Hemispherical and aspheric WGM dielectric resonators with conducting plane: radiation and conductivity losses in millimeter wavelength range

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Abstract — **In this work, electromagnetic properties of a number of whispering gallery mode (WGM) dielectric resonators with conducting endplates are studied. They differ in the shape of the body of rotation. Radiation and conductivity losses are studied in Ka-band. For the hemispherical resonator the results of analytical calculation and experimental measurements of radiation losses are correlated, although the experimental values are higher compared with the calculated ones. Experimental data show that a resonator of a combined geometry, i. e. in the form of aspheric+cylindrical disk, is the most promising for surface impedance characterization of unconventional superconductors as well as for other types of conductors, which is important for contactless measurement of conductivity.**

Index Terms — **Conductivity measurement, high temperature superconductors, impedance measurement, millimeter wave devices, microwave sensors, passive circuits.**

I. Introduction

For quite a long time, approaches for measuring the surface impedance $Z_S=R_S+iX_S$ (where R_S and X_S are the surface resistance and reactance, respectively) were developed based on whispering gallery mode (WGM) dielectric resonators in a form of cylindrical disk with conducting endplates (CEP) [1,2]. Such resonators are in fact quasi-optical dielectric resonators because several wavelengths confined along at least one of the coordinate axes. Whispering gallery mode resonator is body of rotation. WGMs propagate along a curved surface. Thus, these waves have a character of surface waves pressed to this surface. As a rule the resonators are made of single crystal sapphire as dielectric material of very small loss tangent. Both factors, namely, WGM excited in the resonator and sapphire material allow achieving high Q-factor in millimeter wavelength range that provides high accuracy and sensitivity of measuring Z_S.

Recently WGM dielectric resonators were proposed in the forms of truncated cone [3] and hemisphere [4] have recently been proposed. It should be noted, that the electromagnetic properties of isotropic hemispherical resonator were considered analytically earlier, however only for the case of perfect conductor plate [5]. A comparison of three types of the resonators, cylindrical disk, truncated cone and hemisphere, was performed and niche was defined for each of them as the sensors of surface impedance for HTS films [6]. From the viewpoint of maximum sensitivity the hemispherical resonator is the most appropriate because of the lowest radiation loss. However, the interaction of WGM microwave fields with CEP, i.e. with HTS film, is weaker compared with the case of the truncated cone. Thus the question arises concerning the strengthening of the interaction while maintaining the same radiation loss using transformation of the hemispherical surface in the surface of another shape. The answer to this question will allow in principle the possibility of further increasing the sensitivity of impedance measurements.

With this purpose in the present work a number of WGM resonators with the transformation of the hemispherical surface as well as cylindrical surface in aspheric surface are considered. Because electromagnetic problems concerning the proposed resonators have not been solved, the authors propose following. At first, the analytical solution [4] of electromagnetic problem concerning an isotropic hemisphere on the conducting plane is used. In this case the characteristic equation allows finding the frequency spectrum and quality factor, including the radiation quality factor. Then the radiation quality factor values obtained experimentally as a function of frequency (or azimutal index) are compared with calculated ones. After that, analysis of radiation losses depending on the surface evolution (and evolution of eigen frequencies) is performed.

II. Theoretical Description of Isotropic Hemispherical WGM Resonator with Conducting Endplate

Analytical solution of the isotropic dielectric hemisphere placed on the conducting plate obtained in [6] allows determining eigen frequencies and radiation Q-factor, Q_{rad}. For the purpose of describing the microwave fields and calculating the frequency spectrum and quality factor of the resonator, the special program product was developed. The advantage of the product is sufficiently increased performance. Results obtained using this program product are presented in section IV together with the experimental data. Only modes with both zero radial components of electric field, i.e. HE (in cylindrical dick)- and H (in hemisphere)-modes, and one radial variation of field are considered.

