period of a drive,

$$\mathcal{T} \gg \Delta\tau_L, \Delta\tau_R \gg \Gamma_{\tau L}, \Gamma_{\tau R}\,. \tag{B.113}$$

Then there is no a single-particle interference effect which could result in magnetic-flux dependence of a current cross-correlator. On the other hand if,

$$\Delta\tau_L \pm \Delta\tau_R \;=\; 0\,, \tag{B.114}$$

then, the interference of two-particle amplitudes makes $\mathcal{P}_{12}$ dependent on $\Phi_L \pm \Phi_R$.

B.6.2 The scattering matrix elements

Calculating the scattering matrix elements we take into account that an electron can follow to a given contact along the different trajectories. For instance, we have:

$$\begin{aligned}
S_{F,11}(E_n, E) &= S_{F,1U1}(E_n, E) + S_{F,1D1}(E_n, E)\,,\\
S_{F,1U1}(E_n, E) &= \sqrt{R_C R_{L1} R_{L2}}\, e^{i2\pi\frac{\Phi_{LU}}{\Phi_0}}\, e^{i\varphi_{1UL}(E_n)} S_{L,n}\,,\\
&= \sqrt{R_C R_{L1} R_{L2}}\, e^{i2\pi\frac{\Phi_{LU}}{\Phi_0}}\, e^{i\varphi_{1UL}(E)} e^{in\Omega_0\tau_{1UL}} S_{L,n}\,,\\
S_{F,1D1}(E_n, E) &= -\sqrt{R_C T_{L1} T_{L2}}\, e^{-i2\pi\frac{\Phi_{LD}}{\Phi_0}}\, e^{i\varphi_{1DL}(E_n)} S_{L,n}\,,\\
&= -\sqrt{R_C T_{L1} T_{L2}}\, e^{-i2\pi\frac{\Phi_{LD}}{\Phi_0}}\, e^{i\varphi_{1DL}(E)} e^{in\Omega_0\tau_{1DL}} S_{L,n}\,.
\end{aligned}$$

where the dependence $\varphi_{\mathcal{L}}(E)$ is given in Eq. (B.107). After the inverse Fourier transformation, we arrive at the following,

$$S_{in,11}(t,E) = \sqrt{R_C}\Big\{ \sqrt{R_{L1}R_{L2}}\, e^{i2\pi\frac{\Phi_{LU}}{\Phi_0}}\, e^{i\varphi_{1UL}(E)} S_L(t-\tau_{1UL},E) \tag{B.115a}$$
$$- \sqrt{T_{L1}T_{L2}}\, e^{-i2\pi\frac{\Phi_{LD}}{\Phi_0}}\, e^{i\varphi_{1DL}(E)} S_L(t-\tau_{1DL},E)\Big\},$$

By analogy we find other scattering matrix elements we need to calculate Eqs. (B.105):

$$S^*_{in,11}(t,E_q) = \Big\{ \sqrt{R_{L1}R_{L2}}\, e^{-i2\pi\frac{\Phi_{LU}}{\Phi_0}}\, e^{-i\varphi_{1UL}(E)} e^{-iq\Omega_0\tau_{1UL}} S^*_L(t-\tau_{1UL},E) \tag{B.115b}$$
$$- \sqrt{T_{L1}T_{L2}}\, e^{i2\pi\frac{\Phi_{LD}}{\Phi_0}}\, e^{-i\varphi_{1DL}(E)} e^{-iq\Omega_0\tau_{1DL}} S^*_L(t-\tau_{1DL},E)\Big\} \sqrt{R_C}\,,$$

$$S_{in,12}(t,E) = i\sqrt{T_C}\Big\{ \sqrt{R_{L1}R_{L2}}\, e^{i2\pi\frac{\Phi_{LU}}{\Phi_0}}\, e^{i\varphi_{1UR}(E)} S_R(t-\tau_{1UR},E) \tag{B.115c}$$
$$- \sqrt{T_{L1}T_{L2}}\, e^{-i2\pi\frac{\Phi_{LD}}{\Phi_0}}\, e^{i\varphi_{1DR}(E)} S_R(t-\tau_{1DR},E)\Big\},$$

$$S^*_{in,12}(t,E_q) = \Big\{ \sqrt{R_{L1}R_{L2}}\, e^{-i2\pi\frac{\Phi_{LU}}{\Phi_0}}\, e^{-i\varphi_{1UR}(E)} e^{-iq\Omega_0\tau_{1UR}} S^*_R(t-\tau_{1UR},E) \tag{B.115d}$$
$$- \sqrt{T_{L1}T_{L2}}\, e^{i2\pi\frac{\Phi_{LD}}{\Phi_0}}\, e^{-i\varphi_{1DR}(E)} e^{-iq\Omega_0\tau_{1DR}} S^*_R(t-\tau_{1DR},E)\Big\} \left(-i\sqrt{T_C}\right),$$

