NATIONAL TECHNICAL UNIVERSITY
“KHARKIV POLYTECHIC INSTITUTE”

DEPARTMENT OF PHYSICS

OLENA LYUBCHENKO

STUDY GUIDE
“ATOMIC AND NUCLEAR PHYSICS”

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1. MATTER WAVES. DE BROGLIE WAVELENGTH

In 1924, French physicist Louis de Broglie (1892-1987, Nobel Prize in Physics in 1929) made the suggestion that since light waves could exhibit particle-like behavior, particles of matter should exhibit wave-like behavior. De Broglie proposed that all moving matter has a wavelength associated with it, just as a wave does, and wavelength are applicable to particles as well as to waves. According to his theory the wavelength λ of a particle is given by the same relation that applies to a photon:

$$\lambda = \frac{h}{p},$$

where $h = 6.63 \cdot 10^{-34}$ J·s is Plank's constant, p is the linear momentum of the particle, and λ is known as *the de Broglie wavelength* of the particle. Depending on the speed of the particle, its linear momentum may be calculated using the classical formula $p = mv$ or relativistic formula $p = \frac{m_0 \beta c}{\sqrt{1-\beta^2}}$, where $\beta = v / c$.

Confirmation of de Broglie's suggestion came in 1927 from the experiments of the American physicists Clinton J. Davisson (1881–1958) and Lester H. Germer (1896–1971) and, independently, those of the English physicist George P. Thomson (1892–1975). Davisson and Germer directed a beam of electrons onto a crystal of nickel and observed that the electrons exhibited a diffraction behavior; analogous to that seen when X-rays are diffracted by a crystal. The wavelength of the electrons revealed by the diffraction pattern matched that predicted by de Broglie's hypothesis, $\lambda = h/p$. Particles other than electrons can also exhibit wave-like properties. For instance, neutrons are sometimes used in diffraction studies of crystal structure. Although all moving particles have a de Broglie wavelength, the effects of this wavelength are observable only for particles whose masses are very small, on the order of the mass of an electron or a neutron.

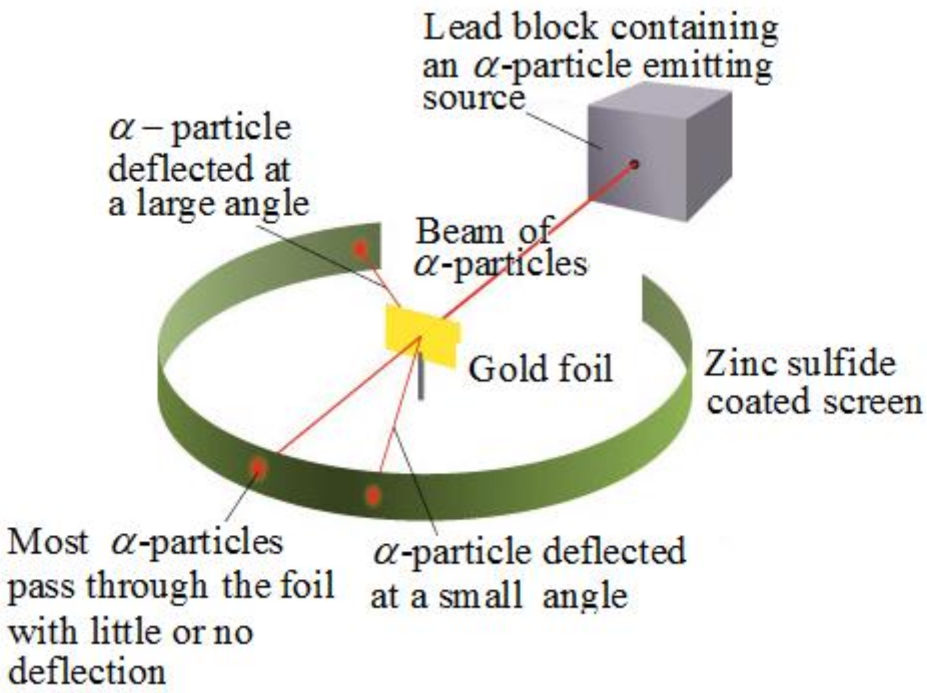
2. BOHR'S ATOM

2.1. Rutherford's scattering experiment. Models of atom

At the end of the 19th century a pattern of chemical properties of elements had begun to emerge and this was fully recognized by Dmitry Mendeleev when he constructed his Periodic Table. Immediately it was apparent that there must be common properties and similar behavior among atoms of different elements and the long process of atomic structure understanding had begun. The idea that matter is made up of atoms, was accepted by most scientists by 1900. The discovery of electron in 1897 by J.J. Thomson made scientists to think that atom is having a structure and electrons are part of it.

The typical model of the atom (*plum-pudding model*) suggested by J.J. Thomson in 1890s visualized the atom as a homogeneous sphere of positive charge inside of which there were tiny negatively charged electrons, a little like plums in a pudding.

Around 1911, Ernest Rutherford and his colleagues performed these famous experiments whose results contradicted Thomson's model of atom. By scattering



fast-moving α-particles (charged nuclei of helium atoms emitted spontaneously in radioactive decay processes) from metal foil targets, Rutherford established that atoms consist of a compact positively charged nucleus (diameter $\sim 10^{-14} - 10^{-15}$ m) surrounded by a swarm of orbiting electrons (electron cloud diameter $\sim 10^{-10}$).

Rutherford’s *planetary model* proposed that negative electrons orbit around a dense positive nucleus. The positive charge on the nucleus was taken to be equal to the sum of the electron charge so that the atom was electrically neutral. Rutherford's model, however, had several major problems, including the fact that it could not account for the appearance of discrete emission line spectra. In Rutherford's model the electrons continuously orbited around the nucleus. This circular, "accelerated" motion should produce a continuous band of electromagnetic radiation, but it did not. Additionally, the predicted orbital loss of energy would cause an atom to disintegrate in a very short time and thus break apart all matter. This phenomenon, too, did not occur. Clearly Rutherford's model was not sufficient. Some sort of modification was necessary, and it was Niels Bohr who provided it by adding an essential idea.

2.2. Bohr’s postulates

Bohr had studied in Rutherford's Laboratory for several months in 1912 and was convinced that Rutherford's planetary model of atom was valid. But in order to make it work, he felt that the newly developed quantum theory would somehow have to be incorporated in it. Perhaps, Bohr argued, the electrons in an atom cannot loose energy continuously, but must do so in quantum "jumps". He formulated his reasons in the form of *postulates*:

1. *Stationary states postulate*. Only certain electron orbits are stable and allowed. In these orbits, no energy in the form of electromagnetic radiation is emitted, so the total energy of the atom remains constant.

2. *Orbits quantizing postulate*. The angular momentum L of an electron in the stationary orbit satisfies a particular equation:

$$L = mvr = n\hbar = n\frac{h}{2\pi},$$

where $n = 1, 2, 3, \ldots$ is the number of state (orbit, shell) – *quantum number*, h and $\hbar = h/2\pi$ are the *Planck's constants*.

In other words, the circumference of an electron's orbit must contain an integral number of de Broglie wavelengths: $2\pi r = n\lambda$. Taking into account that $\lambda = h/p = h/mv$, it gives $2\pi r = nh/mv$, and finally, $mvr = n\hbar = n\frac{h}{2\pi}$.

3. *Frequency postulate*. Electromagnetic radiation is emitted when the electron "jumps" from a more energetic stationary state to a less energetic state; and radiation is absorbed at the "jump" from the less to more energetic state. The frequency of the radiation emitted (absorbed) in the transition is related to the change in the atom's energy, given by

$$h\upsilon = \hbar\omega = E_{n_i} - E_{n_k},$$

where n_i, n_k E_{n_i}, E_{n_k} are the numbers of states and their energies, respectively.

$E_{n_i} > E_{n_k}$ related to the *emission* of photon, and $E_{n_i} < E_{n_k}$ - to the *absorption* of photon.

2.3. Bohr's model of hydrogen atom. Hydrogen spectrum. Balmer's formula

In a stationary orbit of radius *r* the electric force between the negative electron and the positive nucleus is balanced by the centripetal force on the electron due to its circular motion. According to the Newton's 2 Law $ma_n = F$ and

$$m\frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2}.$$

The 2nd Bohr's postulate is

$$mvr = n\hbar.$$

Divide the first equation by the square of the second equation

$$\frac{mv^2}{rmv^2r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2n^2\hbar^2}.$$

Bohr's radii

$$r_n = 4\pi\varepsilon_0 \frac{\hbar^2}{me^2} n^2 = r_1 \cdot n^2,$$

where $n = 1, 2, 3, \ldots$.

According to Bohr's model, an electron can exist only in certain allowed orbits determined by the integer n. The orbit with the smallest radius, called the Bohr radius corresponds to $n = 1$ and has value

$r_1 = 0.53 \cdot 10^{-10}$ m.

The electric potential energy of the atom consisting of the proton (e) and electron ($-e$) separated by distance r is

$$PE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \cdot (-e)}{r} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}.$$

where $\frac{1}{4\pi\varepsilon_0}$ is the Coulomb constant.

Assuming the nucleus is at rest, the total energy E of the atom is the sum of the kinetic and potential energies

$$E = KE + PE = \frac{mv^2}{2} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}.$$

By Newton's second law, the electric force of attraction on the electron, $\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$, must equal ma, where $a = \frac{v^2}{r}$ is the centripetal acceleration of the electron, so

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}.$$

Therefore, $\frac{mv^2}{2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2r}$, and

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2r} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2r}.$$

Substitution of the $r_n = 4\pi\varepsilon_0 \dfrac{\hbar^2}{me^2} n^2$ into the expression for the energy gives

$$E_n = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2r} = -\frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{me^4}{2\hbar^2} \cdot \frac{1}{n^2} = -\frac{2.17 \cdot 10^{-17}}{n^2}\,\text{J} = -\frac{13.6}{n^2}\text{eV}.$$

For the first energy level (*ground level*) of hydrogen ($n = 1$):

$E_1 = -2{,}17 \cdot 10^{-17}$ J $= -13{,}6$ eV .