TABLE I
DIMENSIONS AND FORMS OF THE STUDIED RESONATORS

N	*r*, mm	*2a*, mm	*h*, mm	Shape
1	39	0	39	hemisphere
2	30	18	30	aspheric disk
3	20	38	20	aspheric disk
4	7	64	7	aspheric disk $r=r_o=h$
5	3.5	64	7	aspheric+ cylindrical disk
6	0	78	7	cylindrical disk

III. EXPERIMENTAL DETAILS

A number of WGM resonators were fabricated of Teflon because this material is easily mechanically processed. Their shapes changed from the hemispherical of a radius R to the aspheric of a spherical part radius r and an eccentricity $2a$ (Fig.1) and from cylindrical disk of a radius R and a height h (Fig.2) to the same aspheric resonator. In this case the resonator forms varied in such a way that the same resonator of a radius R=78mm and a height h=7mm was obtained as a result of both evolutions. It is clear that $r_o = h$ in the resonator. Here r_o is a maximum meaning of radius at evolution from cylindrical disk and minimum meaning at evolution from hemisphere (Table I).

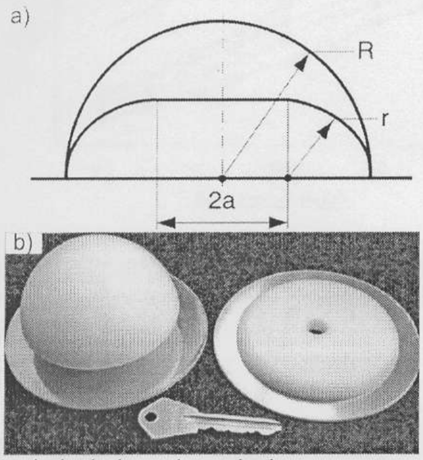

Fig. 1. Hemispherical and aspheric resonators with CEP: schematic drawing (a) and photo (b).

Experimental study was performed in Ka-band using network analyzer PNA-L N5230A.

IV. RESULTS AND CONSIDERATIONS

Fig.3 displays experimental frequency spectra of hemispherical (a) and aspheric (b) WGM resonators. Fig.4 shows understandable change in the resonators frequency. The radiation loss was found using the experimental data and known relationship (see, e.g. [1]).

$$Q_0^{-1} = k\tan\delta + A_s^{CEP} R_s^{CEP} + Q_{rad0}^{-1} \qquad (1)$$

where k and A_s are coefficients describing interaction of fields with dielectric and conducting endplate, and $\tan\delta$ is dielectric loss tangent. Using known k, A_s and the measured value of Q_o Q_{rad} can be found accordingly (1).

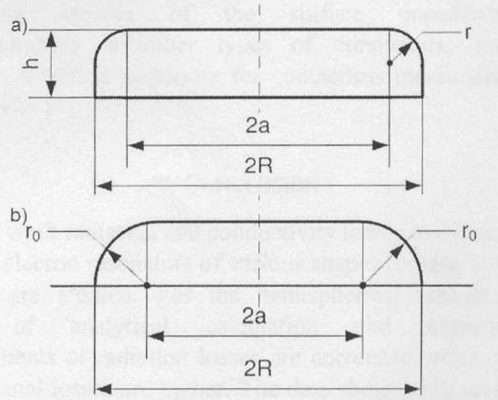

Fig. 2. Aspheric+cylindrical disk(a) and aspheric resonator(b) with CEP.

A coefficient k is very close to 1 and $k\tan\delta \ll A_s R_s$, therefore we can leave k=1 for all resonators. However As is unknown for aspheric resonators. It can be found using two equations (1) and two measured Q_o values for two conductors with the known surface resistance. We used copper with R_s=0.05 Ohm and steel with R_s=0.12 Ohm pre-measured by means of cylindrical disk WGM resonator [1]. Such an approach enables us to determine A_s and Q_{rad} for all resonators presented in Table I.

