MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL TECHNICAL UNIVERSITY
"KHARKIV POLYTECHNIC INSTITUTE"

METHODICAL INSTRUCTIONS

to perform practical classes

in the course
"Mechanics of Viscous Fluid and Drilling Fluids"
for full-time and part-time students
in the specialty "Sectoral Engineering",
educational program "Sectoral Engineering"

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PRACTICAL CLASS 1
Physical properties of the liquid

Task #1

Determine the volume of water that must be additionally supplied to a water pipe with a diameter of $d = 500$ mm and a length of $l = 1$ km to increase the pressure to $\Delta p = 5 \cdot 10^6$ Pa. The pipeline is prepared for hydraulic testing and filled with water at atmospheric pressure. Deformation of the pipeline can be neglected.

Solution. Capacity of the water conduit

$$W_w = \frac{\pi d^2}{4} l = \frac{3,14 \cdot 0,5^2}{4} \cdot 1000 = 196,2 \text{ m}^3$$

Compressibility is a measure of a material's ability to decrease in volume under applied pressure. It describes how easily a material can be compressed when subjected to external forces [1]. The bulk modulus is measured in pascals (Pa) and is an indicator of the stiffness of a material under compression. For example: Gases typically have low bulk moduli, indicating high compressibility. Liquids have higher bulk moduli, showing lower compressibility. Solids exhibit the highest bulk moduli, reflecting their rigidity [2]. Together, compressibility and bulk modulus are essential for understanding and modeling the mechanical and acoustic behavior of materials in various scientific and engineering applications [3].

The volume of water ΔW that must be supplied to the water supply system to increase the pressure is calculated from the equation:

$$\beta_p = \frac{dW}{Wdp} = \frac{\Delta W}{W \Delta p} = \frac{\Delta W}{(W_w + \Delta W)\Delta p}.$$

Then

$$\beta_p (W_w + \Delta W)\Delta p = \Delta W;$$

$$\beta_p W_w \Delta p + \beta_p \Delta W_w \Delta p = \Delta W;$$

$$\beta_p W_w \Delta p = \Delta W - \beta_p \Delta W_w \Delta p = \Delta W (1 - \beta_p \Delta p);$$

$$\beta_p = \frac{1}{K} = \frac{1}{2000 \cdot 10^6} = 5 \cdot 10^{-10} \text{ 1/Pa}$$

$$\Delta W = \frac{W_w \beta_p \Delta p}{1 - \beta_p \Delta p} = \frac{196,2 \cdot 5 \cdot 10^{-10} \cdot 5 \cdot 10^6}{1 - 5 \cdot 10^{-10} \cdot 5 \cdot 10^6} = \frac{0,4905}{0,9975} = 0,49 \text{ m}^3$$

Thus, when the pressure increases by 5 MPa, the volume changes by only 0.25%.

Task #2

The viscosity of oil determined by the Engler viscometer is 8.5 ºE. Calculate the dynamic viscosity of oil if its density is 850 kg/m^3.

Solution.

Viscosity is a measure of a fluid's resistance to flow. It quantifies the internal friction between adjacent layers of fluid as they move relative to one another. There are two types of viscosity commonly discussed: dynamic viscosity and kinematic viscosity [4].

Dynamic viscosity, also known as absolute viscosity, describes a fluid's inherent resistance to shear stress. It relates the shear stress in a fluid to the rate of strain (velocity gradient) across its layers.

Kinematic viscosity accounts for the dynamic viscosity and the fluid's density. It describes how fast momentum diffuses through a fluid [5].

The Engler viscometer is an instrument used to measure the viscosity of a liquid relative to the viscosity of water. It provides a practical method for determining the flow properties of oils, lubricants, and other fluids by comparing the time required for a specific volume of the fluid to flow through an orifice with the time required for the same volume of water at a standard temperature [6].

$$\nu = (0{,}0731°E - \frac{0{,}0631}{°E}) \cdot 10^{-4} =$$

$$= (0{,}0731 \cdot 8{,}5 - \frac{0{,}0631}{8{,}5}) \cdot 10^{-4} = 0{,}614 \cdot 10^{-4} \ m^2 / s$$

$$\mu = \nu\rho = 0{,}614 \cdot 10^{-4} \cdot 850 = 0{,}052 \text{ Pa·s} = 0{,}52 \text{ Poise.}$$

PRACTICAL CLASSES 2-3
Hydrostatics

Task #1

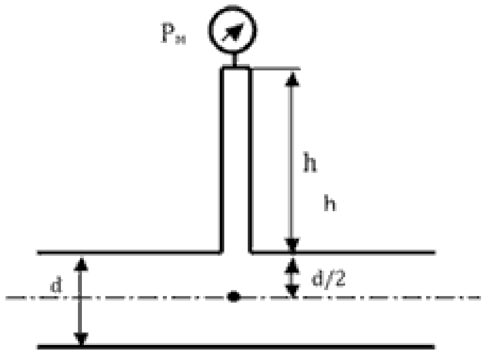

Figure 2.1 - Scheme for the task 1

A vertical metal pipe with a height of $h = 2{,}8$ m is installed on a pipeline of diameter $d = 0{,}6$ m filled with water, to which a pressure manometer is connected, the reading of which is $P_M = 3{,}6$ atm. Determine the pressure on the axis of the pipeline

Solution

The hydrostatic law describes the distribution of pressure in a static fluid at rest. It is a fundamental principle in fluid mechanics and states that the pressure in a fluid increases with depth due to the weight of the fluid above [7]. The hydrostatic law provides a reliable framework for understanding pressure distribution in fluids and is essential for analyzing fluid systems in various fields.

$$p_M = p + \rho g\left(h + \frac{d}{2}\right),$$

$$p = p_M - \rho g\left(h + \frac{d}{2}\right) = 3{,}6 \cdot 98 \cdot 10^3 - 1000 \cdot 9{,}8 \cdot 3{,}1 = 383000 \text{ Pa}.$$

Pressure is a measure of force exerted per unit area. The SI unit of pressure is the Pascal (Pa), defined as one Newton per square meter. Other common units include the bar, atmosphere (atm), millimeters of mercury (mmHg), Torr, and pounds per square inch (psi). Each unit serves specific applications, such as meteorology, medicine, or engineering. For example, 1 atm equals 101,325 Pa, representing standard atmospheric pressure at sea level. Understanding these units and their conversions is crucial for accurately measuring and interpreting pressure across various fields [8].