$$S_{in,21}(t,E) = i\sqrt{T_C}\Big\{ \sqrt{R_{R1}R_{R2}}\, e^{i2\pi\frac{\Phi_{RU}}{\Phi_0}}\, e^{i\varphi_{2UL}(E)} S_L(t-\tau_{2UL},E) \tag{B.115e}$$
$$- \sqrt{T_{R1}T_R}\, e^{-i2\pi\frac{\Phi_{RD}}{\Phi_0}}\, e^{i\varphi_{2DL}(E)} S_L(t-\tau_{2DL},E)\Big\},$$

$$S^*_{in,21}(t,E_q) = \Big\{ \sqrt{R_{R1}R_{R2}}\, e^{-i2\pi\frac{\Phi_{RU}}{\Phi_0}}\, e^{-i\varphi_{2UL}(E)} e^{-iq\Omega_0\tau_{2UL}} S^*_L(t-\tau_{2UL},E)$$

$$- \sqrt{T_{R1}T_{R2}}\, e^{i2\pi\frac{\Phi_{RD}}{\Phi_0}} e^{-i\varphi_{2DL}(E)} e^{-iq\Omega_0\tau_{2DL}} S^*_L(t-\tau_{2DL},E) \Big\} \left(-i\sqrt{T_C}\right), \tag{B.115f}$$

$$S_{in,22}(t,E) = \sqrt{R_C} \Big\{ \sqrt{R_{R1}R_{R2}}\, e^{i2\pi\frac{\Phi_{RU}}{\Phi_0}} e^{i\varphi_{2UR}(E)} S_R(t-\tau_{2UR},E)$$

$$- \sqrt{T_{R1}T_{R2}}\, e^{-i2\pi\frac{\Phi_{RD}}{\Phi_0}} e^{i\varphi_{2DR}(E)} S_R(t-\tau_{2DR},E) \Big\}, \tag{B.115g}$$

$$S_{in,22}(t,E_q) = \Big\{ \sqrt{R_{R1}R_{R2}}\, e^{i2\pi\frac{\Phi_{RU}}{\Phi_0}}\, e^{i\varphi_{2UR}(E)} e^{iq\Omega_0\tau_{2UR}} S_R(t-\tau_{2UR},E)$$

$$- \sqrt{T_{R1}T_{R2}}\, e^{-i2\pi\frac{\Phi_{RD}}{\Phi_0}} e^{i\varphi_{2DR}(E)} e^{iq\Omega_0\tau_{2DR}} S_R(t-\tau_{2DR},E) \Big\} \sqrt{R_C}\,. \tag{B.115h}$$

Using given above equations we calculate the cross-correlator $\mathcal{P}_{12}$ and analyze its dependence on magnetic fluxes Φ_L and Φ_R.

B.6.3 Current cross-correlator

We consider separately quantities A_q, B_q. C_q, and D_q entering Eq. (B.105a).

B.6.3.1 Partial contributions

First we calculate quantities A_q, D_q and corresponding contributions $\mathcal{P}^{(A)}_{12}$, $\mathcal{P}^{(D)}_{12}$ to the cross-correlator. Substituting Eqs. (B.115) into Eq. (B.105b) we find for $q \neq 0$:

$$A_q = R_C T_C \zeta_L \zeta_R \sum_{i=1}^{4} A_{i,q}\,, \quad \zeta_j = \sqrt{R_{j1}R_{j2}T_{j1}T_{j2}}\,, \quad j = L, R\,, \tag{B.116}$$

where

$$A_{1,q} = e^{i2\pi\frac{\Phi_L+\Phi_R}{\Phi_0}} e^{i\frac{E}{\hbar}(\Delta\tau_L+\Delta\tau_R)} \left\{S_L(t-\Delta\tau_L)S_L^*(t)\right\}_q \left\{S_L(t+\Delta\tau_R)S_L^*(t)\right\}_q^* ,$$

$$A_{2,q} = e^{-i2\pi\frac{\Phi_L+\Phi_R}{\Phi_0}} e^{-i\frac{E}{\hbar}(\Delta\tau_L+\Delta\tau_R)} \left\{S_L(t+\Delta\tau_L)S_L^*(t)\right\}_q \left\{S_L(t-\Delta\tau_R)S_L^*(t)\right\}_q^* ,$$

$$A_{3,q} = e^{i2\pi\frac{\Phi_L-\Phi_R}{\Phi_0}} e^{i\frac{E}{\hbar}(\Delta\tau_L-\Delta\tau_R)} \left\{S_L(t-\Delta\tau_L)S_L^*(t)\right\}_q \left\{S_L(t-\Delta\tau_R)S_L^*(t)\right\}_q^* ,$$

$$A_{4,q} = e^{-i2\pi\frac{\Phi_L-\Phi_R}{\Phi_0}} e^{-i\frac{E}{\hbar}(\Delta\tau_L-\Delta\tau_R)} \left\{S_L(t+\Delta\tau_L)S_L^*(t)\right\}_q \left\{S_L(t+\Delta\tau_R)S_L^*(t)\right\}_q^* ,$$