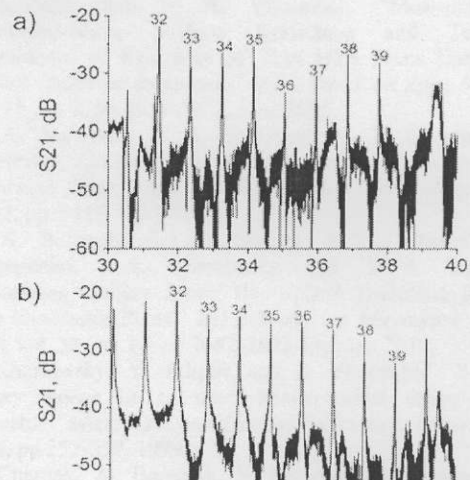

Fig. 3. Experimental spectra of hemispherical (a) and aspheric (b) WGM resonators; numbers at resonances are azimuthal indexes at one variation along radius.

Fig.5 shows that Q_{rad} values in hemispherical resonator determined experimentally are lower than the calculated analytically. On the other hand, Fig.5 displays experimental data concerning Q_{rad} in hemispherical, aspheric, aspheric+cylindrical and cylindrical disks. One can see that contrary expectation a radiation loss is the largest for aspheric resonator. In addition, losses in cylindrical disk and aspheric+cylindrical disks are almost the same within the measurement error.

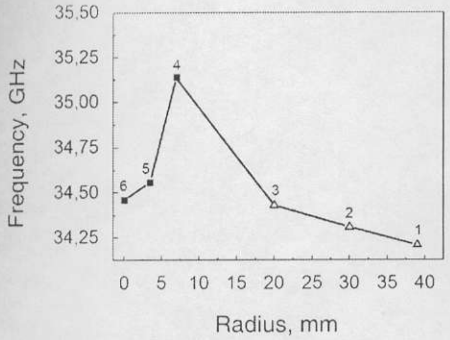

Fig. 4. Resonance frequency as function of radius r in WGM resonators (see Fig. 1, Fig. 2 and Table I) at azimuthal index n=35.

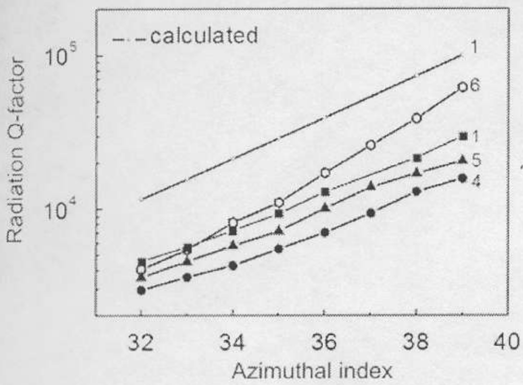

Fig. 5. Experimental and calculated data on radiation Q-factor for number of WGM resonators. 1,4, 5 and 6 are numbers in Table I.

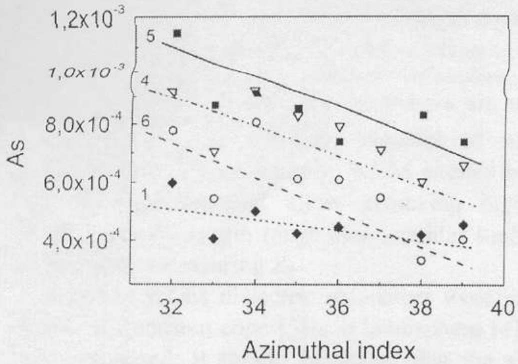

Fig. 6. Coefficient As for a number of resonators depending on azimuthal index. 1,4, 5 and 6 are numbers in Table I.

Fig.6 presents a coefficient A_s as a function of azimutal number n for hemispherical, aspheric, aspheric+cylindrical and cylindrical disks. The data demonstrate that interaction of microwave fields with conductor endplate is the strongest in the aspheric+cylindrical disk. This result is rather unexpected and demands the further studying. Nevertheless, it is useful for developing sensors of the surface impedance of superconductors or other types of conductors, such as graphene, which is important for contactless measurement of conductivity [7].

V. Conclusion

In this work radiation and conductivity losses in a number of WGM dielectric resonators of various shapes (of rotation body surface) are studied. For the hemispherical resonator the results of analytical calculation and experimental measurements of radiation losses are correlated, although the experimental losses are higher. The data show that a resonator of a combined surface, i.e. aspheric+cylindrical disk, is the most promising for surface impedance characterization of unconventional superconductors and other types of conductors, such as a graphene.

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