Task #2

The hole in the bottom of the vessel containing oil with a relative density $\delta = 0{,}83$ is closed with a conical plug with dimensions $D = 100$ mm, $d = 50$ mm and $a = 100$ mm, fixed on a rod with a diameter of $d_1 = 25$ mm (Fig. 2.2). The oil

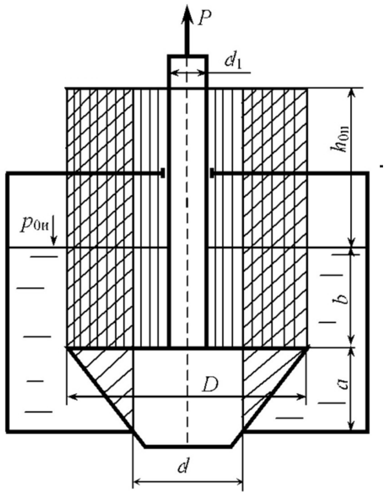

Figure 2.2 - Scheme for the task 2

level is located above the plug at a distance of $b = 50$ mm. Determine the initial force P required to lift the plug at an gauge pressure in the vessel of $p_{0ex} = 10$ kPa. Neglect the mass of the plug and the friction in the oil seal.

Solution. The piezometric plane (in Fig. 2.2, the piezometric plane is the upper limit of the hatching) passes above the free surface of the liquid in the vessel at a height:

$$h_{0ex} = \frac{p_{0ex}}{\rho g} = \frac{10 \cdot 10^3}{0{,}83 \cdot 10^3 \cdot 9{,}8} = 1{,}23 \text{ m.}$$

The upper end surface of the plug is subjected to a downward force of fluid pressure on a flat wall $P_1 = \rho g h_c S$, i.e.

$$P_1 = \rho g \left(h_{oex} + b \right) \frac{\pi \left(D^2 - d_1^2 \right)}{4}.$$

On the side surface of the cork, the horizontal component of the water pressure is zero, since the vertical projections are subject to equal and oppositely directed forces, respectively, and the vertical component

$$P_1 = \rho g W_{md} = \rho g \left(\frac{\pi D^2}{4} \left(b + h_{oex} \right) - \frac{\pi d^2}{4} \left(a + b + h_{oex} \right) + \frac{1}{12} \pi a \left(D^2 + Dd + d^2 \right) \right),$$

and is directed vertically upward. Then the initial force

$$P = P_1 - P_2 = \rho g \left(h_{oex} + b \right) \frac{\pi \left(D^2 - d_1^2 \right)}{4} -$$
$$- \rho g \left(\frac{\pi D^2}{4} \left(b + h_{oex} \right) - \frac{\pi d^2}{4} \left(a + b + h_{oex} \right) + \frac{1}{12} \pi a \left(D^2 + Dd + d^2 \right) \right) =$$

$$= 0,83 \cdot 10^3 \cdot 9,81(1,23 + 0,05)\frac{3,14(0,1^2 - 0,025^2)}{4} -$$

$$-0,83 \cdot 10^3 \cdot 9,81\begin{pmatrix} 0,785 \cdot 0,01 \cdot 1,28 - 0,785 \cdot 0,05^2 \cdot 1,38 + \\ + \frac{3,14 \cdot 0,1}{12}(0,1^2 + 0,1 \cdot 0,05 + 0,05^2) \end{pmatrix} =$$

$$= 76,7 - 8140(0,01 - 0,0027 + 0,00046) = 13,5\ N$$

Task #3

Determine the equivalent force of pressure of a liquid (water) on a vertical rectangular wall of width $b = 5$ m under bilateral pressure, if $H = 2$ m, $h = 0,5$ m.

Solution. The balancing force of pressure is the difference between the gauge pressure forces acting on the left and right sides of the wall.

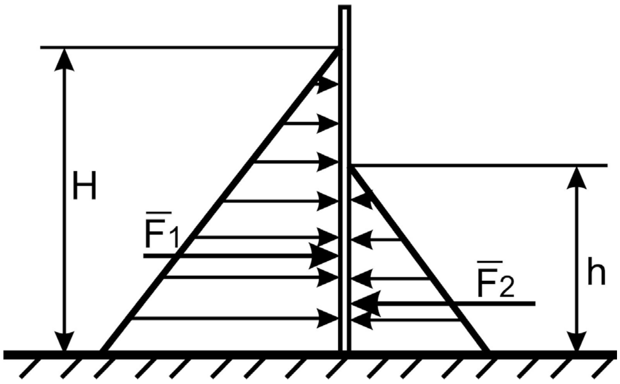

Figure 2.3 - Scheme for the task 3

$$F = F_1 - F_2$$

$$F_1 = p_{c1}S_1 = \frac{\rho g H}{2} bH = \frac{\rho g b}{2} H^2$$

$$F_2 = p_{c1}S_2 = \frac{\rho g h}{2} bh = \frac{\rho g b}{2} h^2$$

$$F = F_1 - F_2 = \frac{\rho g b}{2}(H^2 - h^2) = \frac{1000 \cdot 9,81 \cdot 5}{2}(2^2 - 0,5^2) = 92000\ N$$

The center of pressure is determined using the theorem of moments for an equilibrating force. The moment of the force is calculated relative to an axis perpendicular to the plane of the figure and passing through a point on the Earth's surface:

$$Fx = F_1 \frac{H}{3} - F_2 \frac{h}{3}$$

Hence

$$Fx = \frac{F_1}{F}\frac{H}{3} - \frac{F_2}{F}\frac{h}{3} = \frac{98100}{92000}\frac{2}{3} - \frac{6131}{92000}\frac{0,5}{3} = 0,711 - 0,011 = 0,7\ \text{m}$$

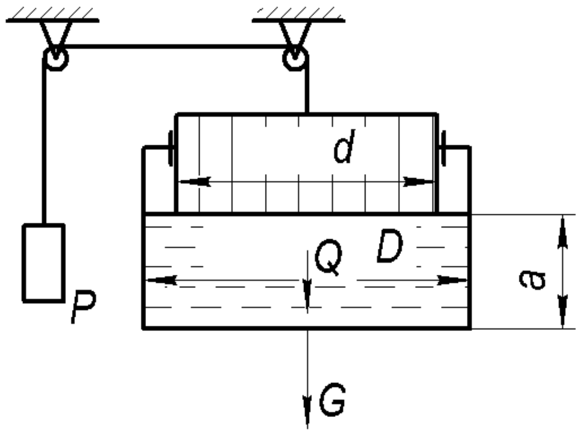

Figure 2.4 - Scheme for the task 4

Task #4

A cylindrical tank with a diameter of $D = 900$ mm and a weight of $G = 0,2$ kN, filled with water to a height of $a = 0,5$ m, is suspended from a piston with a diameter of $d = 850$ mm. A weight is suspended from the piston through blocks to keep the system balanced. Determine the vacuum in the vessel that ensures equilibrium in the cylinder. Neglect the friction in the system.