For other levels the *total energies are*

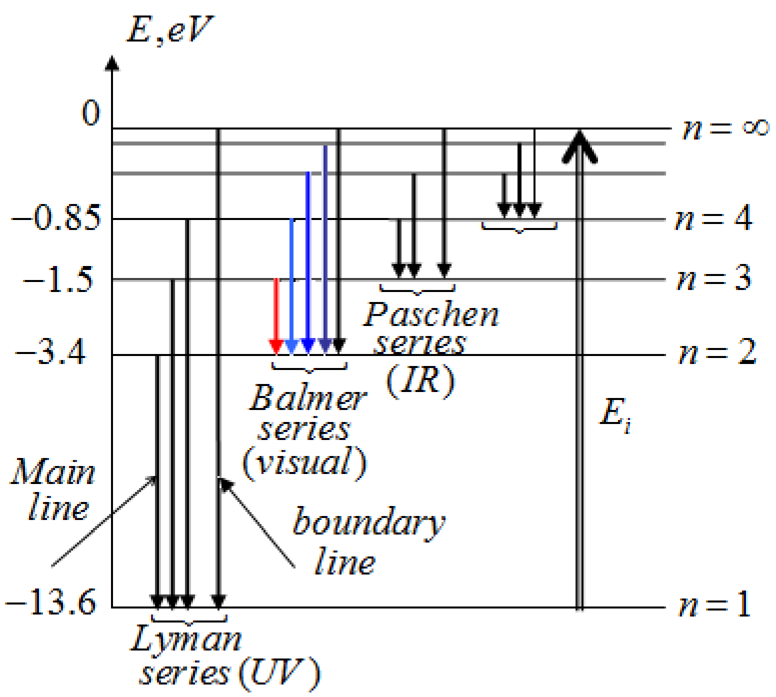
$E_2 = -3.4\text{eV}$, $E_3 = -1.51\text{eV}$, $E_4 = -0.85\text{eV}$.

Relationship between energies is

$$E_n = -KE_n = \frac{PE_n}{2}.$$

Note that energies of *excited states* (n = 2, 3, 4 …) are greater (smaller negative numbers) than those of the *ground state* (n = 1).

The minimum energy required to remove an electron from the ground state of an atom is called the *binding energy* or the *ionization energy*. The ionization energy for hydrogen is E_i=13.6 eV to remove an electron from the lowest state E_1 = - 13.6 eV up to $E = 0$ where it can be free.



Excitation energy is the energy required to remove an electron from the ground state to exited states ($n = 2, 3, 4, \ldots$) of an atom.

For *hydrogen-like ions* (an ion with one electron and nucleus charge $+Ze$, where Z is the number of the element in the Periodic Table) radii and energies are

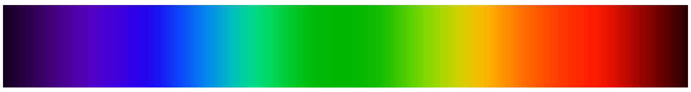
$$r_n = 4\pi\varepsilon_0 \frac{\hbar^2}{Zme^2} n^2 = \frac{\varepsilon_0 h^2}{\pi e^2 m} \cdot \frac{n^2}{Z},$$

$$E_n = -\frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{me^4}{2\hbar^2} \cdot \frac{Z^2}{n^2} = -\frac{me^4}{8\varepsilon_0^2 h^2} \cdot \frac{Z^2}{n^2}.$$

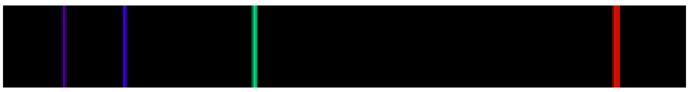
According to Bohr's frequency postulate $(\hbar\omega = \Delta E)$ the *frequencies* of spectrum lines are described by *Rydberg formula*

$$\omega = \frac{E_{n_i} - E_{n_k}}{\hbar} = \frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{me^4}{2\hbar^3} \left(\frac{1}{n_i^2} - \frac{1}{n_k^2} \right) = R' \left(\frac{1}{n_i^2} - \frac{1}{n_k^2} \right),$$

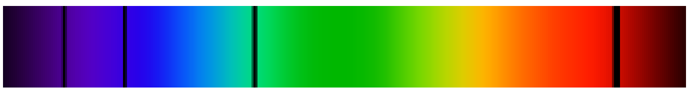
where $R' = 2.07 \cdot 10^{16}$ s^{-1} is the *Rydberg constant (for frequency),* n_i, n_k are the numbers of levels.



Continuous spectrum



Emission spectrum of Hydrogen



Absorption spectrum of Hydrogen

The *wavelengths* of hydrogen spectrum lines are described by *Rydberg formula* (taking into account that $\omega = \dfrac{2\pi c}{\lambda}$):

$$\frac{1}{\lambda} = R \cdot \left(\frac{1}{n_i^2} - \frac{1}{n_k^2} \right),$$

where $R = 1.1 \cdot 10^7$ m^{-1} is the *Rydberg constant (for wavelength).*

For hydrogen-like ions the *frequencies of spectrum lines* are:

$$\omega = Z^2 R' \left(\frac{1}{n_i^2} - \frac{1}{n_k^2} \right),$$

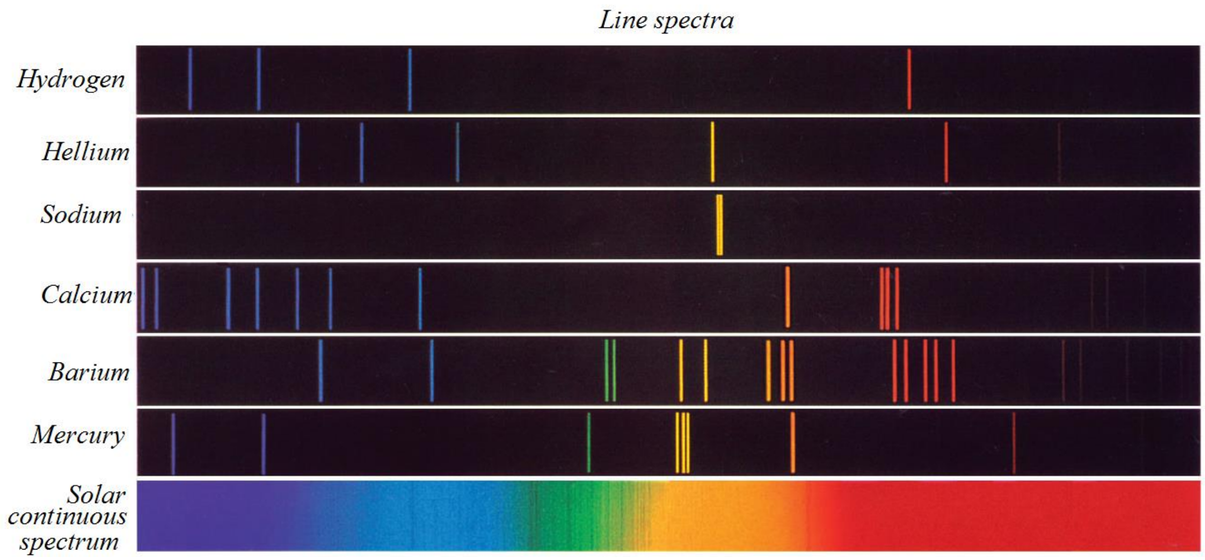
and the wavelengths *of spectrum lines:*

$$\frac{1}{\lambda} = Z^2 R \cdot \left(\frac{1}{n_i^2} - \frac{1}{n_k^2} \right),$$

These results correspond well to Balmer's experiments, in which it was found that the hydrogen spectrum consists of discrete spectrum lines which wavelengths may be described by $\lambda = \lambda_0 \cdot \frac{n^2}{n^2 - 4}$, where $\lambda_0 = const, \quad n = 3, 4, 5, \ldots$

At room temperature, almost all hydrogen atoms are in the ground state (n = 1). At higher temperature or during electric discharge as the energy is externally supplied, the electrons can be excited to higher energy levels E_2, E_3, etc. Once in an excited state, an atom's electron can jump down to a lower state (not necessarily ground state) and give off a photon in the process. This is, according to the Bohr model, the *origin of emission spectra* of excited gases.

In the energy level diagram the vertical arrows represent transitions that correspond to various observed spectral lines.



The *boundary of series* ($n_k = \infty$) corresponds to the frequency (or to the wavelength)

$$\omega_{max} = \frac{R'}{n_i^2}, \quad \lambda_{min} = \frac{n_1^2}{R}.$$

The *main (head) line* of series ($n_k = n_i + 1$) corresponds to the ω_{min} and λ_{max} in this series.

Transitions to

($n_i = 1$) - Lyman series (ultraviolet);

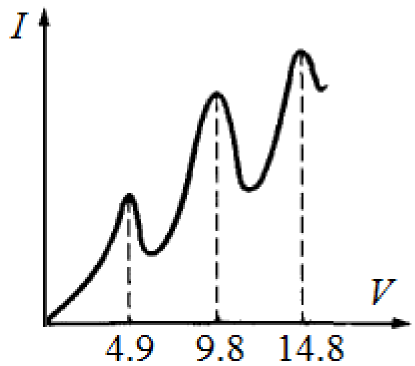
($n_i = 2$) - Balmer series (visible);

($n_i = 3$) - Paschen series (infrared);

($n_i = 4$) - Bracket series (infrared);

($n_i = 5$) - Pfund series (infrared).

The first experimental verification of the existence of discrete energy states in atoms was performed in 1914 by the German-born physicists James Franck and Gustav Hertz. They directed low-energy electrons through a gas enclosed in an electron tube. As the energy of the electrons was slowly increased, a certain critical electron energy was reached at which the electron stream made a change from almost undisturbed passage through the gas to nearly complete stoppage. The gas atoms were able to absorb the energy of the electrons only when it reached a certain critical value, indicating that within the gas atoms themselves the atomic electrons make an abrupt transition to a discrete higher energy level. As long as the bombarding electrons have less than this discrete amount of energy, no transition is possible and no energy is absorbed from the stream of electrons. When they have this precise energy, they lose it all at once in collisions to atomic electrons, which store the energy by being promoted to a higher energy level.



Bohr's model of the atom was both a success and a failure. The success of Bohr Theory is not only because that it can successfully explain the problem of hydrogen atom and hydrogen-like ion, but also embodied in following aspects.

1. Bohr correctly pointed out the existence of the atom energy level, i.e. the energy of atom is quantitative, and it only took some certain discrete values, which was not only testified by the hydrogen atom and hydrogen-like ion, but also proved by the Frank-Hertz experiment. That shows Bohr's hypothesis about the energy quantization has more general meaning than his theory of hydrogen atom.

2. Bohr correctly proposed the concept of the stationary state, i.e. the atom in certain energy state E_n didn't radiate the electromagnetic wave, and only when the atom transited from certain one energy state E_{n_i} to another energy state E_{n_k}, it could emit the photons, and the frequency of photons is

$$\upsilon = \left(E_{n_i} - E_{n_k}\right)/h .$$

The facts indicate that this conclusion is universally correct for various atoms, and his physical idea of the quantum jump has been accepted by the modern science.

