NATIONAL TECHNICAL UNIVERSITY
“KHARKIV POLYTECHICAL INSTITUTE”

DEPARTMENT OF PHYSICS

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PROBLEM SOLVING GUIDE

“DYNAMICS”

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PROBLEMS

Problem 1

Two forces $\vec{F}_1 = \left(3\vec{i} - 5\vec{j}\right)$ *and* $\vec{F}_2 = \left(2\vec{i} + \vec{j}\right)$ *N are applied to the particle of a mass* m*=1.5 kg. Find the acceleration* $\vec{a}$ *of this particle.*

Solution

Newton's 2 Law gives

$m\vec{a} = \vec{F}$,

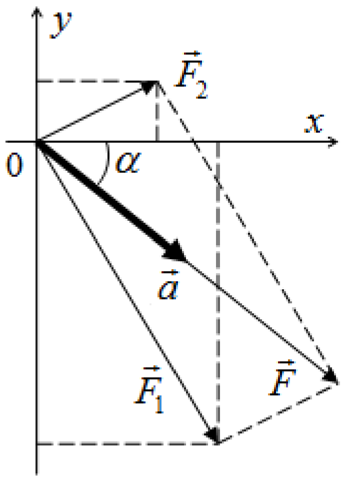
where $\vec{F}$ is a net force (the vector sum of all forces that act on the particle).

$$\vec{a} = \frac{\vec{F}}{m} = \frac{\left(3\vec{i} - 5\vec{j}\right) + \left(2\vec{i} + \vec{j}\right)}{1.5} = \frac{\left(5\vec{i} - 4\vec{j}\right)}{1.5} = \left(3.33\vec{i} - 2.67\vec{j}\right) \text{ m/s}^2.$$

The magnitude of the acceleration is

$a = \sqrt{3.33^2 + 2.67^2} = 4.27$ m/s².

It is directed at the angle $\alpha = \arctan\left(\frac{2.67}{3.33}\right) = \arctan 0.8 = 38.7^0$ below the positive x-axis.



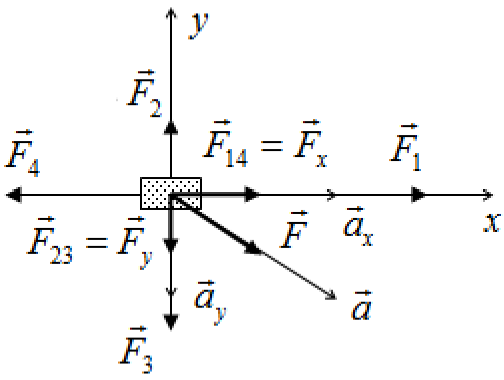
Problem 2

Forces of F_1*= 85 N to the east,* F_2 *=25 N to the north,* F_3 *=45 N to the south,* F_4*= 55 N to the west are simultaneously applied to a box of mass 14 kg. Find the magnitude of the box's acceleration?*

Solution

Two forces $\vec{F}_1$ and $\vec{F}_4$ act in the x-direction. Their vector sum is $\vec{F}_x = \vec{F}_{14} = \vec{F}_1 + \vec{F}_4$, and the magnitude is

$F_x = F_{14} = F_1 - F_4 = 85 - 45 = 40$N.



Two other forces are directed along y-axis.

$\vec{F}_y = \vec{F}_{23} = \vec{F}_2 + \vec{F}_3$,

$F_y = F_{23} = F_3 - F_2 = 45 - 25 = 20$ N.

The net applied to the box is $\vec{F} = \vec{F}_{14} + \vec{F}_{13} = \vec{F}_x + \vec{F}_y$. Its magnitude using Pythagorean Theorem is $F = \sqrt{F_x^2 + F_y^2} = \sqrt{40^2 + 20^2} = 44.7$ N.

According Newton's Second Law $\vec{F} = m\vec{a}$, therefore,

$$a = \frac{F}{m} = \frac{44.7}{14} = 3.2 \text{ m/s}^2.$$

Problem 3

A 5-kg object undergoes an acceleration given by $\vec{a} = \left(3\vec{i} + 5\vec{j}\right)$ *m/s². Find the magnitude and direction of the resultant force acting on it.*

Solution

Using Newton's 2 Law, we obtain $\vec{F} = m\vec{a} = 5 \cdot \left(3\vec{i} + 5\vec{j}\right) = \left(15\vec{i} + 25\vec{j}\right)$ N.

The magnitude of the net force is

$F = \sqrt{15^2 + 25^2} = 29.1$N

Direction of the force is determined by the angle α that the acceleration makes with the positive direction of the x-axis

$\alpha = \arctan\left(F_x / F_y\right) = \arctan(15/25) = 30.9^0$.

Problem 4

A hockey puck having a mass of 0.15 kg slides on the horizontal, frictionless surface of an ice rink. Two hockey-players strike the puck simultaneously by their sticks, exerting the forces on the puck shown in Figure. The force F_1 *has a magnitude of 6 N, and it makes the angle* 60^0 *above the* $+x$*-axis. The force* F_2 *has a magnitude of 9 N and direction* 20^0 *below the* $+x$*-axis. Determine both the magnitude and the direction of the puck's acceleration assuming that the puck slides on the horizontal, frictionless surface of the ice rink.*

Solution

Firstly resolve the force vectors into components. The net force acting on the puck in the x-direction is

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cdot \cos\alpha + F_2 \cdot \cos\beta = 6 \cdot \cos 60^0 + 9 \cdot \cos(-20^0) = 11.5\,\text{N}.$$

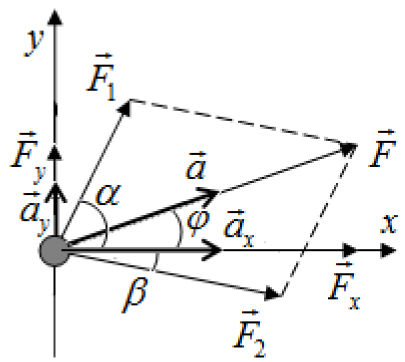
The net force acting on the puck in y-direction is

$$\sum F_y = F_{1y} + F_{2y} = F_1 \cdot \sin\alpha + F_2 \cdot \sin\beta = 6 \cdot \sin 60^0 + 9 \cdot \sin(-20^0) = 2.1\ \text{N}.$$

Now we use Newton's 2 Law in component form to find the x- and y-components of the puck acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{11.5}{0.15} = 76.7\ \text{m/s}^2,$$

$$a_y = \frac{\sum F_y}{m} = \frac{2.1}{0.15} = 14\ \text{m/s}^2.$$



The acceleration has a magnitude of

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{76.7^2 + 14^2} = 77.9\,\text{m/s}^2,$$

and its direction relative to the positive x-axis is

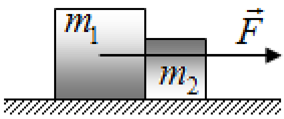
$$\varphi = \arctan\left(a_y / a_x\right) = \arctan\left(14/76.7\right) = 7^0\ .$$

Problem 5

Two blocks of masses m_1=5 kg and m_2=3 kg are place in contact with each other on a frictionless, horizontal surface. A constant force F =20 N is applied to the block m_1 in horizontal direction. (a) Find the magnitude of the acceleration of the system, and (b) the magnitude of the contact force between the two blocks.

Solution

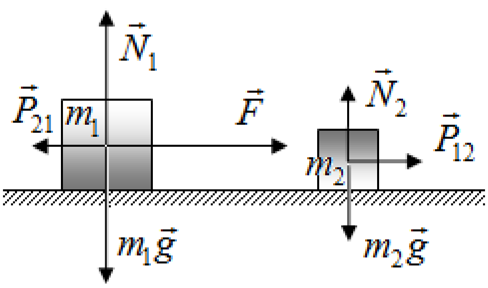
(a) Here we have a force applied to a system consisting of two masses m_1 and m_2.



$$F = M \cdot a = (m_1 + m_2) \cdot a,$$

$$a = \frac{F}{m_1 + m_2} = \frac{20}{5+3} = 2.5\,\text{m/s}^2.$$

(b) The contact force is internal to the system of two blocks. Thus, we cannot find this force by modeling the whole system (the two blocks) as a single particle. We must now treat each of the two blocks individually by categorizing each as a particle subject to a net force. The only horizontal force acting on m_2 is the contact force $\vec{P}_{12}$ (the force exerted by m_1 on m_2), which is directed to the right. Applying Newton's second law to m_2 gives



$$P_{12} = m_2 \cdot a$$

Substituting the value of the acceleration obtained in the previous part into expression for P_{12} gives

$$P_{12} = m_2 a = \left(\frac{m_2}{m_1 + m_2}\right) F = \left(\frac{3}{5+3}\right) 20 = 7.5 \text{ N}.$$

We see from this result that the contact force P_{12} is less than the applied force F. This is consistent with the fact that the force required to accelerate the second block alone must be less than the force required to produce the same acceleration for the two-block system. The horizontal forces acting on m_1 are the applied force $\vec{F}$ to the right and the contact force P_{21} to the left (the force exerted by m_2 on m_1). From Newton's third law, P_{21} is the reaction to P_{12}, so $P_{21} = P_{12}$. Applying Newton's second law to $m1$ gives

$$m_1 a = F - P_{21} = F - P_{12}.$$

Substituting the expression for acceleration, we obtain

$$P_{12} = F - m_1 a = F - m_1 \left(\frac{F}{m_1 + m_2} \right) = \left(\frac{m_2}{m_1 + m_2} \right) F$$

Thus, magnitudes of contact forces are equaled.

Problem 6

The load of mass 1 kg is suspended by the thread. Find the tension $\vec{T}$ *if*

(a) the thread is at rest; (b) the thread is moving downwards at acceleration a *=5 m/s*^{2}*; (c) the thread is moving upwards at acceleration* a *=5 m/s*^{2}*.*

Solution

This problem deals with forces (gravity, tension) and accelerations. That suggests we should apply Newton's Second Law. To apply this law we draw the forces and accelerations for all cases that are under consideration. Since the load has mass, there is gravity $m\vec{g}$. The load is suspended by the thread, so there is the tension $\vec{T}$.

$$m\vec{a} = m\vec{g} + \vec{T}.$$

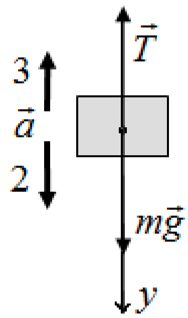
Let y–axis is directed downwards. Then the projections of this equation for different cases are following.

(a) At rest acceleration $\vec{a} = 0$, so

$0 = mg - T$,

$T = mg = 1 \cdot 9.8 = 9.8\,\text{N}$;

(b) Acceleration $\vec{a}$ is directed downwards.



$ma = mg - T$,

$T = m(g - a) = 1(9.8 - 5) = 4.8\,\text{N}$;

(c) Assume that the acceleration $\vec{a}$ is upwards.

$-ma = mg - T$,

$T = m(g + a) = 1(9.8 + 5) = 14.8$ N.

Problem 7

A tension of 6000 N is experienced by the elevator cable of an elevator moving upwards with an acceleration of 2 m/s^2. What is the mass of the elevator?

Solution

From the second Newton's law we have

$m\vec{a} = m\vec{g} + \vec{T}$.

Let’s use the figure to previous problem (the third case) for finding projections

$-ma = mg - T$,

Then

$$m = \frac{T}{g + a} = \frac{6000}{9.8 + 2} = 508\,\text{kg}.$$

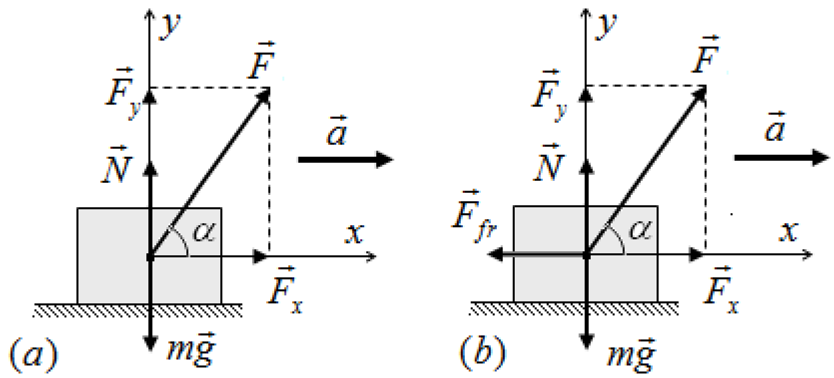
Problem 8

A box is pulled with 20 N force making angle $\alpha = 60^0$ *with horizontal. Mass of the box is 2 kg. Find the acceleration of the box if (a) the surface is frictionless, and (b) coefficient of friction is* $\mu = 0.1$*.*

Solution

We show the forces acting on the box with following free body diagram (FBD). A free body diagram (or force diagram) is a pictorial representation used to analyze the forces acting on a body of interest. Drawing such a diagram can aid in solving for the unknown forces or the equation of motion of the body.

(a) The body is moving under the action of following forces: gravitational force $m\vec{g}$, normal force $\vec{N}$, and force $\vec{F}$.