Solution. Write down the equilibrium condition of the system:

$$P = G + Q.$$

The force of liquid pressure on the bottom wall of the tank:

$$Q = pS = \rho g a \frac{\pi D^2}{4} = 1000 \cdot 9,81 \cdot 0,5 \cdot \frac{3,14 \cdot 0,9^2}{4} = 3120 \text{ N.}$$

The force of pressure on the upper wall associated with the vacuum p_v in the vessel:

$$P = p_v S = p_v \frac{\pi d^2}{4} = p_v \cdot \frac{3,14 \cdot 0,85^2}{4} = 0,567 p_v.$$

Then,

$$0,567 p_v = 3120 + 200 = 3320, \; p_v = 5860 \text{ Pa.}$$

PRACTICAL CLASSES 4-5
Bernoulli's equation

Bernoulli's equation states as follows: "In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line."

Task #1

Water moves through a siphon pipeline. Determine the flow rate Q and pressure of water in the section $x-x$ (Fig. 4.1), neglecting the pressure loss. The upper point of the pipeline axis is located above the water level in the tank by $H=1$ m, and the lower point is located below by $h=3$ m. The internal diameter of the pipeline is $d=20$ mm.

Solution. Let's write the Bernoulli's equation for sections 1-1 and 2-2 with respect to the reference plane 0-0. It is advisable to choose a horizontal plane passing through the lowest point of the pipeline as the reference plane. Section 1-1 coincides with the liquid level in the feed tank, and section 2-2 coincides with the liquid outlet from the pipeline [9]. Bernoulli's equation has the form

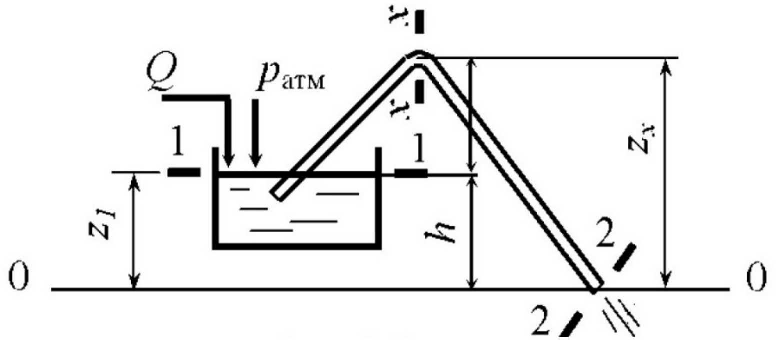

Figure 4.1 - Scheme for the task 1

$$z_1 + \frac{p_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + \sum h_{П} .$$

Here $z_1 = h,\ z_2 = 0,\ \sum h_{П} = 0$. Atmospheric pressure p_{atm} acts on the surface of the liquid in the feed tank and at the outlet of the pipeline, so $p_1 = p_2 = p_{atm}$. We accept $\alpha_1 = \alpha_2 \approx 1$. The rate of change of the level in the tank is $V_1 = 0$, since water enters the tank at a flow rate Q and the water level in the tank is constant. Substituting into the original equation, we obtain the Bernoulli equation in the form

$$h=\alpha_2\frac{V_2^2}{2g},\quad V_2=\sqrt{2gh}=\sqrt{2\cdot 9{,}81\cdot 3}=7{,}67 \text{ m/s}.$$

The flow rate is determined by the formula

$$Q=VS=V_2\frac{\pi d^2}{4}=7{,}67\cdot\frac{3{,}14\cdot 0{,}02^2}{4}=0{,}0024 \text{ m}^3\text{/s}.$$

To calculate the absolute pressure at the top of the pipeline, we will draw up the Bernoulli's equation for sections 1-1 and $x-x$ relative to the reference plane 0-0:

$$z_1+\frac{p_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}=z_x+\frac{p_x}{\rho g}+\alpha_x\frac{V_x^2}{2g}.$$

Here $z_1=h,\ z_x=h+H,\ p_1=p_{atm}$. We accept $\alpha_1=\alpha_2\approx 1;\ V_1=0$. The velocity of the fluid in a pipeline of constant cross section is the same $V_x=V_2$. Then the Bernoulli`s equation will take the form

$$h+\frac{p_{am}}{\rho g}=h+H+\frac{p_x}{\rho g}+\frac{V_2^2}{2g}.$$

Express the pressure p_x:

$$p_x=p_{am}-\rho gH-\rho\frac{V_2^2}{2}.$$

Assuming a normal atmospheric pressure of $p_{atm}=101\,\text{kPa}$ and a water density of $\rho=1000\,\text{kg/m}^3$, we have

$$p_x=101000-1000\cdot 9{,}81\cdot 1-1000\frac{7{,}67^2}{2}=61800 \text{ Pa}.$$

In the cross section $x-x$, the absolute pressure p_x is less than atmospheric pressure. So, the vacuum pressure in the section $x-x$ is

$$p_v=p_{atm}-p_x=101-61{,}8=39{,}2 \text{ kPa}.$$

Task #2

Given the known diameters of the pipeline ($d_1 = 32$ mm, $d_2 = 40$ mm) and the gauge pressure in the tank $p_0 = 20$ kPa, determine the geometric head at which the water flow rate of $Q = 11$ l/s will be provided. Construct the pressure and piezometric lines.