3. The angular momentum quantization derived from Bohr's quantization condition $L = n\hbar$ is universally correct. It successfully predicted the frequencies of the lines in the hydrogen spectrum, so it seemed to be valid.

4. Bohr's theory successfully explained the line spectrum of the hydrogen atom and definitely pointed out the classical physics was inapplicable in the interior phenomena of the atom.

Nevertheless the model was a total failure when it tried to predict energy levels for atoms with more than one electron. It could not explain the fine-structure (two or more closely spaced spectral lines) of emission lines, and why some spectral lines were brighter than others. However, Bohr's theory is very meaningful in the history of physics.

3. NUCLEAR PHYSICS

3.1. Some properties of nuclei

All nuclei are composed of two types of particles: protons and neutrons. The only exception is the ordinary hydrogen nucleus, which is a single proton. The constituents of the nucleus that is protons and neutrons are called *nucleons*.

Proton (p). The nucleus of the simplest atom, hydrogen. It has a positive charge of $q_p = e = 1.6 \cdot 10^{-19}$ C and a mass of $m_p = 1.67 \cdot 10^{-27}$ kg. In nuclear physics the mass is often expressed in *unified atomic mass units* (which is defined as $1/12$ of mass of neutral atom of carbon $^{12}_{6}C$) or in *energy equivalent* (MeV)

$m_p = 1.007276$ u = 938.28 MeV.

Neutron (n). A particle found in the nucleus that is electrically neutral and that has a mass almost identical to the proton. It was discovered by James Chadwick in 1932. Thus, its charge is $q = 0$; its mass is $m_n = 1.6749 \cdot 10^{-27}$ = 1.00898 u = 939.55 MeV.

In the free state the neutron is unstable and spontaneously decays into a proton, an electron and a neutrino:

$$n \to p + e + \nu .$$

In describing some of the properties of nuclei, such as their charge, mass, and radius, we make use of the following quantities:

the atomic (charge) number Z, which equals the number of protons in the nucleus;

the atomic mass number A, which equals the number of nucleons in the nucleus). The mass of a nucleus is very close to A times the mass of one nucleon;

the neutron number N, which equals the number of neutrons in the nucleus. The number of neutrons $N = A - Z$.

Conventional symbols for nuclear species, or *nuclides*, is

$$^{A}_{Z}X ,$$

where X is the chemical symbol of the element.

For example, ${}^{2}_{1}H$, ${}^{2}_{2}He$, ${}^{238}_{92}U$.

The nuclei of all atoms of a particular element must contain the same number of protons, but they may contain different numbers of neutrons. Nuclei that are related in this way are called *isotopes*. The isotopes of an element have the same Z value, but different N and A values.

For example, ${}^{235}_{92}U$ and ${}^{238}_{92}U$; ${}^{1}_{1}H$ - ordinary hydrogen – protium – stable (one proton); ${}^{1}_{1}H$ – heavy hydrogen – deuterium (D) stable (one proton + one neutron); ${}^{3}_{1}H$ - tritium (T) unstable (one proton + two neutrons).

Isobars: nuclei that have the same atomic mass number A.

For example, ${}^{40}_{18}Ar$ and ${}^{40}_{20}Ca$.

Isotones: nuclei that have the same number of neutrons $N = A - Z$ (the number of protons is different).

For example, ${}^{13}_{6}C$ and ${}^{14}_{7}N$.

Isomers: nuclei that have the same number Z and A, but different life time periods.

For example, there are two isomers ${}^{80}_{35}Br$ with half-lives 18 min and 4.4 hours.

Size and shape of nuclei

Most nuclei are nearly spherical. A few, principally nuclei with Z between 56 and 71, have ellipsoidal shape with eccentricities of less than 0.2.

Assuming nucleus to be spherical in shape with nuclear radius R, the corresponding volume is $\frac{4}{3}\pi R^3$. And so R^3 is proportional to A. This relationship is expressed in inverse form as

$$R = R_0 A^{\frac{1}{3}} = 1.3 \cdot 10^{-15} \cdot A^{\frac{1}{3}} \text{ (m)} = 1.3 \cdot A^{\frac{1}{3}} \text{(Fm)}.$$

It should be noted that R_0 is expressed in an indefinite way because nuclei do not have sharp boundaries. Despite this the value of R is the representative of the effective nuclear size.

The value of R_0, as deduced by electron scattering experiments is slightly less than 1.3 Fm. This implies that nuclear matter and nuclear charges are not identically distributed through a nucleus.

We can also look at a nucleus in terms of the forces that hold it together. The electric force described by Coulomb predicts that the nucleus should fly apart (since positive charges repel other positive charges). Another short-range attractive force must be acting within the nucleus. This force must be stronger than the electric force.

Strong Nuclear Force. An attractive force that acts between all nucleons. Protons attract each other via the strong nuclear force while they repel each other via the electric force. The strong nuclear force is the strongest force, but it acts only over very short distances (less than 10^{-10} m). It is a short range force that is essentially zero when nucleons are separated by more than 10^{-15} m.

Stable nuclei tend to have equal numbers of protons and neutrons for nuclei with Z equal to about 30 or 40. If there are too many or too few neutrons relative to the number of protons, the nuclei tend to be unstable. For nuclei with Z greater than 30 or 40, stable nuclei have more neutrons than protons. There are no stable nuclei with Z greater than 83. They are all radioactive. As Z increases, the electric repulsion increases. Nuclides with large numbers of protons need more neutrons (which only exert the attractive strong nuclear force) to overcome the electric repulsion between protons. For these very large nuclei, no number of neutrons can overcome the electric repulsion between protons. All elements with Z greater than 92 do not occur naturally.

3.2. Mass defect. Binding energy

The hydrogen isotope deuterium ${}_1^2H$ has one electron, one proton and one neutron in its nucleus. Thus we would expect the mass of deuterium atom to be equal to that of an ordinary ${}_1^1H$ atom (which has one proton and one electron) plus the mass of the neutron:

mass of ${}_1^1H$ atom	1.007825 u
mass of neutron	1.008665 u
expected mass of ${}_1^2H$ (deuterium)	2.016490 u
measured mass of ${}_1^2H$ (deuterium)	2.014102 u
mass defect Δm	0.002388 u

The measured mass is less than the combined mass of ${}_1^1H$ and neutron. This loss in mass is known as *mass defect* Δm.

What comes into mind is that the "missing" mass might correspond to the energy given off when a stable atom is formed from its constituents. In case of deuterium, the energy equivalent of mass defect (the missing mass) is $\Delta E = \Delta m(\mathrm{u}) \cdot 931.49(\mathrm{MeV/u}) = 2.224\,\mathrm{MeV}$.

In fact, it was experimentally observed that the energy required for breaking deuterium nucleus apart into a separate neutron and a proton is 2.224 MeV. When the energy less than 2.224 MeV is given to ${}_1^2H$ nucleus, the nucleus stays together. When the energy supplied externally is more than 2.224 MeV, the exceed energy goes into kinetic energy of the neutron and proton as they fly apart.

It should be noted that mass defect (“missing mass”) is not peculiar to deuterium atoms only rather all atoms have it. The energy equivalent of the missing mass of a nucleus is called the *binding energy* of the nucleus. The greater the binding energy of a given nucleus is, the more the energy that must be supplied to break it up.

The *binding energy* is

$$E_b = c^2\left\{\left[Zm_p + (A-Z)m_n\right] - m_{nucleus}\right\}.$$

Substitute $m_p = m_H, m_{nucleus} = m_a$ and rewrite:

$$E_b = c^2\left\{\left[Zm_H + (A-Z)m_n\right] - m_a\right\}.$$

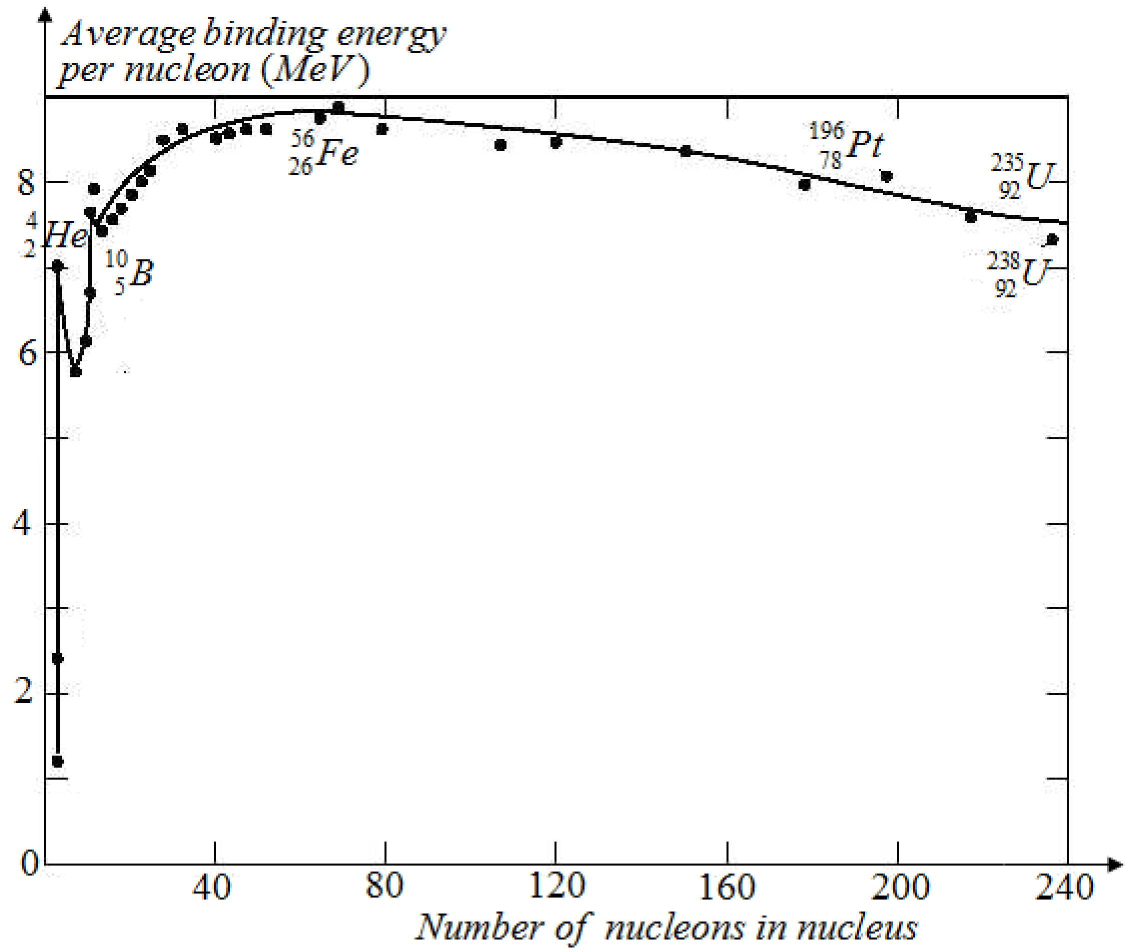
This expression is more convenient as there are atomic masses in different manuals but not the nucleus masses.