The equation of motion according the II Newton's Law is

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}.$$

Let's find the projections of these forces on *x*– and *y*–axes:

$$\begin{cases} ma = F_x, \\ 0 = -mg + N + F_y, \end{cases}$$

or

$$ma = F_x = F\cos\alpha.$$

Finally,

$$a = \frac{F\cos\alpha}{m} = \frac{20 \cdot 0.5}{2} = 5\,\text{m/s}^2.$$

(b) When friction has to be taken into account, the equation of motion according the II Newton's Law is

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F} + \vec{F}_{fr}.$$

Projections of this equation on x– and y–axes are

$$\begin{cases} ma = F_x - F_{fr}, \\ 0 = -mg + N + F_y. \end{cases}$$

Since $F_{fr} = \mu N$, and $N = mg - F_y = mg - F \cdot \sin\alpha$, acceleration is

$$a = \frac{F_x - F_{fr}}{m} = \frac{F\cos\alpha - \mu N}{m} = \frac{F\cos\alpha - \mu\left(mg - F \cdot \sin\alpha\right)}{m} = \frac{F\left(\cos\alpha + \mu\sin\alpha\right)}{m} - \mu g.$$

Substituting the numerical values, we'll obtain the acceleration

$$a = \frac{20 \cdot \left(\cos 60^0 + 0.1 \cdot \sin 60^0\right)}{2} - 0.1 \cdot 9.8 = 4.87 \text{ m/s}^2.$$

Problem 9

The body under the effect of applied force F = 10 N is moving according the dependence $s = A - Bt + Ct^2$*, where C = 1 m/s²*. *Find the mass of the body.*

Solution

Let find the acceleration of the body by differentiation of $s = A - Bt + Ct^3$.

$$v = -B + 2Ct,$$

$$a = 2C = 2 \cdot 1 = 2\,\text{m/s}^2.$$

Using the Second Law $\vec{F} = m\vec{a}$, we can find the mass of the body.

$$m = \frac{F}{a} = \frac{10}{2} = 5\text{kg}.$$

Problem 10

A 0.01 kg object is moving in a plane. The x and y coordinates of the object are given by $x(t) = 2t^3 - t^2$ *and* $y(t) = 4t^3 + 2t$. *Find the linear momentum and the net force acting on the object at* *t = 2 s.*

Solution

The velocity of the object may be determined by differentiating of the $x(t)$ and $y(t)$ dependencies.

$$v_x(t) = \frac{dx}{dt} = 6t^2 - 2t\Big|_{t=2} = 6 \cdot 2^2 - 2 \cdot 2 = 20\,\text{m/s},$$

$$v_y(t) = \frac{dy}{dt} = 12t^2 + 2\Big|_{t=2} = 12 \cdot 2^2 + 2 = 50\,\text{m/s}.$$

The magnitude of velocity is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 50^2} = 53.9$ m/s,

the linear momentum is $p = mv = 0.01 \cdot 53.9 = 0.539\,\text{kg}\cdot\text{m/s}$.

To find the net force we need to find acceleration of the object. The acceleration is the second derivative of coordinate of the object. Then the *x* and *y* components of acceleration are given by the following expressions

$$a_x(t) = \frac{d^2x(t)}{dt^2} = 12t - 2\Big|_{t=2} = 12 \cdot 2 - 2 = 22\ \text{m/s}^2,$$

$$a_y(t) = \frac{d^2y(t)}{dt^2} = 24t\Big|_{t=2} = 24 \cdot = 48\,\text{m/s}^2$$

Then the magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{22^2 + 48^2} = 53\,\text{m/s}^2.$$

The net force is

$$F = ma = 0.01 \cdot 53 = 0.53\ \text{N}.$$

Problem 11

A body with a mass of 1.0 kg is accelerated by a force F = 2.0 N. What is velocity of this body after 5.0 s of motion?

Solution

From Newton's Second Law of motion $m\vec{a} = \vec{F}$ we get expression for acceleration

$$a = \frac{F}{m}.$$

Velocity, according to general formula $v = v_0 + at$, where $v_0 = 0$, is

$$v = at = \frac{F \cdot t}{m}.$$

Substituting numbers given in the problem we get

$$v = \frac{2 \cdot 5}{1} = 10 \text{ m/s.}$$

Problem 12

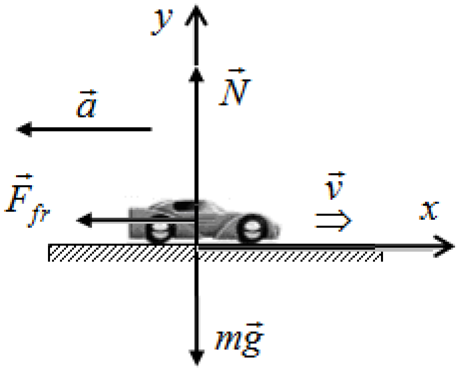
The coefficient of friction between the tires of a car and a horizontal road is 0.55. (a) Find the magnitude of the maximum acceleration of the car when it is braked; (b) What is the shortest distance in which the car can stop if it is initially traveling at 17 m/s? Neglect air resistance and rolling friction.

Solution

(a) During braking the forces acting on the car are: the gravity, the normal force and the friction force. If the velocity of the car is to the right the acceleration is directed to the left. Applying Newton's 2 Law of motion gives

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{fr} .$$

Its x and y projections are



$$ma = F_{fr} ,$$

$$0 = N - mg .$$

Friction force is $F_{fr} = \mu \cdot N = \mu \cdot mg$, therefore,

$$ma = F_{fr} = \mu \cdot mg ,$$

Solving for a and substituting numerical values, we obtain

$$a = \mu \cdot g = 0.55 \cdot 9.8 = 5.39 \text{ m/s}^2.$$

(b) Using a constant-acceleration equation, relate the stopping distance s of the car to its initial speed v_0 and final speed $v = 0$, and its acceleration a:

$$s = \frac{v_0^2 - v^2}{2a} = \frac{v_0^2}{2a} .$$

The stopping distance is

$$s = \frac{17^2}{2 \cdot 5.39} = 26.8 \text{ m}.$$

Problem 13

Two blocks $m_1 = 15 kg$ and $m_2 = 20 kg$ connected by a rope of negligible mass are being dragged by a horizontal force F =70 N. The coefficient of kinetic friction between each block and the surfaces is μ =0.1. Determine the acceleration a of the system and the tension T in the rope.

Solution

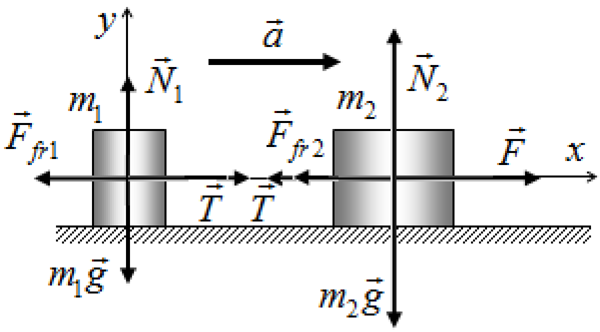
Because the string does not stretch, the blocks will have the same acceleration, and their motions may be described by equations

$$\begin{cases} m_1\vec{a} = m_1\vec{g} + \vec{F}_{fr1} + \vec{N}_1 + \vec{T}, \\ m_2\vec{a} = m_2\vec{g} + \vec{F}_{fr2} + \vec{N}_2 + \vec{T} + \vec{F}. \end{cases}$$

Projections of these equations on the x and y axes, relatively, are given by

$$x: \begin{cases} m_1 a = -F_{fr1} + T, \\ m_2 a = -F_{fr2} - \vec{T} + F. \end{cases}$$

$$y: \begin{cases} 0 = -m_1 g + N_1, \\ 0 = -m_2 g + N_2. \end{cases}$$



Adding x-projections, we obtain

$$a(m_1 + m_2) = F - (F_{fr1} + F_{fr2}).$$

The y-projections gives $N_1 = m_1 g$ and $N_2 = m_2 g$. Taking into account that $F_{fr1} = \mu \cdot N_1 = \mu \cdot m_1 g$, and $F_{fr2} = \mu \cdot N_2 = \mu \cdot m_2 g$, finally we have

$$a = \frac{F - \mu g(m_1 + m_2)}{m_1 + m_2} = \frac{F}{m_1 + m_2} - \mu g = \frac{70}{35} - 0.1 \cdot 9.8 = 1.02 \text{ m/s}^2.$$

The tension determined from the x-projection of the first load equation of motion is equal to

$$T = m_1 a + F_{fr1} = m_1 a + \mu \cdot m_1 g = m_1(a + \mu g) = 15(1.02 + 0.1 \cdot 9.8) = 30 \text{ N}.$$

Problem 14

A block of mass 5 kg is pushed up against a wall by a force $\vec{F}$ that makes angle $\alpha = 40°$ with the horizontal as shown in Figure. The coefficient of static friction between the block and the wall is 0.3. Determine the possible values for the magnitude of F that allow the block to remain stationary.

Solution

According to Newton's 1 Law if this load is at rest the vector sum of the forces applied to it is zero:

$$m\vec{g} + \vec{N} + \vec{F}_{fr} + \vec{F} = 0.$$

Depending on the relationship between the magnitudes of forces $m\vec{g}$ and $\vec{F}$, friction force $\vec{F}_{fr}$ may be directed upwards or downwards preventing the downward and upward motion of the load, respectively. Taking into account that the system is at rest, the projections on the x and y axes are

$$\begin{cases} F_x - N = 0, \\ F_y - mg \pm F_{fr} = 0, \end{cases}$$

$$F_x - N = F_y - mg \pm F_{fr},$$

where $F_x = F \cdot \cos\alpha$, $F_y = F \cdot \sin\alpha$.

Since

$$F(fr) = \mu \cdot N = \mu \cdot F_x = \mu \cdot F \cdot \cos\alpha,$$

$$F \cdot \sin\alpha - mg \pm \mu \cdot F \cdot \cos\alpha = 0$$

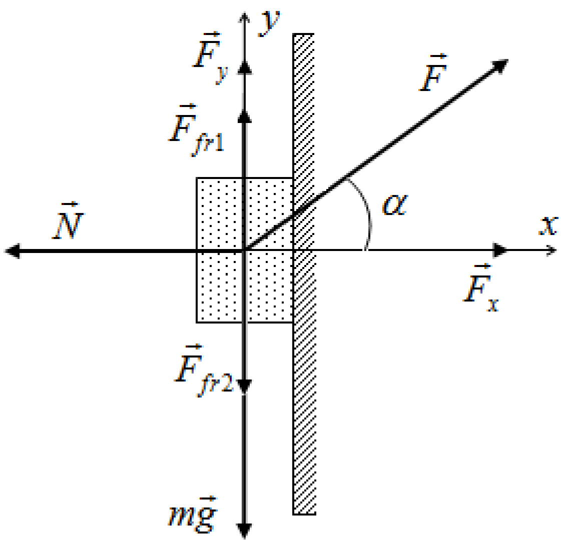
$$F(\sin\alpha \pm \mu \cdot \cos\alpha) = mg,$$

$$F = \frac{mg}{\sin\alpha \pm \mu \cdot \cos\alpha}$$

$$F = \frac{5 \cdot 9.8}{\sin 40^0 \pm 0.3 \cdot \cos 40^0}.$$

$$F_1 = 57.0 \text{ N},$$

$$F_2 = 116.7 \text{ N}.$$



The possible values for the magnitude of F that allow the block to remain stationary are $F_{min} = 57$ N and $F_{max} = 116.7$ N.

Problem 15

A box is placed on a plane with slope angle α = 4^0. (a) What is the static coefficient of friction needed for this box begins to move?(b) Find an acceleration of the box if the coefficient of kinetic friction is μ = 0.03. What time does it take to *the box to cover the distance 100 m. What is its velocity in the terminal point of motion?*

Solution

Friction forces act between two bodies which are in contact but not moving or sliding with respect to each other. The friction in such case is static friction, and the force of static friction is $F_{fr} = \mu_s \cdot N$, where μ_s is the coefficient of static friction.

Newton's II Law for the box motion is

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{fr}$.

Projections on chosen x and y axes of the equation are

$$\begin{cases} ma = mg\sin\alpha - F_{fr}, \\ 0 = -mg\cos\alpha + N. \end{cases}$$

(a) For the first case we have to find the coefficient of static friction μ_s. From the second equation of the system $N = mg\cos\alpha$, and from the first equation

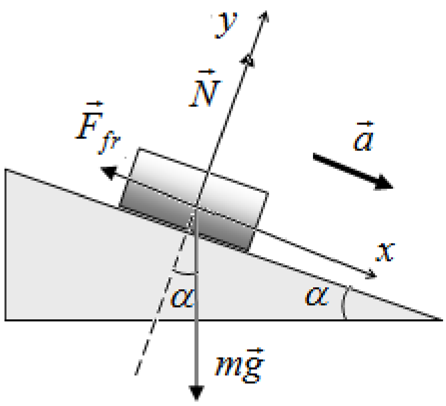
$$ma = mg\sin\alpha - \mu_s \cdot N = mg\sin\alpha - \mu_s \cdot mg\cos\alpha = mg\left(\sin\alpha - \mu_s \cdot \cos\alpha\right).$$

Taking into account that $a = 0$, we obtain

$$0 = g\left(\sin\alpha - \mu_s \cdot \cos\alpha\right).$$

Therefore,

$$\mu_s = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \tan 5^\circ = 0.08.$$



(b) When two surfaces are moving with respect to one another, the friction force depends on the coefficient of kinetic friction μ_k. The coefficient of kinetic friction is typically smaller than the coefficient of static friction. Consequently, the box is sliding along the incline at acceleration

$$a = g(\sin\alpha - \mu_k \cdot \cos\alpha) = 9.8\left(0.087 - 0.02 \cdot 0.996\right) = 0.66 \text{ m/s}^2.$$

Since the object is moving at acceleration from the rest, the time of motion and the terminal speed may be found by means of the kinematical equations

$$\begin{cases} s = v_0 t + \dfrac{at^2}{2} \\ v = v_0 + at \end{cases}$$

Taking into account that $v_0 = 0$, we obtain

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \cdot 10}{0.66}} = 5.5 \text{ s},$$

$$v = at = 0.66 \cdot 5.5 = 3.6 \text{ m/s}.$$

Problem 16

A box is sliding up an incline that makes an angle of 20 degrees with respect to the horizontal. The coefficient of kinetic friction between the box and the surface of the incline is 0.2. The initial speed of the box at the bottom of the incline is 2 m/s. How far does the box travel along the incline before coming to rest?