Solution. Write the Bernoulli`s equation for the sections 0-0 and 2-2, choosing the axis of the pipeline as the plane of comparison:

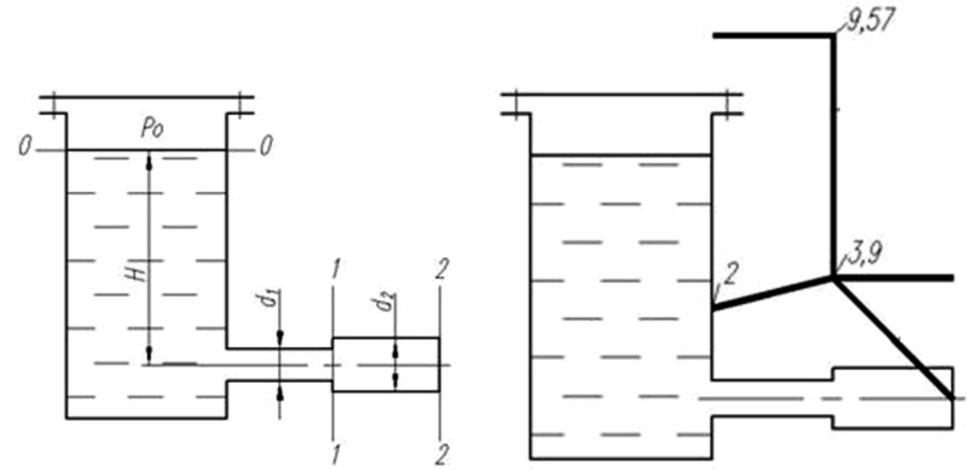

Figure 4.2 - Scheme for the task 2

$$z_0 + \frac{p_0}{\rho g} + \alpha_0 \frac{V_0^2}{2g} = z_2 + \frac{p_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + \sum h_{\Pi}$$

Here $z_0 = H$, $z_2 = 0$, $\sum h_{\Pi} = 0$, $V_0 = 0$, $p_2 = 0$. Accept $\alpha_1 = \alpha_2 \approx 1$. Then

$$H + \frac{p_0}{\rho g} = \frac{V_2^2}{2g}, \ H = \frac{V_2^2}{2g} - \frac{p_0}{\rho g}.$$

From the flow equation, we determine the velocity V_2:

$$Q = V_2 S_2, \ V_2 = \frac{Q}{S_2} = \frac{4Q}{\pi d_2^2} = \frac{4 \cdot 11 \cdot 10^{-3}}{3{,}14 \cdot 0{,}04^2} = 8{,}75 \text{ m/s.}$$

Find $H = \frac{V_2^2}{2g} - \frac{p_0}{\rho g} = \frac{8{,}75^2}{2 \cdot 9{,}81} - \frac{20 \cdot 10^3}{1000 \cdot 9{,}81} = 1{,}87$ m.

A pressure line refers to a conceptual or graphical representation of how pressure varies within a fluid system, often used in fluid mechanics, engineering, and instrumentation to analyze and visualize pressure behavior. In static fluid systems, the pressure line illustrates the variation of pressure with depth, demonstrating the hydrostatic principle that pressure increases linearly as depth increases due to the weight of the fluid. This is commonly seen in applications like water towers, reservoirs, or submarine environments. In engineering systems, a pressure line can represent a pipeline or conduit through which a fluid flows under controlled pressure, as in hydraulic or pneumatic systems, where maintaining specific pressure levels is crucial for efficient operation [10]. In instrumentation, a pressure line may refer to tubing or channels that transfer pressure signals from a source to a measurement device like a gauge or transducer, ensuring accurate monitoring of system performance. Additionally, pressure lines can appear in graphical contexts, such as pressure-time curves in transient analysis or pressure-depth plots in hydrostatics. In meteorology and aerodynamics, pressure lines often represent contours of equal pressure (isobars) to analyze fluid flow or weather systems [11]. Regardless of context, pressure lines are essential tools for understanding and managing pressure-related phenomena in scientific and practical applications, providing insights into system behavior and aiding in design, operation, and safety assessments.

To design a pressure line, let's determine the velocity V_1 and dynamic heads:

$$V_1 S_1 = V_2 S_2, \quad V_1 = \frac{V_2 S_2}{S_1} = \frac{V_2 d_2^2}{d_1^2} = \frac{8,75 \cdot 0,04^2}{0,032^2} = 13,7 \text{ m/c}$$

Dynamic heads: $\frac{V_0^2}{2g} = 0$; $\frac{V_1^2}{2g} = 9,57$ m; $\frac{V_2^2}{2g} = 3,9$ m.

To construct a piezometric line, we determine the piezometric heights:

$$\frac{p_0}{\rho g} = 2,04 \text{ m}; \quad \frac{p_1}{\rho g} = \frac{p_0 + \rho g H}{\rho g} = 3,91 \text{ m}; \quad \frac{p_2}{\rho g} = 0 .$$

PRACTICAL CLASSES 6-7
Real fluid dynamics

Task #1

Finding the head. Find the pressure required to ensure a constant level h in a small watering can in a large watering can H. If known:

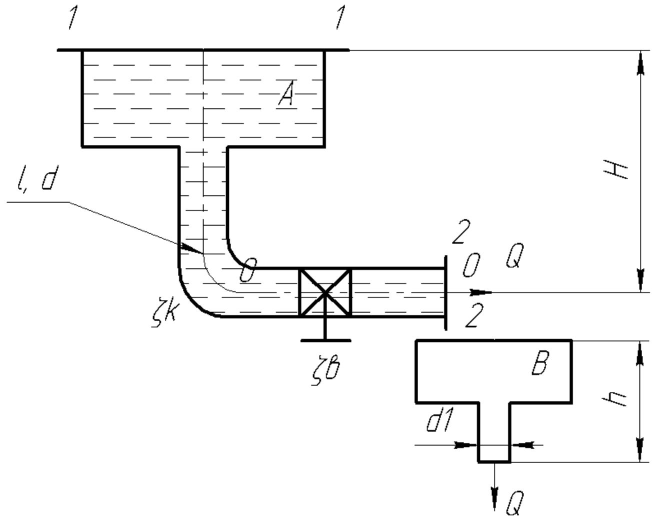

Figure 6.1 - Scheme for the task 1

$d = d_1 = 80\,mm$

$h = 1{,}5\,m$

$L = 2l = 10\,m$

$\mu = 0{,}82$

$\zeta_к = 0{,}3;\ \zeta_в = 4;$

$\frac{\Delta}{d} = 0{,}01$

Solution

Find the flow rate

$$Q = \mu f\sqrt{2gh} = 0{,}82 \cdot \frac{\pi \cdot 0{,}08^2}{4}\sqrt{2 \cdot 9{,}81 \cdot 1{,}5} = 0{,}0223 \text{ m}^3\text{/s}$$