Notice the sums $A_{1,q} + A_{2,-q}$ and $A_{3,q} + A_{4,-q}$ become real (only) after integrating over energy in Eq. (B.105a). The Fourier coefficients are:

$$\left\{S_L(t \mp \Delta\tau_L)S_{0L}^*(t)\right\}_q = -s_{L,q} \begin{cases} e^{iq\Omega_0 t_{0L}^{(-)}} e^{\pm iq\Omega_0\Delta\tau_L} + e^{iq\Omega_0 t_{0L}^{(+)}} , & q > 0 , \\ e^{iq\Omega_0 t_{0L}^{(+)}} e^{\pm iq\Omega_0\Delta\tau_L} + e^{iq\Omega_0 t_{0L}^{(-)}} , & q < 0 , \end{cases}$$

$$\left\{S_L(t \pm \Delta\tau_R)S_{0L}^*(t)\right\}_q^* = -s_{L,q} \begin{cases} e^{-iq\Omega_0 t_{0L}^{(-)}} e^{\pm iq\Omega_0\Delta\tau_R} + e^{-iq\Omega_0 t_{0L}^{(+)}} , & q > 0 , \\ e^{-iq\Omega_0 t_{0L}^{(+)}} e^{\pm iq\Omega_0\Delta\tau_R} + e^{-iq\Omega_0 t_{0L}^{(-)}} , & q < 0 , \end{cases}$$

where $s_{L,q} = 2\Omega_0\Gamma_{\tau L} e^{-|q|\Omega_0\Gamma_{\tau L}}$. And the corresponding products read as follows:

$$\left\{S_L(t-\Delta\tau_L)S_{0L}^*(t)\right\}_q \left\{S_L(t+\Delta\tau_R)S_{0L}^*(t)\right\}_q^* = s_{L,q}^2$$
$$\times \begin{cases} 1 + e^{iq\Omega_0(\Delta\tau_L+\Delta\tau_R)} + e^{iq\Omega_0\left(t_{0L}^{(-)}-t_{0L}^{(+)}+\Delta\tau_L\right)} + e^{-iq\Omega_0\left(t_{0L}^{(-)}-t_{0L}^{(+)}-\Delta\tau_R\right)} , & q > 0 , \\ 1 + e^{iq\Omega_0(\Delta\tau_L+\Delta\tau_R)} + e^{iq\Omega_0\left(t_{0L}^{(-)}-t_{0L}^{(+)}+\Delta\tau_R\right)} + e^{-iq\Omega_0\left(t_{0L}^{(-)}-t_{0L}^{(+)}-\Delta\tau_L\right)} , & q < 0 , \end{cases}$$

$$\left\{S_L(t+\Delta\tau_L)S^*_{0L}(t)\right\}_q \left\{S_L(t-\Delta\tau_R)S^*_{0L}(t)\right\}^*_q = s^2_{L,q}$$

$$\times\begin{cases} 1+e^{-iq\Omega_0(\Delta\tau_L+\Delta\tau_R)}+e^{iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}-\Delta\tau_L\right)}+e^{-iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}+\Delta\tau_R\right)}, & q>0,\\ 1+e^{-iq\Omega_0(\Delta\tau_L+\Delta\tau_R)}+e^{iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}-\Delta\tau_R\right)}+e^{-iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}+\Delta\tau_L\right)}, & q<0,\end{cases}$$

$$\left\{S_L(t-\Delta\tau_L)S^*_{0L}(t)\right\}_q \left\{S_L(t-\Delta\tau_R)S^*_{0L}(t)\right\}^*_q = s^2_{L,q}$$

$$\times\begin{cases} 1+e^{iq\Omega_0(\Delta\tau_L-\Delta\tau_R)}+e^{iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}+\Delta\tau_L\right)}+e^{-iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}+\Delta\tau_R\right)}, & q>0,\\ 1+e^{iq\Omega_0(\Delta\tau_L-\Delta\tau_R)}+e^{iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}-\Delta\tau_R\right)}+e^{-iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}-\Delta\tau_L\right)}, & q<0,\end{cases}$$

$$\left\{S_L(t+\Delta\tau_L)S^*_{0L}(t)\right\}_q \left\{S_L(t+\Delta\tau_R)S^*_{0L}(t)\right\}^*_q = s^2_{L,q}$$

$$\times\begin{cases} 1+e^{-iq\Omega_0(\Delta\tau_L-\Delta\tau_R)}+e^{iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}-\Delta\tau_L\right)}+e^{-iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}-\Delta\tau_R\right)}, & q>0,\\ 1+e^{-iq\Omega_0(\Delta\tau_L-\Delta\tau_R)}+e^{iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}+\Delta\tau_R\right)}+e^{-iq\Omega_0\left(t^{(-)}_{0L}-t^{(+)}_{0L}+\Delta\tau_L\right)}, & q<0.\end{cases}$$

