NATIONAL TECHNICAL UNIVERSITY
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DEPARTMENT OF PHYSICS

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PROBLEM SOLVING GUIDE

"KINEMATICS"

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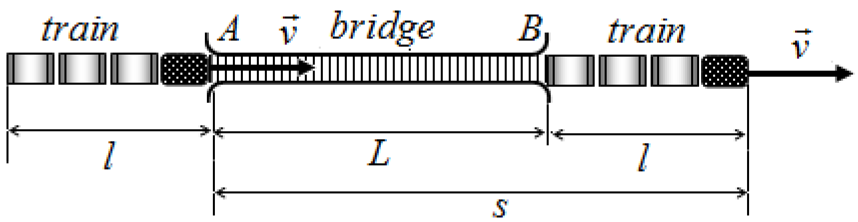
Problem 1

A train 100 m long is moving with a speed of 60 km/hr. In what time shall it cross a bridge 1 km long?

Solution

The train's motion over the bridge begins when the nose of the train pulls into the bridge (point A), and finishes when rear end of the train pull off the bridge (point B), therefore, the total distance to be covered is

$s = L + l = 1000 + 100 = 1100$ m.



If the speed of the train is $v = 60$ km/hr $= \frac{60000}{3600} = 16.67$ m/s, the sought time is

$t = \frac{s}{v} = \frac{1100}{16.67} = 66$ s.

Problem 2

Find the average speed of the bicyclist if he is travelling along a straight road (a) for the first half time with speed v_1 and for the second half time with speed v_2; (b) for the first half distance with speed v_1 and for the second part distance with speed v_2.

Solution

The average speed over any interval of time is defined as the ratio of the distance travelled during that interval divided by the time interval.

(a) Let t be the total time taken.

Distance covered in the first half time is

$$s_1 = v_1 \cdot \left(\frac{t}{2}\right) = \frac{v_1 \cdot t}{2}.$$

Distance covered in the second half is

$$s_2 = v_2 \cdot \left(\frac{t}{2}\right) = \frac{v_2 \cdot t}{2}.$$

The average speed is

$$v_{av} = \frac{s_1 + s_2}{t} = \frac{\frac{v_1 \cdot t}{2} + \frac{v_2 \cdot t}{2}}{t} = \frac{t(v_1 + v_2)}{2 \cdot t} = v_1 + v_2.$$

(b) Let s be the total distance travelled.

Time taken for the first half distance is

$$t_1 = \frac{(s/2)}{v_1} = \frac{s}{2 \cdot v_1}$$

Time taken for the second half distance is

$$t_2 = \frac{(s/2)}{v_2} = \frac{s}{2 \cdot v_2}.$$

The total time taken is

$$t = t_1 + t_2 = \frac{s}{2 \cdot v_1} + \frac{s}{2 \cdot v_2} = \frac{s(v_1 + v_2)}{2 \cdot v_1 \cdot v_2}.$$

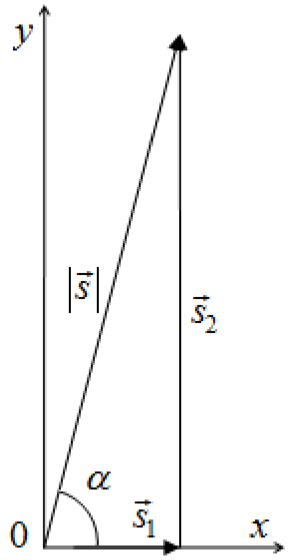
The average speed is

$$v_{av} = \frac{2 \cdot v_1 \cdot v_2 \cdot s}{s \cdot (v_1 + v_2)} = \frac{2 \cdot v_1 \cdot v_2}{v_1 + v_2}.$$

Problem 3

The particle moves for t_1 *=5 s with the velocity* v_1 *=5 m/s due east and for* t_2 *= 36 sec with the velocity* v_2 *=4 m/s due north. Find the distance covered for the time of motion; the displacement of the particle; the average speed; and the average velocity. If the particle moving at the velocity* v_3 *=3 m/s covered the distance to the initial point along the same path, find the average speed and velocity.*

Solution



The distances covered for the first and second periods of motion, respectively, are

$s_1 = v_1 \cdot t_1 = 5 \cdot 5 = 25$ m,

$s_2 = v_2 \cdot t_2 = 4 \cdot 36 = 144$ m.

The total distance is

$s = s_1 + s_2 = 25 + 144 = 169$ m.

The average speed which is the distance covered divided by the time taken for this motion

$$v_{av} = \frac{s}{t} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{25 + 144}{5 + 36} = \frac{169}{41} = 4.1 \text{m/s}.$$

Displacement $\vec{s}$ is the vector connecting the initial and the final points of the motion, therefore, it is $\vec{s} = \vec{s}_1 + \vec{s}_2$.

The magnitude of the displacement may be found using Pythagorean Theorem

$$|\vec{s}| = \sqrt{s_1^2 + s_2^2} = \sqrt{25^2 + 144^2} = \sqrt{21361} = 146.2 \text{ m}.$$

The magnitude of the average velocity is the ratio of displacement and corresponding time interval, therefore,

$$v_{av} = \frac{|\vec{s}|}{t} = \frac{146.2}{5 + 36} = 3.57 \text{ m/s}.$$

The time for return trip is $t_3 = \frac{s_3}{v_3} = \frac{s}{v_3} = \frac{169}{3} = 56.3\text{s}$.

The average speed is

$$v_{av1} = \frac{2 \cdot s}{t_1 + t_2 + t_3} = \frac{2 \cdot 169}{5 + 36 + 56.3} = 3.47 \text{ m/s}.$$

The average velocity is zero because the displacement in this case is zero (the initial point is the final points of the motion).

Problem 4

A boat speed in still water is v_0 *=2 m/s. The river is 100 m wide, and the speed of river current is* u *=1 m/s.*

(a) If the boat starts at the south shore and is to reach north shore just opposite the starting point (as shown in Figure), at what angle must the boat head? Find the time for crossing the river.

(b) If the boat heads directly across the river, find its velocity relative to the shore. How long it take to cross the river and how far downstream will the boat be then?

Solution

(a) If the boat heads straight across the river, the current will drag it downstream (westward). To overcome the river's westward current the boat must acquire an eastward component of velocity as well as northward component. Thus the boat must head in a northeasterly direction. The resultant velocity of the boat

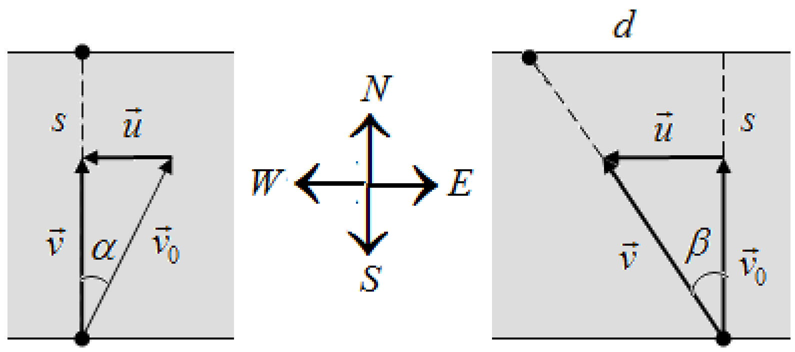
$$\vec{v} = \vec{v}_0 + \vec{u} .$$

The magnitude of this velocity is

$$v = \sqrt{v_0^2 - u^2} = \sqrt{2^2 - 1^2} = \sqrt{3} = 1.73\text{m/s}.$$

The vector $\vec{v}$ points upstream at an angle α as shown in Figure. From the diagram,

$$\sin\alpha = \frac{u}{v_0} = \frac{1}{2},$$



Thus, $\alpha = 30^0$, so the boat must head upstream at a 30^0 angle.

Assuming the uniform motion of the boat, we can find the time for river crossing

$$t = \frac{s}{v} = \frac{100}{1.73} = 58.8\,\text{s}.$$

(b) If the boat heads directly across the river, the current pulls it downstream. The boat's velocity with respect to the shore $\vec{v}$ is the sum of its velocity with respect to water, $\vec{v}_0$, plus the velocity of the water with respect to the shore, $\vec{u}$:

$$\vec{v} = \vec{v}_0 + \vec{u}\ .$$

Now, using the Theorem of Pythagoras, we can find the magnitude of v:

$$v = \sqrt{v_0^2 + u^2} = \sqrt{2^2 + 1^2} = \sqrt{5} = 2.24\,\text{m/s}.$$

The angle we can find from

$$\tan\beta = \frac{u}{v_0} = \frac{1}{2}.$$

Thus, $\beta = 65.9^0$. Note that this angle is not equal to the angle α calculated in the first part of the problem.

The time for getting the opposite shore is

$$t = \frac{s}{v} = \frac{100}{2} = 50\,\text{s}.$$

The boat will have been carried downstream, in this time, a distance

$$d = u \cdot t = 1 \cdot 50 = 50\,\text{m}.$$

Problem 5

A car starts from the rest and accelerates uniformly for t=5 seconds over a distance of 100m. Find the acceleration of the car and the speed at this instant of time.

Solution

This is the motion with constant acceleration. The equations which describe this motion are the following:

$$\begin{cases} x = \dfrac{at^2}{2} \\ v = at \end{cases}$$

We know the travelled time and we know the travelled distance, and then from the first equation we can find acceleration:

$$a = \frac{2x}{t^2} = \frac{2 \cdot 100}{25} = 8\,\text{m/s}^2.$$

The speed calculated using the second equation:

$$v = at = 8 \cdot 5 = 40\ \text{m/s}.$$

Problem 6

A car going at 10 m/s undergoes an acceleration of 5 m/s^2 for 6 seconds. How far did it go when it was accelerating?

Solution

In this problem we just need to use the equation:

$$s = v_0 t + \frac{at^2}{2},$$

where $v_0 = 10$ m/s, a = 5 m/s^2, and t = 6 s.

Then we have: $s = 10 \cdot 6 + \frac{5 \cdot 6^2}{2} = 150$ m.

Problem 7

A car accelerates from rest to the speed v_1 =36 m/s for 4 s, and then decelerates to v_2 = 32 m/s for next 6 s. Find the distance traveled.

Solution

If the car begins to move from rest at the uniform acceleration a_1 its velocity after the time t_1 is

$$v_1 = v_0 + a_1 t_1 = a_1 t_1.$$

$$a_1 = \frac{v_1}{t_1} = \frac{36}{4} = 9 \text{ m/s}^2.$$

The distance traveled for this time is

$$s_1 = \frac{a_1 t_1^2}{2} = \frac{9 \cdot 4^2}{2} = 72\,\text{m}.$$

Acceleration during decelerating motion

$$a_2 = \frac{v_2 - v_1}{t_2} = \frac{32 - 36}{6} = -\,0.67\ \text{m/s}^2.$$

The traveled distance is

$$s_2 = v_1 t_2 - \frac{a_2 t_2^2}{2} = 36 \cdot 6 - \frac{0.67 \cdot 6^2}{2} = 203.9\,\text{m}.$$

The total distance $s = s_1 + s_2 = 72 + 203.9 = 275.9\ \text{m}$

Problem 8

If the position of a particle is given by $x = 20t - 5t^3$, *where* x *is in meters and* t *is in seconds, when if ever is the particle's velocity zero? When is its acceleration a zero? Determine if the motion is accelerated or decelerated.*

Solution

To determine velocity, we have to differentiate x with respect to time

$$v = \frac{dx}{dt} = \frac{d\left(20t - 5t^3\right)}{dt} = 20 - 15 \cdot t^2.$$

If $v = 0$, then $20 - 15 \cdot t^2 = 0$, $t^2 = 1.33$, $t = 1.15\,\text{s}$.

Acceleration is the second-order derivative of position and second derivative of velocity

$$a = \frac{dv}{dt} = \frac{d\left(20 - 15t^2\right)}{dt} = -30 \cdot t.$$

From this, we see that acceleration can be zero only at $t = 0$; we can also see that the acceleration is negative whenever t is positive.

Problem 9

The reaction time for an automobile driver is 0.7 second. If the automobile can be decelerated at 5 m/s^2, calculate the total distance travelled in coming to stop from an initial velocity of 30 km/h after a signal is observed.

Solution

Since the reaction time of the driver is $t_1 = 0.7$ s, therefore, the automobile, during this time, will continue to move with uniform velocity of 30 km/h, i.e., 8.33 m/s.