Let's write the Bernoulli equation for the intersections 1-1 and 2-2:

$$\frac{p_1}{\rho g} + z_1 + \alpha_1\frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \alpha_2\frac{V_2^2}{2g} + \sum h_{1-2}$$

$$p_1 = 0; \quad z_1 = H; \quad V_1 = 0; \ p_2 = 0; \quad z_2 = 0; \ \alpha_1 = \alpha_2 = 1$$

then $H = 0{,}0827\left(\lambda\frac{l}{d} + \sum\zeta\right)\frac{Q^2}{d^4}$

$$\sum\zeta = \zeta_{вх} + \zeta_к + \zeta_в = 0{,}5 + 0{,}3 + 4 = 4{,}8$$

Find the coefficient λ:

$$V = \frac{4Q}{\pi d^2} = \frac{4 \cdot 0{,}0223}{3{,}14 \cdot 0{,}08^2} = 4{,}44 \text{ m/s}$$

$$Re = \frac{Vd}{\nu} = \frac{4,44 \cdot 0,08}{1 \cdot 10^{-6}} = 355200; \qquad Re\frac{\Delta}{d} = 355200 \cdot 0,01 = 355.$$

Then, let's use the formula for advanced turbulent motion (Altschul's formula):

$$\lambda = 0,11\left(\frac{\Delta}{d} + \frac{68}{Re}\right)^{0,25} = 0,11 \cdot \left(0,01 + \frac{68}{355200}\right)^{0,25} = 0,035$$

Find $H = 0,0827\left(0,035\frac{10}{0,08} + 4,8\right)\frac{0,0223^2}{0,08^4} = 9,2$ m.

Task #2

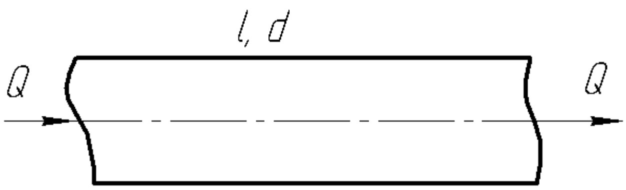

Figure 6.2 - Scheme for the task 2

Finding the flow rate. Find the flow rate Q in the pipe if $l = 200$ m; $d = 100$ mm; $\Delta = 0,1$ mm; $H = 10$ m. Water with a viscosity of 1 cSt flows in the pipe

Solution. Write the equation for determining the pressure loss in the pipeline:

$$H_p = f(Q) = 0,0827\left(\lambda\frac{l}{d} + \sum\zeta\right)\frac{Q^2}{d^4}.$$

Since there is no minor losses, then $H_p = f(Q) = 0,0827\frac{\lambda l Q^2}{d^5}$.

Minor losses in fluid mechanics refer to the additional energy losses in a flow system caused by disturbances such as fittings, bends, valves, expansions, contractions, or other components that disrupt smooth fluid flow. Unlike major losses, which occur due to friction along the length of a pipe, minor losses are associated with localized effects. These losses are typically expressed as a head loss and depend on factors like the flow velocity, fluid properties, and the geometry of the obstruction. They are calculated using empirical coefficients and are crucial in designing efficient fluid systems [12].

Let's find the volumetric flow rate using the graph-analytical method. To do this, we determine the values H_n of for three different flow rates.

The order of the value of the flow rate is determined by the following condition:

$$Q = \mu f \sqrt{2gH} = \mu \frac{\pi d^2}{4} \sqrt{2gH} =$$

$$= 0{,}82 \cdot \frac{\pi \cdot 0{,}1^2}{4} \sqrt{2 \cdot 9{,}81 \cdot 10} = 0{,}09\ m^3/s = 90 l/s.$$

Let's make a table of the calculated values:

Table 6.1 – Pipeline calculation

Q, l/s	V, m/s	Re	$\mathrm{Re}\frac{\Delta}{d}$	λ	H, m
0	0	0	0	-	0
15	1,91	191000	191	0,0211	7,8
30	3,82	382000	382	0,0204	30,4

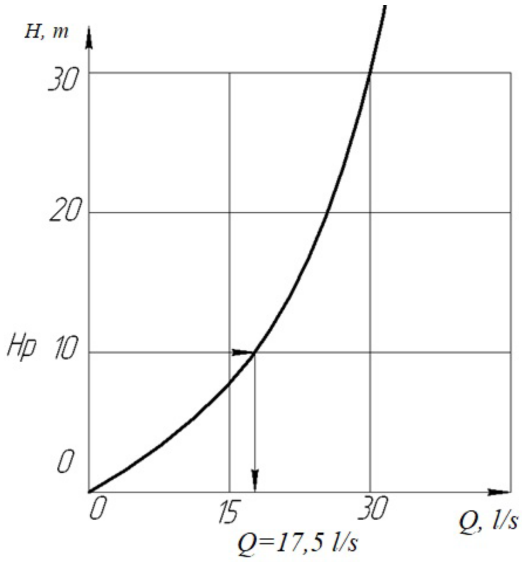

Figure 6.3 - Dependency diagram $H_p = f(Q)$

Calculation of one value of flow rate $Q = 15$ l/s:

$$V = \frac{4Q}{\pi d^2} = \frac{4 \cdot 15 \cdot 10^{-3}}{3{,}14 \cdot 0{,}1^2} = 1{,}91 \text{ m/c};$$

$$\mathrm{Re} = \frac{Vd}{\nu} = \frac{1{,}91 \cdot 0{,}1}{1 \cdot 10^{-6}} = 191000;$$

$$\mathrm{Re}\frac{\Delta}{d} = 191000 \cdot \frac{0{,}1}{100} = 191.$$

Let's use the formula for advanced turbulent motion (Altschul's formula):

$$\lambda = 0{,}11\left(\frac{\Delta}{d} + \frac{68}{\mathrm{Re}}\right)^{0{,}25} = 0{,}11 \cdot \left(0{,}001 + \frac{68}{191000}\right)^{0{,}25} = 0{,}0211$$

$$H = 0{,}0827\frac{\lambda l Q^2}{d^5} = 0{,}0827 \cdot \frac{0{,}0211 \cdot 200 \cdot 0{,}015^2}{0{,}1^5} = 7{,}8 \text{ m.}$$

Plot dependency diagram $H_p = f(Q)$.