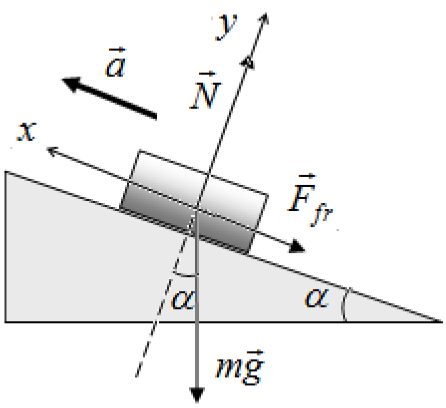
Solution

The first part in the problem is to find an acceleration of the motion. Newton's II Law for the box motion is

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{fr},$$

or, in projections on x and y axes

$$\begin{cases} ma = -mg\sin\alpha - F_{fr}, \\ 0 = -mg\cos\alpha + N. \end{cases}$$



Since

$$N = mg\cos\alpha,$$

the second equation of the system is

$ma = -mg\sin\alpha - F_{fr} = -mg\sin\alpha - \mu \cdot N = -mg\sin\alpha - \mu \cdot mg\cos\alpha$.

Than the acceleration is

$a = -g(\sin\alpha + \mu\cos\alpha) = -9.8(0.342 + 0.2 \cdot 0.94) = -5.2\,\text{m/s}^2$.

The second part is to write down the kinematic equations of motion. In this problem we need to use the relation between the travelled distance and initial and final (the final velocity is 0) velocities:

$v_0 = 2as$,

where s is the travelled distance.

Then

$$s = \frac{v_0^2}{2a} = \frac{4}{2 \cdot 5.2} = 0.32\,\text{m}.$$

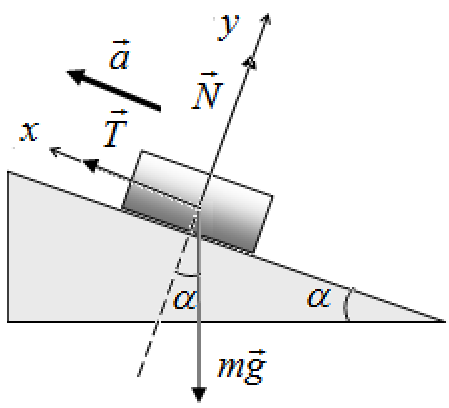
Problem 17

A 10.0 kg block is towed up an inclined at α=30^0 with respect to the horizontal. The rope is parallel to the incline and has a tension of 100 N. Assume that the block starts from rest at the bottom of the hill, and neglect friction. How fast is the block going after moving 40 m up the hill?

Solution

To find the speed of the block we need to find the acceleration of the block.

The motion of the block is described by the Second Law



$m\vec{a} = m\vec{g} + \vec{N} + \vec{T}$.

Projections on chosen axes are

$$\begin{cases} ma = T - mg\sin\alpha, \\ 0 = -mg\cos\alpha + N. \end{cases}$$

The acceleration is given by the first equation of the system $a = \dfrac{T - mg\sin\alpha}{m} = \dfrac{100 - 10\cdot 9.8\cdot \sin 30^0}{10} = 5.1\text{m/s}^2.$

The kinematic equations that describe the accelerated motion of the block are

$$\begin{cases} s = v_0 t + \dfrac{at^2}{2}, \\ v = v_0 + at. \end{cases}$$

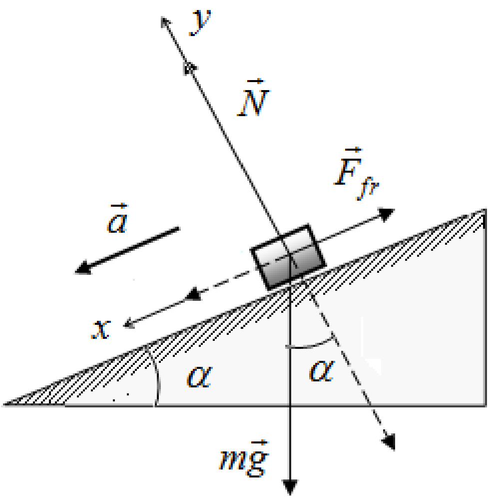
Taking into account that $v_0 = 0$ and excluding time of motion, we obtain relationship between acceleration and travelled distance

$$v = \sqrt{2as} = \sqrt{2\cdot 5.1\cdot 40} = 20.2 \text{ m/s}.$$

Problem 18

A car is going at a speed of v_0 = 25.2 km/h when it encounters a 150 m long slope of angle 30º. The friction coefficient between the road and the tyre is 0.3. Show that no matter how hard the driver applies the brakes; the car will reach the bottom with a speed greater than 100 km/h. Take g =10 m/s².

Solution



The forces acting on the car during downward accelerated motion are: the gravity, normal force and friction force.

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{fr}\ .$$

Projections on x and y axes are

$$\begin{cases} ma = mg\cdot\sin\alpha - F_{fr}, \\ 0 = N - mg\cdot\cos\alpha. \end{cases}$$

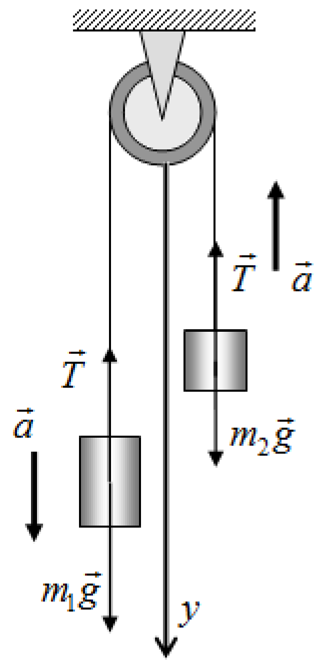
The brake, even the hardest one, will produce the friction force $F_{fr} = \mu \cdot N = \mu \cdot mg \cdot \cos\alpha$, thus the acceleration is

$$a = g(\sin\alpha - \mu \cdot \cos\alpha) = 10\left(\sin 30^0 - 0.3 \cdot \cos 30^0\right) = 2.4 \text{ m/s}^2.$$

The initial speed of the car is $v_0 = 25.2$ km/h = 7 m/s; the covered distance is $s = 150$ m. Using the expression from kinematics $v^2 = v_0^2 + 2as$, we obtain the final speed of the car as

$$v = \sqrt{v_0^2 + 2as} = \sqrt{7^2 + 2 \cdot 2.4 \cdot 150} = 27.7 \text{ = 100 km/h}$$

Problem 19



Two blocks of masses 2 kg and 1 kg are connected by the inextensible rope passing over a small frictionless fixed pulley. If the rope and the pulley are weightless, find the acceleration $\vec{a}$ of the blocks and tension $\vec{T}$ of the rope.

Solution

The acceleration of the block 1 is equal by magnitude to the acceleration of the other block but they are directed oppositely. If the tension of the rope is *T*, then the motion of the blocks are following:

$$\begin{cases} m_1\vec{a} = m_1\vec{g} + \vec{T} \\ m_2\vec{a} = m_2\vec{g} + \vec{T} \end{cases}$$

If the axis *y* is directed downwards, the projections of these equations are

$$\begin{cases} m_1 a = m_1 g - T \\ -m_2 a = m_2 g - T \end{cases}$$

Consequently, the acceleration and tension are

$$a = \frac{g(m_1 - m_2)}{m_1 + m_2} = \frac{9.8(2-1)}{2+1} = 3.27 \text{ m/s}^2,$$

$$T = m_1(g-a) = m_1 g\left(1 - \frac{m_1 - m_2}{m_1 + m_2}\right) = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2\cdot 2\cdot 1\cdot 9.8}{2+1} = 13.07 \text{ N}.$$

Problem 20

A block of mass m_1 =4 kg on the inclined plane of angle α = 30^0 (coefficient of friction μ = 0.1) is connected by a rope over a pulley to another block of mass m_2 =1 kg. What are the magnitude and direction of the acceleration of the second block?

Solution

According to Newton's Second Law the equations of the blocks motion are

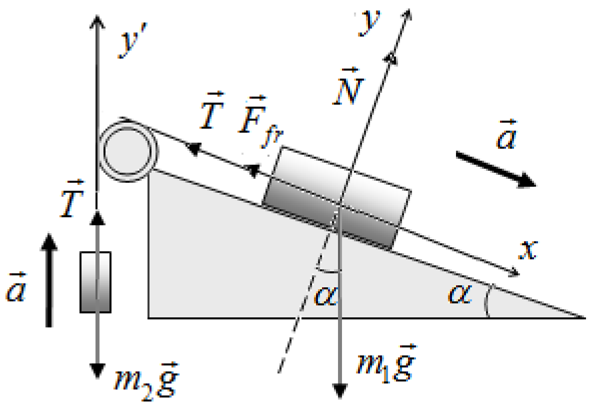
$$\begin{cases} m_1\vec{a} = m_1\vec{g} + \vec{N} + \vec{T} + \vec{F}_{fr}, \\ m_2\vec{a} = m_2\vec{g} + \vec{T}. \end{cases}$$

Let's assume that block m_2 will move with acceleration $\vec{a}$ directed upwards.

If our assumptions are wrong, the calculated value of acceleration will be negative.

The projections on the *x*, *y*, and y' axis are following:

$$\begin{cases} m_1 a = m_1 g\sin\alpha - T - F_{fr}, \\ 0 = -m_1 g\cos\alpha + N, \\ m_2 a = -m_2 g + T. \end{cases}$$



Let's express the normal force from the second equation of system $N = m_1 g\cos\alpha$ and tension from the third equation $T = m_2 g + m_2 a$, and substitute them to the first equation taking into account that $F_{fr} = \mu \cdot N$:

$$m_1 a = m_1 g \sin\alpha - m_2 a - m_2 g - \mu \cdot m_1 g \cos\alpha$$

After a little of elementary algebra we get

$$a = g\,\frac{m_1 \sin\alpha - m_2\left(1 + \mu \cdot \cos\alpha\right)}{m_1 + m_2}.$$

Substituting numbers given in the problem we get

$$a = 9.8 \cdot \frac{4 \cdot 0.5 - 1\left(1 + 0.1 \cdot 0.866\right)}{4 + 1} = 1.79\,\text{m/s}^2.$$

The plus sign tells us, that acceleration has direction to the one chosen by us for writing equations leading to the solution of the problem. This is a general rule in all kinds of problems. Negative numerical value means the direction of the parameter found is opposite to the one which was assumed for writing equations.

Problem 21

Two masses m_1 *= 1 kg and* m_2 *= 10 kg are on inclines (*α *=50º and* β *=30º) and are connected together by a string as shown in the figure. The coefficient of kinetic friction between each mass and its incline is* μ *= 0.25. If* m_1 *moves up, and* m_2 *moves down, determine their acceleration.*

Solution

We define the positive x and x' directions to be the directions of motion for each block. Newton's 2 law for both objects are

$$m_1\vec{a} = m_1\vec{g} + \vec{N}_1 + \vec{T} + \vec{F}_{fr1},$$

$$m_2\vec{a} = m_2\vec{g} + \vec{N}_2 + \vec{T} + \vec{F}_{fr2}\ .$$

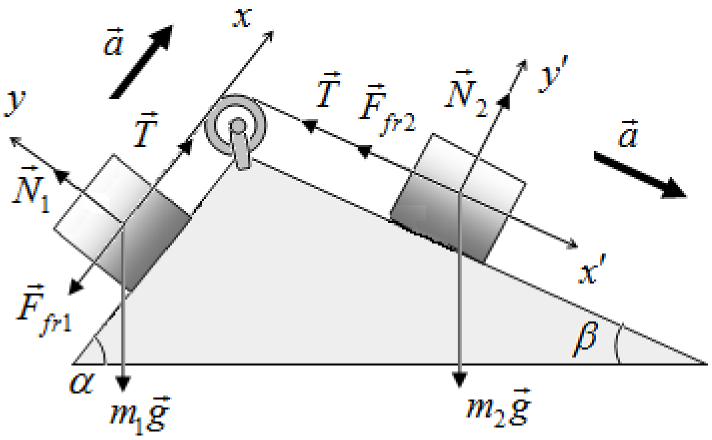
Projections on the chosen axes are

$$x:\ m_1 a = T - m_1 g \cdot \sin\alpha - F_{fr1},$$

$$x': m_2 a = m_2 g \cdot \sin\beta - T - F_{fr2},$$

$$y:\ 0 = -m_1 g \cdot \cos\alpha + N_1,$$

$$y':\ 0 = -m_2 g \cdot \cos\beta + N_2.$$



The friction forces with regard to the last equations are

$$F_{fr1} = \mu \cdot N_1 = \mu \cdot m_1 g \cdot \cos\alpha,$$

$$F_{fr2} = \mu \cdot N_2 = \mu \cdot m_2 g \cdot \cos\beta.$$

Now add the x and x' projections, substitute the friction forces, and solve for acceleration

$$a = g \cdot \frac{m_2(\sin\beta - \mu \cdot \cos\beta) - m_1(\sin\alpha + \mu \cdot \cos\alpha)}{m_1 + m_2}.$$

Substitute the numerical values and obtain

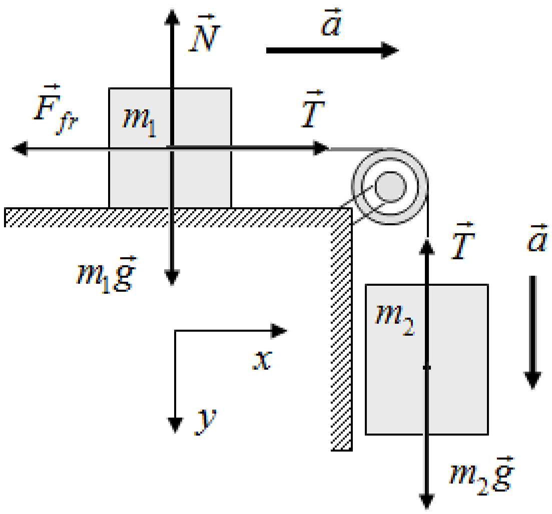
$$a = 9.8 \cdot \frac{10 \cdot (\sin 30 - 0.25 \cdot \cos 30) - 1 \cdot (\sin 50 + 0.25 \cdot \cos 50)}{1 + 10} = 2.67 \text{ m/s}^2.$$

Problem 22

A 2-kg load placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging 3-kg load. (a) Find the acceleration of the two loads and the tension in the string. (b) Solve the problem if the coefficient of kinetic friction between the first load and the table is $k = 0.1$.