Taking into account Eqs. (B.53), (B.54) and the presence of integration over energy in Eq. (B.105a) we conclude that the quantity A_q (after summing up over q) results in a noticeable contribution to the cross-correlator only if it does not oscillate in energy. This is the case under conditions given in Eq. (B.114). Then we calculate,

$$\mathcal{P}^{(A)}_{12} = 4\mathcal{P}_0\zeta_L\zeta_R\cos\left(2\pi\frac{\Phi_L\pm\Phi_R}{\Phi_0}\right). \tag{B.117}$$

The quantity D_q leads to the same contribution, $\mathcal{P}^{(D)}_{12} = \mathcal{P}^{(A)}_{12}$. Then the corresponding part of a cross-correlator, $\mathcal{P}^{(A+D)}_{12} = \mathcal{P}^{(A)}_{12} + \mathcal{P}^{(D)}_{12}$, is:

$$\mathcal{P}^{(A+D)}_{12} = 8\mathcal{P}_0\zeta_L\zeta_R\cos\left(2\pi\frac{\Phi_L\pm\Phi_R}{\Phi_0}\right). \tag{B.118}$$

Next we calculate B_q. Using Eqs. (B.115) we calculate the corresponding products of scattering amplitudes entering Eq. (B.105c):

$$S_{in,11}(E)S^*_{in,12}(E_q) = -i\sqrt{T_C R_C}\, e^{i\frac{E\Delta\tau_{LR}}{\hbar}} \Big\{ R_{L1}R_{L2}\, e^{-iq\Omega_0\tau_{1UR}} S_L(t-\tau_{1UL})\, S^*_R(t-\tau_{1UR}) + T_{L1}T_{L2}\, e^{-iq\Omega_0\tau_{1DR}} S_L(t-\tau_{1DL})\, S^*_R(t-\tau_{1DR}) - -\zeta_L\, e^{i\frac{E\Delta\tau_L}{\hbar}}\, e^{i2\pi\frac{\Phi_L}{\Phi_0}} e^{-iq\Omega_0\tau_{1DR}} S_L(t-\tau_{1UL})\, S^*_R(t-\tau_{1DR}) - \zeta_L\, e^{-i\frac{E\Delta\tau_L}{\hbar}}\, e^{-i2\pi\frac{\Phi_L}{\Phi_0}} e^{-iq\Omega_0\tau_{1UR}} S_L(t-\tau_{1DL})\, S^*_R(t-\tau_{1UR}) \Big\},$$

$$S_{in,21}(E)S^*_{in,22}(E_q) = i\sqrt{T_C R_C}\, e^{i\frac{E\Delta\tau_{LR}}{\hbar}} \Big\{ R_{R1}R_{R2}\, e^{-iq\Omega_0\tau_{2UR}} S_L(t-\tau_{2UL})\, S^*_R(t-\tau_{2UR}) + T_{R1}T_{R2}\, e^{-iq\Omega_0\tau_{2DR}} S_L(t-\tau_{2DL})\, S^*_R(t-\tau_{2DR}) - \zeta_R\, e^{i\frac{E\Delta\tau_R}{\hbar}}\, e^{i2\pi\frac{\Phi_R}{\Phi_0}} e^{-iq\Omega_0\tau_{2DR}} S_L(t-\tau_{2UL})\, S^*_R(t-\tau_{2DR}) - \zeta_R\, e^{-i\frac{E\Delta\tau_R}{\hbar}}\, e^{-i2\pi\frac{\Phi_R}{\Phi_0}} e^{-iq\Omega_0\tau_{2UR}} S_L(t-\tau_{1DL})\, S^*_R(t-\tau_{1UR}) \Big\}.$$

The Fourier coefficients are:

$$\left\{S_{in,11}(E)S^*_{in,12}(E_q)\right\}_q = -i\sqrt{T_C R_C}\, e^{i\frac{E\Delta\tau_{LR}}{\hbar}} \Big\{ \{R_{L1}R_{L2} + T_{L1}T_{L2}\} \left\{S_L(t-\Delta\tau_{LR})\, S^*_R(t)\right\}_q -$$

$$-\zeta_L\, e^{i\frac{E\,\Delta\tau_L}{\hbar}}\, e^{i2\pi\frac{\Phi_L}{\Phi_0}} \left\{S_L(t-\Delta\tau_L-\Delta\tau_{LR})\, S_R^*(t)\right\}_q$$

$$-\zeta_L\, e^{-i\frac{E\,\Delta\tau_L}{\hbar}}\, e^{-i2\pi\frac{\Phi_L}{\Phi_0}} \left\{S_L(t+\Delta\tau_L-\Delta\tau_{LR})\, S_R^*(t)\right\}_q \Bigg\},$$

$$\left\{S_{in,21}(E)S_{in,22}^*(E_q)\right\}_q^* = -i\,\sqrt{T_C R_C}\, e^{-i\frac{E\,\Delta\tau_{LR}}{\hbar}} \Bigg\{$$