Distance covered during 0.7 s is

$s_1 = v_1 t_1 = 8.33 \cdot 0.7 = 5.83\,\text{m}.$

After this the automobile begins to decelerate. Relationship between initial velocity $v_1 = 8.33$ m/s and final velocity $v_2 = 0$, acceleration $a = -5\,\text{m/s}^2$, and covered distance s_2 is

$$s_2 = \frac{v_2^2 - v_1^2}{2a} = \frac{0 - 8.33^2}{-2 \cdot 5} = 6.94\,\text{m}$$

Total distance travelled is

$s = s_1 + s_2 = 5.83 + 6.94 = 12.77$ m.

Problem 10

A particle moves along a horizontal path with a velocity of $v = 3t^2 - 6t$ (m/s), where t is time in seconds. If it is initially located at the origin O, determine the distance travelled in 3.5 s, and the particle's average velocity and average speed during the time interval.

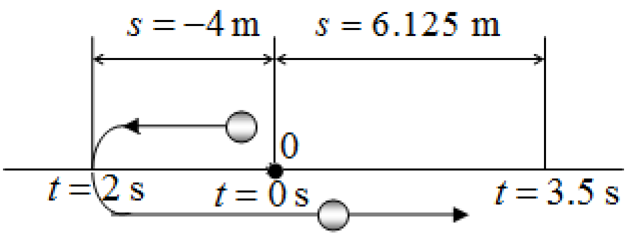
Solution

Since $v = f(t)$, the position as a function of time may be found by integrating $v = \frac{ds}{dt}$. Assume that at $t = 0$, $s = 0$, we obtain

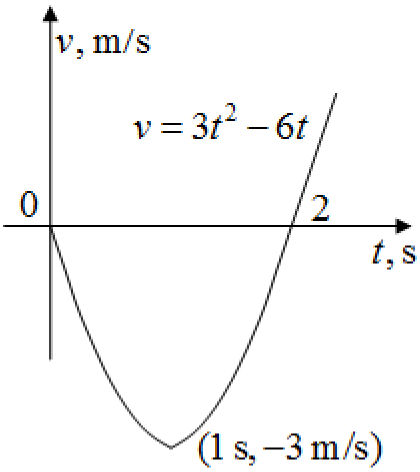
$$ds = vdt = \left(3t^2 - 6t\right)dt,$$

$$\int_0^s ds = \int_0^t \left(3t^2 - 6t\right)dt,$$

$$s = \left(t^3 - 3t^2\right) \text{ m}.$$



In order to determine the distance travelled in 3.5 s, it is necessary to investigate the path of motion. The function $v = 3t^2 - 6t$ is the parabola. From $v = 3t^2 - 6t = 0$, $t = 2$, therefore, the graph intersects the x-axis at the point $t = 2$. To determine the minimum of the function find the magnitude of t when derivative of the function is zero.



$$\frac{dv}{dt} = \left(3t^2 - 6t\right)' = 6t - 6 = 0,$$

$$t = 1.$$

Thus, for $0 < t < 2$ s the velocity is negative, and the particle is travelling to the left, for $2\text{ s} < t < 3.5\text{ s}$ the velocity is positive, and the particle is travelling to the right. Using $s = \left(t^3 - 3t^2\right)$ m we can determine the particle's positions:

$$s\big|_{t=0} = 0, \quad s\big|_{t=2s} = -4\text{ m}, \quad s\big|_{t=3.5s} = 6.125\text{ m}.$$

The distance traveled in 3.5 s is

$$s = 4 + 4 + 6.125 = 14.125 \text{ m}.$$

The displacement from $t = 0$ to $t = 3.5$s is

$$\Delta s = s\big|_{t=3.5s} - s\big|_{t=0} = 6.125 - 0 = 6.125 \text{ m}.$$

The average velocity which is equal to the displacement divided by the time of motion is

$$|\vec{v}|_{avg} = \frac{\Delta s}{\Delta t} = \frac{6.125}{3.5} = 1.75 \text{ m/s.}$$

The average speed is defined in terms of the distance traveled for the time of motion.

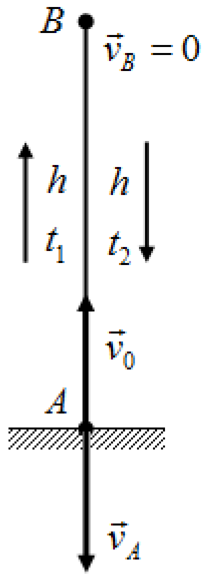
$$v_{avg} = \frac{s}{\Delta t} = \frac{14.1}{3.5} = 4.04 \text{ m/s.}$$

Problem 11

A body thrown upwards strikes the ground after $t = 3$ s. Find its initial velocity v_0 and the height h.

Solution

The upwards motion of the body during the time period t_1 is decelerated (uniform acceleration $\vec{g}$ directed downwards meanwhile the initial velocity v_0 directed upwards), and the downward motion during the time period t_2 is accelerated (free fall). The equations describing these motions form the system:



$$\begin{cases} h = v_0 t_1 - \dfrac{g t_1^{\,2}}{2}, \\ v_B = v_0 - g t_1, \\ h = \dfrac{g t_2}{2}, \\ v_A = g t_2, \\ t = t_1 + t_2. \end{cases}$$

As $v_B = 0$ then $v_0 = g t_1$. Substituting v_0 in the first equation of the system we obtain $h = \frac{gt_2^2}{2}$. Comparison of this equation with the third equation of the system allows making the conclusion that the time of ascent is equal to the time of descent $t_1 = t_2 = \frac{t}{2} = 1.5$ s. Therefore, the initial velocity is equal to the velocity at touchdown $v_0 = v_A = gt_1 = 9.8 \cdot 1.5 = 14.7$ m/s.

The height is

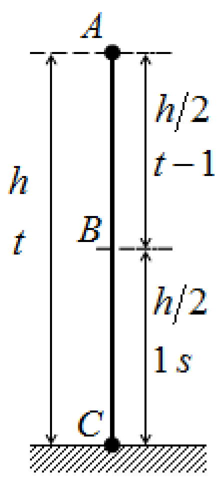
$$h = \frac{gt_1^2}{2} = \frac{9.8 \cdot 1.5^2}{2} = 22.05 \text{ m.}$$

Problem 12

Free falling body covered the second half the height during 1 *s. How long does the body take to strike the ground? What was the height?*

Solution

The dependence of the distance covered by free falling object on time is $h = \frac{gt^2}{2}$. As the section BC (the half of the whole distance *h*) is covered for the time period 1 s, then the first half of the distance AC is covered for $(t-1)$ s. Therefore, the motion along the section AB may be described as $\frac{h}{2} = \frac{g(t-1)^2}{2}$. Solving the system



$$\begin{cases} h = \frac{gt^2}{2}, \\ \frac{h}{2} = \frac{g(t-1)^2}{2}, \end{cases}$$

we obtain $t^2 - 4t + 2 = 0$. The roots of this equation are

$t_1 = 3.41$ s and $t_2 = 0.59$ s. The second root has to be rejected as the time of motion has to be greater than 1 s. Therefore, the time of free fall is $t_1 = 3{,}41$ s and the height is

$$h = \frac{gt^2}{2} = \frac{9.8 \cdot 3.41^2}{2} \approx 57 \text{ m.}$$

Problem 13

A body dropped from the top of a tower falls through 40 m during the last two seconds of its fall. What is the height of the tower?

Solution

Let *h* be the height of the tower and *t* be the time of fall.

Now, initial velocity, $v_0 = 0$; acceleration, $a = g \approx 10$ m/s^2.

Using

$$s = v_0 t + \frac{at^2}{2},$$

we get

$$h = \frac{gt^2}{2} = 5t^2.$$

Since the body travels 40 m during the last two seconds, therefore, the body covers $(h - 40)$ m in time $(t - 2)$ second.

Again, using $s = v_0 t + \frac{at^2}{2}$, we get $h - 40 = \frac{g(t-2)^2}{2}$,

$$h - 40 = 5(t-2)^2.$$

Subtracting obtained equations, we get

$$40 = 5\left[t^2 - (t-2)^2\right],$$

$2t - 2 = 4$,

$t = 3$ s.

Then the height of the tower is

$h = 5t^2 = 5 \cdot 3^2 = 45\,\text{m}$.

Problem 14

A body is thrown up with velocity of 78.4 m/s. Find how high it will rise and how much time it will take to return to its point of projection.

Solution

Let use the equations describing accelerated motion

$$\begin{cases} s = v_0 t \pm \dfrac{at^2}{2} \\ v = v_0 \pm at \end{cases}$$

Taking into account that upward motion is decelerated and velocity at the maximum height is zero, rewrite this system

$$\begin{cases} h = v_0 t - \dfrac{gt^2}{2} \\ 0 = v_0 - gt \end{cases}$$

We can find the time using the second equation and $g = 9.8\,\text{m/s}^2$

$$t = \frac{v_0}{g} = \frac{78.4}{9.8} = 8 \text{ s}.$$

The height is

$$h = v_0 t - \frac{gt^2}{2} = v_0 \cdot \frac{v_0}{g} - \frac{g}{2}\left(\frac{v_0}{g}\right)^2 = \frac{v_0^2}{2g} = \frac{(78.4)^2}{2 \cdot 9.8} = 313.6\,\text{m}.$$

Problem 15

A ball is dropped from a height of h metre above the ground and at the same instant another ball is projected upwards from the ground. The two balls meet when upper ball falls through a distance h/3. Prove that the velocities of two balls when they meet are in ratio 2:1.

Solution

For the first ball

$$\frac{h}{3} = \frac{gt^2}{2}.$$

For the second ball

$$h - \frac{h}{3} = v_0 t - \frac{gt^2}{2}.$$

Adding these two equations, we obtain

$h = v_0 t$, and $t = \dfrac{h}{v_0}$.

From the first equation

$$\frac{h}{3} = \frac{gt^2}{2} = \frac{g}{2}\left(\frac{h}{v_0}\right)^2,$$

$$v_0^2 = \frac{g}{2} \cdot 3h,$$

$$v_0 = \sqrt{\frac{3gh}{2}}.$$

Velocity of the first ball at the position where it meets the second ball is given by

$$v_1^2 - 0 = \frac{2gh}{3} \quad \text{or} \quad v_1 = \sqrt{\frac{2gh}{3}}.$$

The velocity of the second ball at the place where it meets the first ball is given by

$$v_2^2 - v_0^2 = -2g\left(h - \frac{h}{3}\right),$$

$$v_2^2 = v_0^2 - 2g \cdot \frac{2h}{3},$$

$$v_2^2 = \frac{3gh}{2} - \frac{4gh}{3} = \left(\frac{3}{2} - \frac{4}{3}\right)gh,$$

$$v_2 = \sqrt{\frac{gh}{6}}.$$

Now, $\dfrac{v_1}{v_2} = \sqrt{\dfrac{2gh \cdot 6}{3 \cdot gh}} = 2.$

Problem 16

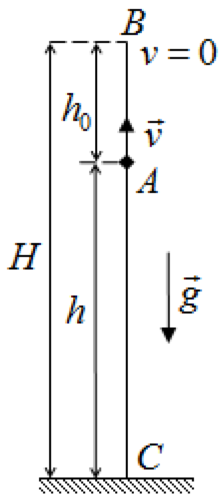
A balloon starts rising from the ground with an acceleration of 1. 5 m/s. After 10 second, a stone is released from the balloon. Starting from the release of stone, find the displacement and distance traveled by the stone on reaching the ground. Also, find the time taken to reach the ground.

Solution

When the balloon is rising at constant acceleration $a = 1.5$ m/s^2, this acceleration is the resultant acceleration allowing downward acceleration due to gravity. This means that the balloon rises with this net vertical acceleration of 1.5 m/s^2 in the upward direction.

Let the balloon rises to a height “h” during this time, then (considering origin on ground and upward direction as positive) the displacement of the balloon after $t = 10$ seconds is

$$h = \frac{at^2}{2} = \frac{1.5 \cdot 10^2}{2} = 75 \text{ m.}$$



At this instant of time the velocity of the balloon is

$v = at = 1.5 \cdot 10 = 15$ m/s.