Solution

(a) Since we neglect the masses of the cable and the pulley, and the pulley is frictionless, the magnitudes of tension T at both ends of the cable are the same. Load m_2 accelerates downward with magnitude a. The load m_1 connected by the string to the first load moves at the same acceleration to the right. The equations according to the Newton's 2 Law for each load are



$$\begin{cases} m_1\vec{a} = \vec{T} + \vec{N} + m_1\vec{g}, \\ m_2\vec{a} = \vec{T} + m_2\vec{g}. \end{cases}$$

The projections of these equations on the x and y axes are

$$\begin{cases} m_1 a = T, \\ 0 = m_1 g - N, \\ m_2 a = m_2 g - T. \end{cases}$$

Substituting the tension T from the first equation to the third equation gives

$$m_2 a = m_2 g - m_1 a.$$

It follows that

$$a = \frac{m_2 g}{m_1 + m_2} = \frac{3 \cdot 9.8}{2+3} = 5.88\,\text{m/s}^2.$$

(b) If the friction is between the first load and the surface of the table, the equations of motion are given by

$$\begin{cases} m_1\vec{a} = \vec{T} + \vec{N} + m_1\vec{g} + \vec{F}_{fr}, \\ m_2\vec{a} = \vec{T} + m_2\vec{g}. \end{cases}$$

As a consequence, the projections on the on the x and y axes are

$$\begin{cases} m_1 a = T - F_{fr}, \\ 0 = m_1 g - N, \\ m_2 a = m_2 g - T. \end{cases}$$

Allow for the fact that $F_{fr} = \mu \cdot N = \mu \cdot m_1 g$, we obtain

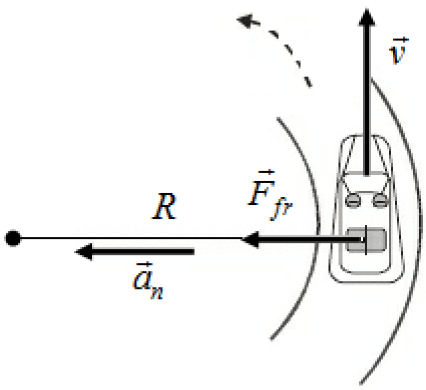
$$m_2 a = m_2 g - m_1 a - F_{fr} = m_2 g - m_1 a - \mu \cdot m_1 g ,$$

$$a = \frac{(m_2 - k \cdot m_1) \cdot g}{m_1 + m_2} = \frac{(3 - 0.1 \cdot 2) \cdot 9.8}{2 + 3} = 5.49 .$$

Problem 24

A car of the mass m *=1000 kg moves on a flat, horizontal road negotiates along the circular path. If the radius of the curve is 40 m and the coefficient of static friction between the tires and dry pavement is 0.55, find the maximum speed the car can have and still make the turn successfully.*

Solution



The force that enables the car to remain in the circular way is the force of static friction. (Static, because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero - for example, if the car were on an icy road -the car would continue in a straight line and slide off the road.) The maximum speed the car can have around the curve is the speed at which it is on the verge of skidding outward.

According Newton’s 21 Law the equation of the car’s motion is

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{fr} , \qquad (1)$$

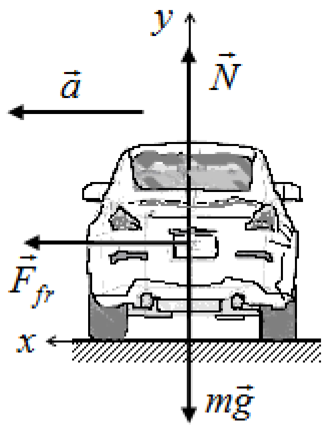
and x and y projections of (1) are

$$ma = F_{fr} , \qquad (2)$$

$$0 = N - mg .$$

The maximum friction force is $F_{fr} = \mu \cdot N = \mu \cdot mg$. Since the car moves along the curvilinear path its normal (or centripetal) acceleration is $a_n = v^2/R$, and equation (2) is given by

$$\frac{mv^2}{R} = \mu \cdot mg\ .$$



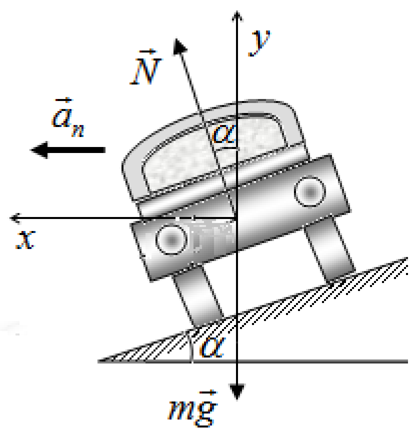
The maximum speed of the car is determined by the maximum friction force is equal to

$$v = \sqrt{\mu \cdot R \cdot g} = \sqrt{0.55 \cdot 40 \cdot 9.8} = 14.7 \text{ m/s} = 52.9 \text{ km/h}.$$

Problem 25

The designated speed for the ramp is to be 13.4 m/s; the radius of the curve is 40 m. At what angle should the curve be banked?

Solution



The curved parts of the roads have to be designed in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually banked; this means the roadway is tilted toward the inside of the curve.

On a level (unbanked) road the force that causes the centripetal acceleration is the force of static friction between car and road, as we saw in the previous Problem. However, if the road is banked at an angle α, the equation of the motion according Newton's 2 Law is

$$m\vec{a} = m\vec{g} + \vec{N},$$

Its x and y projections are

$ma = N \cdot \sin\alpha$,

$0 = N \cdot \cos\alpha - mg$.

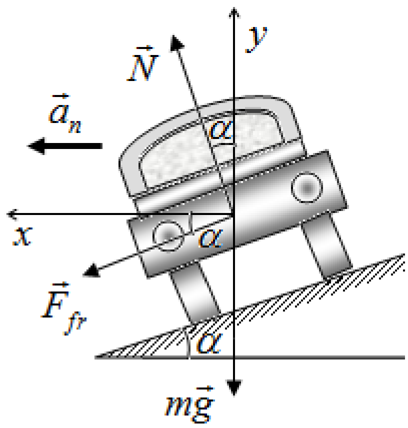
Now, the normal force $\vec{N}$ has a horizontal component $N\sin\alpha$, pointing toward the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $N\sin\alpha$ causes the centripetal acceleration.

$$m\frac{v^2}{R} = N \cdot \sin\alpha = \frac{mg \cdot \sin\alpha}{\cos\alpha} = mg \cdot \tan\alpha,$$

$$\tan\alpha = \frac{v^2}{gR}.$$

$$\alpha = \arctan\frac{v^2}{gR} = \arctan\frac{17^2}{9.8 \cdot 40} = \arctan 0.74 = 36.4^0.$$

Problem 26



The freeway off-ramp is circular with 60-m radius. The off-ramp has a slope $\alpha = 15^0$. If the coefficient of static friction between the tires of a car and the road is $\mu = 0.55$, what is the maximum speed at which it can enter the ramp without losing traction?

Solution

The equation that describes the motion of the car according to Newton's 2 Law is

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{fr},$$

and its x and y projections are

$$\begin{cases} ma = N \cdot \sin\alpha + F_{fr}\cos\alpha, \\ 0 = N \cdot \cos\alpha - mg - F_{fr} \cdot \sin\alpha. \end{cases}$$

Since the friction force is $F_{fr} = \mu \cdot N$, these equations takes the form

$$\begin{cases} ma = N \cdot \sin\alpha + \mu \cdot N \cdot \cos\alpha, \\ mg = N \cdot \cos\alpha - \mu \cdot N \cdot \sin\alpha, \end{cases}$$

$$\begin{cases} ma = N \cdot \left(\sin\alpha + \mu \cdot \cos\alpha\right), \\ mg = N \cdot \left(\cos\alpha - \mu \cdot \sin\alpha\right). \end{cases}$$

Dividing the first equation of the system by the second one gives

$$\frac{\cancel{m}a}{\cancel{m}g} = \frac{\cancel{N} \cdot \left(\sin\alpha + \mu \cdot \cos\alpha\right)}{\cancel{N} \cdot \left(\cos\alpha - \mu \cdot \sin\alpha\right)},$$

Finally, taking into account that $a = a_n = \dfrac{v^2}{R}$, we obtain

$$v = \sqrt{gR \cdot \left(\frac{\sin\alpha + \mu \cdot \cos\alpha}{\cos\alpha - \mu \cdot \sin\alpha}\right)} \text{ m/s}$$

Substitution of the numerical values gives the maximum speed

$$v = \sqrt{\frac{9.8 \cdot 60 \cdot \left(\sin 15^0 + 0.55 \cdot \cos 15^0\right)}{\cos 15^0 - k \cdot \sin 15^0}} = 23.9 \text{ m/s} = 85.9 \text{ km/h}.$$

Problem 27

The car is moving along the convex bridge with radius of curvature R =100 m at the speed v =36 km/h. Find the force of its pressure on the middle of the bridge? Find the force of pressure on concave bridge of the same radius of curvature.

Solution

Since the car moves along the curvilinear path it has normal (centripetal) acceleration

$$a = a_n = \frac{v^2}{R}.$$

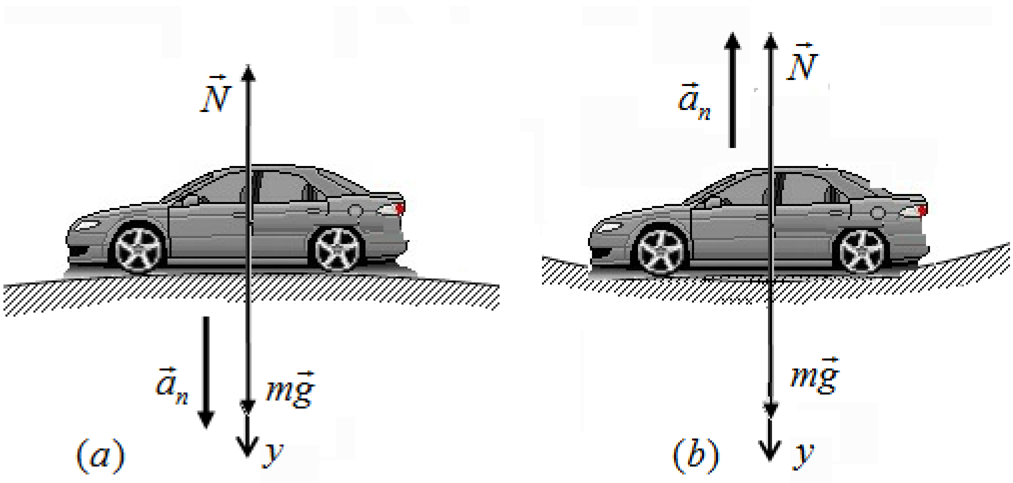
Newton's 2 Law gives the following equation of the car's motion $m\vec{a} = m\vec{g} + \vec{N}$.

According to Newton's 3 Law the magnitude of the force of car's pressure on the bridge is equal to the magnitude of the normal force *N*.

If the *y*–axis is directed downwards, the projections of the equation are:

(a) for the case of the convex bridge

$$ma_n = mg - N$$



$$F = N = m(g - a_n) = m\left(g - \frac{v^2}{R}\right) = 5\cdot10^3\left(9.8 - \frac{10^2}{100}\right) = 4.4\cdot10^4\text{N}.$$

(b) for the case of concave bridge

$$-ma_n = mg - N$$

$$F = N = m(g + a_n) = m\left(g + \frac{v^2}{R}\right) = 5\cdot10^3\left(9.8 + \frac{10^2}{100}\right) = 5.4\cdot10^4\ \text{N}.$$

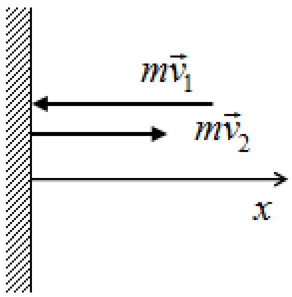
Problem 28

A steel ball of mass m=10 g moving at a speed v=100 m/s along the normal to the wall hits it and bounces at the same speed. Find the linear momentum obtained by the wall.

Solution

According to the law of conservation of linear momentum for the "ball – wall" closed (isolated) system the magnitude of the linear momentum obtained by the wall (p_w) is equal to the magnitude of ball's linear momentum increment (Δp_b):

$$\vec{p}_w = \Delta\vec{p}_b = \vec{p}_2 - \vec{p}_1 = m\vec{v}_2 - m\vec{v}_1.$$



The projection of this equation on x – axis is

$$p_w = mv_2 - (-mv_1) = mv_2 + mv_1.$$

Since $v_1 = v_2 = v$, therefore,

$$p_w = 2mv = 2\cdot 10^{-2}\cdot 10^2 = 2\,\text{kg·m/s}.$$

Problem 29

(a) What is the impulse of a force of 10 N acting on a ball for 2 seconds?

(b) A 2 kg-ball is initially at rest. What is the velocity of the ball after the force has acted on it?

Solution

(a) The definition of impulse is force over a time, so we have to do a simple calculation:

$$F\Delta t = 10\cdot 2 = 20\ \text{N·s}.$$

(b) Recall that an impulse causes a change in linear momentum. Because the particle starts with zero velocity, it initially has a zero momentum. Thus:

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\frac{\Delta v}{\Delta t},$$

$$F\Delta t = m\Delta v = mv_2 - mv_1 = mv_2,$$

$$v_2 = \frac{F\Delta t}{m} = \frac{20}{2} = 10\,\text{m/s}.$$

Thus the ball has a final velocity of 10 m/s. This problem is the simplest form of the impulse–momentum theorem.

Problem 30

A 3–kg particle has a velocity of $\left(3\vec{i} - 4\vec{j}\right)$ *m/s. Find its x and y components of linear momentum and the magnitude of its total momentum.*

Solution

Using the definition of momentum and the given values of mass m and velocity we have:

$$\vec{p} = m\vec{v} = 3\left(3\vec{i} - 4\vec{j}\right) = \left(9\vec{i} - 12\vec{j}\right)\ \text{kg·m/s}.$$

So the particle has momentum components

$p_x = +\,9$ kg·m/s, and $p_y = -12$ kg·m/s.