The *mass defect* is

$$\Delta = \left[Zm_p + (A-Z)m_n\right] - m_{nucleus} \cong \left[Zm_H + (A-Z)m_n\right] - m_a$$

or

$$\Delta = \frac{E_b}{c^2}.$$



The *binding energy per nucleon* refers to the average energy associated with every single constituent of the nucleus. For a given nucleus it can be calculated by dividing its total binding energy by the number of nucleons (the sum of numbers of protons and neutrons) it contains.

The greater the binding energy per nucleon is the more stable the nucleus is. The graph has its maximum of $8.8\,\mathrm{MeV/nucleon}$ for $^{56}_{26}Fe$, making it more stable of them all.

Two remarkable conclusions can be drawn from the above curve.

1. If we can somehow split a heavy nucleus into two medium-sized ones, each of the new nuclei will have more energy per nucleon than the original nucleus did. The extra energy will be given off, and it can be a lot. For instance, if the heavy uranium nucleus $^{235}_{92}U$ is broken into two smaller medium sized nuclei, the binding energy difference per nucleon is about $0.8\,\mathrm{MeV}$. The total energy given off is therefore

$$\left(0.8\,\mathrm{MeV/nucleon}\right)\cdot\left(235\,\mathrm{nucleons}\right)=188\,\mathrm{MeV}.$$

This is a truly enormous amount of energy. Splitting a heavy nucleus into smaller nucleus is called *nuclear fission*. We will study this in more detail later on.

2. Joining two light nuclei together to give a single nucleus of a medium size also means more binding energy. For instance, if two $^{2}_{1}H$ deuterium nuclei combine to form a $^{4}_{2}He$ helium nucleus, over $23\,\mathrm{MeV}$ is released. Such a process, called *nuclear fusion*, is a very effective way to obtain energy. This in fact is the process that powers the sun and other stars.

3.3. Radioactivity. Radioactive decay modes

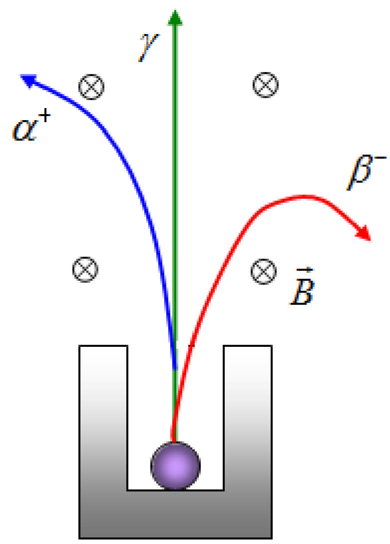
Many isotopes are radioactive. Radioactive nuclei are not stable; they decay into other nuclei after a certain amount of time. *Radioactivity* is the property exhibited by certain nuclei and it refers to the spontaneous emission of energy and subatomic particles by nuclei. Most of the elements are stable and have no radioactivity isotopes but still there are many of them that are unstable and spontaneously change into other nuclei by radioactive decay process. Of course, all nuclei can be transformed by reactions with nucleons or other nuclei that collide with them.

Radioactivity was first reported by the French physicist Henri Becquerel in 1896, for a double salt of uranium and potassium. Soon thereafter it was found that all uranium compounds and the metal itself were similarly radioactive. In 1898, French physicists Pierre and Marie Curie discovered two other strongly radioactive elements, radium and polonium, that occur in nature. Although Becquerel's discovery was accidental, he realized its importance at once and explored various aspects of radioactivity of uranium for the rest of his life. He was awarded 1903 Nobel Prize in physics for his work on radioactivity.

There are two types of radioactivity: *natural* radioactivity (for elements which exist in nature) and *artificial* radioactivity (induced by nuclear reactions). There is no difference between them from the point of view of the laws describing them.

The early experimenters distinguished three components in the radiation. The radiations deflected in a horizontal magnetic field to the left are positively charged α-particles, those deflected to the right are negatively charged β-particles and those which remained undeflected were γ-rays.

Later two more decay modes (the positron emission and the electron capture) were added to the list of decay modes.



i. α***-decay.*** If a nucleus emits an alpha particle $\left({}^4_2He\right)$, it loses two protons and two neutrons.

The decay can be written symbolically as

$${}^A_Z X \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2He.$$

Here X is called the *parent nucleus*, while Y is the *daughter nucleus*. Note that the number of neutrons and the number of protons is conserved in the reaction. This is not a real requirement in the nuclear reaction, only the total number of nucleons and the total charge must be conserved. Since no other charged particles are present in the reaction this implies the separate conservation of neutron and proton numbers.

The decay happens because by this decay the system goes into a lower energy state. The energy of the state is lower because, for nuclei with $A > 80$ the binding energy per nucleon increases if the nucleus becomes lighter.

The typical examples of alpha-decay are

$^{238}_{92}U \to {}^{234}_{90}Th + {}^{4}_{2}He$,

$^{226}_{88}Ra \to {}^{222}_{86}Rn + {}^{4}_{2}He$.

ii. ***β-decay***. This process is connected with emitting the electron (positron) by the nucleus or with the capture of the electron of the lowest levels by the nucleus. The production of a β- particle in the nucleus involves the action of so called *weak nuclear forces*.

- The general form of the electron β^-- *decay* is

$^{A}_{Z}X \to {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \tilde{\nu}$.

Note that the number of nucleons is conserved and the charge is conserved, because one of the neutrons is transformed into a proton and an electron ($^{1}_{0}n \to {}^{1}_{1}p + {}^{0}_{-1}e$). For the accomplishment of the conservation laws it is necessary for the nuclei to emit the massless and electrically neutral particle (*antineutrino* $\tilde{\nu}$).

The typical example: $^{234}_{90}Th \to {}^{234}_{91}Pa + {}^{0}_{-1}e + \tilde{\nu}$.

- The general form for positron β^+-*decay* is

$^{A}_{Z}X \to {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \nu$.

A *positron* is a positive electron produced in the nucleus by the decay of a proton into a positron and a neutron. It has a charge of $q = +e$ and essentially no mass. They are emitted by nuclei that have too few neutrons relative to their number of protons.For the accomplishment of the conservation laws it is necessary for the nuclei to emit a massless and electrically neutral particle (*neutrino* ν).

The typical example:

$$ {}^{13}_{7}N \to {}^{13}_{6}C + {}^{0}_{+1}e + \nu . $$

Electron capture (K-capture, L-capture, etc). Sometimes nuclei decay through electron capture which is a type of radioactive decay where the nucleus of an atom absorbs the inner electrons of K -shell $(n=1)$ or L-shell $(n=2)$ and converts a proton into a neutron. A neutrino is emitted from the nucleus. Another electron falls into the empty energy level and so on causing a cascade of electrons falling. One free electron, moving about in space, falls into the outermost empty level.

$$ {}^{A}_{Z}X + {}^{0}_{-1}e \to {}^{A}_{Z-1}Y + \nu $$

The typical example:

$$ {}^{40}_{19}K + {}^{0}_{-1}e \to {}^{40}_{18}Ar + \nu . $$

iii. γ-decay. In these decays neither Z nor A changes. This is the preferred decay mode of excited states of nuclei. $\gamma-$particles are photons, only much more energetic than those emitted in atomic or molecular decays. They have energy of 1 MeV. Many times a γ-decay follows a $\beta-$decay, which lands the nucleus in an excited state

$$ {}^{Z}_{A}X^{*} \to {}^{Z}_{A}X + \gamma , $$

where X^{*} denotes an excited nuclear state.

The typical example: $\quad {}^{87}_{38}Sr^{*} \to {}^{87}_{38}Sr + \gamma .$

The spectrum of $\gamma-$radiation is discrete that confirms the discreteness of nucleus energy levels.

There are many isotopes especially of heavy elements that are naturally radioactive. All isotopes of elements with $Z > 83$ (heavier then Pb and Bi) are radioactive. All of them follow chains of decays that end up in stable isotopes of either Pb or Bi. There three natural radioactive series, called the thorium, uranium-radium and actinium series (see Table).

Also included in this table is the neptunium series, the longest member of which has the half-life of $2.2\cdot10^{6}$ years. This is much less than the age of the Earth and so the series has long since decayed. However, Neptunium is produced artificially in nuclear reactors and can be important in some solutions.

Series name	Final stable element	Longest-lived member
Thorium	${}^{208}_{82}Pb$	${}^{232}_{90}Th$ $(T=1.39\cdot10^{10}$ years)
Uranium-radium	${}^{206}_{82}Pb$	${}^{238}_{92}U$ $(T=4.5\cdot10^{9}$ years)
Actinium	${}^{207}_{82}Pb$	${}^{235}_{92}U$ $(T=8.52\cdot10^{8}$ years)
Neptunium	${}^{209}_{83}Bi$	${}^{237}_{93}U$ $(T=2.2\cdot10^{8}$ years)

The term "series" is used because an atom undergoes a succession of radioactive transformations until it reaches a stable state. In Thorium series, the atom is initially ${}^{232}_{90}Th$ and undergoes a series of radioactive decays as follows:

$$
{}^{232}_{90}Th\to{}^{228}_{88}Ra\to{}^{228}_{89}Ac\to{}^{228}_{90}Th\to{}^{224}_{88}Ra\to
$$
$$
{}^{220}_{86}Ra\to{}^{216}_{84}Po\to{}^{212}_{82}Pb\to{}^{212}_{83}Bi\to{}^{212}_{84}Po\to{}^{208}_{82}Pb
$$

The half-life of the members of the decay series range from 0.15 s for ${}^{216}_{84}Po$ to about $1.4\cdot10^{10}$ years for ${}^{232}_{90}Th$.

The summary of the information about decays is in the following table.