Task #3

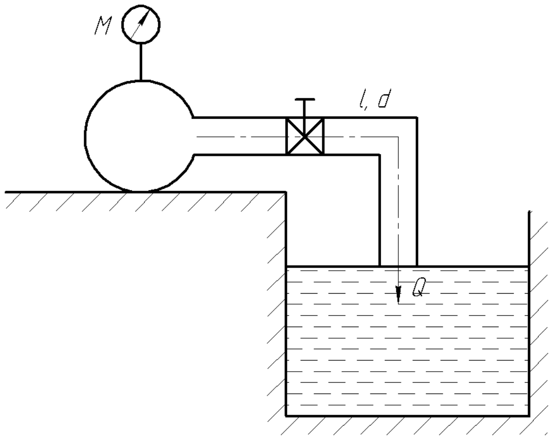

Figure 6.4 - Scheme for the task 3

Find the diameter of the pipeline. Using the pump, fill the tank with a volume of $W = 36\,\text{m}^3$, per $t = 30\,\text{min}$. The length of the supply pipeline, $l = 45$ m. Find the diameter of the pipeline if: $p_M = 245$ kPa; $\zeta_в = 4$. Determine the coefficient of friction λ by the following equation:
$\lambda = \dfrac{0,02}{d^{0,33}}$

Solution. Problems of this type are solved graphically and analytically according to the scheme:

$$d \to V \to \text{Re} \to \lambda \to H\,.$$

Given three different diameters, then calculate H, and plot the relationship $H = f(d)$. Let's write down the equation for determining the pressure loss in the pipeline:

$$H = 0,0827\left(\lambda \frac{l}{d} + \sum \zeta\right)\frac{Q^2}{d^4}\,.$$

Determine the volumetric flow rate at which the tank should be filled:

$$Q = \frac{W}{t} = \frac{36}{30 \cdot 60} = 0,02\ \text{m}^3/\text{s}.$$

Let's define the head:

$$H_p = \frac{p_M}{\rho g} = \frac{245 \cdot 10^3}{1000 \cdot 9,81} = 25\ \text{m}.$$

Let's make a table of the calculated values:

Table 6.2 – Calculation of the pipeline

d, mm	λ	H, m
20	0,074	35000
60	0,051	110
80	0,0464	25,1
100	0,043	8

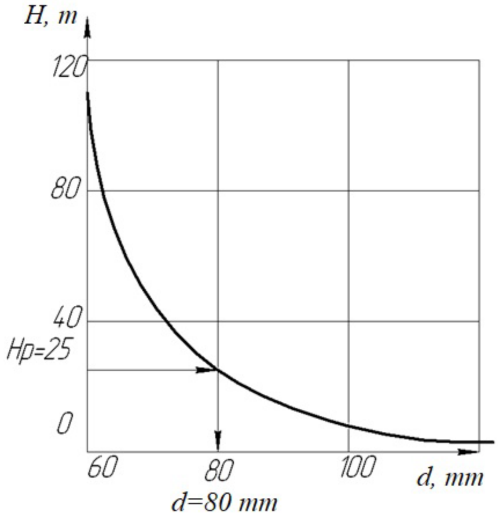

Figure 6.5 - Dependency diagram $H_p = f(d)$

Calculation of one flow rate value $d = 20$ mm

$$\sum \zeta = \zeta_{в} + \zeta_{вых} = 4 + 1 = 5; \qquad \lambda = \frac{0,02}{d^{0,33}} = \frac{0,02}{0,02^{0,33}} = 0,074.$$

Then, $H = 0,0827\left(0,074\dfrac{45}{0,02} + 5\right)\dfrac{0,02^2}{0,02^4} = 35000$ m.

Plot dependency diagram $H_p = f(d)$.

PRACTICAL CLASS 8
Complex task

Cylindrical tank 1 (Fig. 8.1) with hemispherical upper and lower covers of radius $R = 2{,}0$ m, has a side hatch with a diameter of $d_o = 0{,}70$ m and a pipeline located at a distance of $h_o = 2{,}0$ m from the axis, closed with a flat cover, is connected to tank 2 by a pipeline made of old steel with a length of $L = 12$ m and a diameter of $d = 40$ mm and a valve resistance coefficient $\zeta = 5{,}5$. Water under a pressure head of $H_o = 3{,}0$ m and an air pressure on the free surface of $p_M = 0{,}5$ MPa flows from tank 1 into tank 2, and from it, at a constant pressure head of $H_1 = 4{,}0$ m, flows out into the atmosphere through a cylindrical nozzle with a diameter d_H, hitting a flat barrier 3.