This velocity is the initial velocity of the stone and it is directed upward as that of the velocity of balloon. Once released, the stone is acted upon by the force of gravity alone. The role of the acceleration of the balloon is over. Now, the acceleration for the motion of stone is equal to the acceleration due to gravity, $\vec{g}$.

The path of motion of the stone is depicted in the figure. Stone rises due to its initial upward velocity to a certain height above 75 m where it was released till its velocity is zero. From this highest vertical point, the stone falls freely under gravity and hits the ground.

$$0 = v - gt_1,$$

$$t_1 = \frac{v}{g} = \frac{15}{9.8} = 1.53 \text{ s.}$$

In order to describe motion of the stone once it is released, we realize that it would be easier for us if we shift the origin to the point where the stone is released. Considering origin at the point of release and upward direction as positive as shown in the figure, we obtain

$$h_0 = v \cdot t_1 - \frac{gt_1^2}{2} = 15 \cdot 1.53 - \frac{9.8 \cdot 1.53^2}{2} = 11.48 \text{ m.}$$

The height of the point B is

$$H = h + h_0 = 75 + 11.48 = 86.48.$$

The time of the free fall is

$$t_2 = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \cdot 86.48}{9.8}} = 4.2 \text{ s.}$$

The total time of the stone motion is

$t = t_1 + t_2 = 1.53 + 4.2 = 5.73\,\text{s}.$

The distance travelled by the stone is

$s = AB + BC = h_0 + H = 11.48 + 86.48 = 97.96$ m.

The magnitude of displacement is the separation between the initial (A) and final (C) points of the motion, therefore, ii is

$AC = h = 75\,\text{m}.$

Problem 17

If a particle's position is given by $x = 6 - 12t + 4t^2$ *(where t is in seconds, and x is in meters), (a) what is it's velocity at t=1s? (b) what is it's speed at t=1s? (c) Is there ever an instant when the velocity is 0? If so, give the time.*

Solution

The velocity is the derivative of $x(t)$ with respect to time. Then

$$v(t) = \frac{dx}{dt} = -12 + 8t$$

(a) at t =1 s we get:

$v\big|_{t=1s} = -12 + 8 = -4$ m/s

What we calculate here is an *x*-component of velocity. The negative sign means that the direction of velocity is opposite to the direction of axis *x*.

(b) the speed is the magnitude of velocity. Then at *t* = 1 s the speed is 4 m/s

(c) to find time at which the velocity is 0 we just need to solve an equation:

$v(t) = -12 + 8t = 0$

From this equation we find time:

$$t = \frac{12}{8} = 1{,}5\,\text{s}.$$

At this moment of time the velocity is 0.

Problem 18

A rock is dropped from rest into a well. (a) The sound of the splash is heard 4 s after the rock is released from rest. How far below to top of the well is the surface of the water? (The speed of sound in air at ambient temperature is 336 m/s).(b) If the travel time for the sound is neglected, what % error is introduced when the depth of the well is calculated?

Solution

(a) Let h be the depth of the well. To find the time of the rock's fall we use the equation $h = \frac{gt^2}{2}$.

The time of racks falling down is

$$t = \sqrt{\frac{2h}{g}}\,.$$

After the rock hits the water the sound of splash propagates the distance h with speed v_s = 336 m/s. The sound reaches the ground level for the time period

$$t_s = \frac{h}{v_s}\,.$$

The total time is

$$t_0 = t + t_s = \sqrt{\frac{2h}{g}} + \frac{h}{v_s}.$$

This time is equal to t_0 = 4 s. Then

$$4 = \sqrt{\frac{2h}{9.8}} + \frac{h}{336}$$

The height found from this equation is $h = 70{,}4\,\text{m}$.

(b) If we neglect the sound velocity then t = 4 s and we can find the height from the equation:

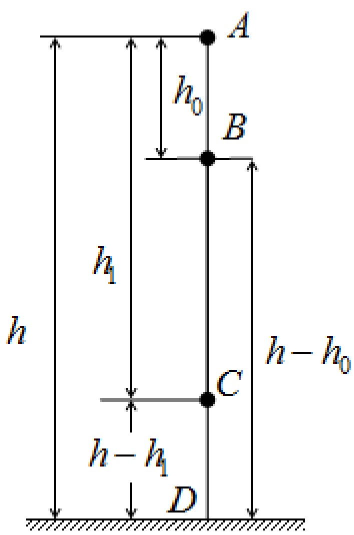
$$h = \frac{gt^2}{2} = \frac{9.8 \cdot 4^2}{2} = 78.4\,\text{m}$$

If we compare this result with the result from part (a), we can find an error:

Problem 19

A ball is released from a top. Another ball is dropped from a point $h_1 = 15$ m below the top, when the first ball reaches a point $h_0 = 5$ m below the top. Both balls reach the ground simultaneously. Determine the height h of the top.

Solution



The first ball is free falling from the top and covered the distance $AB = h_0 = \frac{gt_0^2}{2}$ for the time $t_0 = \sqrt{\frac{2h_0}{g}}$.

Therefore, the velocity of the ball at the point B is equal to $v_B = gt_0 = g \cdot \sqrt{\frac{2h_0}{g}} = \sqrt{2gh_0}$.

Now, the first ball covered the distance $BD = h - h_0$ moving with initial velocity $v_B = \sqrt{2gh_0}$ during the time

t. This is accelerated motion with acceleration due to gravity

$$h - h_0 = v_B t + \frac{gt^2}{2}.$$

The second ball during free fall covered the distance $CD = h - h_1$ for the same time t

$$h - h_1 = \frac{gt^2}{2}.$$

Combining last two equations we obtain

$$h - h_0 = v_B \cdot t + \frac{gt^2}{2} = v_B \cdot t + h - h_1.$$

$$h_1 - h_0 = v_B \cdot t = \sqrt{2gh_0} \cdot t,$$

$$t = \frac{h_1 - h_0}{\sqrt{2gh_0}} = \frac{15 - 5}{\sqrt{2 \cdot 9.8 \cdot 5}} = 1.01\,\mathrm{s}.$$

$$h = \frac{gt^2}{2} + h_1 = \frac{9.8 \cdot (1.01)^2}{2} + 15 = 20\,\mathrm{m}.$$

Problem 20

A rocket is fired vertically with an upward acceleration of 25 m/s². After 20 s, the engine shuts off and the rocket continues to move as a free particle until it reaches the ground. Calculate (a) the highest point the rocket reaches, (b) the total time the rocket is in the air, (c) the speed of the rocket just before it hits the ground.

Solution

The first period of the rocket motion is upward accelerated motion with acceleration a =20 m/s^2 directed upwards. Since the rocket starts from rest, its initial velocity is zero, and the distance covered for this time is

$$h_1 = v_0 t_1 + \frac{a t_1^2}{2} = \frac{a t_1^2}{2} = \frac{25 \cdot 20^2}{2} = 5000\,\text{m}.$$

The velocity of the rocket at the end of this time interval is

$$v = v_0 + at = at = 25 \cdot 20 = 500\,\text{m/s}.$$

The second period of the rocket motion is upward decelerated motion with acceleration due to gravity $\vec{g}$ directed downwards. It takes until the velocity becomes zero ($v_2 = 0$) at the top point of the trajectory. The time interval from

$$0 = v - g t_2,$$

is equal to

$$t_2 = \frac{v}{g} = \frac{500}{9.8} = 51\,\text{s}.$$

The distance covered for this time is

$$h_2 = \frac{v_2^2 - v_1^2}{2a} = \frac{-v_1^2}{2 \cdot (-g)} = \frac{500^2}{2 \cdot 9.8} = 12755 \text{ m}.$$

The third period of the rocket motion is the free fall, i.e., the downward motion from rest with acceleration due to gravity $\vec{g}$ directed downwards. The height of the free fall is

$$H = h_1 + h_2 = 5000 + 12755 = 17755 \text{ m}.$$

The time of free fall is

$$t_3 = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \cdot 17755}{9.8}} = 60.2 \text{ s}.$$

The total time the rocket is in the air

$$t = t_1 + t_2 + t_3 = 20 + 51 + 60.2 = 131.2\,\text{s}.$$

The velocity of the rocket just before it hits the ground is

$$v = gt = 9.8 \cdot 60.2 = 599 \text{ m/s}.$$

Problem 21

Ball A is dropped from the top of a building at the same instant that ball B is thrown vertically upward from the ground. When the balls collide, they are moving in opposite directions, and the speed of A is twice the speed of B. (a) At what fraction of the height of the building does the collision occur? (b) Solve this problem if the collision occurs when the balls are moving in the same direction and the speed of A is 4 times that of B.

Solution

(a) Take $x = 0$ at ground, upward direction is positive.

The distance covered by the first ball during its free falling to the point A for the time t is

$$x_1 = \frac{gt^2}{2}.$$

The coordinate of the collision point and the velocity of the first ball at this point are

$$x_A = H - x_1 = H - \frac{gt^2}{2},$$

$$v_A = -gt.$$

The coordinate of the collision point (x_B) determined for the second ball is separated from the origin by distance x_2,

$$x_B = x_2 = v_0 t - \frac{gt^2}{2},$$

and the velocity of the second ball at this point is

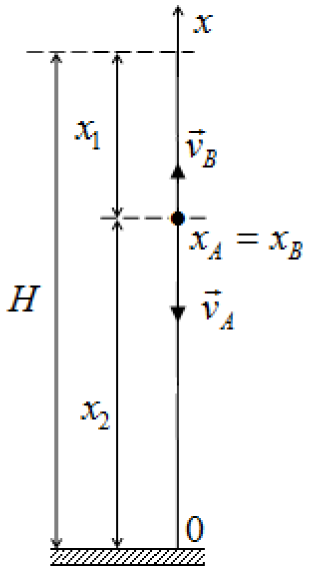
$$v_B = v_0 - gt.$$

Since the balls are moving in the opposite directions and the speed of the first ball is twice the speed of the second ball $v_A = -2v_B$, and

$$-gt = -2(v_0 - gt),$$

$$t = \frac{2v_0}{3g}.$$

From $x_A = x_B$,



$$H - \frac{gt^2}{2} = v_0 t - \frac{gt^2}{2},$$

$$H = v_0 t = \frac{2v_0^2}{3g}.$$

$$x_A = H - \frac{gt^2}{2} = \frac{2v_0^2}{3g} - \frac{g}{2}\left(\frac{2v_0}{3g}\right)^2 = \frac{4v_0^2}{9g}.$$

$$\frac{x_A}{H} = \frac{4v_0^2 \cdot 3g}{9g \cdot 2v_0^2} = \frac{2}{3}.$$

$$x_A = \frac{2H}{3}$$

(b) Now, for the second part of the problem, $v_A = 4v_B$. It gives

$$-gt = 4(v_0 - gt),$$

$$t = \frac{4v_0}{3g}.$$

From $x_A = x_B$, $H - \frac{gt^2}{2} = v_0 t - \frac{gt^2}{2}$,

$$H = v_0 t = \frac{4v_0^2}{3g}.$$

$$x_A = H - \frac{gt^2}{2} = \frac{4v_0^2}{3g} - \frac{g}{2}\left(\frac{4v_0}{3g}\right)^2 = \frac{4v_0^2}{9g}.$$

$$\frac{x_A}{H} = \frac{4v_0^2 \cdot 3g}{9g \cdot 4v_0^2} = \frac{1}{3}.$$

$$x_A = \frac{H}{3}.$$

Problem 22

An object is thrown vertically and has an upward velocity of 5 m/s when it reaches three fourths of its maximum height above its launch point. What is the initial (launch) speed of the object?

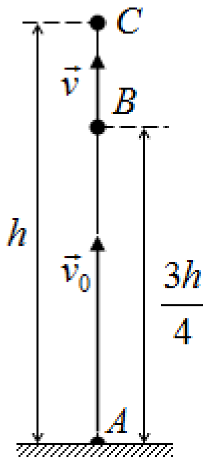
Solution

The velocity of the decelerated motion of the object depends on time as

$v = v_0 - gt$.

At the highest point C of the trajectory velocity of the object is zero

$0 = v_0 - gt$, and the time of the motion is $t = \dfrac{v_0}{g}$.