$$\{R_{R1}R_{R2} + T_{R1}T_{R2}\} \left\{S_L(t-\Delta\tau_{LR})\, S_R^*(t)\right\}_q^* -$$

$$-\zeta_R\, e^{-i\frac{E\,\Delta\tau_R}{\hbar}}\, e^{-i2\pi\frac{\Phi_R}{\Phi_0}} \left\{S_L(t-\Delta\tau_R-\Delta\tau_{LR})\, S_R^*(t)\right\}_q^*$$

$$-\zeta_R\, e^{i\frac{E\,\Delta\tau_R}{\hbar}}\, e^{i2\pi\frac{\Phi_R}{\Phi_0}} \left\{S_L(t+\Delta\tau_R-\Delta\tau_{LR})\, S_R^*(t)\right\}_q^* \Bigg\}.$$

Then the quantity B_q, Eq. (B.105c), can be represented as follows:

$$B_q = -R_C T_C \left\{B_{0,q} + \zeta_L\zeta_R \sum_{i=1}^{4} B_{i,q} + \sum_{i=5}^{6} B_{i,q}\right\}, \tag{B.119}$$

where

$$B_{0,q} = T_{MZI}^{(L,0)} T_{MZI}^{(R,0)} \left|\left\{S_L(t-\Delta\tau_{LR})\, S_R^*(t)\right\}_q\right|^2,$$

$$B_{1,q} = e^{i2\pi\frac{\Phi_L+\Phi_R}{\Phi_0}}\, e^{i\frac{E}{\hbar}(\Delta\tau_L+\Delta\tau_R)} \left\{S_L(t-\Delta\tau_L-\Delta\tau_{LR})S_R^*(t)\right\}_q$$

$$\times \left\{S_L(t+\Delta\tau_R-\Delta\tau_{LR})S_R^*(t)\right\}_q^*,$$

$$B_{2,q} = e^{-i2\pi\frac{\Phi_L+\Phi_R}{\Phi_0}}\, e^{-i\frac{E}{\hbar}(\Delta\tau_L+\Delta\tau_R)} \left\{S_L(t+\Delta\tau_L-\Delta\tau_{LR})S_R^*(t)\right\}_q$$

$$\times \left\{S_L(t-\Delta\tau_R-\Delta\tau_{LR})S_R^*(t)\right\}_q^*,$$

$$B_{3,q} = e^{i2\pi\frac{\Phi_L-\Phi_R}{\Phi_0}} e^{i\frac{E}{\hbar}(\Delta\tau_L-\Delta\tau_R)} \left\{S_L(t-\Delta\tau_L-\Delta\tau_{LR})S_R^*(t)\right\}_q \times \left\{S_L(t-\Delta\tau_R-\Delta\tau_{LR})S_R^*(t)\right\}_q^* ,$$

$$B_{4,q} = e^{-i2\pi\frac{\Phi_L-\Phi_R}{\Phi_0}} e^{-i\frac{E}{\hbar}(\Delta\tau_L-\Delta\tau_R)} \left\{S_L(t+\Delta\tau_L-\Delta\tau_{LR})S_R^*(t)\right\}_q \times \left\{S_L(t+\Delta\tau_R-\Delta\tau_{LR})S_R^*(t)\right\}_q^* ,$$

$$B_{5,q} = -T_{MZI}^{(L,0)}\zeta_R \left\{S_L(t-\Delta\tau_{LR})S_R^*(t)\right\}_q \Big\{ e^{-i\frac{E\Delta\tau_R}{\hbar}} e^{-i2\pi\frac{\Phi_R}{\Phi_0}} \times \left\{S_L(t-\Delta\tau_R-\Delta\tau_{LR})\,S_R^*(t)\right\}_q^* + e^{i\frac{E\Delta\tau_R}{\hbar}} e^{i2\pi\frac{\Phi_R}{\Phi_0}} \left\{S_L(t+\Delta\tau_R-\Delta\tau_{LR})\,S_R^*(t)\right\}_q^* \Big\},$$

$$B_{6,q} = -T_{MZI}^{(R,0)}\zeta_L \left\{S_L(t-\Delta\tau_{LR})S_R^*(t)\right\}_q^* \Big\{ e^{i\frac{E\Delta\tau_L}{\hbar}} e^{i2\pi\frac{\Phi_L}{\Phi_0}} \left\{S_L(t-\Delta\tau_L-\Delta\tau_{LR})\,S_R^*(t)\right\}_q + e^{-i\frac{E\Delta\tau_L}{\hbar}} e^{-i2\pi\frac{\Phi_L}{\Phi_0}} \left\{S_L(t+\Delta\tau_L-\Delta\tau_{LR})\,S_R^*(t)\right\}_q \Big\},$$

where $T_{MZI}^{(j,0)} = R_{j1}R_{j2} + T_{j1}T_{j2}$. The quantities $T_{MZI}^{(j,0)}$ and ζ_j [see Eq. (B.116)] define the transmission probability $T_{MZI}^{(j)}(E) = T_{MZI}^{(j,0)} - 2\zeta_j \cos\left(2\pi\Phi_L/\Phi_0 + E\Delta\tau_j/\hbar\right)$ for electrons with energy E through the interferometer j from the central QPC to the contact 1(2) for $j = L(R)$.