The magnitude of its momentum is

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{9^2 + (-12)^2} = 15\ \text{kg·m/s}.$$

Problem 31

A child bounces a superball on the sidewalk. The linear impulse delivered by the sidewalk is 2.00 N·s during the $1.25\cdot10^{-3}$ *s of contact. What is the magnitude of the average force exerted on the ball by the sidewalk.*

Solution

The magnitude of the change in momentum of the ball (*or* impulse delivered to the ball) is $|\Delta p| = 2.00$ N·s. The direction of the impulse is upward, since the initial momentum of the ball was downward and the final momentum is upward.

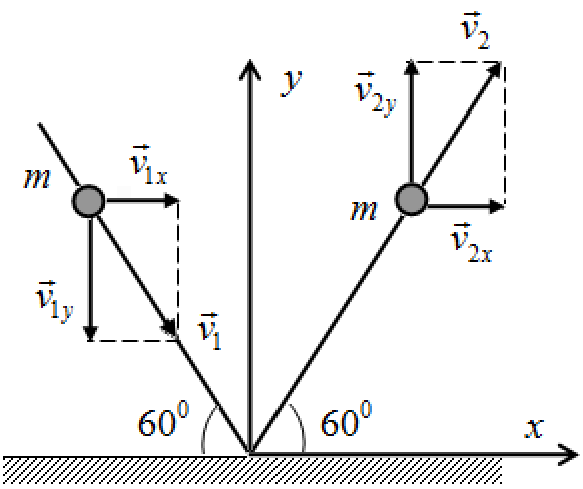
Since the time over which the force was acting was $\Delta t = 1.25 \cdot 10^{-3}$ s then from the definition of average force we get:

$$F = \frac{|\Delta p|}{\Delta t} = \frac{2}{1.25 \cdot 10^{-3}} = 1.6 \cdot 1.25 \cdot 10^{3} \text{ N.}$$

Problem 32

A 3.0 kg steel ball strikes a wall with a speed of 10 m/s at an angle of 60^0 with the surface. It bounces off with the same speed and angle, as shown in Figure. If the ball is in contact with the wall for 0.20 s, what is the average force exerted on the wall by the ball.

Solution



The average force is defined as $\vec{F} = \frac{d\vec{p}}{dt}$, so firstly we find the change in momentum of the ball. Since the ball has the same speed before and after bouncing from the wall, it is clear that its x–velocity (see the coordinate system in Figure) stays the same and so the x–momentum stays the same. But the y–momentum does change. The initial y–velocity is

$$v_{1y} = -10 \cdot \sin 60^0 = 8.7\,\text{m/s},$$

and the final y–velocity is

$$v_{2y} = 10 \cdot \sin 60^0 = +8.7\ \text{m/s}.$$

So the change in *y*–momentum is

$$\Delta p_y = mv_{2y} - mv_{1y} = m\left(v_{2y} - v_{1y}\right) = 3\left[8.7 - \left(-8.7\right)\right] = 52\,\text{kg}\cdot\text{m/s}.$$

The average y force on the ball is

$$F = \frac{\Delta p_y}{\Delta t} = \frac{52}{0.2} = 2.6 \cdot 10^2\,\text{N}.$$

Since F has no *x*-component, the average force has magnitude $2.6 \cdot 10^2\,\text{N}$ and points in the *y*-direction (away from the wall).

Problem 33

A 2 kg-ball is thrown straight up into the air with an initial velocity of 10 m/s. Using the impulse-momentum theorem, calculate the time of flight of the ball (g =10 m/s²).

Solution

Once the ball is thrown up, it is acted on by a constant force $m\vec{g}$. This force causes a change in momentum until the ball has reversed directions, and lands with the velocity of 10 m/s. Thus we can calculate the total change in momentum:

$$\Delta p = p_2 - p_1 = mv_2 - mv_1 = 2 \cdot \left(10\right) - 2\left(-10\right) = 40\,\text{kg}\cdot\text{m/s}.$$

Now we turn to the impulse–momentum theorem to find the time of flight:

$$F\Delta t = mg\Delta t = \Delta p.$$

Thus, taking $g = 10\,\text{m/s}^2$,

$$\Delta t = \frac{\Delta p}{mg} = \frac{40}{2 \cdot 10} = 2\,\text{s}.$$

The ball has a time of flight of 2 seconds.

The kinematics gives that the height of the ball is

$$h = \frac{v_0^2}{2g} = \frac{100}{2 \cdot 10} = 5\text{m}.$$

The velocity at the top point is $v = 0$, therefore, $v = v_0 - gt$ gives

$0 = v_0 - gt$ and $v_0 = gt$.

Then $h = v_0 t - \frac{gt^2}{2} = gt^2 - \frac{gt^2}{2} = \frac{gt^2}{2}$.

Finally

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 5}{10}} = 1\,\text{s}.$$

The time of upward motion is equal to the time of downward motion, then the time of flight is 2 s.

We obtain same result. But the calculation using impulse-momentum theorem was much easier than the one using kinematic equations.

Problem 34

Machine gun fires 35.0 g bullets at a speed of 750 m/s. If the gun can fire 200 bullets/min, what is the average force the shooter must exert to keep the gun from moving?

Solution

The gun interacts with the bullets; it exerts a brief, strong force on each of the bullets which in turn exerts an “equal and opposite” force on the gun. The gun’s force changes the bullet's momentum from zero (as they are initially at rest) to the final value of

$$p = p_2 - p_1 = p_2 = 0.035 \cdot 750 = 26.2 \text{ kg·m/s}.$$

So this is also the change in momentum for each bullet: $p = \Delta p$.

Now, since 200 bullets are fired every minute (60 s), we should count the interaction time as the time to fire one bullet,

$$\Delta t = \frac{60}{200} = 0.3\,\text{s},$$

because every 0.30 s, a firing occurs again, and the average force that we compute will be valid for a length of time for which many bullets are fired. So the average force of the gun on the bullets is

$$F = \frac{\Delta p}{\Delta t} = \frac{26.2}{0.3} = 87.5\,\text{N}.$$

From Newton's Third Law, there must an average backwards force of the bullets on the gun of magnitude 87.5 N. If there were no other forces acting on the gun, it would accelerate backward. To keep the gun in place, the shooter (or the gun's mechanical support) must exert a force of 87.5 N in the forward direction.

We can also work with the numbers as follows. In one minute, 200 bullets were fired, and a total momentum of

$$p_{total} = 200 \cdot 26.2 = 5.24 \cdot 10^3\,\text{kg·m/s}$$

was imparted to them. So during this time period (60 seconds) the average force on the whole set of bullets was

$$F = \frac{\Delta p_{total}}{\Delta t} = \frac{5.24 \cdot 10^3}{60} = 87.5\,\text{N}.$$

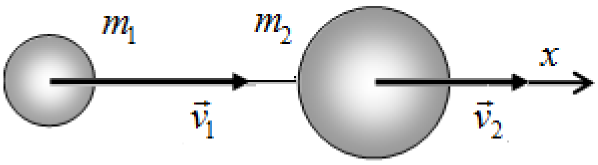
As before, this is also the average backwards force of the bullets on the gun and the force required to keep the gun in place.

Problem 35

A 2 kg ball $\left(m_1\right)$ *with a velocity of* v_1*=3 m/s collides head on in an elastic manner with an 8 kg ball* $\left(m_2\right)$ *with a velocity* v_2 *=1 m/s. What are the velocities after the collision?*

Solution

If the head on (central) and elastic collision takes place the laws of conservation of linear momentum and energy are carried out:



$$\begin{cases} m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2, \\ \dfrac{m_1\vec{v}_1^{\,2}}{2} + \dfrac{m_2\vec{v}_2^{\,2}}{2} = \dfrac{m_1\vec{u}_1^{\,2}}{2} + \dfrac{m_2\vec{u}_2^{\,2}}{2}, \end{cases}$$

$$\begin{cases} m_1(\vec{v}_1 - \vec{u}_1) = m_2(\vec{u}_2 - \vec{v}_2), \\ \left[m_1(\vec{v}_1 - \vec{u}_1)\right](\vec{v}_1 + \vec{u}_1) = \left[m_2(\vec{u}_2 - \vec{v}_2)\right](\vec{u}_2 + \vec{v}_2) \end{cases}$$

Therefore,

$$\vec{v}_1 + \vec{u}_1 = \vec{u}_2 + \vec{v}_2.$$

After two mathematical operations: multiplication of $\vec{v}_1 + \vec{u}_1 = \vec{u}_2 + \vec{v}_2$ by m_2 and subtraction of the product from $m_1(\vec{v}_1 - \vec{u}_1) = m_2(\vec{u}_2 - \vec{v}_2)$; and multiplication of $\vec{v}_1 + \vec{u}_1 = \vec{u}_2 + \vec{v}_2$ by m_1 and addition of the result to $m_1(\vec{v}_1 - \vec{u}_1) = m_2(\vec{u}_2 - \vec{v}_2)$, we obtain velocities of the balls after collision:

$$\vec{u}_1 = \frac{2m_2\vec{v}_2 + \left(m_1 - m_2\right)\vec{v}_1}{m_1 + m_2},$$

$$\vec{u}_2 = \frac{2m_1\vec{v}_1 + \left(m_2 - m_1\right)\vec{v}_2}{m_1 + m_2}.$$

The projection of the velocities on *x*–axis and substitution of the data gives:

$$u_1 = \frac{2\cdot 8\cdot 1 + \left(2-8\right)\cdot 3}{2+8} = -0,2 \text{ m/s},$$

$$u_2 = \frac{2\cdot 2\cdot 3 + (8-2)\cdot 1}{2+8} = 1,8 \text{ m/s}.$$

The sign “minus” in the first expression means that due to collision the first ball moves in opposite direction, meanwhile the second ball moves in its previous direction.

Problem 36

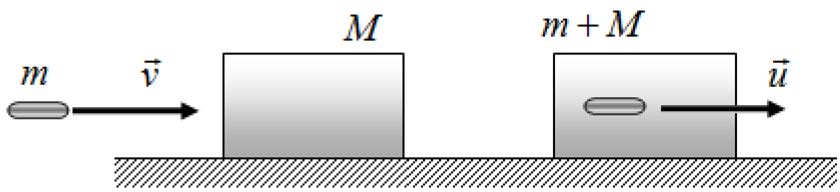
A 10.0 g bullet is stopped in a block of wood (m = 5.00 kg). The speed of the “bullet–wood” combination immediately after the collision is 0.600 m/s. What was the original speed of the bullet?

Solution

A picture of the collision just before and after the bullet (quickly) embeds itself in the wood is given in Figure. The bullet has some initial speed v (we don’t know what it is.)

The collision (and embedding of the bullet) takes place very rapidly; for that brief time the bullet and block essentially form an isolated system because any external forces (say, friction) will be of no importance compared to the enormous forces between the bullet and the block. So the total momentum of the system will be conserved; it is the same before and after the collision.

Just before the collision, only the bullet (with mass m) is in motion and its velocity is $\vec{v}$. So the initial momentum is $\vec{p} = m\vec{v}$. Just after the collision, the



“bullet–block” combination, with its mass of $m+M$ has a velocity $\vec{u}$. So the final momentum is

$$\vec{p}' = (m+M)\vec{u}.$$

In this problem there is only motion along the horizontal axis, so we only need the condition that the total momentum along this axis is conserved:

$\vec{p} = \vec{p}'$.

Consequently,

$mv = (m + M)u$.

The speed of the bullet was

$$v = \frac{(m+M)u}{m} = \frac{(10^{-2}+5)\cdot 0.6}{10^{-2}} = 301\,\text{m/s}.$$

Problem 37

The platform of mass m_1=10 tonnes is on the rails. The gun of mass m_2=5 tonnes fixed on the platform shoot by the shell of mass m_3=100 kg. The shot is directed along the rails. Find the velocity of the platform with the gun immediately after the shot if the velocity of the shell is v_0 =500 m/s respectively the gun. Solve the problem if the platform with gun on it at the instant of firing (a) was at rest; (b) moved at the velocity v = 18 km/h and the shot was made in the direction o f its motion; and (c) moved at the velocity v = 18 km/h and the shot was made in the opposite direction.

Solution

The solution of this problem is based on the law of conservation of linear momentum for the closed system consisting of the platform$(\vec{p}_1)$, the gun$(\vec{p}_2)$, and the shell$(\vec{p}_3)$. The linear momentum of the system is vector sum of the linear momenta of the objects of the system.

(a) The linear momentum of the system before the shot (the platform was at rest $v = 0$) is

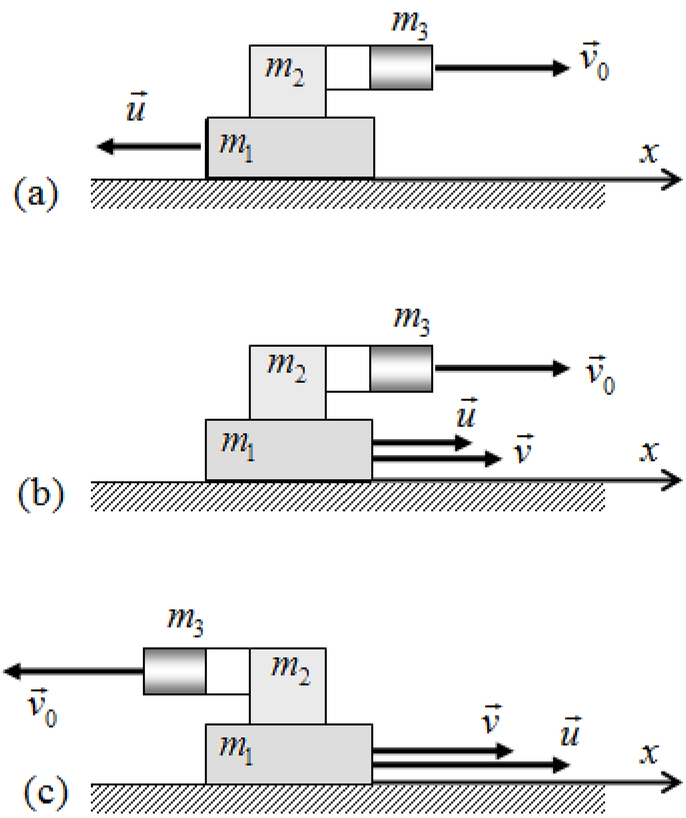
$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = m_1\vec{v} + m_2\vec{v} + m_3\vec{v} = (m_1 + m_2 + m_3)\vec{v} = 0.$$

The linear momentum of the system after the shot

$$\vec{p}' = \vec{p}_1' + \vec{p}_2' + \vec{p}_3' = m_1\vec{u} + m_2\vec{u} + m_3\vec{v}_0 = (m_1 + m_2)\vec{u} + m_3\vec{v}_0.$$

According the law of conservation of linear momentum $\vec{p} = \vec{p}'$, consequently,

$$0 = (m_1 + m_2)\vec{u} + m_3\vec{v}_0.$$



After projecting this equation on x–axis we obtain

$$0 = -(m_1 + m_2)u + m_3 v_0.$$

We believe that the motion of the platform with the gun ($\vec{u}$) after firing occurs in the opposite direction relatively the motion of the shell ($\vec{v}_0$) we have chosen the sign "minus" for this velocity.

$$(m_1 + m_2)u = m_3 v_0$$

$$u = \frac{m_3 v_0}{m_1 + m_2} = \frac{100 \cdot 500}{10000 + 5000} = 3,33 \text{ m/s}.$$

In the cases when we are not sure as for direction of the motion after the change of the linear momentum, we can choose any sign of the velocity, but the result of calculation shows the correctness of our choice.