The maximum height of the object is

$$h = v_0 t - \frac{gt^2}{2} = \frac{v_0^2}{g}.$$

The height of the point B is

$$h_B = \frac{3}{4}h = \frac{3}{4}\frac{v_0^2}{g}.$$

On the other hand, velocity of the object at the point B is

$v = v_0 - gt_1$,

and the time for reaching this point is

$$t_1 = \frac{v_0 - v}{g}.$$

Thus, the height of the point B is

$$h_B = v_0 t_1 - \frac{gt_1^2}{2} = \frac{v_0(v_0 - v)}{g} - \frac{g(v_0 - v)^2}{2g^2} = \frac{v_0^2 - v^2}{2g}.$$

Equating the expressions for the height of the point B, we obtain

$$\frac{3}{4}\frac{v_0^2}{g} = \frac{v_0^2 - v^2}{2g},$$

$$v_0 = 2 \cdot v = 2 \cdot 5 = 10 \text{ m/s}.$$

Problem 23

The object is dropped from a helicopter which is at the height h. *Plot the acceleration-time, speed-time and distance-time graphs of the object motion if the helicopter (a) is at rest; (b) is ascending at uniform speed* v_0*; and (b) is descending at the uniform velocity* v_0.

Solution

(a) The motion of the object dropped from the helicopter poised at the height h is the free fall, i.e., the accelerated motion with constant acceleration g. Therefore, the acceleration-time graph is the straight line parallel to the horizontal axis. The velocity-time graph for uniformly accelerated motion is a straight line sloping upwards. Slope of velocity-time graph gives acceleration which is constant during the motion. The distance depends upon time in a quadratic way, therefore, the distance-time graph of a particle moving with uniformly accelerated rectilinear motion is a parabola, more properly, rising branch of a parabola beginning in the origin of the coordinate system.

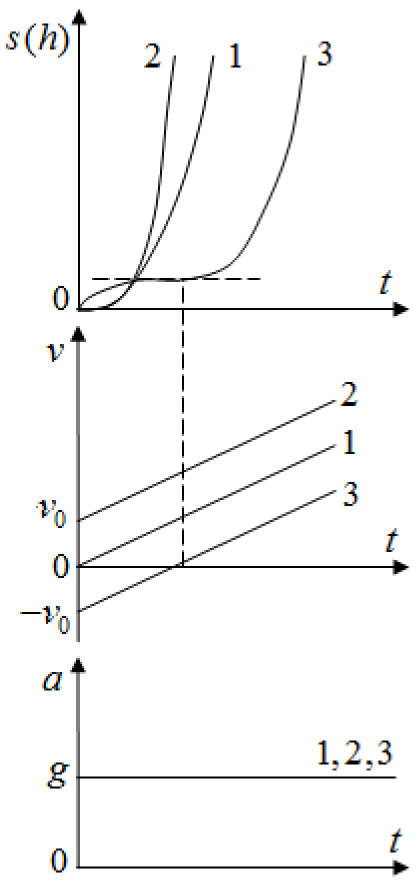
The equations that describe this motion are

$$\begin{cases} a = g, \\ v = gt, \\ h = \dfrac{gt^2}{2}. \end{cases}$$

(b) The object that was dropped from the helicopter descending with the constant velocity v_0, directed downwards, has the same downward initial velocity v_0. The equations describing this motion are

$$\begin{cases} a = g, \\ v = v_0 + gt, \\ h = v_0 t + \dfrac{gt^2}{2}. \end{cases}$$

The appropriate graphs are shown in the Figure and marked by number 2. Since the acceleration is $\vec{g}$ the slope of the velocity-time graph is the same as in the case #1, but the line $v(t)$ is displaced by magnitude v_0 up from the origin. Parabolic dependence $s(t)$ becomes steeper with respect to the vertical axis in comparison to the free falling object.



(c) The motion of the object dropped from the helicopter ascending with the constant velocity v_0 consists of two stages. Firstly, the object with velocity v_0 directed upwards takes part in decelerated motion until its velocity becomes zero. After travelled the distance h_0, the object starts free falling from the height $h + h_0$. The equations for these motions are

$$\begin{cases} a = g, \\ v = -v_0 + gt, \\ h_0 = -v_0 t + \dfrac{gt^2}{2}. \end{cases} \qquad \begin{cases} a = g, \\ v = gt, \\ h + h_0 = \dfrac{gt^2}{2}. \end{cases}$$

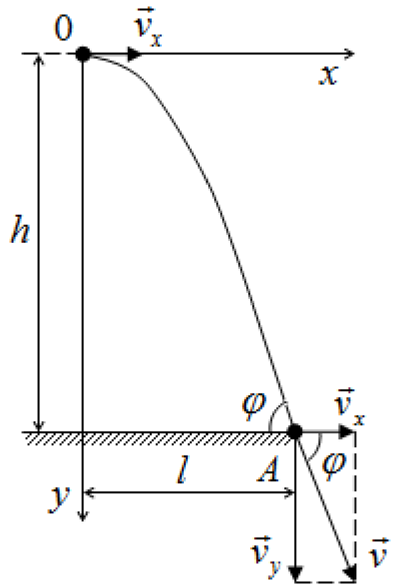
The acceleration-time dependence (labeled by number 3) is the same as in two previous cases. The dependence $v(t)$ now is displaced by magnitude v_0 down from the origin (to the point $-v_0$). The rectilinear dependence intersects the horizontal axis at the instant of time when the object is at its highest point and its velocity is zero. The distance-time dependence consists of two parts with different curvatures: convex branch of parabola for decelerated (upward) motion and concave branch of parabola for accelerated (downward) motion. The flex point where the inflectional tangent to the curve is parallel top the horizontal axis corresponds to the zero velocity at the top point of trajectory.

Problem 24

A stone is released from a height of 25 m above the ground at the horizontal velocity v_x =10 m/s.

1) *How long does the stone take to strike the ground?*
2) *With what velocity does the stone strike the ground?*
3) *What is its horizontal displacement?*
4) *What is the angle between the trajectory of the stone and the horizontal? Neglect the air resistance.*

Solution



A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile. The motion of projectile is a two-dimensional motion. So, it can be discussed in two parts: (i) horizontal motion, (ii) vertical motion. These two motions take place independent on each other. This called the principle of physical independence of motions.

The case described in this problem is *horizontal projectile*, i.e., a body is thrown horizontally from point O with velocity $\vec{v}_x$. The point O is at certain height

h above ground. Through the point O, take two axes – x-axis and y-axis. Let x and y be the horizontal and vertical distances respectively covered by the projectile in time t. The horizontal motion of the projectile is uniform motion. This is because the only force acting on the projectile is force of gravity. This force acts in the vertically downward direction, so the vertical motion is accelerated motion. The initial velocity in the vertically downward direction is zero, i.e., $v_{y0} = 0$. Since y-axis is taken downwards, therefore, the downward direction will be regarded as positive direction. So, the acceleration in vertical direction is g.

The equations describing the motion are

$$\begin{cases} x = v_x t, \\ v_x = const, \\ y = \dfrac{gt^2}{2}, \\ v_y = gt. \end{cases}$$

A body thrown horizontally from certain height above the ground follows a parabolic trajectory till it hits the ground in the point A.

For the point A these equations are

$$\begin{cases} l = v_x t, \\ h = \dfrac{gt^2}{2}, \\ v_y = gt. \end{cases}$$

From the second equation the time t of the motion OA is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 25}{9.8}} = 2.26 \text{ s}.$$

Note that in the vertical motion, the displacement, velocity and acceleration are all vectors and thus have no effect in the perpendicular horizontal direction. Similarly the horizontal displacement and velocity have no effect in the perpendicular vertical direction. The only scalar quantity used to link the two independent motions is time. For as long as the body is falling, it is also moving horizontally and vice versa.

Therefore, we have the possibility of finding the distance l and the vertical component of velocity v_y at the point A:

$l = 10 \cdot 2.26 = 22.6$ m, and

$v_y = 9.8 \cdot 2.26 = 22.15$ m/s.

The velocity and its magnitude are

$\vec{v} = \vec{v}_x + \vec{v}_y$,

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{100 + 490.5} = 24.3$ m/s.

The angle φ, which the resultant velocity $\vec{v}$ makes with the horizontal, may be found from the relationship

$$\tan\varphi = \frac{v_y}{v_x} = \frac{22.15}{10} = 2.215,$$

$\varphi = 68.7°$.

Problem 25

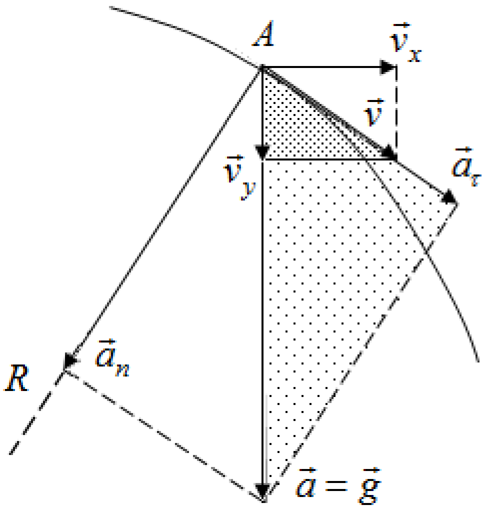
For the object thrown at horizontal velocity v_x = 10 m/s find the normal, tangential and total acceleration after time t = 2 s after the beginning of its motion and the radius of the curvilinear trajectory at this point.

Solution

Let us calculate the resultant velocity of the projectile at any point on the trajectory. The projectile reaches point A in time *t*. Let v_x and v_y be the horizontal and vertical components of $\vec{v}$. Since the horizontal motion of the projectile is uniform motion, v_x is not changed. The vertical motion is an accelerated motion. We know that

$v_y = v_{y0} + gt$.

As $v_{y0} = 0$, the vertical component of the velocity is $v_y = gt = 9.8 \cdot 2 = 19.6$ m/s.



At any point of trajectory $\vec{v} = \vec{v}_x + \vec{v}_y$.

The magnitude of resultant velocity $\vec{v}$ at the point A is given by

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{10^2 + 19.6^2} = 22 \text{ m/s}.$$

The resultant vector $\vec{v}$ and its components $\vec{v}_x$ and $\vec{v}_y$ form the triangle of velocities.

As the horizontal motion is uniform motion, the acceleration is only in vertical motion. This acceleration is $\vec{g}$ - acceleration due to gravity. It is directed downwards. Let resolve it into two components which are normal to each other: tangential acceleration $\vec{a}_\tau$ directed along the resultant velocity, and normal acceleration $\vec{a}_n$ directed at right angle to the resultant velocity.

The resultant acceleration $\vec{g}$ and its components $\vec{a}_\tau$ and $\vec{a}_n$ form the triangle of accelerations, which is similar to the triangle of velocities (see Figure to the Problem). Therefore, their sides are proportional to each other

$$\frac{a}{v} = \frac{a_\tau}{v_y} = \frac{a_n}{v_x}.$$

From this proportion we can find

$$a_\tau = a\frac{v_y}{v} = g\frac{v_y}{v} = 9.8 \cdot \frac{19.6}{22} = 8.73 \text{ m/s}^2,$$

$$a_n = a\frac{v_x}{v} = g\frac{v_x}{v} = 9.8 \cdot \frac{10}{22} = 4.45 \text{ m/s}^2.$$

As the normal acceleration is $a_n = \dfrac{v^2}{R}$, the radius of the curvilinear trajectory is

$$R = \frac{v^2}{a_n} = \frac{22^2}{4.45} = 108.8 \text{ m}.$$

Problem 26

A ball is projected horizontally from a height at a speed of 30 m/s. Find the time after which the vertical component of velocity becomes equal to horizontal component of velocity? (g = 10 m/s^2)

Solution

The ball moves at the constant horizontal component of velocity v_x. The vertical component of velocity since the ball is accelerated downward and gains speed in vertical direction is changing as $v_y = g \cdot t$.

At certain instant of time, the vertical component of velocity equals horizontal component of velocity $v_x = v_y$, and $v_x = g \cdot t$,

$$t = \frac{v_x}{g} = \frac{30}{10} = 3\text{s}.$$

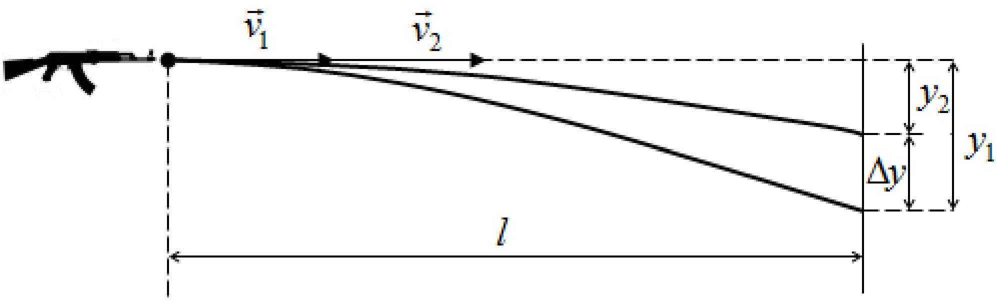
Problem 27

Find the distance between the places of bullet hitting the target if the velocities of the bullets were v_1=320 m/s and v_2 = 350 m/s and they were fired horizontally. The distance between the gun and the target is l = 60 m.