We see from Eq. (B.119) that the term $B_{0,q}$ always contributes leading to the cross-correlator similar to what we got in Sec. B.4.1. The difference is only in an additional factor $T_{MZI}^{L,0}T_{MZI}^{R,0}$ due to interferometers and in the time delay $\Delta t_{L,R}$ appeared in the non-adiabatic regime. Other terms in Eq. (B.119) do contribute in the case they lose oscillating dependence on energy E: Some of terms $B_{1,q} - B_{4,q}$ do contribute if $\Delta\tau_L = \pm\Delta\tau_R$. While the terms $B_{5,q}$ and $B_{6,q}$ (both or either of them) do contribute to current cross-correlator in the case of symmetrical interferometers $\Delta\tau_L = 0$ and/or $\Delta\tau_R = 0$. Alternatively, all the terms do contribute in the adiabatic regime. Therefore, with Eqs. (B.113), (B.114) the relevant coefficients are $B_{0,q}$ and $B_{1,q}$ - $B_{4,q}$ which we combine as:

$$
\begin{aligned}
B_{\pm,q} &= e^{i2\pi\frac{\Phi_L \pm \Phi_R}{\Phi_0}} \left| \left\{ S_L(t - \Delta\tau_L - \Delta\tau_{LR}) S_R^*(t) \right\}_q \right|^2 \\
&+ e^{-i2\pi\frac{\Phi_L \pm \Phi_R}{\Phi_0}} \left| \left\{ S_L(t + \Delta\tau_L - \Delta\tau_{LR}) S_R^*(t) \right\}_q \right|^2 .
\end{aligned} \tag{B.120}
$$

Note the sign "+" or "−" is chosen depending on which sign ("+" or "−") is in Eq. (B.114). For the geometry given in Fig. B.8 it is $\Delta\tau_L = \Delta\tau_R < 0$. Therefore, in Eq. (B.114) the sign "−" should be kept.

Taking into account that after all $C_q^* = B_q$ we can write the relevant coefficients as follows,

$$
B_{0,q} + C_{0,q} = 2T_{MZI}^{(L,0)} T_{MZI}^{(R,0)} \left| \left\{ S_L(t - \Delta\tau_{LR})\, S_R^*(t) \right\}_q \right|^2 , \tag{B.121a}
$$

$$
B_{\pm,q} + C_{\pm,q} = 2\cos\left(2\pi\frac{\Phi_L \pm \Phi_R}{\Phi_0}\right) \Big\{ \tag{B.121b}
$$

$$
\left| \left\{ S_L(t - \Delta\tau_L - \Delta\tau_{LR}) S_R^*(t) \right\}_q \right|^2 + \left| \left\{ S_L(t + \Delta\tau_L - \Delta\tau_{LR}) S_R^*(t) \right\}_q \right|^2 \Big\}.
$$

Let us consider the part $\mathcal{P}_{12}^{(B+C,0)}$ of a cross-correlator due to $B_{0,q}$ and $C_{0,q}$. With Eq. (B.121a) the integration in Eq. (B.105) is trivial. Then summing up over q by analogy with how we did in Sec. B.4.1 we calculate:

$$
\mathcal{P}_{12}^{(B+C,0)} = -2\mathcal{P}_0 T_{MZI}^{(L,0)} T_{MZI}^{(R,0)} \left\{ \gamma\left(\Delta t_{L,R}^{(-,-)} + \Delta\tau_{LR}\right) + \gamma\left(\Delta t_{L,R}^{(+,+)} + \Delta\tau_{LR}\right) \right\}, \tag{B.122}
$$

where the damping factor $\gamma(\Delta t)$ is given in Eq. (B.57).

Similarly we calculate the part due to $B_{\pm,q}$ and $C_{\pm,q}$:

$$\begin{aligned}\mathcal{P}_{12}^{(B+C,\Phi)} &= -2\mathcal{P}_0\zeta_L\zeta_R\cos\left(2\pi\frac{\Phi_L\pm\Phi_R}{\Phi_0}\right)\Bigg\{ \\ &\gamma\left(\Delta t_{L,R}^{(-,-)}-\Delta\tau_L+\Delta\tau_{LR}\right)+\gamma\left(\Delta t_{L,R}^{(+,+)}-\Delta\tau_L+\Delta\tau_{LR}\right) \\ &+\gamma\left(\Delta t_{L,R}^{(-,-)}+\Delta\tau_L+\Delta\tau_{LR}\right)+\gamma\left(\Delta t_{L,R}^{(+,+)}+\Delta\tau_L+\Delta\tau_{LR}\right)\Bigg\}.\end{aligned} \tag{B.123}$$