(b) If the platform is moving at the speed $v = 18$ km/h = 5 m/s, the linear momentum of the system before shooting is distinct from zero, and the law of conservation is

$$(m_1 + m_2 + m_3)\vec{v} = (m_1 + m_2)\vec{u} + m_3(\vec{v}_0 + \vec{v}).$$

Then the projection of the equation on x–axis is

$$(m_1 + m_2 + m_3)v = -(m_1 + m_2)u + m_3(v_0 + v).$$

The speed of the platform with the gun is

$$u = \frac{m_3\left(v_0 + v\right) - (m_1 + m_2 + m_3)v}{m_1 + m_2} =$$

$$= \frac{10^2 \cdot (500 + 5) - (10^5 + 5000 + 10^2) \cdot 5}{10^5 + 5000} = -1,67 \text{ m/s.}$$

In our mathematical consideration we have supposed that the platform after shooting was moving in the opposite direction relatively its previous direction, but the negative result of calculation shows that it is moving in the same direction but at smaller speed.

(c) The law of the conservation of linear momentum is the same as in the second case, but the projection on *x*–axis is

$$(m_1 + m_2 + m_3)v = \left(m_1 + m_2\right)u + m_3\left(-v_0 + v\right).$$

The speed of the platform with the gun is

$$u = \frac{-m_3\left(-v_0 + v\right) + (m_1 + m_2 + m_3)v}{m_1 + m_2} =$$

$$\frac{-10^2(-500 + 5) + (10^5 + 5000 + 10^2) \cdot 5}{10^5 + 5000} = -8.33 \text{ m/s.}$$

We see that the platform is moving in the same direction as before shooting.

Problem 38

The man of mass m_1*=60 kg running at the velocity* v_1*=2 m/s jumps onto the carriage of mass* m_2 *=80 kg moving at the velocity* v_2 *=1 m/s. Find the velocity of the carriage with the man if (a) the man catches up the carriage; and (b) the man moves towards the carriage.*

Solution

The law of the conservation of linear momentum is

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{u}$$

(a) If the man catches up the carriage, the velocities of both objects are of the same directions, therefore, the projection on *x*–axis is

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)u,$$

and the velocity of the carriage with man is

$$u = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{60 \cdot 2 + 80 \cdot 1}{60 + 80} = 1.43 \text{ m/s.}$$

(b) If the man moves towards the carriage, the velocities have the different signs. The projection of the equation of the law of conservation of linear momentum is

$$m_1 v_1 - m_2 v_2 = (m_1 + m_2)u,$$

and the velocity of the carriage with man is

$$u = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2} = \frac{60 \cdot 2 - 80 \cdot 1}{60 + 80} = 0.29 \text{ m/s.}$$

The carriage is moving in the direction of previous motion of the man.

Problem 39

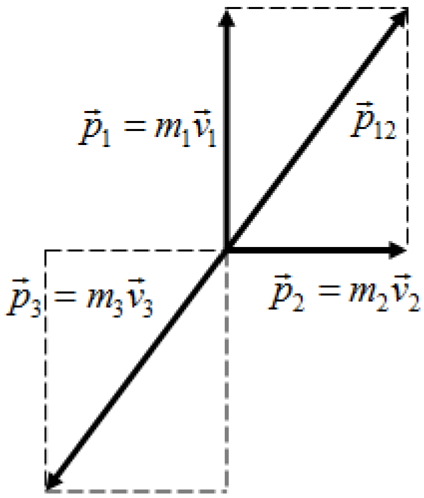
The shell that has been shot upwards blew up at the highest point of its trajectory. The first fragment of mass m_1 *= 1 kg has the horizontal velocity* v_1 *=400 m/s. The second fragment of mass* m_2 *=1.5 kg has the vertical velocity* v_2 *=200 m/s. Find the velocity of the third fragment of mass* m_3 *=2 kg.*

Solution

As the shell has been blew up in the highest point of trajectory, its velocity, and thus the linear momentum before the burst was zero. According to the law of conservation of linear momentum the vector sum of the momenta after the burst has to be zero as well. Therefore,

$$0 = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 .$$

Let's find the linear momenta of the first and the second fragments



$$p_1 = m_1 v_1 = 1 \cdot 400 = 400 \text{ kg·m/s},$$

$$p_2 = m_2 v_2 = 1{,}5 \cdot 200 = 300 \text{ kg·m/s}$$

and add them graphically (see Figure).

The vector sum $\vec{p}_{12} = \vec{p}_1 + \vec{p}_2$ has to be equal to the linear momentum of the third fragment in magnitude and opposite in direction, i.e., $p_{12} = p_3$.

Since

$$p_{12} = \sqrt{p_1^2 + p_2^2} = \sqrt{400^2 + 300^2} = 500 \text{ kg·m/s},$$

$$p_3 = m_3 v_3 = p_{12},$$

and

$$v_3 = \frac{p_{12}}{m_3} = \frac{500}{2} = 250 \text{ kg·m/s}.$$

Problem 40

A particle of mass 2 kg moves under the action of the constant force $\vec{F} = \left(5\vec{i} - 2\vec{j}\right)$*N. If its displacement is* $6\vec{j}$ *m. What is the work done by this force?*

Solution

The work done by force is

$$A = \left(\vec{F}, \vec{s}\right) = \left(5\vec{i} - 2\vec{j}\right)\left(6\vec{j}\right) = -12 \text{ J}.$$

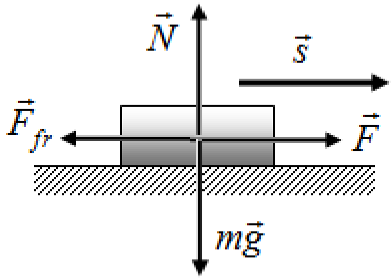
Problem 41

A 40 kg box initially at rest is pushed 5.0 m along a rough horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between the box and floor is 0.30, find (a) the work done by the applied force, (b) the energy lost due to friction, (c) the change in kinetic energy of the box, and (d) the final speed of the box.

Solution

(a) The motion of the box and the forces which do work on it are shown in Figure. The (constant) applied force $\vec{F}$ points in the same direction as the displacement $\vec{s}$. Our formula for the work done by a constant force gives

$$A=\left(\vec{F},\vec{s}\right)=F\cdot s\cdot\cos\alpha=130\cdot5\cdot\cos0^0=6.5\cdot10^2\text{ J.}$$



The applied force does $6.5\cdot10^2$ J of work.

(b) Figure shows all forces acting on the box.

The vertical forces acting on the box are gravity ($m\vec{g}$, downward) and the floor's normal force ($\vec{N}$, upward). It follows that $N=mg$ and so the magnitude of the friction force is

$$F_{fr}=\mu N=\mu mg=0.3\cdot40\cdot9.8=1.2\cdot10^2\text{ N.}$$

The friction force is directed opposite the direction of motion (α = 180^0) and so the work that it does is

$$A_{fr}=\left(\vec{F}_{fr},s\right)=F_{fr}s\cos180^0=1.2\cdot10^2\cdot5\cdot\left(-1\right)=-5.9\cdot10^2\text{ J.}$$

or we might say that $5.9\cdot10^2$ J is *lost* to friction.

(c) Since the normal force and gravity do no work on the box as it moves, the net work done is

$$A_{net}=A+A_{fr}=6.5\cdot10^2-5.9\cdot10^2=62\text{ J.}$$

By the Work–Kinetic Energy Theorem, this is equal to the change in kinetic energy of the box:

$\Delta W_k = W_{k2} - W_{k1} = A_{net} = 62\,\mathrm{J}.$

(d) Here, the initial kinetic energy W_{k1} was zero because the box was initially at rest. So we have $W_{k2} = 62\,\mathrm{J}$. From the definition of kinetic energy $W_k = \dfrac{mv^2}{2}$, we get the final speed of the box: $v = \sqrt{\dfrac{2W_k}{m}} = \sqrt{\dfrac{2\cdot 62}{40}} = \sqrt{3.1} = 1.8\,\mathrm{m/s}.$

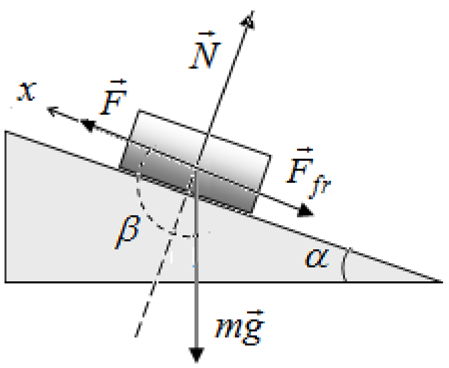
Problem 42

A crate of mass 10 kg is pulled up a rough incline with an initial speed of 1.5 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of $\alpha = 20^0$ *with the horizontal. The coefficient of kinetic friction is 0.4, and the crate is pulled 5 m. (a) How much work is done by gravity?(b) How much energy is lost due to friction?(c) How much work is done by the 100 N force? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5 m.*

Solution

(a) Let calculate the work done by gravity using in the definition: $A = \left(m\vec{g}, \vec{d}\right)$.

The magnitude of the gravity force is $mg = 10\cdot 9.8 = 98\,\mathrm{N}$ and the displacement has magnitude 5 m. We see from geometry (see Figure) that the angle between the force and displacement (along x–axis) vectors is 110^0. Then the work done by gravity is



$A_{gr} = mgd\cos\beta = 98\cdot 5\cos 110^0 = -168\,\text{J}.$

(b) To find the work done by friction, we need to know the force of friction. The forces on the block are shown in Figure. As we have seen before, the normal force between the slope and the block is $N = mg\cos\alpha = mg\cos 20^0$ so as to cancel the normal component of the force of gravity. Then the force of kinetic friction on the block points down the slope (opposite the motion) and has magnitude

$F_{fr} = \mu\cdot N = \mu\cdot mg\cdot\cos 20^0 = 0.4\cdot 10\cdot 9.8\cdot 0.94 = 36.8$ N.

This force points exactly opposite the direction of the displacement d, so the work done by friction is

$A_{fr} = F_{fr}\cdot d\cdot\cos 180^0 = 36.8\cdot 5\cdot(-1) = -184\,\text{J}.$

(c) The 100 N applied force pulls in the direction up the slope, which is along the direction of the displacement *d*. So the work that is does is

$A_{appl} = \left(\vec{F},\vec{d}\right) = F\cdot d\cdot\cos 0^0 = 100\cdot 5\cdot 1 = 500\,\text{J}.$

(d) We have now found the work done by each of the forces acting on the crate as it moved: gravity, friction and the applied force. (We should note the normal force of the surface also acted on the crate, but being perpendicular to the motion, it did no work.) The net work done was:

$A_{net} = A_{gr} + A_{fr} + A_{appl} = -168 - 184 - 500 = 148\,\text{J}.$

From the work–energy theorem, this is equal to the change in kinetic energy of the box: $\Delta W_k = A_{net} = 148\,\text{J}.$

(e) The initial kinetic energy of the crate was $W_{k1} = \dfrac{mv_1^2}{2} = \dfrac{10\cdot 1.5^2}{2} = 11.2$ J.

If the final speed of the crate is v_2, then the change in kinetic energy was:

$$\Delta W_k = W_{k2} - W_{k1} = \frac{mv_2^2}{2} - W_{k1}.$$

Using our answers from previous parts $\Delta W_k = A_{net} = 148\,\text{J}$ and $W_{k1} = 11.2\,\text{J}$, we get:

$$v_2 = \sqrt{\frac{2(\Delta W_k + W_{k1})}{m}} = \sqrt{\frac{2(148+11.2)}{10}} = 5.64\,\text{m/s}.$$

The final speed of the crate is 5.64 m/s.

Problem 43

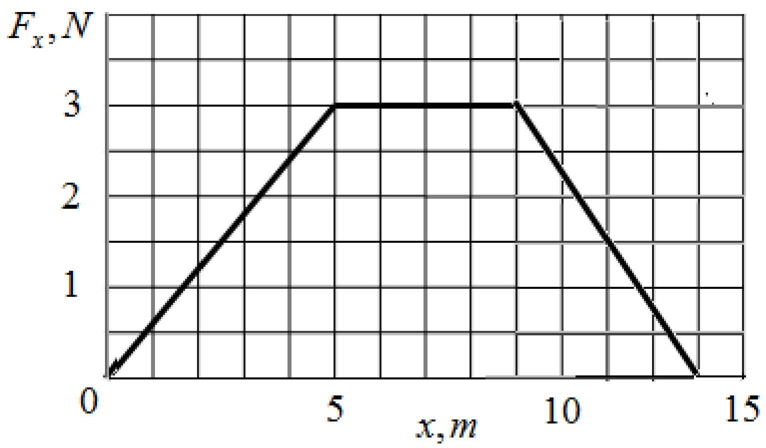
A particle is subject to a force F_x that varies with position. Find the work done by the force on the body as it moves (a) from x =0 to x =5 m,(b) from x = 5 m to x = 10 m and (c) from x = 10 m to x = 14 m. (d) What is the total work done by the force over the distance x = 0 to x = 14 m?