Solution

If a bullet that fired horizontally aimed directly at a target, then, for the time that elapses between the bullet leaving the muzzle and its hitting the target, the bullet falls from rest accelerating under gravity like any other object that is dropped.

The time of bullet motion may be determined as $t = \dfrac{l}{v_x}$ and is equal for the first bullet $t_1 = \dfrac{l}{v_{x1}} = \dfrac{50}{320} = 0.156$ s and for the second bullet $t_2 = \dfrac{l}{v_{x2}} = \dfrac{50}{350} = 0.143$ s.



The deviation from the vertical direction is $y = \dfrac{gt^2}{2}$. Therefore, the distance between holes is

$$\Delta y = \frac{g}{2}\left(t_1^2 - t_2^2\right) = \frac{9.8}{2}\left(0.156^2 - 0.143^2\right) = 1.89 \cdot 10^{-2} \text{ m.}$$

Problem 28

A tourist kicks a little stone horizontally off a cliff $h = 40$ m high into a river. If the tourist hears the sound of the splash $t_0 = 3$ s later, what was the initial speed given to the rock? Assume the speed of sound in air to be $v_s = 343$ m/s.

Solution

The stone's motion is described by the following equations

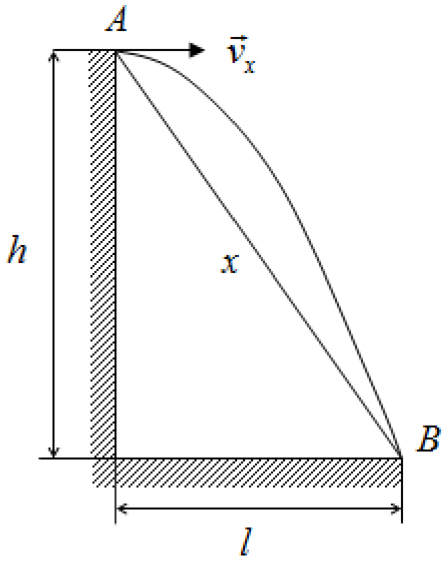
$$\begin{cases} l = v_x \cdot t, \\ h = \dfrac{gt^2}{2}. \end{cases}$$

The time of stone falling down is $t = \sqrt{2h/g}$.

$$l = v_x \cdot t = v_x \cdot \sqrt{2h/g}\ .$$

The sound is propagating along the straight line BA = x, where

$$x = \sqrt{l^2 + h^2} = \sqrt{\frac{v_x^2 \cdot 2h}{g} + h^2}\ .$$



On the other hand, this is the distance which is covered by sound travelling with the velocity v_x for the time t_s

$$x = v_s \cdot t_s\ .$$

Combining two previous equations, we obtain

$$\sqrt{\frac{v_x^2 \cdot 2h}{g} + h^2} = v_s \cdot t_s,$$

$$t_s = \frac{1}{v_s} \cdot \sqrt{\frac{v_x^2 \cdot 2h}{g} + h^2}\ .$$

Now, the total time t_0 is

$$t_0 = t + t_s = \sqrt{\frac{2h}{g}} + \frac{1}{v_s} \cdot \sqrt{\frac{v_x^2 \cdot 2h}{g} + h^2},$$

$$v_s \cdot \left(t_0 - \sqrt{\frac{2h}{g}} \right) = \sqrt{\frac{v_x^2 \cdot 2h}{g} + h^2},$$

$$v_x^2 = \frac{g}{2h} \left(v_s^2 \cdot \left(t_0 - \sqrt{\frac{2h}{g}} \right)^2 - h^2 \right)$$

$$v_x = \sqrt{\frac{g}{2h} \cdot \left(v_s^2 \cdot \left(t - \sqrt{\frac{2h}{g}} \right)^2 - h^2 \right)} =$$

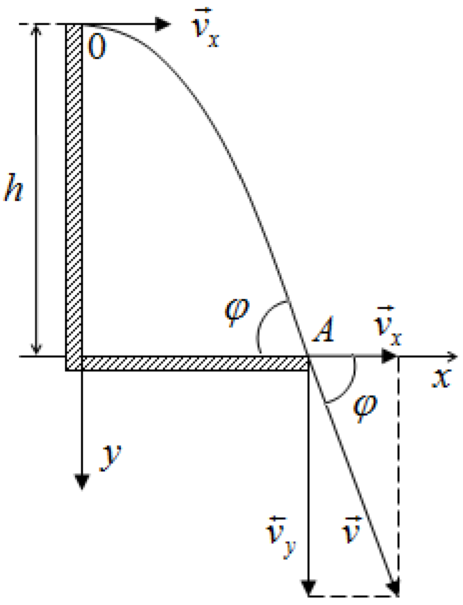
$$= \sqrt{\frac{9.8}{2 \cdot 40} \cdot \left(343^2 \cdot (3 - 2.86)^2 - 40^2 \right)} = 9.3 \text{ m/s.}$$

Problem 29

A marble is thrown horizontally with a speed of 10 m/s from the top of a building. When it strikes the ground, the marble has a velocity that makes an angle of 64° with the horizontal. From what height above the ground was the marble thrown?

Solution

Velocity triangle at the point where the marble strikes the ground is formed by three vectors: the velocity $\vec{v}$ and its vertical $\vec{v}_y$ and horizontal $\vec{v}_x$ components.



$$\tan\varphi = \frac{v_y}{v_x},$$

$$v_y = v_x \cdot \tan\varphi.$$

On the other hand, during the downward motion the vertical component of velocity is changed as $v_y = gt$. The time of falling down is $t = \frac{v_y}{g}$.

The distance traveled for this time and equaled to the height of the building is

$$h=\frac{gt^2}{2}=\frac{g}{2}\cdot\left(\frac{v_y}{g}\right)^2=\frac{v_y^2}{2g}=\frac{\left(v_x\cdot\tan\varphi\right)^2}{2g}=\frac{\left(10\cdot\tan 64^0\right)^2}{2\cdot 9.8}=21.4\,\text{m}.$$

Problem 30

A stone is thrown at the initial velocity v_0 *=10 m/s at the angle* α *=30*^{0} *with the horizontal. Find the time of flight, the horizontal range of the projectile, and its maximum height.*

Solution

If a body is projected at a certain angle with the horizontal, then the body is called an *oblique projectile*. Consider a projectile thrown with velocity $\vec{v}_0$ at an angle α with horizontal. The velocity can be resolved into two rectangular components along x-axis and y-axis, respectively,

$$\begin{cases} v_{0x}=v_0\cos\alpha, \\ v_{0y}=v_0\sin\alpha. \end{cases}$$

The motion of the projectile is a two-dimensional motion. It can be supposed to be made up of two motions – horizontal motion (along x-axis) and vertical motion (along y-axis). The horizontal motion of the projectile is uniform motion. This is because the only force acting on the projectile is the force of gravity. This force acts in vertical downward direction and its horizontal component is zero. Thus, the equations of motion of the projectile for horizontal direction are simply the equations of uniform motion with constant velocity $v_{0x}=v_0\cos\alpha$,

$$x=v_{0x}t_1.$$

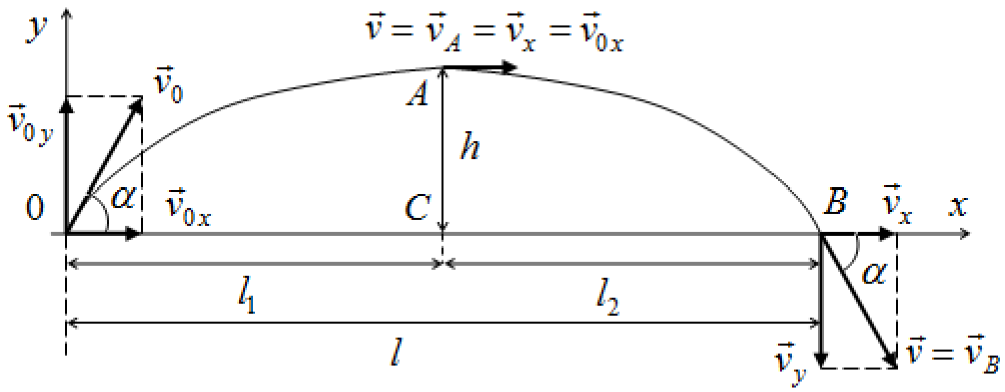
The vertical motion of the projectile is controlled by the force of gravity. The projectile increases its height up to a maximum where its vertical velocity

v_y becomes zero. After this, the projectile reverses its vertical direction and returns to earth striking the ground with a speed $\vec{v}_B$.

Let y be vertical distance covered by the projectile in time t_1. Let us now consider the vertical motion of the projectile.

$$y = v_{0y}t - \frac{gt_1^2}{2}.$$

Let velocity $\vec{v}$ be the resultant velocity of the projectile at time t. This velocity is along the tangent to the trajectory at point. Since the horizontal motion of the projectile is uniform motion therefore the horizontal component of velocity will remain uncharged $v_x = v_{0x}$. The vertical component of the velocity $v_y = v_{0y} - gt_1$.



That way, the decelerated motion of the projectile along the path OA is described by the following system of equations

$$\begin{cases} x = v_{0x}t_1, \\ v_x = v_{0x}, \\ y = v_{0y}t - \dfrac{gt_1^2}{2}, \\ v_y = v_{0y} - gt_1. \end{cases}$$

This system at the point A is

$$\begin{cases} l_1 = v_{0x}t_1, \\ h = v_{0y}t - \dfrac{gt_1^2}{2}, \\ 0 = v_{0y} - gt_1. \end{cases}$$

The time of motion from point O to the point A is

$$t_1 = \frac{v_{0y}}{g} = \frac{v_0 \sin\alpha}{g}.$$

The maximum height *h* and the distance l_1 are

$$h = \frac{gt_1^2}{2} = \frac{v_0^2 \sin^2\alpha}{2g},$$

$$l_1 = \frac{v_{0x} v_y}{g} = \frac{v_0^2 \cdot \sin\alpha \cdot \cos\alpha}{g} = \frac{v_0^2 \cdot \sin 2\alpha}{2g}.$$

Time of flight is the total time taken by the projectile to return to the same level from where it was thrown. Time of flight is equal to twice the time taken by the projectile to reach the maximum height. This is because the time of ascent is equal to the time of descent. This fact is also clear from symmetry of the parabolic curve along which the projectile is followed. So, time of flight

$$t = 2t_1 = \frac{2v_0 \sin\alpha}{g} = \frac{2 \cdot 10 \cdot 0.5}{9.8} = 1.02 \text{ s.}$$

The horizontal range is the total horizontal distance from the point of projection to the point where the projectile comes back to the plane of projection. This distance is covered for time $t = t_1 + t_2 = 2t_1$. Therefore, the horizontal range of the projectile is

$$l = 2l_1 = \frac{v_0^2 \cdot \sin 2\alpha}{g} = \frac{10^2 \sin 60°}{9,8} = 8,84 \text{ m.}$$

The height of the projectile flight is

$$h = \frac{v_0^2 \sin^2\alpha}{2g} = \frac{10^2 \sin^2 30°}{2 \cdot 9,8} = 1,28 \text{ m.}$$