Decay		Transformation	Example
$\alpha-$decay		${}^{A}_{Z}X\to{}^{A-4}_{Z-2}Y+{}^{4}_{2}He$	${}^{238}_{92}U\to{}^{234}_{90}Th+{}^{4}_{2}He$
$\beta-$decay	β^{-}-decay	${}^{A}_{Z}X\to{}^{A}_{Z+1}Y+{}^{0}_{-1}e+\tilde{\nu}$	${}^{234}_{90}Th\to{}^{234}_{91}Pa+{}^{0}_{-1}e+\tilde{\nu}$
	β^{+}-decay	${}^{A}_{Z}X\to{}^{A}_{Z-1}Y+{}^{0}_{+1}e+\nu$	${}^{13}_{7}N\to{}^{13}_{6}C+{}^{0}_{+1}e+\nu$
	K-capture	${}^{A}_{Z}X+{}^{0}_{-1}e\to{}^{A}_{Z-1}Y+\nu$	${}^{40}_{19}K+{}^{0}_{-1}e\to{}^{40}_{18}Ar+\nu$
$\gamma-$decay		${}^{Z}_{A}X^{*}\to{}^{Z}_{A}X+\gamma$	${}^{87}_{38}Sr^{*}\to{}^{87}_{38}Sr+\gamma$

The * denotes an excited nuclear state.

3.4. Radioactive decay law

Let us consider the radioactive decay statistically. Suppose that at a given moment we have N nuclei in a given state. Every nucleus in the excited state has the same chance to decay at any instant. This probability depends on the excited state itself; different excited states have different probabilities of decay. Suppose that at the initial moment we have N_0 nuclei in a given state. After the decay these nuclei transform into another nuclei, so the number of nuclei in this state will decrease. The change of the nuclei number in time dt is dN. Since every nucleus has the same chance to decay and the probability of decay is proportional to dt and the amount of nuclei N at the moment t we have

$$dN = -\lambda N dt,$$

where λ is the constant of proportionality, the *decay constant*, characterizing the decay process. Sign "minus" was taken to show that dN is the increment of the number of undecayed nuclei.

This equation is separable, giving

$$\frac{dN}{N} = -\lambda \cdot dt,$$

or, after integration

$$\ln N = \ln N_0 - t\lambda,$$

where $\ln N_0$ is an integration constant.

Exponentiation gives the ***exponential decay law***

$$N = N_0 e^{-\lambda t},$$

where N_0 is the original number of nuclei and N is the number of undecayed nuclei at the instant t.

The number of nuclei decayed to the instant t is

$$N_0 - N = N_0\left(1 - e^{-\lambda t}\right).$$

Half- life is the time it takes half of the number of nuclei to decay. The half-lives of radioactive nuclides vary from small fractions of a second to billions of years.

Using the radioactive decay equation, it's easy to show that the half-life and the decay constant are related by:

$$N_0/2 = N_0 e^{-\lambda T},$$

$$T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda},$$

Decay time (mean life-time) τ is the time interval in which the number of radioactive nuclei of a sample has diminished by a factor of e.

$$\frac{N_0}{N} = \frac{N_0}{N_0 e^{-\lambda\tau}} = e^{\lambda\tau} = e \quad \Rightarrow \quad \lambda\tau = 1 \quad \Rightarrow \quad \tau = \frac{1}{\lambda} = \frac{\ln 2}{T} = \frac{0.693}{T}.$$

The activity, $A(t)$ of a radioactive sample is defined as the number of decays per second. Thus,

$$A(t) = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = \lambda N.$$

The activity is proportional to the decay constant λ and to the number of radioactive nuclei that are present and, hence, decreases at the same exponential rate as $N(t)$.

SI unit of activity is the Becquerel (Bq): 1 Bq = 1 disintegration/second.

The Becquerel is an extremely small unit so normally it is employed as kilo- Mega- or GigaBequerel (kBq, MBq or GBq).

The special unit of activity for activity radiation is the Curie, abbreviated Ci.
1 Ci = $3.7 \cdot 10^{10}$ Bq.

SOLVED PROBLEMS

1. MATTER WAVES. DE BROGLIE WAVELENGTH

Problem 1.1

Find the de Broglie wavelengths for electrons that passed across the potential difference $U_1 = 1$ *V and* $U_2 = 100$ *kV.*

Solution

The de Broglie wavelength is determined by the expression

$$\lambda = \frac{h}{p} ,$$

where $h = 6.63 \cdot 10^{-34}$ J·s is the Plank's constant, and p is a linear momentum of the particle.

1. The kinetic energy of the particle is the result of the work of the accelerating electric field.

$$eU_1 = \frac{mv^2}{2} .$$

The speed of the electrons is

$$v = \sqrt{\frac{2eU_1}{m}} = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 1}{9.1 \cdot 10^{-31}}} = 5.93 \cdot 10^5 \text{ m/s.}$$

The linear momentum of the particle is equal to

$$p = mv = 9.1 \cdot 10^{-31} \cdot 5.93 \cdot 10^5 = 5.4 \cdot 10^{-25} \text{ kg·m·s}^{-1}.$$

The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63 \cdot 10^{-34}}{5.4 \cdot 10^{-25}} = 1.23 \cdot 10^{-9} \text{ m.}$$

The same result may be obtained using the relationship $p = \sqrt{2m \cdot KE} = \sqrt{2m \cdot eU_1}$ and

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2emU_1}} = \frac{6.63 \cdot 10^{-34}}{\sqrt{2 \cdot 1.6 \cdot 10^{-19} \cdot 9.1 \cdot 10^{31} \cdot 1}} = 1.23 \cdot 10^{-9} \text{ m.}$$

2. The said method used for the potential difference $U_2 = 100$ kV $= 10^5$ V gives the calculated speed

$$v = \sqrt{\frac{2eU_2}{m}} = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 10^5}{9.1 \cdot 10^{-31}}} = 1.8 \cdot 10^8 \text{ m/s.}$$

This speed is closed to the speed of light which is to say that the relativistic formulas have to be used.

$$eU_2 = m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right),$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{eU_2}{m_0 c^2} + 1.$$

After substitution of the numbers we obtain

$$\frac{1}{\sqrt{1-\beta^2}} = 1.2,$$

$$\beta = 0.55$$

The linear momentum makes

$$p = \frac{m_0 \beta c}{\sqrt{1-\beta^2}} = 9.1 \cdot 10^{-31} \cdot 0.55 \cdot 3 \cdot 10^8 \cdot 1.2 = 1.8 \cdot 10^{-22} \text{ kg·m·s}^{-1}.$$

The de Broglie wavelength for electrons is

$$\lambda = \frac{h}{p} = \frac{6.63 \cdot 10^{-34}}{1.8 \cdot 10^{-22}} = 3.68 \cdot 10^{-12} \text{ m.}$$

This problem may be solved by another method using the relationship between the linear momentum p, kinetic energy ($KE = eU_2$) and the rest energy ($\varepsilon_0 = m_0 c^2$) $p = \frac{\sqrt{KE\left(KE + 2\varepsilon_0\right)}}{c}$.

The wavelength is

$$\lambda = \frac{hc}{\sqrt{KE\left(KE + 2\varepsilon_0\right)}} = \frac{hc}{\sqrt{eU_2\left(eU_2 + 2m_0c^2\right)}} =$$

$$= \frac{6.63\cdot10^{-34}\cdot3\cdot10^{8}}{\sqrt{1.6\cdot10^{-19}\cdot10^{5}\left(1.6\cdot10^{-19}\cdot10^{5} + 2\cdot9.1\cdot10^{-31}\cdot\left(3\cdot10^{8}\right)^{2}\right)}} = 3.68\cdot10^{-12}\,\text{m}.$$

Problem 1.2

The alpha-particle is rotating in the magnetic field H =18.9 kA/m along the circular path of radius R =8.3 mm. Find the de Broglie wavelength of this particle.

Solution

When electric charges move through a magnetic field, there is the Lorentz force acting on the charges $\vec{F} = q\left[\vec{v},\vec{B}\right]$. The magnitude of the Lorenz force is equal to $F = qvB\sin\varphi$. If the speed of the particle is perpendicular the magnetic field $\sin\varphi = 1$. The equation of the motion of the particle is $ma_n = F$, then

$$m\cdot\frac{v^2}{R} = qvB\ .$$

The speed of the α-particle, taking into account that $q = 2e, m = 4m_p$, will be

$$v = \frac{qBR}{m} = \frac{q\mu_0HR}{m} = \frac{2\cdot1.6\cdot10^{-19}\cdot1.26\cdot10^{-6}\cdot18.9\cdot10^{3}\cdot8.3\cdot10^{-3}}{4\cdot1.67\cdot10^{-27}} = 9.5\cdot10^{3}\ \text{m/s}$$

The de Broglie wavelength for α-particle is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\cdot10^{-34}}{4\cdot1.67\cdot10^{-27}\cdot9.5\cdot10^{3}} = 1.05\cdot10^{-11}\ \text{m}.$$

Problem 1.3

Find the change in de Broglie wavelength of the electron in the hydrogen atom if it emits the photon of the head line of Paschen series.

Solution

The head line of the Paschen series corresponds to the electron transition from the 4th to the 3rd level. The speeds of electrons on these orbits according to the formula

$$v_n = \frac{e^2}{4\pi\varepsilon_0 \cdot \hbar} \cdot \frac{1}{n} = \frac{2.19 \cdot 10^6}{n}$$

are $v_3 = 7.29 \cdot 10^5$ m/s and $v_3 = 5.48 \cdot 10^5$ m/s.

The de Broglie wavelengths are

$$\lambda_3 = \frac{h}{mv_3} = \frac{6.62 \cdot 10^{-34}}{9.1 \cdot 10^{-31} \cdot 7.29 \cdot 10^5} = 9.98 \cdot 10^{-10} \text{ m},$$

$$\lambda_4 = \frac{h}{mv_4} = \frac{6.63 \cdot 10^{-34}}{9.1 \cdot 10^{-31} \cdot 5.48 \cdot 10^5} = 1.33 \cdot 10^{-9} \text{m}.$$

The change in de Broglie wavelength is

$$\Delta\lambda = \lambda_4 - \lambda_4 = 1.33 \cdot 10^{-9} - 9.98 \cdot 10^{-10} = 3.29 \cdot 10^{-10} \text{ m}.$$

Problem 1.4

The charged particle after the acceleration in the electric field U =200 V has the de Broglie wavelength of 2.02 pm. Find the mass of the particle if its charge is equal to the elementary charge e.