B.6.3.2 Total equation and its analysis

Using Eqs. (B.118), (B.122), and (B.123) we find the total current cross-correlation function, $\mathcal{P}_{12}=\mathcal{P}_{12}^{(A+D)}+\mathcal{P}_{12}^{(B+C,0)}+\mathcal{P}_{12}^{(B+C,\Phi)}$:

$$\begin{aligned}\mathcal{P}_{12} &= -2\mathcal{P}_0T_{MZI}^{(L,0)}T_{MZI}^{(R,0)}\left\{\gamma\left(\Delta t_{L,R}^{(-,-)}+\Delta\tau_{LR}\right)+\gamma\left(\Delta t_{L,R}^{(+,+)}+\Delta\tau_{LR}\right)\right\} \\ &+2\mathcal{P}_0\zeta_L\zeta_R\cos\left(2\pi\frac{\Phi_L\pm\Phi_R}{\Phi_0}\right)\Bigg\{4 \\ &-\gamma\left(\Delta t_{L,R}^{(-,-)}-\Delta\tau_L+\Delta\tau_{LR}\right)-\gamma\left(\Delta t_{L,R}^{(-,-)}+\Delta\tau_L+\Delta\tau_{LR}\right) \\ &-\gamma\left(\Delta t_{L,R}^{(+,+)}-\Delta\tau_L+\Delta\tau_{LR}\right)-\gamma\left(\Delta t_{L,R}^{(+,+)}+\Delta\tau_L+\Delta\tau_{LR}\right)\Bigg\}.\end{aligned} \tag{B.124a}$$

Notice, the suppression of a magnetic-flux independent contribution and the appearance of a contribution dependent on a magnetic flux occur at different conditions.

If particles emitted by the sources S_L and S_R propagate through the circuit without collisions between themselves then it is,

$$\mathcal{P}_{12} = -4\mathcal{P}_0 T_{MZI}^{(L,0)} T_{MZI}^{(R.0)}\,, \tag{B.125}$$

(compare to Eq. (B.73) in an adiabatic regime without interferometers). Here the factor 4 reflects the presence of four particles (two electrons and two holes) emitted by the two sources during a pumping period. The factor $T_{MZI}^{(j,0)} = R_{j1}R_{j2} + T_{j1}T_{j2}$ is a probability for an electron (a hole) to pass through the interferometer j. In the non-adiabatic regime under consideration, Eq. (B.113), this probability is a sum of probabilities to pass through each arm of an interferometer: The probability $R_{j1}R_{j2}$ is for the arm U, and the probability $T_{j1}T_{j2}$ is for the arm D, see Fig. B.8.

To analyze the effect of particle collisions we consider the sources emitting wave-packets with the same shape, $\Gamma_{\tau L} = \Gamma_{\tau R}$. If two emitted during a period electrons collide at the central QPC, $\Delta t_{L,R}^{(-,-)} + \Delta\tau_{LR} = 0$, then the correlator is suppressed: $\mathcal{P}_{12} = -2\mathcal{P}_0 T_{MZI}^{(L,0)} T_{MZI}^{(R.0)}$. If in addition the two holes collide, $\Delta t_{L,R}^{(+,+)} + \Delta\tau_{LR} = 0$, then it is suppressed down to zero: $\mathcal{P}_{12} = 0$. We already discussed this effect in previous sections.

B.6.3.3 Magnetic-flux dependent correlator

An interesting effect arises if two electrons (or two holes) can collide at the interferometer's exit, i.e., at the quantum point contact $L1$ ($R2$) for the interferometer L (R), see Fig. B.8. Because of Eq. (B.114) the collision conditions are satisfied for both interferometers simultaneously. For definiteness we consider an electron contribution and assume the following condition:

$$\Delta t_{L,R}^{(-,-)} - \Delta\tau_L + \Delta\tau_{LR} = 0\,, \qquad \Delta\tau_L = \Delta\tau_R\,. \tag{B.126}$$

Then we find,

$$\mathcal{P}_{12}^{(e)} = -2\mathcal{P}_0 \left\{ T_{MZI}^{(L,0)} T_{MZI}^{(R.0)} - \zeta_L \zeta_R \cos\left(2\pi \frac{\Phi_L - \Phi_R}{\Phi_0}\right)\right\}\,, \tag{B.127}$$

that the current cross-correlator depends on magnetic fluxes of distant interferometers. This non-local effect is due to two-particle correlations being a consequence of erasing which-path information for electrons arriving simultaneously at contacts 1 and 2. As it was shown in Ref. [117], these correlations are quantum, since they violate the Bell inequalities [125].

To clarify the origin of this effect and to relate the magnetic-flux dependent part of $\mathcal{P}_{12}^{(e)}$ to the two-electron probability $\mathcal{N}_{12}$ we consider in detail propagating of two electrons through the circuit.