Solution

Since the force depends on time, the work done when a particle moves along a straight line

$$W = \int_{x_1}^{x_2} F_x dx.$$

Geometrically this is just the “area under the curve” of $F_x(x)$ from x_1 to x_2.



(a) The figure (from x = 0 to x = 5.0 m) is the triangle which “area” is equal to the half of a rectangle of base 5.0m and height 3.0 N. So the work done is

$$W_1 = \frac{3 \cdot 5}{2} = 7.5\,\text{J}.$$

It is important that when we evaluate the “area”, we just keep the units which go along with the base and the height; here they were meters and Newtons, the product of which is a Joule.

So the work done by the force for this displacement is 7.5 J.

(b) The region under the curve from x = 5 m to x = 10 m is a full rectangle of base 5 m and height 3 N. The work done for this movement of the particle is

$W_2 = 3\cdot 5 = 15$ J.

(c) For the movement from x = 10 m to x = 14 m the region under the curve is a half rectangle of base 5 m and height 3 N. The work done is

$$W_3 = \frac{3\cdot 4}{2} = 6 \text{ J.}$$

(d) The total work done over the distance x = 0 to x = 14 m is the sum of the three separate “areas”,

$W = W_1 + W_2 + W_3 = 7.5 + 15 + 6 = 28.5$J.

Problem 44

What work is done by a force $\vec{F} = \left(2x\vec{i} + 3\vec{j}\right)N$*, with* x *in meters, that moves a particle from position* $\vec{r}_1 = \left(2\vec{i} + 3\vec{j}\right)m$ *to position* $\vec{r}_2 = \left(-4\vec{i} - 3\vec{j}\right)m$*?*

Solution

According to the general definition of work (for a two-dimensional problem),

$$W = \int_{x_1}^{x_2} F_x\left(\vec{r}\right)dx + \int_{y_1}^{y_2} F_y\left(\vec{r}\right)dy \,.$$

If $F_x = 2x$ and $F_y = 3$(we mean that the force is in Newtons when x is in meters, and the work will come out in Joules), we obtain

$$W = \int_2^{-4} 2x\,dx + \int_3^{-3} 3dy = x^2 \Big|_2^{-4} + 3x \Big|_3^{-3} = (16-4) + (-9-9) = -6\text{J.}$$

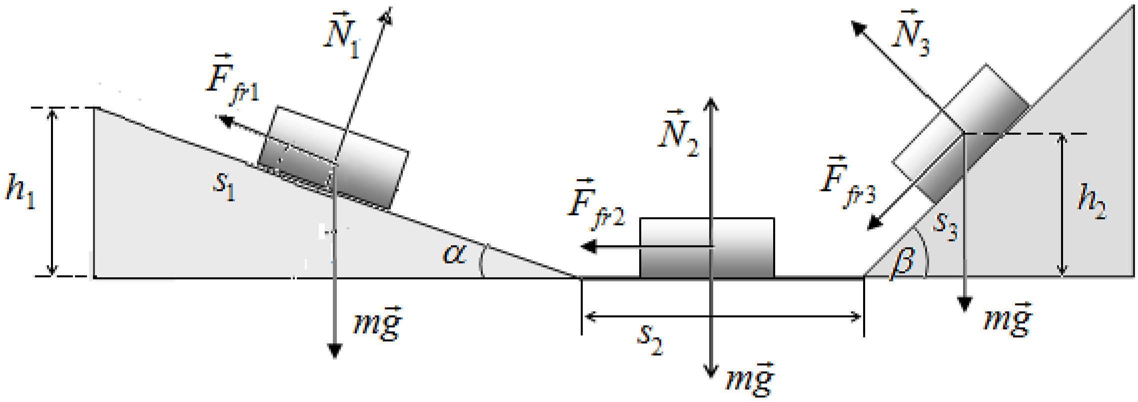
Problem 45

The sledge after motion down from the hill of height h_1 =10 m and slope angle α =30^0, has covered 3 m of horizontal way, and begins to move upwards along the incline with angle β = 45^0. If the coefficient of friction doesn't vary along all sectors of motion and equals μ = 0.1, find the height h_2 of the sledge ascent along the hill.

Solution

According to the law of conservation of energy the potential energy of the sledge (W_p) at its initial point is expended in kinetic energy (W_{k1}) and the work (A_{fr1}) done by friction force.

$$W_p = W_{k1} + A_{fr1}.$$



Due to the kinetic energy (W_{k1}) the sledge has a possibility of motion along the horizontal way, at that, a part of this energy is expended in the work (A_{fr2}) of friction force

$$W_{k1} = A_{fr2} + W_{k2} \ .$$

And finally, the kinetic energy W_{k2} allows the ascent of the sledge to the height h_2, with that,

$W_{k2} = A_{fr_3} + W_{p2}$.

Combining all previous equations, we obtain

$W_p = A_{fr1} + A_{fr2} + A_{fr_3} + W_{p2}$.

Friction force at the first segment of the distance is equal to $F_{fr1} = \mu \cdot N_1 = \mu \cdot mg \cdot \cos\alpha$, at the second segment: $F_{fr2} = \mu \cdot N_2 = \mu \cdot mg$, and at the third segment: $F_{fr3} = \mu \cdot N_3 = \mu \cdot mg \cdot \cos\beta$. Accordingly, the works of friction forces are:

$A_{fr1} = \mu \cdot mg \cdot \cos\alpha \cdot s_1$, $A_{fr2} = \mu \cdot mg \cdot s_2$, and $A_{fr3} = \mu \cdot mg \cdot \cos\beta \cdot s_3$.

Taking into account that

$$s_1 = \frac{h_1}{\sin\alpha},\ s_2 = 3\text{m},\ s_3 = \frac{h_2}{\sin\beta},$$

we write down the equation

$$mgh_1 = \mu \cdot mg \cdot \cos\alpha \cdot s_1 + \mu \cdot mg \cdot s_2 + \mu \cdot mg \cdot \cos\beta + mg \cdot h_2,$$

$$h_1 = \mu\left(\cos\alpha \cdot s_1 + s_2 + \cos\beta \cdot s_3\right) + h_2,$$

$$\frac{h_1}{\mu} = \cos\alpha \cdot \frac{h_1}{\sin\alpha} + s_2 + \cos\beta \cdot \frac{h_2}{\sin\beta} = \frac{h_1}{\tan\alpha} + s_2 + \frac{h_2}{\tan\beta} + \frac{h_2}{\mu},$$

$$\frac{h_1}{\mu} - \frac{h_1}{\tan\alpha} - s_2 = \frac{h_2}{\tan\beta} + \frac{h_2}{\mu},$$

$$h_2 = \frac{h_1\left(\tan\alpha - \mu\right) - \mu \cdot s_2 \cdot \tan\alpha}{\tan\beta + \mu} \cdot \frac{\tan\beta}{\tan\alpha} =$$

$$= \frac{10\left(0.577 - 0.1\right) - 0.1 \cdot 3 \cdot 0.577}{1 + 0.1} \cdot \frac{1}{0.577} = 7.24\,\text{m}.$$

Problem 46

A particle has linear momentum of 10 kg·m/s, and a kinetic energy of 25 J. What is the mass of the particle?

Solution

Recall that kinetic energy and momentum are related according to the following equations: $W_k = \frac{mv^2}{2}$ and $p = mv$.

Since $v = \frac{p}{m}$, then $W_k = \frac{p^2}{2m}$. Solving for m we see that

$$m = \frac{p^2}{2W_k} = \frac{100}{2 \cdot 25} = 2\,\text{kg}.$$

From our knowledge of energy and momentum we can state the mass of the ball from these two quantities. This method of finding the mass of a particle is commonly used in particle physics, when particles decay too quickly to be massed, but when their momentum and energy can be measured.

Problem 47

A 2 kg bouncy ball is dropped from a height of 10 meters, hits the floor and returns to its original height. What was the change in momentum of the ball upon impact with the floor? What was the impulse provided by the floor?

Solution

To find the change in momentum of the ball we must find first the velocity of the ball just before it hits the ground. To do so, we must rely on the conservation of mechanical energy. The ball was dropped from a height h = 10 meters, and so had a potential energy of *mgh*. This energy is converted completely to kinetic energy by the time the ball hits the floor. Thus:

$$\frac{mv^2}{2} = mgh,$$

$$v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 10} = 14 \text{ m/s}.$$

Thus the ball hits the ground with a velocity of 14 m/s.

The same argument can be made to find the speed with which the ball bounced back up. When the ball is at ground level, all of the energy of the system is kinetic energy. As the ball bounces back up, this energy gets converted to gravitational potential energy. If the ball reaches the same height it was dropped from, then, we can deduce that the ball leaves the ground with the same speed with which it hit the ground, though in a different direction. Thus the change in momentum,

$$\Delta p = p_2 - p_1 = mv_2 - mv_1 = 2(14) - 2(-14) = 56 \text{ kg·m/s}.$$

The ball's momentum changes by 56 kg·m/s.

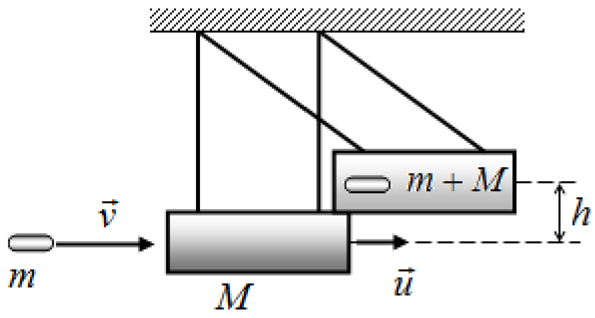
We are next asked to find the impulse provided by the floor. By the impulse–momentum theorem, a given impulse causes a change in momentum. Since we have already calculated our change in momentum, we already know our impulse. It is simply 56 kg·m/s.

Problem 48

A bullet of mass m = 3 g, travelling at a velocity of v = 500 m/s, imbeds itself in the wooden block of a ballistic pendulum. If the wooden block has a mass of M = 0.3 kg, to what height does the “bullet–block” combination rise?

Solution

Since the bullet and the block “stick together” after the collision, this is an example of a perfectly inelastic collision. Momentum is conserved but kinetic energy is not.



Conserve momentum:

$$mv + Mv_{block} = (m+M)u.$$

Since $v_{block}=0$,

$$mv = (m+M)u$$

$$u = \frac{mv}{m+M}.$$

After the collision energy is conserved as long as non–conservative forces do not act on the system, hence:

$$W_k = W_p,$$

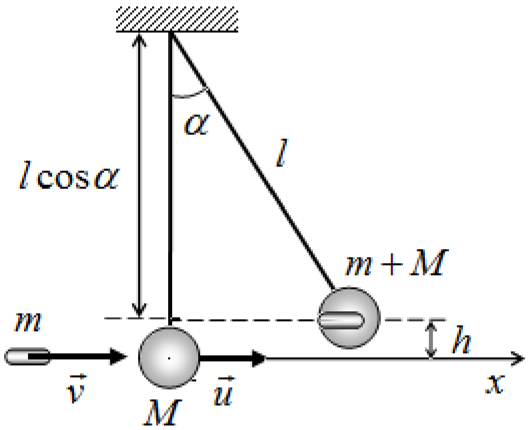
$$\frac{(m+M)u^2}{2} = (m+M)gh,$$

$$h = \frac{u^2}{2g} = \frac{(mv)^2}{2g(m+M)^2} = \frac{(3\cdot 10^{-3}\cdot 500)^2}{2\cdot 9.8\cdot (3\cdot 10^{-3}+0.3)^2} = 1.25\,\text{m}.$$

Problem 49

The bullet moving horizontally hits the sphere suspended by weightless rigid rod. The bullet mass m is 1000 times less than the sphere mass M. The distance between the sphere centre and the pivot point is l =1 m. Find the velocity of the bullet if the rod makes an angle α =10^0 with vertical.

Solution



We'll use the laws of conservation of energy and linear momentum for the closed system "sphere–bullet" taking into account that bullet hitting the sphere is the example of inelastic collision of two objects the result of which is the motion of two interacted objects as a whole:

$$m\vec{v}+M\vec{V}=\left(m+M\right)\vec{u},$$

where $\vec{v}$ and $\vec{V}$ are the velocities of the bullet and the sphere before collision, and $\vec{u}$ is the velocity of the sphere with the bullet inside it after the collision.

As the sphere was at rest before collision, i.e. $V=0$, the projection of this equation on horizontal *x*–axis is

$$mv=\left(m+M\right)u,$$

$$u=\frac{mv}{m+M}.$$

Using the law of conservation of energy

$$\frac{\left(m+M\right)u^2}{2}=\left(m+M\right)gh.$$

As $h=l-l\cos\alpha=l\left(1-\cos\alpha\right)$, then $u=\sqrt{2gl\left(1-\cos\alpha\right)}$ and

$$v=\frac{\left(m+M\right)u}{m}=\frac{\left(m+M\right)\sqrt{2gl\left(1-\cos\alpha\right)}}{m}.$$

Taking into account that $M=1000m$ we obtain

$$v=1001\sqrt{2gl\left(1-\cos\alpha\right)}=1001\sqrt{2\cdot 9.8\cdot 1\left(1-\cos 10^\circ\right)}=546 \text{ m/s}.$$

Problem 50

The skater of mass M = 70 kg standing on the ice throws a stone of m =3 kg in a horizontal direction with a speed v = 8 m/s. Find the distance of recoil if the coefficient of friction between the ice and the skates μ *= 0.02.*

Solution

This problem may be solved by two methods based on dynamics and kinematics, and on conservation laws. Let’s consider the two.