Solution

Relationship between the kinetic energy of the particle and the work of the electric field done for the acceleration of electrons is

$$eU = KE = \frac{mv^2}{2} = \frac{p^2}{2m}.$$

The linear momentum of the particle is equal to

$$p = \sqrt{2emU}.$$

The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2emU}}$$

and the mass of the charged particle is

$$m = \frac{h^2}{2eU\lambda^2} = \frac{\left(6.63 \cdot 10^{-34}\right)^2}{2 \cdot 1.6 \cdot 10^{-19} \cdot 200 \cdot \left(2.02 \cdot 10^{-12}\right)^2} = 1.68 \cdot 10^{-27} \text{ kg.}$$

2. BOHR'S ATOM

Problem 2.1

For a hydrogen atom, find the radii of the first three orbitals and the speeds of electrons on them.

Solution

The radius of n-th orbital is

$$r_n = 4\pi\varepsilon_0 \cdot \frac{\hbar^2}{m \cdot e^2} \cdot n^2 = r_1 \cdot n^2 = 5.29 \cdot 10^{-11} \cdot n^2.$$

Then the radii of the orbitals are

$n = 1$ $\qquad r_1 = 5.29 \cdot 10^{-11}$ m;

$n = 2$ $\qquad r_2 = 2.117 \cdot 10^{-10}$ m;

$n=3 \qquad r_3 = 4.76\cdot 10^{-10}$ m.

The speed of electron on the n-th orbital is given by

$$v_n = \frac{\hbar}{m\cdot r_n}\cdot n = \frac{e^2}{4\pi\varepsilon_0\cdot\hbar}\cdot\frac{1}{n} = \frac{v_1}{n} = \frac{2.19\cdot 10^6}{n} \text{ m/s.}$$

The speeds of electron are

$n=1 \qquad v_1 = 2.19\cdot 10^6$ m/s;

$n=2 \qquad v_2 = 1.09\cdot 10^6$ m/s;

$n=3 \qquad v_3 = 7.29\cdot 10^5$ m/s.

Problem 2.2

Find the kinetic, potential and total energy of electron on the first ($n=1$) and n-th orbitals.

Solution

The total energy of electron depends on the number of the orbital n

$$E_n = -\frac{1}{(4\pi\varepsilon_0)^2}\cdot\frac{me^4}{2\hbar^2}\cdot\frac{1}{n^2} = -\frac{E_1}{n^2},$$

$E_1 = 2.18\cdot 10^{-18}$ J = 13.6 eV.

The total energy of the electro may be expressed as

$$E_n = -\frac{13.6}{n^2} \text{ eV.}$$

Kinetic energy of electron on the n-th orbital is

$KE_n = -E_n$,

Therefore,

$$KE_n = \frac{13.6}{n^2}\ \text{eV},$$

$$KE_1 = 13.6\ \text{eV}.$$

The potential energy of electron on the n-th orbital is equal to

$$PE_n = 2E_n.$$

For the 1st orbital electron

$$PE_1 = 2E_1 = -27.2\ \text{eV}$$

Problem 2.3

Find the frequency and the period of the electron on the first ($n=1$) and n-th orbitals.

Solution

The period of electron is equal to the time of one revolution $T = \frac{2\pi r}{v}$. Taking into account that

$$r_n = 4\pi\varepsilon_0 \cdot \frac{\hbar^2}{m \cdot e^2} \cdot n^2 = 5.29 \cdot 10^{-11} \cdot n^2\ \text{ m}$$

and

$$v_n = \frac{e^2}{4\pi\varepsilon_0 \cdot \hbar} \cdot \frac{1}{n} = \frac{2.19 \cdot 10^6}{n}\ \text{m/s},$$

we obtain

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi \cdot 5.29 \cdot 10^{-11}}{2.19 \cdot 10^6} \cdot \frac{1}{n^3} = \frac{1.5 \cdot 10^{-16}}{n^3}\ \text{s}.$$

For the first orbit, the period of electron rotation is

$$T_1 = \frac{2\pi r_1}{v_1} = \frac{1.5 \cdot 10^{-16}}{n^3} = \frac{1.5 \cdot 10^{-16}}{1^3} = 1.5 \cdot 10^{-16}\,\text{s}.$$

The corresponding expressions for the frequency and the angular frequency are

$$\upsilon_n = \frac{1}{T_n} = \frac{v_n}{2\pi r_n} = \frac{n^3}{1.5\cdot 10^{-16}} \text{ Hz},$$

$$\upsilon_1 = \frac{1^3}{1.5\cdot 10^{-16}} = 6.67\cdot 10^{15}\,\text{Hz}.$$

$$\omega_n = 2\pi\upsilon_n = \frac{2\pi v_n}{2\pi r_n} = \frac{v_n}{r_n} = \frac{2\pi\cdot n^3}{1.5\cdot 10^{-16}} = 4.19\cdot 10^{16}\cdot n^3 \text{ rad/s}.$$

$$\omega_1 = 4.19\cdot 10^{16} \text{ rad/s}.$$

Problem 2.4

Find the longest and the shortest wavelengths of the photons emitted by hydrogen in the ultraviolet and visible range.

Solution

The Lyman series for the hydrogen atom corresponds to electron transitions that end up in the state with quantum number $n=1$(ground state). The longest wavelength photon is emitted at the transition from the $n=2$ level to the $n=1$ level. The shortest-wavelength photon corresponds to the transition of electron from $n=\infty$ to the $n=1$ state.

$$\frac{1}{\lambda} = R\cdot\left(\frac{1}{1^2} - \frac{1}{n^2}\right),$$

where $R=1.1\cdot 10^7$ m^{-1} is the Rydberg constant.

$$\frac{1}{\lambda_{\max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 8.25\cdot 10^6 \text{ m}^{-1},$$

$$\lambda_{\max} = 1.21\cdot 10^{-7}\,\text{m}.$$

$$\frac{1}{\lambda_{\min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = 1.1\cdot 10^7 \text{ m}^{-1},$$

$$\lambda_{\min} = 9.1\cdot 10^{-8} \text{ m}.$$

Both wavelengths are in the ultraviolet spectrum.

The Balmer series for the hydrogen atom corresponds to electron transitions that ends up in the state with quantum number $n = 2$.

$$\frac{1}{\lambda} = R \cdot \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

$$\frac{1}{\lambda_{\max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 1.53 \cdot 10^6 \text{ m}^{-1},$$

$$\lambda_{\max} = 6.55 \cdot 10^{-7}\,\text{m},$$

$$\frac{1}{\lambda_{\min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty} \right) = 2.75 \cdot 10^7 \text{ m}^{-1},$$

$$\lambda_{\min} = 3.64 \cdot 10^{-7}\,\text{m}.$$

This value is out of the visible range ($0.4 \cdot 10^{-6} ... 0.76 \cdot 10^{-6}$ m). Let's calculate wavelengths for the possible transitions onto the second level from the higher levels

$n = 4$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2} \right) = 2.0625 \cdot 10^6 \text{ m}^{-1},$$

$$\lambda = 4.85 \cdot 10^{-7}\,\text{m};$$

$n = 5$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{5^2} \right) = 2.31 \cdot 10^6 \text{ m}^{-1},$$

$$\lambda = 4.33 \cdot 10^{-7}\,\text{m};$$

$n = 6$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{6^2} \right) = 2.44 \cdot 10^6 \text{ m}^{-1},$$

$$\lambda = 4.09 \cdot 10^{-7} \text{ m},$$

$n = 7$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{7^2} \right) = 2.53 \cdot 10^6 \text{ m}^{-1},$$

$$\lambda = 3.96 \cdot 10^{-7}\,\text{m}.$$

This line is in the ultraviolet range. Therefore, only four spectral lines are in the visible range of the Hydrogen spectrum. They correspond to the transitions of electrons on the second level from the 3rd, 4th, 5th, and 6th levels. In the visible range, the longest-wavelength photon has $\lambda_{max} = 6.55 \cdot 10^{-7}$ m and the shortest-wavelength photon has $\lambda = 4.09 \cdot 10^{-7}$ m.

In ultraviolet range, the longest wavelength is $\lambda_{max} = 1.21 \cdot 10^{-7}$ m and the shortest wavelength is $\lambda_{min} = 9.1 \cdot 10^{-8}$ m. Both lines are contained in Lyman series.

Problem 2.5

Electron in the hydrogen atom transited from the 3rd to the 2nd level. Find the wavelength, linear momentum and energy of the emitted photon. Determine the velocity of the recoiled atom.

Solution

The transition from the 3rd to the 2nd level is related to the Balmer series. The photon of visual range is emitted at this transition. Its wavelength may be found using the Rydberg relation

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.53 \cdot 10^6 \text{ m}^{-1},$$

$$\lambda_{max} = 6.55 \cdot 10^{-7} \text{ m}.$$

The linear momentum and the energy of the photon are

$$p = \frac{h}{\lambda} = \frac{6.62 \cdot 10^{-34}}{6.55 \cdot 10^{-7}} = 1.01 \cdot 10^{-27} \text{ kg·m·s}^{-1},$$

$$\varepsilon = \frac{hc}{\lambda} = pc = 1.01 \cdot 10^{-27} \cdot 3 \cdot 10^{8} = 3.03 \cdot 10^{-19} \text{ J}.$$

The atom was initially at rest and its linear momentum was zero. According to the law of conservation of linear momentum

$$0 = m_H v - p,$$

$$v = \frac{m_H}{p} = \frac{m_p + m_e}{p} = \frac{1.66 \cdot 10^{-27} + 9.1 \cdot 10^{-31}}{1.01 \cdot 10^{-27}} = 1.65 \text{ m/s.}$$

Problem 2.6

Find the ionization potential and energy, and the first excitation potential and energy for the hydrogen atom.

Solution

Electrons are excited to higher energy levels when they absorb a photon's energy. This process is called excitation and the atom is said to be in the excited state. The energy absorbed to move from one orbit to the other is called excitation energy. If the energy supplied is large enough to remove an electron from the atom, then the atom is said to be ionized. The minimum energy needed to ionize an atom is called ionization energy. Here the removed electron will have zero energy. The potential difference which accelerated the electron that can excite or ionize atom is excitation or ionization potential, respectively.