Problem 31

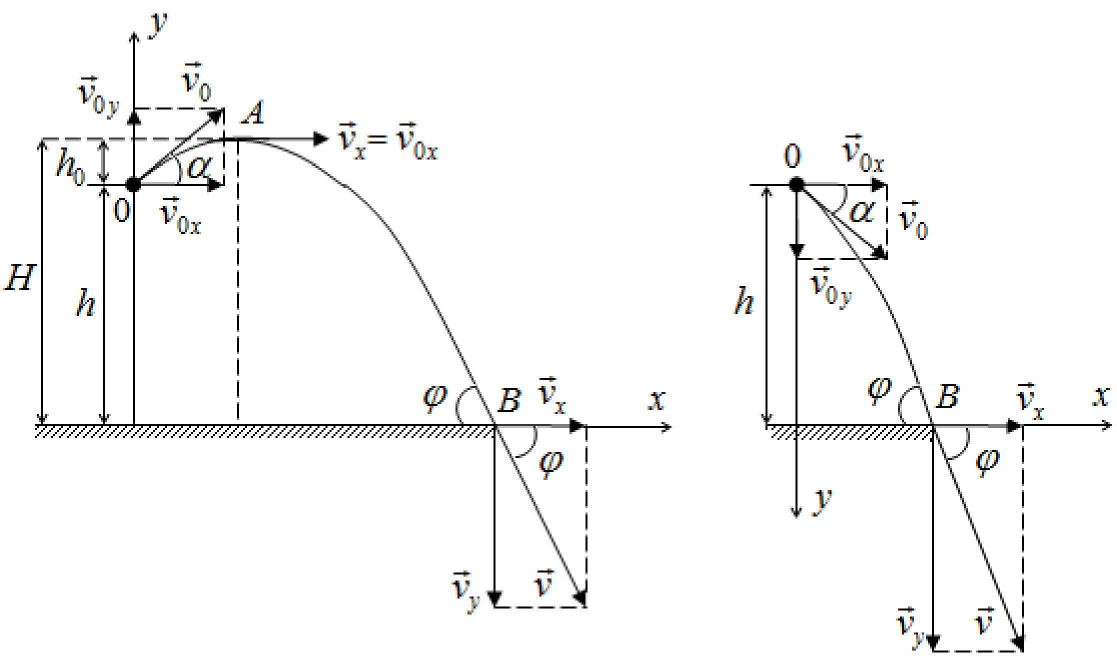
Two planes are about to drop an empty fuel tank. At the moment of release each plane has the same speed of 135 m/s, and each tank is at the same height of around 2 km above the ground. Although the speeds are the same, the velocities are *different at the instant of release, because the first plane is flying at an angle of 15^0 above the horizontal and the second is flying at an angle of 15^0 below the horizontal. Find the magnitude and the direction of the velocity with which the fuel tank hits the ground if it is from the first and from the second plane. Give the directional angles with respect to the horizontal.*

Solution

The velocities of the fuel tanks at the instant of release are the same as the velocities of the planes from which they are released; therefore, their paths are different.

(a) For the first plane the components of the velocity are

$$\begin{cases} v_{0x} = v_0 \cdot \cos\alpha = 135 \cdot \cos 15^0 = 130.4 \text{ m/s}, \\ v_{0y} = v_0 \cdot \sin\alpha = 135 \cdot \sin 15^0 = 34.9 \text{ m/s}. \end{cases}$$



The vertical component v_{0y} is directed upwards. The motion of the tank is decelerated with acceleration $\vec{g}$ directed downwards.

$v_y = v_{0y} - gt$.

This motion takes place until this vertical component becomes zero.

$0 = v_{0y} - gt$.

The time for this motion is $t = \dfrac{v_{0y}}{g} = \dfrac{34.9}{9.8} = 3.56$ s.

The tank during this time ascended to the height

$$h_0 = v_{0y}t - \frac{gt^2}{2} = 34.9 \cdot 3.56 - \frac{9.8 \cdot 3.56^2}{2} \approx 62 \text{ m.}$$

The motion from the highest point (the point A) is accelerated motion from the height

$H = h + h_0 = 2000 + 62 = 2062$ m.

The time of this motion is $t = \sqrt{\dfrac{2H}{g}} = \sqrt{\dfrac{2 \cdot 2062}{9.8}} = 20.5$ s.

the vertical component of the velocity before hitting the ground is

$v_y = gt = 9.8 \cdot 20.5 = 201$ m/s.

Taking into account that the horizontal component of velocity is not changed due to absence of air resistance ($v_x = v_{0x} = 130.4$ m/s), the velocity of the tank at its landing is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{130.4^2 + 201^2} = 239.6 \text{ m/s.}$$

The angle that the path makes with horizontal we can find from

$$\tan\varphi = \frac{v_y}{v_x} = \frac{201}{130.4} = 1.54,$$

$$\varphi = \arctan 1.54 = 57^0.$$

(b) The motion of the second tank is accelerated motion with the initial vertical velocity $v_{0y} = v_0 \cdot \sin\alpha = 135 \cdot \sin 15^0 = 34.9$ m/s directed downwards. The horizontal component of velocity is constant due to absence of the air resistance $v_x = v_{0x} = v_0 \cdot \cos\alpha = 135 \cdot \cos 15^0 = 130.4$ m/s.

The tank starts at the height $h = 2000$ m and its flight continues during

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 2000}{9.8}} = 20.2\,\text{s}.$$

The vertical component of tank velocity is

$$v_y = v_{0y} + gt = 34.9 + 9.8 \cdot 20.2 = 232.9 \text{ m/s}.$$

The velocity of the tank from the second plane near the ground is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{130.4^2 + 232.9^2} = 266.9 \text{ m/s}.$$

The angle that the path makes with horizontal we can find from

$$\tan\varphi = \frac{v_y}{v_x} = \frac{266.9}{130.4} = 2.05,$$

$$\varphi = \arctan 2.05 = 64^0.$$

Problem 32

By trial and error, a frog learns that it can leap a maximum horizontal distance of 1.4 m. If, in the course of an hour, the frog spends 32% of the time resting and 68% of the time performing identical jumps of that maximum length, in a straight line, what is the distance traveled by the frog?

Solution

The range of each projectile (including the frog) is

$$L = \frac{v_0^2 \cdot \sin 2\alpha}{g}.$$

The maximum range is at $\alpha = 45^0$ and is equal to

$$L_{\max} = \frac{v_0^2}{g}.$$

The initial velocity of the jumping frog has to be

$$v_0 = \sqrt{\frac{L_{\max}}{g}} = \sqrt{\frac{1.4}{9.8}} = 3.7 \text{ m/s.}$$

The time of one jump is

$$t = \frac{2v_0 \sin\alpha}{g} = \frac{2 \cdot 3.7 \cdot \sin 45^0}{9.8} = 0.53 \text{ s}$$

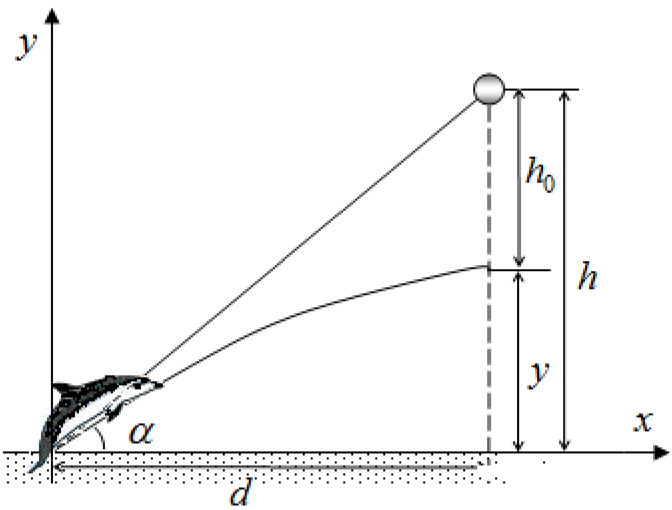
Taking into account that the frog is jumping during 1 hour $0.68 \cdot 3600 = 2448$ seconds, we can determine the number of its jumps as $N = 2448 / 0.53 = 4619$.

The total distance is $s = N \cdot L_{\max} = 4619 \cdot 1.4 = 6467$ m.

Problem 33

A trained dolphin leaps from the water with an initial speed of 12 m/s. It jumps directly towards a ball held by the trainer a horizontal distance of 5.50 m away and a vertical distance of 4.10 m above the water. In the absence of gravity the dolphin would move in a straight line to the ball and catch it, but because of gravity the dolphin follows a parabolic path well below the ball's initial position. If the trainer releases the ball the instant the dolphin leaves the water, show that the dolphin and the falling ball meet.

Solution



The angle that the dolphin leaves the water is

$$\tan\alpha = \frac{h}{d} = \frac{4.1}{5.5} = 0.745,$$

$$\alpha = \arctan 0.745 = 36.7^0.$$

The horizontal and vertical components of the initial velocity of the dolphin are

$v_{0x} = v_0 \cdot \cos\alpha = 12 \cdot \cos 36.7^0 = 9.62$ m/s,

$v_{0y} = v_0 \cdot \sin\alpha = 12 \cdot \sin 36.7^0 = 7.17$ m/s.

Since the air resistance is neglected, horizontal component $v_x = v_{0x}$.

The distance in horizontal direction is $x = v_x \cdot t$, and time of dolphin motion is

$$t = \frac{x}{v_x} = \frac{5.5}{9.62} = 0.572 \text{ s}.$$

The vertical displacement of the dolphin is

$$y = v_{0y} \cdot t - \frac{gt^2}{2} = 7.17 \cdot 0.572 - \frac{9.8 \cdot 0.572^2}{2} = 2.5 \text{ m}.$$

The ball's location at this instant of time is

$$y = h - h_0 = h - \frac{gt^2}{2} = 4.1 - \frac{9.8 \cdot 0.572^2}{2} = 2.5 \text{ m}.$$

The calculation shows that after 0.572 s the dolphin and the ball are at the same point at the height 2.5 m above the water surface.

Problem 34

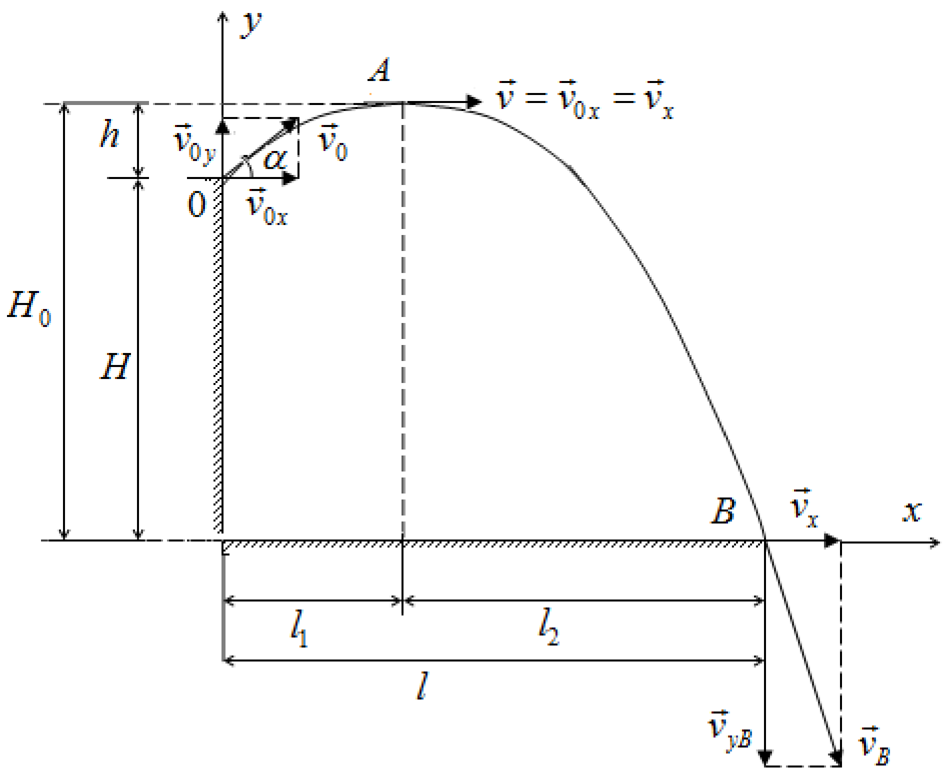
A projectile is thrown from the top of a building 30 m high, at an angle of 40° with the horizontal speed of 20 m/s. Find (a) time of the flight;(b) horizontal distance covered at the end of journey; (c) the maximum height of the projectile above the ground; (d) Find the magnitude of the final velocity.

Solution

Let's take the origin of the coordinate system at the projectile starting point.