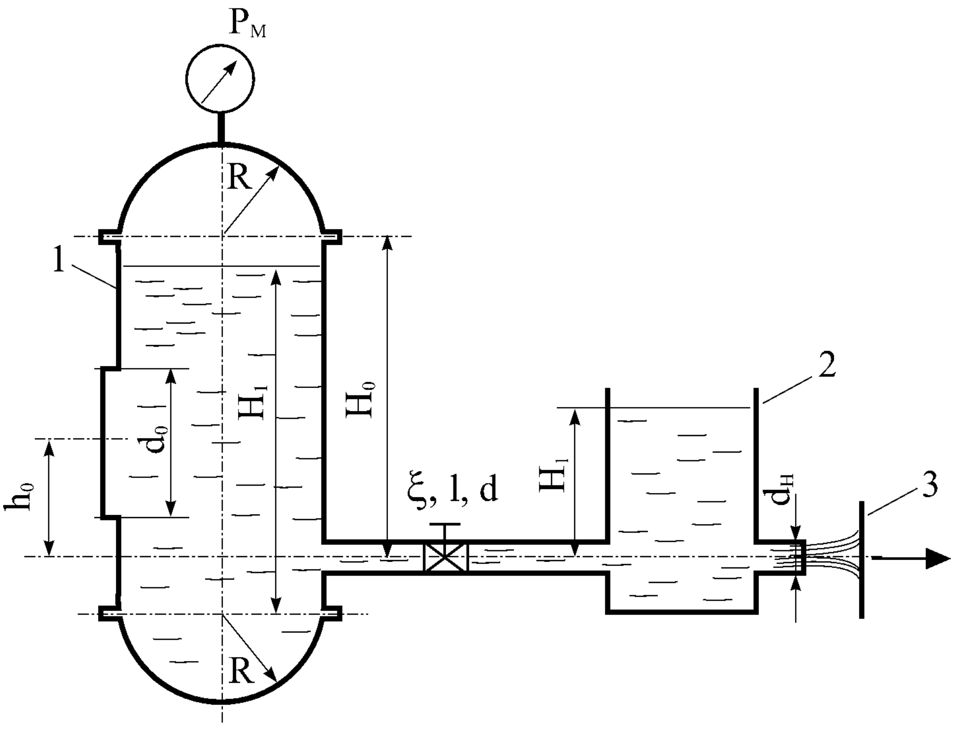

Figure 8.1 - Scheme of the hydraulic system

Solution

1. Determine the pressure forces on the top, bottom, and side covers of the tank 1. The pressure at any point in a fluid at rest can be determined by the basic hydrostatic equation:

$$p = p_o + \rho g h.$$

The force on any flat surface can be defined as: $P = p_c F$. The force of pressure on a flat hatch:

$$P = p_c F = \left(p_m + \rho g (H_o - h_o)\right) \cdot \frac{\pi d_o^2}{4} =$$

$$= \left(0,5 \cdot 10^6 + 1000 \cdot 9,81 \cdot (3,0 - 2,0)\right) \cdot \frac{\pi \cdot 0,7^2}{4} = 196\ kN$$

The pressure force on a curved surface can be calculated by first decomposing it into horizontal P_Γ and vertical components P_B: $P = \sqrt{P_\Gamma^2 + P_B^2}$. The pressure force on the top hatch, given that there is air there and the pressure on the lid will be the same at all points:

$$P = p_M F = p_M \pi R^2 = 0,5 \cdot 10^6 \cdot 3,14 \cdot 2,0^2 \cdot 10^{-3} = 6280\ \text{kN}$$

On the bottom hatch: $P_\Gamma = 0$;

$$P_B = \rho g W$$

$$= \rho g \left(\left(\frac{p_M}{\rho g} + H_1 \right) \cdot \pi R^2 + \frac{2}{3} \pi R^3 \right) =$$

$$= 1000 \cdot 9,81 \left(\left(\frac{0,5 \cdot 10^6}{1000 \cdot 9,81} + 4,0 \right) \cdot \pi \cdot 2,0^2 + \frac{2}{3} \pi \cdot 2,0^3 \right) \cdot 10^{-3} =$$

$$= 6940\ kN$$

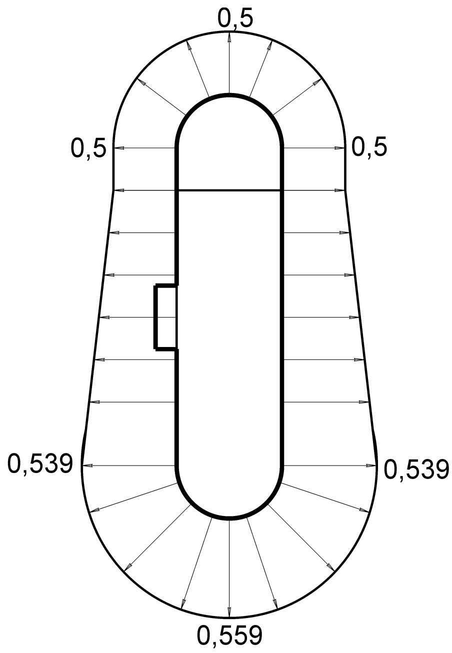

Figure 8.2 - Hydrostatic pressure diagram on the internal surfaces of the tank 1 (in MPa)

2. Determine the pressure at the pipeline inlet with the valve closed.

$$p = p_M + \rho g H_o =$$
$$= 0,5 \cdot 10^6 + 1000 \cdot 9,81 \cdot 2,0 = 520000\ \text{Pa} =$$
$$0,52\ \text{MPa}$$

3. Draw hydrostatic pressure diagrams on the internal surfaces of the tank.

Knowing the law of pressure change with depth, you can graphically plot the pressure on the surface of the tank. To do this, find the pressure at the extreme points, plot its value on a scale normal to the surface, and connect it with lines

4. Determine the flow rate of water entering tank 2.

The flow rate of the liquid flowing from tank 1 to tank 2 can be determined by graphical and analytical method by plotting the dependence of the required head H_n on the flow rate Q, and using the head H_h to find the desired flow rate.

To graph this equation (i.e., a graph of the form $H_n = f(Q)$), several flow rates are set and all the values included in the equation are determined. The maximum value of the specified flow rate can be approximately determined by the formula:

$$Q_{\max} = \mu f \sqrt{2gH_h}$$

The pressure at the inlet to the pipeline is determined by the basic hydrostatic equation:

$$H_h = \frac{p_m}{\rho g} + H_o = \frac{0,5 \cdot 10^6}{1000 \cdot 9,81} + 2,0 = 53,0 \text{ m}$$

$$Q_{\max} = 0,6 \cdot \frac{\pi \cdot 0,040^2}{4} \sqrt{2 \cdot 9,81 \cdot 53,0} = 0,024 \text{ m}^3\text{/s=24 l/s}$$

Having calculated the maximum flow rate $Q_{\max}$, several values of the current flow rate Q (usually three) are set and the other values are determined in the following sequence:

The current values of the fluid velocity in the pipeline are calculated:

$$V = \frac{Q}{f} = \frac{4Q}{\pi d^2}.$$

Calculate the Reynolds criterion Re and determine the mode of fluid motion $\mathrm{Re} = Vd / \nu$. Having determined the mode of fluid movement, calculate the value of the coefficient of friction λ.

In general, the coefficient of friction λ is a function of two parameters - the Reynolds number Re and the relative roughness $\frac{\Delta_e}{d}$.

The static head H_{st} is determined by the outlet conditions.