Let us consider two electrons going to the same, say L, interferometer. From Eq. (B.126) we have, $\tau_{1DC} + \tau_{CL} + t_{0L}^{(-)} = \tau_{1UC} + \tau_{CR} + t_{0R}^{(-)}$. This means that an electron emitted by the source S_L and going along the down arm of the left interferometer, the path $\mathcal{L}_{1DL}$, does meet an electron emitted by the source S_R and going along the upper arm of the same interferometer, the path $\mathcal{L}_{1UR}$. Therefore, after the quantum point contact $L1$ we do not know where an electron came from. The same happens if two electrons go to the interferometer R: Again due to Eq. (B.126) there is, $\tau_{2DC}+\tau_{CL}+t_{0L}^{(-)} = \tau_{2UC}+\tau_{CR}+t_{0R}^{(-)}$. Therefore, an electron emitted by the source S_L and going along the down arm of the right interferometer, the path $\mathcal{L}_{2DL}$, and an electron emitted by the source S_R and going along the upper arm of the same interferometer, the path $\mathcal{L}_{2UR}$, lose their which-path information after the quantum point contact $R2$. We stress these events do not manifest themselves in the cross-correlator $\mathcal{P}_{12}$. We considered them only with a purpose to show an existence of two pairs of single-particle trajectories, $\mathcal{L}_{1DL}$, $\mathcal{L}_{1UR}$ and $\mathcal{L}_{2DL}$, $\mathcal{L}_{2UR}$, responsible for losing of which-path information.

From these single-particle trajectories one can compose two-particle trajectories corresponding to particles going to different interferometers. These trajectories are the following, $\mathcal{L}_a^{(2)} = \mathcal{L}_{1DL}\mathcal{L}_{2UR}$ and $\mathcal{L}_b^{(2)} = \mathcal{L}_{2DL}\mathcal{L}_{1UR}$. With Eq. (B.126) the trajectories $\mathcal{L}_a^{(2)}$ and $\mathcal{L}_b^{(2)}$ correspond to two-particle indistinguishable events: They have the same initial and final states. The final state is characterized by the places where electrons are appeared and the times when electrons are appeared at these places. Electrons going along these trajectories are responsible for magnetic-flux dependence of the cross-correlator $\mathcal{P}_{12}^{(e)}$. Since there are a number of different two-particle trajectories, the amplitude $\mathcal{A}^{(2)}$ for mentioned trajectories defines only a part of the two-particle probability which we denote as $\mathcal{N}_{12}^{(2)} = \left|\mathcal{A}^{(2)}\right|^2$. Since the amplitude $\mathcal{A}^{(2)}$ comprises contributions from two indistinguishable trajectories, it is the Slater determinant,

$$\mathcal{A}^{(2)} = \det \begin{vmatrix} \mathcal{A}_{1DL} & \mathcal{A}_{1UR} \\ \mathcal{A}_{2DL} & \mathcal{A}_{2UR} \end{vmatrix}, \tag{B.128}$$

with following single-particle amplitudes,

$$\begin{aligned}
\mathcal{A}_{1DL} &= -\sqrt{R_C T_{L2} T_{L1}}\, e^{-i2\pi\frac{\Phi_{LD}}{\Phi_0}}\, e^{ik_F L_{1DL}}, \\
\mathcal{A}_{1UR} &= i\sqrt{T_C R_{L2} R_{L1}}\, e^{i2\pi\frac{\Phi_{LU}}{\Phi_0}}\, e^{ik_F L_{1UR}}, \\
\mathcal{A}_{2DL} &= -i\sqrt{T_C T_{R1} T_{R2}}\, e^{-i2\pi\frac{\Phi_{RD}}{\Phi_0}}\, e^{ik_F L_{2DL}}, \\
\mathcal{A}_{2UR} &= \sqrt{R_C R_{R1} R_{R2}}\, e^{i2\pi\frac{\Phi_{RU}}{\Phi_0}}\, e^{ik_F L_{2UR}}.
\end{aligned}$$

After squaring we find,

$$\begin{aligned}
\mathcal{N}_{12}^{(2)} &= R_C^2 T_{L1} T_{L2} R_{R1} R_{R2} + T_C^2 R_{L1} R_{L2} T_{R1} T_{R2} \\
&+2R_C T_C \zeta_L \zeta_R \cos\left(2\pi\frac{\Phi_L - \Phi_R}{\Phi_0}\right).
\end{aligned} \tag{B.129}$$

Note here it is a difference of magnetic fluxes since we chose $\Delta\tau_L = \Delta\tau_R$, Eq. B.126, see explanation after Eq. (B.120).

Using Eq. (B.31) one can check that the magnetic-flux dependence of the two-electron probability $\mathcal{N}_{12}^{(2)}$, Eq. (B.129), completely explains the magnetic-flux dependence of the current cross-correlator $\mathcal{P}_{12}^{(e)}$, Eq. (B.127).

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