The components of the initial velocity $\vec{v}_0$ are

$$\begin{cases} v_{0x} = v_0 \cdot \cos\alpha = 20 \cdot \cos 40^0 = 15.32 \quad \text{m/s}, \\ v_{0y} = v_0 \cdot \sin\alpha = 20 \cdot \sin 40^0 = 12.86 \quad \text{m/s}. \end{cases}$$



Firstly we examine the motion along 0A path, which can be resolved by to simple motions: horizontal uniform motion at the velocity $v_x = v_{0x}$, and decelerated vertical motion, where the vertical component of velocity and height are given by the equations for a motion with uniform acceleration g.

$$\begin{cases} x = v_x \cdot t, \\ y = v_{0y} \cdot t - \dfrac{g \cdot t^2}{2}, \\ v_y = v_{0y} - g \cdot t. \end{cases}$$

For the point A these equations give

$$\begin{cases} l_1 = v_x \cdot t_1, \\ h = v_{0y} \cdot t_1 - \dfrac{g \cdot t_1^2}{2}, \\ 0 = v_{0y} - g \cdot t_1. \end{cases}$$

Now, $t_1 = \frac{v_{0y}}{g} = \frac{12.86}{9.8} = 1.31$ s,

$$h = v_{0y} \cdot \frac{v_{0y}}{g} - \frac{g \cdot v_{0y}^2}{2 \cdot g^2} = \frac{v_{0y}^2}{2 \cdot g} = \frac{12.86^2}{2 \cdot 9.8} = 8.44 \text{m},$$

$$l_1 = v_x \cdot t_1 = \frac{v_{0x} \cdot v_{0y}}{g} = \frac{v_0^2 \cdot \sin 2\alpha}{2 \cdot g} = \frac{12.86^2 \cdot \sin 80^0}{2 \cdot 9.8} = 8.31 \text{ m.}$$

For the accelerated motion along AB path

$$\begin{cases} x = v_x \cdot t_2, \\ y = \dfrac{g \cdot t_2{}^2}{2}, \\ v_y = g \cdot t_2. \end{cases}$$

The height of the point A is $H_0 = h + H = 8.44 + 30 = 38.44$ m.

For the point B these equation are transformed in

$$\begin{cases} l_2 = v_x \cdot t_2, \\ H_0 = \dfrac{g \cdot t_2{}^2}{2}, \\ v_{yB} = g \cdot t_2. \end{cases}$$

The time of descending is $t_2 = \sqrt{\frac{2H_0}{g}} = \sqrt{\frac{2 \cdot 38.44}{9.8}} = 2.8$ s,

$$l_2 = v_x \cdot t_2 = 15.32 \cdot 2.8 = 42.9 \text{ m,}$$

$$v_{yB} = g \cdot t_2 = 9.8 \cdot 2.8 = 27.44 \text{ m/s.}$$

Since the final velocity at the point B is $\vec{v}_B = \vec{v}_x + \vec{v}_{yB}$, its magnitude is equal to

$$v_B = \sqrt{v_x^2 + v_{yB}^2} = \sqrt{15.32^2 + 27.44^2} = 31.43 \text{ m/s.}$$

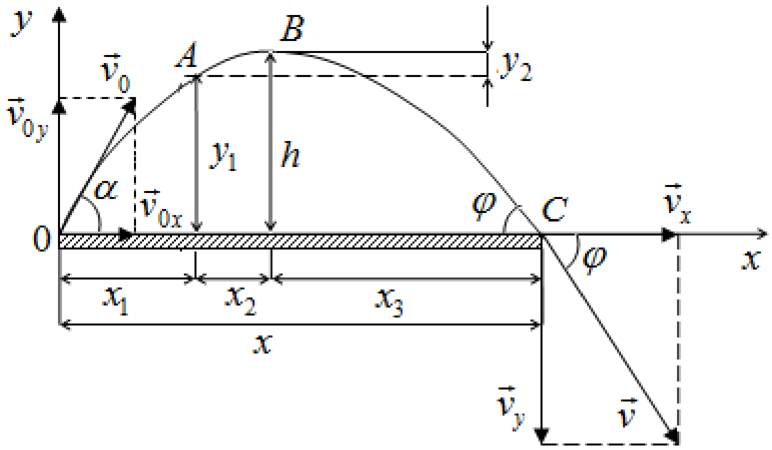
Problem 35

A catapult launches a rocket at an angle of 50° above the horizontal with an initial speed of 110 m/s. The rocket engine immediately starts a burn, and for 2.5 s the rocket moves along its initial line of motion with an acceleration of 35 m/s^2. Then its engine fails, and the rocket proceeds to move in free-fall.

Find (a) the maximum altitude reached by the rocket; (b) the total time of flight; (c) the horizontal range; (d) the velocity of the rocket at hitting the ground; (e) the angle that the path makes with horizontal axis at the point of the rocket landing.

Solution

The projectile moves in a curved path (parabola) and it leaves the origin (the point O) with velocity $\vec{v}$ which horizontal and vertical components are



$$\begin{cases} v_{0x} = v_0 \cdot \cos\alpha = 110 \cdot \cos 50^0 = 70.7 \text{ m/s}, \\ v_{0y} = v_0 \cdot \sin\alpha = 110 \cdot \sin 50^0 = 84.26 \text{ m/s}. \end{cases}$$

The motion of the rocket consists of three parts: upwards motion with engine (OA); upward motion without engine (AB); and downward motion (BC).

(a) During this stage the rocket is moving at acceleration $\vec{a}$. The components of acceleration are

$$\begin{cases} a_x = a \cdot \cos\alpha = 35 \cdot \cos 50^0 = 22.5 \text{ m/s}^2, \\ a_y = a \cdot \sin\alpha = 35 \cdot \sin 50^0 = 26.8 \text{ m/s}^2. \end{cases}$$

At the instant when the engine fails the components of the rocket velocity are

$$\begin{cases} v_x = v_{0x} + a_x t = 70.7 + 22.5 \cdot 2.5 = 126.95 \text{ m/s}, \\ v_y = v_{0y} + a_y t = 84.26 + 26.8 \cdot 2.5 = 113.56 \text{ m/s}. \end{cases}$$

Coordinates of the rocket at this instant of time (point A) are

$$\begin{cases} x_1 = v_{0x} \cdot t + \dfrac{a_x \cdot t^2}{2} = 70.7 \cdot 2.5 + \dfrac{22.5 \cdot 2.5^2}{2} = 247.05 \text{ m}, \\ y_1 = v_{0y} \cdot t + \dfrac{a_y \cdot t^2}{2} = 84.26 \cdot 2.5 + \dfrac{26.8 \cdot 2.5^2}{2} = 294.4 \text{ m}. \end{cases}$$

(b) The upward motion along parabolic path to the highest point B. This is decelerated motion with acceleration due to gravity - $\vec{g}$ (directed downwards). For this motion $a_x = a_y = a = 0$; the velocity in horizontal direction $v_x = 126.95$ m/s and it is not changed during motion; vertical component of velocity

$v_{y1} = v_y - g t_1$.

At the highest point of trajectory (point B) $v_{y1} = 0$, therefore,

$$t_1 = \frac{v_y}{g} = \frac{113.56}{9.8} = 11.59 \text{ s}.$$

$$\begin{cases} x_2 = v_x \cdot t = 126.95 \cdot 11.59 = 1471 \text{ m}, \\ y_2 = v_y \cdot t_1 - \dfrac{g \cdot t_1^2}{2} = 113.56 \cdot 11.59 - \dfrac{9.8 \cdot 11.59^2}{2} = 658 \text{ m}. \end{cases}$$

(c) Downward accelerated motion from the height

$h = y_1 + y_2 = 294.4 + 658 = 952.4$ m.

$v_{y2} = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 952.4} = \sqrt{18666} = 136.6$ m/s,

Since $v_x = 126.95$ m/s, and $\vec{v} = \vec{v}_x + \vec{v}_{y2}$, the magnitude of velocity of the rocket at the point C where it hits the ground is

$$v = \sqrt{v_x^2 + v_{y2}^2} = \sqrt{126.95^2 + 136.6^2} = 186.5 \text{ m/s.}$$

$$v_{y2} = g \cdot t_2 .$$

$$t_2 = \frac{v_{y2}}{g} = \frac{136.6}{9.8} = 13.94 \text{ s.}$$

$$x_3 = v_x \cdot t_3 = 126.95 \cdot 13.94 = 1769.5 \text{ m.}$$

The total time of the rocket motion is

$$t_0 = t + t_1 + t_2 = 2.5 + 11.59 + 13.94 = 28.03 \text{ s.}$$

The horizontal range of the projectile is equal to

$$x_0 = x_1 + x_2 + x_3 = 247.05 + 1471 + 1769.5 = 3487.55 \text{ m.}$$

(d) The angle that the path makes with horizontal axis at rocket landing is

$$\tan\varphi = \frac{v_{y2}}{v_x} = \frac{136.6}{126.95} = 1.08,$$

$$\varphi = 47^0 .$$

Problem 36

Find the angular speed (a) of the second hand on clock; (b) of the minute hand on clock; (c) of the hour hand on clock; (d) of the Earth's rotation about its axis.

Solution

(a) The angular speed of the second hand on clock is

$$\omega = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/s.}$$

(b) The angular speed of the minute hand on clock is

$$\omega = \frac{2\pi}{60 \cdot 60} = \frac{\pi}{1800} \text{ rad/s.}$$

(c) The angular speed of the hour hand on clock is

$$\omega = \frac{2\pi}{12 \cdot 3600} = \frac{\pi}{21600} \text{ rad/s.}$$

(d) The Earth is rotating about its axis. It takes 1 day to complete one rotation, during which the angular displacement is 2π radians. So the angular speed of the Earth is 2π per 1 day.

1 day = 24 hours =24·3600 s =86400 s

The angular speed of the earth's rotation about its axis is

$$\omega = \frac{2\pi}{86400} = \frac{\pi}{43200} = 7.27 \cdot 10^{-5} \text{ rad/s.}$$

Problem 37

The axis with two disks located 0.5 m apart from each other, rotates at a frequency 1600 rev/min. The bullet flying along an axis perforates both disks; thus the bullet hole in the second disk is misaligned with respect to the first disk by the angle $\varphi = 12^0$. Find the bullet velocity.

Solution

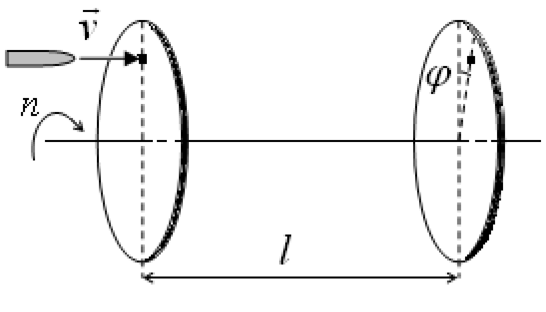
To find the bullet velocity we have to know the time of covering the distance l.

Let us try to calculate this time estimating the rotation of discs with the angular velocity (frequency) n.

$\varphi = 2\pi n t$,

or

$$t = \frac{\varphi}{2\pi n}.$$



Substituting $\varphi = 12^0 = \frac{\pi}{15}$ rad,

$n = 1600 \text{rev/min} = \frac{1600}{60}$ rev/s, we obtain

$$t = \frac{\cancel{\pi} \cdot 60}{15 \cdot 2\cancel{\pi} \cdot 1600} = 1.25 \cdot 10^{-3}\text{ s.}$$

Then the velocity of the bullet is

$$v = \frac{l}{t} = \frac{0.5}{1.25 \cdot 10^{-3}} = 400 \text{ m/s.}$$

Problem 38

A wheel begins to rotate with uniform acceleration $\varepsilon =$ *3 rad/s²*. *Find the angular velocity which the wheel reaches after 3 s of rotation. Determine the number of revolutions for this period of time*.

Solution

If the wheel takes part in the rotation with uniform acceleration its motion may be described by the system of equations

$$\begin{cases} \varphi = \omega_0 \cdot t + \dfrac{\varepsilon \cdot t^2}{2}, \\ \omega = \omega_0 + \varepsilon \cdot t. \end{cases}$$

Firstly, as the wheel was at rest, its initial angular velocity is $\omega_0 = 0$, therefore,

$$\begin{cases} \varphi = \dfrac{\varepsilon \cdot t^2}{2}, \\ \omega = \varepsilon \cdot t. \end{cases}$$

Consequently, the angular velocity is

$$\omega = \varepsilon \cdot t = 3 \cdot 3 = 9 \text{ rad/s,}$$

and the number of revolutions for this period of time is

$$N = \frac{\varphi}{2\pi} = \frac{\varepsilon t^2}{4\pi} = \frac{3 \cdot 3^2}{4\pi} = 2.15 \text{ rev.}$$

Problem 39

A fan blade spins at the frequency of 900 rev/min. After switching off it was stopped after $N = 75$ revolutions. What is its angular acceleration? How long does the fan take to stop?