Actually, ionization is the transition of electron from the ground state $n = 1$ to $n = \infty$. Therefore, the ionization energy and ionization potential are, respectively,

$$\varepsilon_i = \frac{hc}{\lambda} = hcR\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = 6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8 \cdot 1.1 \cdot 10^7 = 2.18 \cdot 10^{-18} \text{ J} = 13.6 \text{ eV},$$

$$eU_i = \varepsilon_i, \qquad U_i = \varepsilon_i / e = 13.6 \text{ V.}$$

The first excitation energy and the first excitation potential which are related to the transition from the $n = 1$ level to the $n = 2$ level are given by

$$\varepsilon_1 = \frac{hc}{\lambda} = hcR\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3\left(6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8 \cdot 1.1 \cdot 10^7\right)}{4} = 1.63 \cdot 10^{-18} \text{ J} = 10.2 \text{ eV},$$

$$eU_1 = \varepsilon_1, \qquad U_1 = \varepsilon_1 / e = 10.2 \text{ V.}$$

Problem 2.7

Find the minimum energy of electrons that necessary for excitation of the hydrogen atom which gives all lines of all spectrum series. Find the speed of these electrons.

Solution

All spectral lines of all hydrogen series appear when atom obtains energy equaled the ionization energy

$$\varepsilon = \frac{hc}{\lambda} = hcR\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = hcR = 2.19 \cdot 10^{-18} \text{ J} = 13.6 \text{ eV}.$$

This energy is imparted by electron with kinetic energy $KE = \frac{mv^2}{2}$. The electron obtained this energy in the electric field that did the work eU .

Therefore, $\varepsilon = eU = \frac{mv^2}{2}$, and the speed of electron has to be not less than

$$v = \sqrt{\frac{2\varepsilon}{m}} = \sqrt{\frac{2 \cdot 2.19 \cdot 10^{-18}}{9.1 \cdot 10^{-31}}} = 2.2 \cdot 10^6 \text{ m/s}.$$

Problem 2.8

Find the range for the wavelengths of the visual light photons that excite three spectral lines of the hydrogen atom.

Solution

Three spectral lines are observed a when the electrons after the transition from the first to the third energy level realize three types of transitions: from 3rd to the

2nd level, from the 3rd to the 1st level, and from the 2nd to the 1st level. As a result, one line of the Balmer series (λ_{32}) and two lines of the Lyman series ($\lambda_{31}, \lambda_{21}$) emerge.

The wavelengths of these emerged lines according to the Rydberg formula $\frac{1}{\lambda_{ik}} = R\left(\frac{1}{n_i^2} - \frac{1}{n_k^2}\right)$ are

$$\frac{1}{\lambda_{32}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.1\cdot 10^7 \cdot \frac{5}{36} = 1.53\cdot 10^6 \text{ m}^{-1}, \quad \lambda_{32} = 6.55\cdot 10^{-7} \text{ m};$$

$$\frac{1}{\lambda_{31}} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.1\cdot 10^7 \cdot \frac{8}{9} = 9.78\cdot 10^6 \text{ m}^{-1}, \quad \lambda_{31} = 1.023\cdot 10^{-7}$$

$$\frac{1}{\lambda_{21}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.1\cdot 10^7 \cdot \frac{3}{4} = 8.25\cdot 10^6 \text{m}^{-1}, \quad \lambda_{21} = 1.21\cdot 10^{-7} \text{ m}.$$

Therefore, the range for photon wavelengths is $102.3 \le \lambda \le 121$ nm.

Problem 2.9

The hydrogen atom in its ground state absorbs the photon of the wavelength $\lambda = 121.5$ *nm. Determine the electron orbit radius of the excited atom and the angular momentum of the electron on it.*

Solution

Let's find the number of the energy level where the electron gets after the absorption of the photon by the Rydberg formula $\frac{1}{\lambda_{ik}} = R\left(\frac{1}{n_i^2} - \frac{1}{n_k^2}\right)$, taking into account that the $n_i = 1$ for the ground state

$$n_k = n_i\sqrt{\frac{\lambda R}{\lambda R - n_i^2}} = 1\cdot\sqrt{\frac{121.5\cdot 10^{-9}\cdot 1.1\cdot 10^7}{121.5\cdot 10^{-9}\cdot 1.1\cdot 10^7 - 1}} = 2.$$

This means that as a result of the photon absorption the electron made the upward transition from the 1st to the 2nd level.

Since the radius of the n-th orbit is given by

$$r_n = 4\pi\varepsilon_0 \frac{\hbar^2}{me^2} n^2 = r_1 \cdot n^2 = 0.53 \cdot 10^{-10} \cdot n^2 \ ,$$

the radius of the 2nd orbit is

$$r_2 = 0.53 \cdot 10^{-10} \cdot 2^2 = 2.12 \cdot 10^{-10} \text{ m.}$$

The angular momentum of the electron may be calculated in two ways.

1. The speed of electron according to the expression $v_n = \frac{v_1}{n} = \frac{2.19 \cdot 10^6}{n}$ (m/s) is $v_2 = \frac{2.19 \cdot 10^6}{2} = 1.09 \cdot 10^6$ m/s. Then, from the knowledge of mass of the electron $m_0 = 9.1 \cdot 10^{-31}$ kg and the radius of the electron orbit $r_2 = 2.12 \cdot 10^{-10}$ m, the angular moment is

$$L_2 = mv_2r_2 = 9.1 \cdot 10^{-31} \cdot 1.09 \cdot 10^6 \cdot 2.12 \cdot 10^{-10} = 2.1 \cdot 10^{-34} \text{ kg·m}^2\text{·s}^{-1}.$$

2. The angular momentum according the Bohr postulate $L_n = mv_nr_n = n\hbar$ is equal to $L_2 = 2\hbar = 2 \cdot \frac{h}{2\pi} = \frac{6.63 \cdot 10^{-34}}{3.14} = 2.1 \cdot 10^{-34}$ kg·m^2·s^{-1}.

Problem 2.10

Photon with energy 15 eV dislodges the electron that was in the ground state in the hydrogen atom. Find the speed of the electron at its motion from the atom.

Solution

The photon energy is spent to the ionization of the atom and the kinetic energy of the electron $E = \varepsilon_i + KE$.

Ionization is the process in which one electron is removed from an atom, i.e. the transition of the electron from the ground level ($n_i = 1$) to the level $n_k = \infty$. The ionization energy is

$$\varepsilon_i = \frac{hc}{\lambda} = hcR\left(\frac{1}{n_i^2} - \frac{1}{n_k^2}\right) = 13.6\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = 13.6 \text{ eV}.$$

The kinetic energy of the electron is equal to

$$KE = E - \varepsilon_i = 15 - 13.6 = 1.4 \text{ eV} = 2.24 \cdot 10^{-19} \text{ J}.$$

From $KE = \frac{mv^2}{2}$, the speed of electron is

$$v = \sqrt{\frac{2 \cdot KE}{m_0}} = \sqrt{\frac{2 \cdot 2.24 \cdot 10^{-19}}{9.1 \cdot 10^{-31}}} = 7 \cdot 10^5 \text{ m/s}.$$

Problem 2.11

Find the change of the kinetic energy of the electron in hydrogen atom after emitting the photon with the wavelength λ =102.3 nm.

Solution

The given value of the wavelength is related to the ultraviolet range, therefore, the photon of the Lyman series ($n_i = 1$). The Rydberg formula implies that

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right),$$

$$\frac{1}{n^2} = 1 - \frac{1}{\lambda R}.$$

The number of the level of the electron upward transition is

$$n = \sqrt{\frac{\lambda R}{\lambda R - 1}} = \sqrt{\frac{102.3 \cdot 10^{-9} \cdot 1.1 \cdot 10^7}{102.3 \cdot 10^{-9} \cdot 1.1 \cdot 10^7 - 1}} = 3.$$

The total energy of the electron on the 3rd orbit is equal to

$$E_3 = -\frac{13.6}{n^2} = -\frac{13.6}{3^2} = -1.51 \text{ eV}.$$

The kinetic energy of this electron is $KE_3 = -E_3 = 1.51$ eV.

For the ground state,

$$KE_1 = -E_1 = 13.6 \text{ eV}.$$

As a result, the change of kinetic energy is

$$\Delta KE = KE_1 - KE_3 = 13.6 - 1.51 = 12.09 \text{ eV}.$$

Problem 2.12

The light beam from the discharge tube filled by hydrogen is normally incident on the diffraction grating with grating constant d =5 μm. Find the type of electron transition for the spectral line that is observed in the 5-th order spectrum at the angle ψ =41°.

Solution

From the diffraction maximum condition for the diffraction grating $d \sin\psi = k\lambda$ the wavelength is

$$\lambda = \frac{d \sin\psi}{k} = \frac{5 \cdot 10^{-6} \cdot \sin 41^0}{5} = 0.656 \cdot 10^{-6} \text{ m}.$$

This line is related to the visual range, hence, to the Balmer series. Therefore, $n_i = 2$ in the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_k^2}\right),$$

$$\frac{1}{n_k} = \sqrt{\frac{1}{4} - \frac{1}{\lambda R}} = \sqrt{\frac{1}{4} - \frac{1}{0.656 \cdot 10^{-6} \cdot 1.1 \cdot 10^7}} = 0.333,$$

$$n_k = 3.$$

And it is clear that the electron transited from the 3rd to the 2nd level.

Problem 2.13

Determine the ionization potential and ionization energy for the doubly-ionized Lithium Li^{++}.

Solution

The Rydberg formula for the hydrogen like ions is given by

$$\frac{1}{\lambda} = Z^2 R \cdot \left(\frac{1}{n_i^2} - \frac{1}{n_k^2} \right),$$

where Z is the number of the element in the Periodic Table ($Z = 3$ for Lithium)

Ionization energy is

$$\varepsilon_i = \frac{hc}{\lambda} = Z^2 hcR \left(\frac{1}{1^2} - \frac{1}{\infty} \right) = 9 \cdot 13.6 \text{ eV} = 122.4 \text{ eV} = 2.4 \cdot 10^{-15} \text{ J}.$$

The electrons obtained the kinetic energy due to the work of electric field with the potential difference $\Delta\varphi_i$:

$$e \cdot \Delta\varphi_i = KE = \varepsilon_i,$$

$$\Delta\varphi_i = \frac{\varepsilon_i}{e} = \frac{2.4 \cdot 10^{-15}}{1.6 \cdot 10^{-19}} = 122.4 \text{ V}.$$

Problem 2.14

Is it possible for the photon related to the transition between the first and the second levels in the doubly-ionized Lithium to dislodge the electron from the ground state of the singly-ionized Helium?

Solution

For the ionization of the Helium ion it is necessary that the energy of the photon emitted by Li^{++} has to be more than the ionization energy for Helium.