After determining all the values included in the equation for the required head H_n, several values are calculated for all the specified flow rates.

$$H_n = H_{st} + 0,0827\left(\lambda \frac{l}{d} + \sum \zeta\right)\frac{Q^2}{d^4}$$

Table 8.1 – Calculation of the pipeline

Q, m³/s	V, m/s	Re	λ	$\sum\zeta$	H_{st},m	H_n,m
0	0	0	-	7,0	4,0	4,0
0,004	3,18	127000	0,0368	7,0	4,0	13,3
0,08	6,36	254000	0,0368	7,0	4,0	41,3
0,012	9,54	381000	0,0368	7,0	4,0	87,9

Calculation of one flow rate value

$Q = 0{,}004$ m³/c

$$V = \frac{Q}{f} = \frac{4 \cdot 0{,}004}{\pi \cdot 0{,}04^2} = 3{,}18 \text{ m/s}; \quad \mathrm{Re} = \frac{Vd}{\nu} = \frac{3{,}18 \cdot 0{,}04}{1 \cdot 10^{-6}} = 127000$$

$$\mathrm{Re} \cdot \frac{\Delta_e}{d} = 127000 \cdot \frac{0{,}0005}{0{,}04} = 1590 > 500,$$

$$\lambda = 0{,}11\left(\frac{\Delta_e}{d}\right)^{0,25} = 0{,}11 \cdot (0{,}0125)^{0,25} = 0{,}0368$$

$$\sum\zeta = \zeta + \zeta_{вх} + \zeta_{вых} = 5{,}5 + 0{,}5 + 1 = 7{,}0$$

$$H_n = H_{st} + 0{,}0827\left(\lambda\frac{l}{d} + \sum\zeta\right)\frac{Q^2}{d^4} =$$

$$= 4{,}0 + 0{,}0827 \cdot \left(0{,}0368 \cdot \frac{12}{0{,}04} + 7{,}0\right)\frac{0{,}004^2}{0{,}04^4} = 13{,}3\ m$$

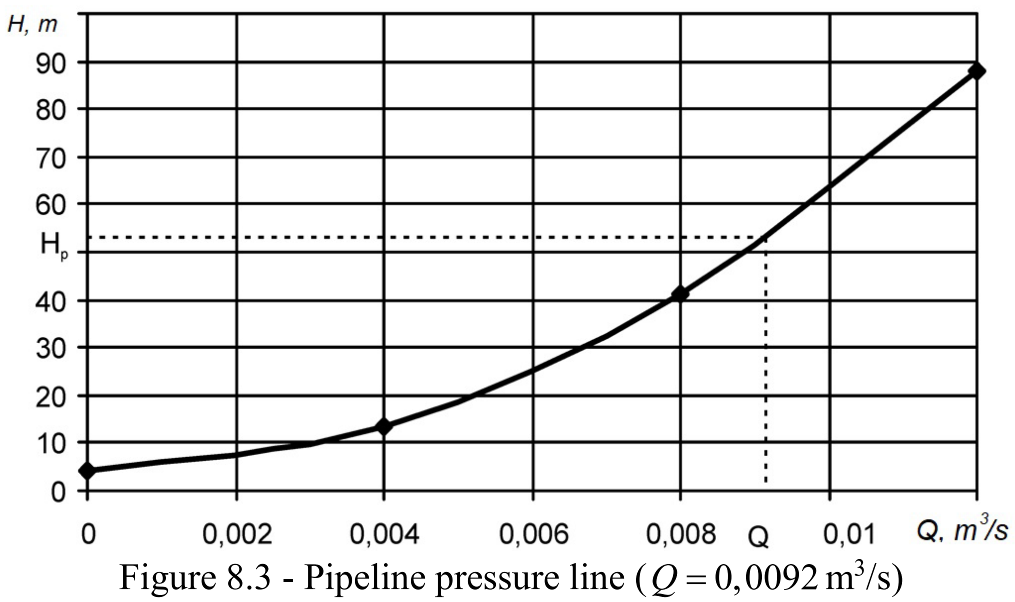

Figure 8.3 - Pipeline pressure line ($Q = 0{,}0092$ m³/s)

5. Find the diameter of the nozzle d_H that ensures a constant level H_1 in tank 2.

The diameter of the nozzle that ensures a constant level in tank 2 can be found using the formulas for the flow of liquid through the holes and nozzles. The flow rate through them is determined by the formula:

$$Q = \mu f \sqrt{2gH} \, .$$

Knowing the flow rate Q out of tank 2 (since the level H_1 is constant under the conditions of the problem, the flow rate in and the flow rate out are equal), you can find the area of the nozzle and, therefore, its diameter.

$$f = \frac{Q}{\mu \cdot \sqrt{2gH}} = \frac{0,0092}{0,82 \cdot \sqrt{2g \cdot 4,0}} = 0,00127 \ \text{m}^2$$

$$d = \sqrt{\frac{4f}{\pi}} = \sqrt{\frac{4 \cdot 0,00127}{\pi}} = 0,04 \ \text{m} = 40 \ \text{mm}.$$

6. Find the force of interaction between the jet flowing from the nozzle and the obstacle.

$$R = \rho \cdot QV(1 - \cos\beta) = \rho \cdot V^2 f (1 - \cos\beta) =$$

$$= 1000 \cdot \left(\frac{0,0092}{0,00127} \right)^2 \cdot 0,0092 \cdot (1 - \cos 90) = 483 \ N$$

7. Find the pressure increase in the pipeline at the instantaneous closure of the valve at its end, taking the wall thickness $\delta = 0,05d$.

The pressure increase in the pipeline during the instantaneous closure of the valve can be found using the formulas for calculating the hydraulic shock that occurs when the pipeline cross-section is instantly closed. The value of this pressure increase is equal to: $\Delta p = \rho \cdot c \cdot V$. The speed of shock wave propagation can be found using the Zhukovsky formula:

$$c = \frac{\sqrt{\dfrac{K}{\rho}}}{\sqrt{1 + \dfrac{K \cdot d}{E \cdot \delta}}} = \frac{\sqrt{\dfrac{2 \cdot 10^9}{1000}}}{\sqrt{1 + \dfrac{2 \cdot 10^9 \cdot 0,04}{2 \cdot 10^{11} \cdot 0,05 \cdot 0,04}}} = 1292, m/s \, .$$

Then $\Delta p = 1000 \cdot 1292 \cdot \dfrac{4 \cdot 0,0092}{3,14 \cdot 0,040^2} \cdot 10^{-6} = 9,46 \ \text{MPa}.$

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FOR NOTES

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Методичні вказівки до практичних занять з навчальної дисципліни
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за спеціальністю «Галузеве машинобудування»
освітня програма «Галузеве машинобудування»

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