Solution

The decelerated motion of the fan is described by

$$\begin{cases} 2\pi \cdot N = 2\pi \cdot n_0 \cdot t - \dfrac{\varepsilon \cdot t^2}{2}, \\ 2\pi \cdot n = 2\pi \cdot n_0 - \varepsilon \cdot t. \end{cases}$$

The final velocity is $n = 0$, then the second equation of system is

$0 = 2\pi \cdot n_0 - \varepsilon \cdot t$,

and the time of the motion is $t = \dfrac{2\pi \cdot n_0}{\varepsilon}$. Substituting it in the first equation and taking into account that $n_0 = 900$ rev/min = 15 rev/s, we obtain

$$\varepsilon = \frac{\pi \cdot n_0^2}{N} = \frac{\pi \cdot 15^2}{75} = 9.42 \text{ rad/s}^2.$$

The time of motion is

$$t = \frac{2\pi \cdot n_0}{\varepsilon} = \frac{2\pi \cdot 15}{9.42} = 10 \text{ s}.$$

Problem 40

A wheel starting from rest is uniformly accelerated at 4 rad/s^2 for 5 seconds. It is allowed to rotate uniformly for next 50 seconds and is finally brought to rest in the next 10 seconds. Find the total angle rotated by the wheel.

Solution

There are three periods of the wheel rotation. Accelerated motion during $t_1 = 5\,\mathrm{s}$ is described as

$$\begin{cases} \omega = \varepsilon_1 \cdot t_1 = 4 \cdot 5 = 20 \quad \text{rad/s}, \\ \varphi_1 = \dfrac{\varepsilon_1 \cdot t_1^2}{2} = \dfrac{4 \cdot 5^2}{2} = 50 \quad \text{rad}. \end{cases}$$

The angle rotated during $t_2 = 50$ seconds of the uniform motion at the angular velocity $\omega = 20$ rad/s is

$$\varphi_2 = \omega \cdot t_2 = 20 \cdot 50 = 1000 \text{ rad}.$$

During next 10 seconds of decelerated motion the magnitude of acceleration may be calculated using following equation:

$$0 = \omega - \varepsilon_2 \cdot t_3,$$

$$\varepsilon_2 = \frac{\omega}{t_3} = \frac{20}{10} = 2 \text{ rad/s}^2.$$

The angle rotated for this time is

$$\varphi_3 = \omega \cdot t_3 - \frac{\varepsilon_2 \cdot t_3}{2} = 20 \cdot 10 - \frac{2 \cdot 10^2}{2} = 100 \text{ rad}.$$

The total angle is

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 = 50 + 1000 + 100 = 1150 \text{ rad}.$$

Problem 41

The point is rotating along the circular path of radius R = 20 cm with the uniform tangential acceleration a_τ *=5 cm/s*^{2}*. Find the period of time until the instant when the normal acceleration is twice greater than the tangential acceleration.*

Solution

The angular velocity of the point during the accelerated motion may be calculated from the relationship $\omega = \omega_0 + \varepsilon \cdot t$. As $\omega_0 = 0$, then $\omega = \varepsilon \cdot t$. The normal acceleration $a_n = \omega^2 \cdot R = (\varepsilon \cdot t)^2 \cdot R$. The tangential acceleration is $a_\tau = \varepsilon \cdot R$. Since normal acceleration is twice greater than the tangential acceleration $a_n = 2a_\tau$ we may write

$$(\varepsilon t)^2 R = 2\varepsilon \cdot R.$$

Consequently,

$$t = \sqrt{\frac{2}{\varepsilon}} = \sqrt{\frac{2R}{a_\tau}} = \sqrt{\frac{2 \cdot 0.2}{0.05}} = 2.83 \text{ s.}$$

Problem 42

The point is moving along the circular path of radius $R = 2$ cm. The dependence of the distance on time is $s(t) = Ct^3$, where C = 0.1 cm/s^3. Find the normal and tangential accelerations of the point in the instant of time when the linear velocity of the point is $v = 0.3$ m/s.

Solution

Using the dependence of the distance on time $s(t) = Ct^3$, we can find the dependencies of velocity and tangential acceleration on time

$$v = \frac{ds}{dt} = 3Ct^2,$$

$$a_\tau = \frac{dv}{dt} = 6Ct.$$

Therefore,

$$t = \sqrt{\frac{v}{3C}} = \sqrt{\frac{0.3}{3 \cdot 0.1 \cdot 10^{-2}}} = 10 \text{ s.}$$

The tangential acceleration is

$$a_\tau = 6 \cdot Ct = 6 \cdot 0.1 \cdot 10^{-2} \cdot 10 = 0.06 \text{ m/s}^2.$$

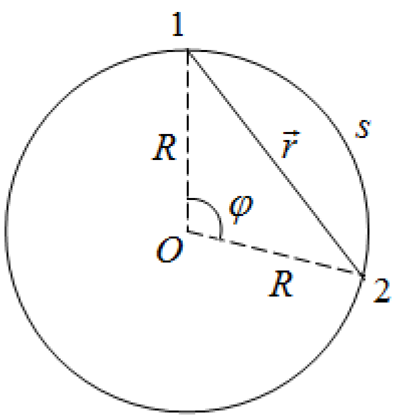
The normal acceleration

$$a_n = \frac{v^2}{R} = \frac{0.3^2}{2 \cdot 10^{-2}} = 4.5 \text{ m/s}^2.$$

Problem 43

The point is moving along the circular path of radius $R = 4$ m. The initial velocity of the point is $v_0 = 3$ m/s, tangential acceleration $a_\tau = 1$ m/s^2. Find for the time instant $t = 2$ s: (a) the distance covered for this time; (b) the magnitude of displacement; (c) the linear and angular velocities; (d) the normal, total and angular accelerations.

Solution



The dependence of the distance covered by the point on time is

$$s(t) = v_0 \cdot t + \frac{a_\tau \cdot t^2}{2} \text{ m.}$$

It allows to find the distance $s = 3 \cdot 2 + \dfrac{1 \cdot 2^2}{2} = 8$ m. Taking into account that during one revolution the point covers the distance which is equal to the length of the circle $s_1 = 2\pi \cdot R = 8\pi$ (m), we can find the angular displacement from the proportion $\frac{2\pi}{\varphi} = \frac{8\pi}{8}$, $\varphi = 2$ (rad) = 114.7^0.

Therefore, the magnitude of displacement as the chord related to the angle φ may be calculated according to cosine theorem:

$$|\vec{r}| = \sqrt{2R^2 - 2R^2 \cdot \cos\varphi} = R\sqrt{2(1-\cos\varphi)} = 4\sqrt{2(1+0.418)} = 6.73 \text{ m.}$$

The linear velocity of the point is

$$v = v_0 + a_\tau \cdot t = 3 + 1 \cdot 2 = 5 \text{ m/s.}$$

The angular velocity is

$$\omega = v \cdot R = 5 \cdot 4 = 20 \text{ rad/s.}$$

The normal acceleration is

$$a_n = \frac{v^2}{R} = \frac{5^2}{4} = 6,25 \text{ m/s}^2.$$

The total acceleration is

$$\vec{a} = \vec{a}_n + \vec{a}_\tau,$$

and its magnitude is

$$a = \sqrt{a_n^2 + a_\tau^2} = \sqrt{6.25^2 + 1^2} = 6.33 \text{ m/s}^2.$$

The angular acceleration is

$$\varepsilon = \frac{a_\tau}{R} = \frac{1}{4} = 0.25 \text{ rad/s}^2.$$

Problem 44

The car is moving at the velocity 36 km/h along the curvilinear road of radius R = 200 m. It begins to decelerate with the acceleration 0,3 m/s^2. Find the normal acceleration of the car. Find the angle φ between the vector of total acceleration and the vector of the velocity. Find the angular velocity and acceleration of the car at the beginning of turning.

Solution

If the velocity of the car is $v = 36$ km/h = 10 m/s, the normal acceleration is

$$a_n = \frac{v^2}{R} = \frac{10^2}{200} = 0.5 \text{ m/s}^2.$$

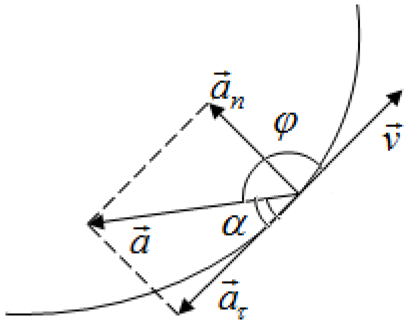
The total acceleration of the car $a = \sqrt{a_n^2 + a_\tau^2} = \sqrt{0.5^2 + 0.3^2} = 0.58 \text{ m/s}^2$.

The angular acceleration

$$\varepsilon = \frac{a_\tau}{R} = \frac{0.3}{200} = 1.5 \cdot 10^{-3} \text{ m/s}^2.$$

The angular velocity

$$\omega = \frac{v}{R} = \frac{10}{200} = 0.05 \text{ rad/s}.$$



As the car takes part in decelerated motion, the vectors of velocity $\vec{v}$ and tangential acceleration $\vec{a}_\tau$ are directed in the opposite directions, and total acceleration and velocity make the obtuse angle φ. Therefore, after finding the supplementary angle α

$$\tan\alpha = \frac{a_n}{a_\tau} = \frac{0.5}{0.3} = 1.67 \quad \Rightarrow \quad \alpha = 59°,$$

we can determine the required angle

$$\varphi = 180° - \alpha = 121°.$$

Problem 45

The point is moving along the circle of radius R = 20 cm with constant tangential acceleration a_τ. Find normal a_n, tangential a_τ and total acceleration a of the point after t =20 s of the motion if it is known that in the end of the fifth revolution after the beginning of the motion the linear velocity of the point was v = 10 cm/s.

Solution

There are two instants of time that are considered in the problem: the first instant t_1 when the point has made 5 revolutions, and the second instant - t_2 after 20 s of motion.

The point takes part in accelerated motion with initial velocity equal to zero, and then its motion is described by the equations

$$\begin{cases} 2\pi \cdot N = \dfrac{\varepsilon \cdot t_1^2}{2}, \\ 2\pi \cdot n = \varepsilon \cdot t_1, \end{cases}$$

or, taking into account that $2\pi \cdot n = \omega = \dfrac{v}{R}$,

$$\begin{cases} 2\pi \cdot N = \dfrac{\varepsilon \cdot t_1^2}{2}, \\ \dfrac{v}{R} = \varepsilon \cdot t_1. \end{cases}$$

Let us take the square of the second equation of the system, and divide the result termwise by the first equation

$$\frac{2\pi \cdot N \cdot R^2}{v^2} = \frac{\cancel{\varepsilon} \cdot \cancel{t_1^2}}{\varepsilon^{\cancel{2}} \cdot \cancel{t_1^2} \cdot 2}.$$

Then the angular acceleration is

$$\varepsilon = \frac{v^2}{4\pi \cdot N \cdot R^2},$$

and the tangential acceleration which is independent on time is

$$a_\tau = \varepsilon \cdot R = \frac{v^2}{4\pi \cdot N \cdot R} = \frac{10^{-2}}{4\pi \cdot 5 \cdot 0.2} = 8 \cdot 10^{-4} \text{ m/s}^2.$$

The normal acceleration which depends on velocity for the instant of time t_2 is

$$a_n = \omega_2^2 \cdot R = \left(\varepsilon \cdot t_2\right)^2 \cdot R = \frac{v^4 \cdot t_2^2 \cdot \cancel{R}}{16\pi^2 \cdot N^2 \cdot R^{\cancel{4}3}} =$$

$$= \frac{v^4 \cdot t_2^2}{16\pi^2 \cdot N^2 \cdot R^3} = \frac{10^{-4} \cdot 400}{16\pi^2 \cdot 25 \cdot 8 \cdot 10^{-3}} = 3.2 \cdot 10^{-2} \text{m/s}^2.$$