The energy of the photon emitted by Lithium at the electron transition from the second to the first level is

$$\varepsilon = \frac{hc}{\lambda} = Z_1^2 hcR\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4} Z_1^2 hcR = \frac{27}{4} hcR = 6.75 \cdot 13.6 \text{ eV} = 91.8 \text{ eV}$$

The ionization energy of Helium ia

$$\varepsilon_i = \frac{hc}{\lambda_2} = Z_2^2 hcR\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = Z_2^2 hcR = 4hcR = 4 \cdot 13.6 \text{ EV} = 54.4 \text{ eV}.$$

Since $\varepsilon > \varepsilon_i >$ the photon emitted by Lithium can ionize Helium.

3. NUCLEAR PHYSICS

Problem 3. 1

Initially a certain radioactive sample contains 10^{10} *of the sodium isotope* ${}^{24}_{11}$ *Na, which has a half-life of 15 h. Find the number of parent nuclei present 90 h later. Find the initial activity of the sample and the activity 90 h later (in Bq).*

Solution

Taking into account that the half-life T and delay (or disintegration) constant λ are related by $T = \frac{\ln 2}{\lambda}$, the number of undecayed nuclei remaining at time t is given by

$$N = N_0 e^{-\lambda t} = N_0 e^{-\frac{\ln 2}{T}t} = 10^{10} \cdot e^{-\frac{\ln 2}{15} 90} = 1.56 \cdot 10^8 .$$

The activity (or rate of decay) is equal to $A = \lambda N$. To get the value of the activity in Becquerel we have to express the decay constant in seconds^{-1}.

$$\lambda = \frac{\ln 2}{T} = \frac{0.693}{15 \cdot 3600} = 4.6 \cdot 10^{-5} \text{ s}^{-1}.$$

Then the activities of the initial sample and after 90 hours are, respectively,

$$A_0 = \lambda N_0 = 4.6 \cdot 10^{-5} \cdot 10^{10} = 4.6 \cdot 10^5 \text{ Bq},$$

$$A = \lambda N = 4.6 \cdot 10^{-5} \cdot 1.56 \cdot 10^8 = 7.18 \cdot 10^3 \text{ Bq}.$$

Problem 3.2

How many Radon nuclei are decayed for the time period of 1 day if the initial amount of nuclei was $N_0 = 10^6$ *and half-life* $T = 3.82$ *days?*

Solution

According to the radioactive decay law the amount of remained (undecayed) nuclei is $N = N_0 e^{-\lambda t}$, where N_0 is the initial amount of nuclei. From this consideration, the amount of decayed nuclei is

$$N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 \left(1 - e^{-\lambda t}\right).$$

Taking into account that the delay constant λ related to the half-life as $\lambda = \dfrac{\ln 2}{T}$, the amount of undecayed nuclei is

$$N_0 - N = N_0 \left(1 - e^{-\lambda t}\right) = 10^6 \cdot \left(1 - e^{-\frac{\ln 2 \cdot 1}{3.82}}\right) = 1.66 \cdot 10^5 .$$

Problem 3.3

The charred bones of a sloth that was found in a cave in Chile represent the earliest evidence of human presence in the southern tip of South America. A sample of the bone has a specific activity of 87 mBq per gram of carbon. If the ${}^{12}_{6}C / {}^{14}_{6}C$ *ratio for living organisms results in a specific activity of 255 mBq per gram, how old are the bones (*$T = 5730$ *years)?*

Solution

Radioisotopic (radiometric) dating developed by Willard F. Libby (Nobel 1960) is the technique that is based on measuring the amount of ${}^{14}_{6}C$ and ${}^{12}_{6}C$ in materials of biological interest. The ratio ${}^{12}_{6}C / {}^{14}_{6}C$ is constant when organism lives as it takes up ${}^{14}_{6}C$ and it stops when it dies. So in dead organisms the ratio ${}^{12}_{6}C/{}^{14}_{6}C$ increases because of the decay of ${}^{14}_{6}C$. The difference in the ${}^{12}_{6}C/{}^{14}_{6}C$ ratio between the living and dead organisms reflects the time elapsed since the organism died.

Activity depends on time as $A = A_0 e^{-\lambda t}$, where $\lambda = \frac{\ln 2}{T}$ is decay constant and A_0 is activity for the living organism. Since $\lambda t = \ln\frac{A_0}{A}$, the age of the sample is $t = \frac{1}{\lambda}\cdot\ln\frac{A_0}{A} = T\cdot\frac{\ln(A_0/A)}{\ln 2}$.

Taking into account that for ${}^{14}_{6}C$ activity $A = 87$ mBq per gram of carbon, $A_0 = 255$ mBq per gram and $T = 5730$ years, the age of the bones from a cave is

$$t = 5730\cdot\frac{\ln(255/87)}{0.693} = 8891 \text{ years.}$$

Problem 3.4

Tellurium ${}^{128}_{52}Te$, the most stable of all radioactive nuclides, has a half-life of about $1.5\cdot 10^{24}$ years. How long would it take for 75 percent of a sample of this isotope to decay?

Solution

After 75% of the original sample decays, 25%, or one-fourth, of the parent nuclei remain. The rest of the nuclei have decayed into daughter nuclei. Therefore, the remained amount of nuclei is $N = 0.25N_0 = \frac{N_0}{4}$. Using the delay law $N = N_0 e^{-\frac{\ln 2}{T}\cdot t}$, we get

$$\frac{N}{N_0} = e^{-\frac{\ln 2}{T}\cdot t}, \qquad \frac{N_0}{N} = e^{\frac{\ln 2}{T}\cdot t},$$

$$\ln \frac{N_0}{N} = \frac{\ln 2}{T} \cdot t,$$

$$t = \frac{\ln\left(N_0/N\right)}{\ln 2} \cdot T = \frac{\ln\left(4N_0/N_0\right)}{\ln 2} \cdot T = \frac{\ln 4}{\ln 2} \cdot T = 2T = 2 \cdot 1.5 \cdot 10^{24} = 3 \cdot 10^{24} \text{ years.}$$

Problem 3.5

Find the decay constant of Radon $_{86}Rn$ if it is known that the amount of its nuclei is decreased by 18% every day.

Solution

The fraction of the decayed nuclei respectively the initial amount of nuclei is $\frac{N_0 - N}{N_0} = 0.182$. Then according to the decay law

$$\frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - \frac{N_0 e^{-\lambda t}}{N_0} = 1 - e^{-\lambda t}.$$

Therefore,

$$1 - e^{-\lambda t} = 0.182, \qquad e^{-\lambda t} = 0.818.$$

Solving for λ we get

$$-\lambda t = \ln 0.818,$$

$$\lambda = -\frac{\ln 0.818}{24 \cdot 3600} = 2.33 \cdot 10^{-6} \text{ s}^{-1}.$$

Problem 3.6

The activity of radioactive sample is $A_0 = 14.8 \cdot 10^9$ Bq. How long will it be before the activity will decrees to the value of $A = 2.22 \cdot 10^9$ Bq if the half-life is $T = 3.82$ days.

Solution

The ratio of activities from the data is $\frac{A_0}{A} = \frac{14.8 \cdot 10^9}{2.22 \cdot 10^9} = 6.67$. Since the activity of the radioactive sample is $A = \lambda N$,

$$\frac{A_0}{A} = \frac{\cancel{\lambda} N_0}{\cancel{\lambda} N} = \frac{\cancel{N_0}}{\cancel{N_0} e^{-\lambda t}} = e^{\lambda t}.$$

Hence, from the above, $e^{\lambda t} = 6.67$, and desired time period

$$t = \frac{\ln 6.67}{\lambda} = \frac{\ln 6.67}{\ln 2} \cdot T = \frac{\ln 6.67 \cdot 3.82}{\ln 2} = 10.45 \text{ days.}$$

Problem 3.7

What nuclide results when a $^{232}_{90}Th$ undergoes a succession of radioactive decays consisting of four α-decays and two β^--decays?

Solution

The equations of the corresponding processes are

$$^{232}_{90}Th \to 4\,^4_2He + {}^{216}_{82}Pb,$$

$$^{216}_{82}Pb \to 2\,^{0}_{-1}e + {}^{216}_{84}Po.$$

This radioactive chain may be written in one line

$$^{232}_{90}Th \to 4\,^4_2He + 2\,^{0}_{-1}e + {}^{216}_{84}Pb.$$

Speed of light in vacuum	c = 2,998·10^8 m/s = $\approx 3 \cdot 10^8$ m/s
Plank’s constant	$h = 2\pi\hbar = 6.62 \cdot 10^{-34}\,\mathrm{J \cdot s}$ $\hbar = h/2\pi = 1.055 \cdot 10^{-34}\,\mathrm{J \cdot s}$
Rydberg constant (for wavelength)	$R = 1.1 \cdot 10^{7}$ m^{-1}
Rydberg constant (for frequency)	$R' = 2.07 \cdot 10^{16}$ s^{-1}
Compton wavelength for the electron	$\lambda_c = h/m_0 c = 2.43 \cdot 10^{-12}$ m =2.43 pm
Electron rest mass	$m_{0e} = 9.1 \cdot 10^{-31}$ kg
Proton rest mass	$m_{0p} = 1.67 \cdot 10^{-27}$ kg
α-particle rest mass	$m_{0\alpha} = 6.64 \cdot 10^{-27}$ kg
Elementary charge (proton/electron)	$e = \pm 1.6 \cdot 10^{-19}$ C
α-particle charge	$q_\alpha = 2e = 3.2 \cdot 10^{-19}$ C
Electric constant (vacuum permittivity)	ε_0 = 8.85·10^{-12} F/m $1/4\pi\varepsilon_0$ = 9·10^9 m/ F
1 electron-Volt	1eV = $1.6 \cdot 10^{-19}$ J
Rest energy of electron	$\varepsilon_{0e} = m_{0e}c^2 = 8.187 \cdot 10^{-14}$ J = = $5.12 \cdot 10^5$ eV = 0.512 MeV
Rest energy of proton	$\varepsilon_{0p} = m_{0p}c^2 = 1.49 \cdot 10^{-10}$ J = =9.315·10^8 eV = 0.93 GeV
Rest energy of α-particle	$\varepsilon_{0\alpha} = m_{0\alpha}c^2 = 5.97 \cdot 10^{-10}$ J = $= 3.72 \cdot 10^9$ eV = 3.72 GeV
	$m_{0e}c = 2.73 \cdot 10^{-22}$ kg·m/s $(m_{0e}c)^2 = 7.46 \cdot 10^{-44}$ kg^2·m^2/s^2
	$hc = 1.986 \cdot 10^{-25}$ J·m
	$(hc)^2 = 3.95 \cdot 10^{-50}$ (J·m)^2
	$hcR = 2.18 \cdot 10^{-18}$ J = 13.6 eV