(a) The net linear momentum of the isolated (closed) system "skater–stone" is conserved, hence.

$$\left(M + m\right)\vec{v}_0 = M\vec{u} + m\vec{v}.$$

Firstly, the skater and the stone were at rest; consequently, the linear momentum of the system was equal to zero. After the fling of the stone, the skater begins to move in the opposite side respectively to the direction of stone motion, so the equation in projections on x–axis:

$$0 = -Mu + mv,$$

and the skater speed is

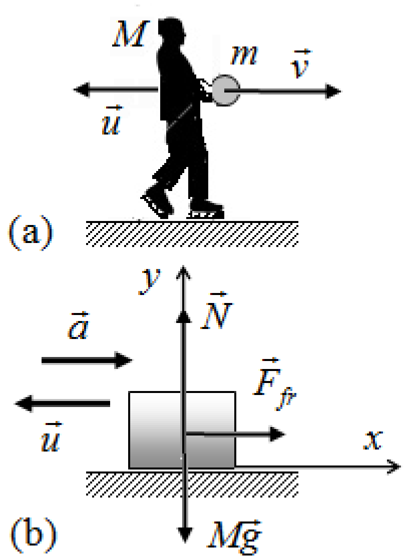
$$u = \frac{mv}{M}.$$

According to the Second Law the equation of the skater motion is

$$M\vec{a} = M\vec{g} + \vec{N} + \vec{F}_{fr},$$

and x and y projections:

$$\begin{cases} Ma = F_{fr}, \\ 0 = N - Mg. \end{cases}$$



Solving this system we can find

$$a = \mu \cdot g.$$

The decelerated skater motion before his stop may be described by kinematic equations

$$\begin{cases} v = 0 = u - at, \\ s = ut - \dfrac{at^2}{2}. \end{cases}$$

Travelled distance is

$$s = \frac{u^2}{2a} = \frac{u^2}{2\mu g} = \frac{m^2 v^2}{2\mu M^2 g} = \frac{9 \cdot 64}{2 \cdot 0.02 \cdot 4900 \cdot 9.8} = 0.3.$$

(b) Using the law of conservation of linear momentum the skater speed is $u = \dfrac{mv}{M}$ (see above).

According the law of conservation of energy, the kinetic energy of the skater is wasted on the work against the friction force

$$A_{fr} = \Delta W_k ,$$

$$A_{fr} = \left(\vec{F}_{fr}, \vec{s}\right) = F_{fr} \cdot s \cdot \cos\alpha = -F_{fr} \cdot s ,$$

where $\cos\alpha = -1$, as the friction force is directed opposite to the skater displacement.

The increment of kinetic energy is

$$\Delta W_k = 0 - \frac{Mu^2}{2} = -\frac{Mu^2}{2} .$$

Then

$$-F_{fr} \cdot s = -\frac{Mu^2}{2} .$$

The distance until the stop is

$$s = \frac{Mu^2}{2F_{fr}} = \frac{Mu^2}{2\mu \cdot Mg} = \frac{u^2}{2\mu g} = \frac{m^2 v^2}{2\mu \cdot M^2 g} = \frac{9 \cdot 64}{2 \cdot 0.02 \cdot 4900 \cdot 9.8} = 0.3 .$$

We obtained the same result as by the first method.

Problem 51

The tangential force F =100 N is applied to the rim of the homogeneous disc of R = 0.2 m. At the rotation the frictional torque M_{fr} =5N·m acts on disc. Find the disc mass if its acceleration is ε =100 rad/s^2.

Solution

The disc motion is described by the Second Law for rotation

$$I\vec{\varepsilon} = \vec{M} ,$$

where I – is a moment of inertia of disc, ε – its angular acceleration, M – net torque of the forces applied to disc.

Moment of inertia of disc respectively the axis passing through its center of mass is

$$I = \frac{mR^2}{2}.$$

Since vectors of the angular acceleration and driving torque are directed along the axis of rotation, and the frictional torque in opposite direction, the net torque is

$$M = M_F - M_{fr} = F \cdot R - M_{fr}.$$

The Second Law for rotation is

$$\frac{mR^2\varepsilon}{2} = F \cdot R - M_{fr}.$$

Thus the disc mass is

$$m = \frac{2(F \cdot R - M_{fr})}{R^2\varepsilon} = \frac{2(100 \cdot 0.2 - 5)}{0.04 \cdot 100} = 7.5 \text{ kg}.$$

Problem 52

If a wheel has moment of inertia of 5 kg·m^2, (a) what angular speed does the wheel attain if 10^5 Joule of work is done in producing rotational kinetic energy? (b) What torque is required to bring the wheel to rest in 25 seconds?

Solution

(a) The work was done for change in rotational kinetic energy, consequently,

$$A = \Delta W_k = W_{k2} - W_{k1} = W_{k2} = \frac{I\omega^2}{2},$$

as $W_{k1} = 0$.

An angular velocity of the disc is

$$\omega = \sqrt{\frac{2A}{I}} = \sqrt{\frac{2 \cdot 10^5}{5}} = 200 \text{ rad/s}.$$

(b) The decelerated motion of the disc is described by kinematic equation

$$\omega_{final} = \omega_{initial} - \varepsilon t\,.$$

where $\omega_{final} = 0$, $\omega_{initial} = \omega$.

Hence,

$$0 = \omega - \varepsilon t\,,$$

or

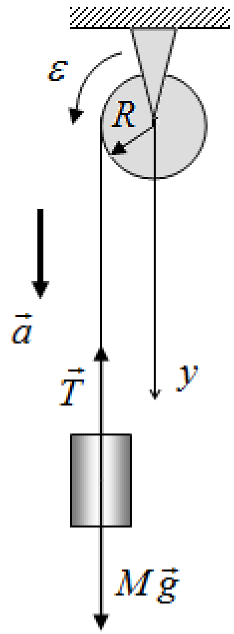
$$\varepsilon = \frac{\omega}{t}.$$

The braking torque is

$$M = I\varepsilon = \frac{I\omega}{t} = \frac{5 \cdot 200}{25} = 40 \text{ N·m}.$$

Problem 53

A rope is wrapped around a solid cylindrical drum. The drum has a fixed frictionless axle. The mass of the drum is 100 kg and it has a radius of R = 50.0 cm. The other end of the rope is tied to a block, M = 10.0 kg. What is the angular acceleration of the drum? What is the linear acceleration of the block? What is the tension in the rope? Assume that the rope does not slip.



Solution

The forces acting directing on the block are gravity $M\vec{g}$ and tension $\vec{T}$. Presumably the block will accelerate downwards. The only force directly acting on the drum which creates a torque is tension. Note that ropes, and therefore tensions, are always tangential to the object and thus normal to the radius. The other forces acting on the drum, the normal from the axle and the weight, both act through the centre of mass and thus do not create torque. The drum accelerates counter clockwise as the block moves down.

Since the rope is wrapped around the drum, we also have the kinematic relationship $a = \varepsilon R$.

Since the problem wants accelerations and forces, and one object rotates, that suggests we must use both the linear and rotational versions of Newton's Second Law.

$$\begin{cases} M\vec{a} = M\vec{g} + \vec{T}, \\ I\vec{\varepsilon} = \vec{M}. \end{cases}$$

Let's find the projections of the first equation on y-axis, and rewrite the second equation substituting the moment of inertia of cylinder $I = \dfrac{mR^2}{2}$.

$$\begin{cases} Ma = Mg - T, \\ \dfrac{mR^2}{2} \cdot \dfrac{a}{R} = T \cdot R. \end{cases}$$

The second equation after simplifying gives $T = \dfrac{ma}{2}$. Putting this result into the first equation yields

$$a = \frac{Mg}{M + 0.5m} = \frac{10 \cdot 9.8}{10 + 50} = 1.63\,\text{m/s}^2.$$

Thus an angular acceleration is

$$\varepsilon = \frac{a}{R} = \frac{1.63}{0.5} = 3.27\,\text{rad/s}^2.$$

As well, tension is

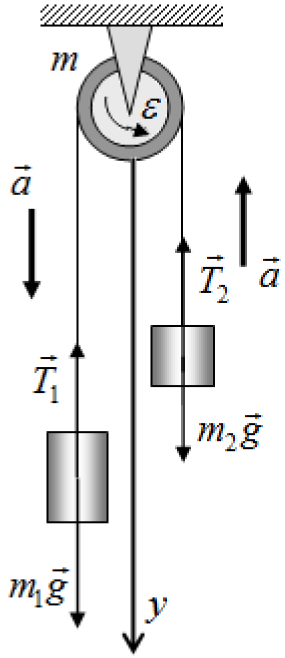
$$T = \frac{ma}{2} = \frac{10 \cdot 1.63}{2} = 81.5\,\text{N}.$$

Problem 54

Two blocks (m_1 = 2 kg and m_2 = 1 kg) are connected by rope over a frictionless pulley as shown in the Figure. The pulley is a cylinder of mass m=1 kg. What is the acceleration of the blocks and the tension in the rope on either side of the pulley?

Solution

In this problem we must use both the linear and rotational versions of Newton's Second Law, because there are two objects that take part in translational motion, and one object rotates. We have to write down three equations. It is need to remember that the tension must be different on either side of the pulley or the pulley would not rotate.



$$\begin{cases} m_1\vec{a} = m_1\vec{g} + \vec{T}_1, \\ m_2\vec{a} = m_2\vec{g} + \vec{T}_2, \\ I\vec{\varepsilon} = \vec{M}. \end{cases}$$

Taking into account that the moment of inertia of solid disc is $I = \dfrac{mR^2}{2}$, and since the rope is strung over the pulley, the linear acceleration of the rope and the angular acceleration of the disc relate as $a = \varepsilon R$, we can write

$$\begin{cases} m_1 a = m_1 g - T_1, \\ -m_2 a = m_2 g - T_2, \\ \dfrac{m\cancel{R}^{\cancel{2}}}{2} \cdot \dfrac{a}{\cancel{R}} = (T_1 - T_2)\cancel{R}. \end{cases}$$

or

$$\begin{cases} m_1 a = m_1 g - T_1, \\ -m_2 a = m_2 g - T_2, \\ \dfrac{ma}{2} = T_1 - T_2. \end{cases}$$

Solution of this system gives

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + m/2} = \frac{(2-1)\cdot 9.8}{2+1+0.5} = 2.8\,\text{m/s}^2,$$

$$T_1 = \frac{m_1 g\left(2m_2 + m/2\right)}{m_1 + m_2 + m/2} = 14\,\text{N},$$

$$T_2 = \frac{m_2 g\left(2m_1 + m/2\right)}{m_1 + m_2 + m/2} = 12.6\ \text{N}.$$

Problem 55

Cylinder of mass m = 20 kg and radius R = 0.5 m rotates about the axis passing through its center according the dependence $\varphi = A + Bt^2 - Ct^3$*, where B = 2 rad/s*^{2}*, C = 2 rad/s*^{3}*. Find the dependence for torque changing, and the magnitude of the torque on instant of time* t *=5 s.*

Solution

According to the Second Law for rotation the torque depends on the moment of inertia and the angular acceleration of cylinder

$$\vec{M} = I\vec{\varepsilon}\,.$$

The moment of inertia equals $I = \dfrac{mR^2}{2}$, and if we differentiate the given dependence for the angle, we obtain

$$\omega = \frac{d\varphi}{dt} = 2Bt - 3Ct^2\,,$$

$$\varepsilon = \frac{d\omega}{dt} = 2B - 6Ct\,.$$

Then

$$M = I\varepsilon = \frac{mR^2}{2}\left(2B - 6Ct\right) = \frac{20\cdot 0.25}{2}\left(2\cdot 2 - 6\cdot 2t\right) = 10 - 30t\Big|_{t=5} = 140\,\text{N}\cdot\text{m}.$$

Problem 56

A circular hoop of mass 50 kg and radius 50 cm is rotating at an angular speed of 120 rotations per minute. Calculate its kinetic energy.

Solution

For kinetic energy calculation we have to know and the angular speed $\omega = 120$rot/min=2 rot/s and the moment of inertia of the hoop

$I = mR^2 = 50 \cdot 0.25 = 12.5\,\text{kg·m}^2$,

Then the kinetic energy of rotating hoop is

$$W_k = \frac{I\omega^2}{2} = \frac{12.5 \cdot 0.04}{2} = 0.25\,\text{J}.$$

Problem 57

A sphere of mass 100 kg and radius 50 cm rolls without slipping with the speed of 5 cm/s. Calculate its kinetic energy.

Solution

Since the sphere is moving, it has kinetic energy. The sphere takes part in two motions simultaneously, i.e., the translational motion with kinetic energy $\left(W_k\right)_{trans} = \frac{mv^2}{2}$, and rotational motion with kinetic energy $\left(W_k\right)_{rot} = \frac{I\omega^2}{2}$, where m and $I = \frac{2mR^2}{5}$ are the mass and the moment of inertia of sphere, respectively; v and $\omega = \frac{v}{R}$ are its linear and angular velocities. The total energy is the sum of these two energies:

$$W_k = \frac{mv^2}{2} + \frac{I\omega^2}{2} = \frac{mv^2}{2} + \frac{\cancel{2}m\cancel{R^2}v^2}{5\cdot\cancel{2}\cdot\cancel{R^2}} = 0.7mv^2 .$$

Problem 58

(a) Calculate the kinetic energy of rotation of a disc of mass 1 kg and radius 0.2 m rotating at 30 rad/min.

(b) Find the kinetic energy of this disc if it rolls without slipping, and the points on the disc surface have the angular velocity 30 rad/min.

Solution

(a) Kinetic energy of rotating disc may be determined according to the expression

$$W_k = \frac{I\omega^2}{2},$$

where $I = \frac{mR^2}{2} = \frac{1\cdot 0.04}{2} = 0.02$ kg·m^2 is moment of inertia of the disc, and ω =30 rad/min = 0.5 rad/s is its angular velocity.

Then the kinetic energy of rotation is

$$W_k = \frac{I\omega^2}{2} = \frac{0.02\cdot 0.25}{2} = 2.5\cdot 10^{-3}\,\text{J}.$$

(b) When disc rolls without slipping its kinetic energy is sum of kinetic energies of translational and rotational motions. Taking into account that the moment of inertia of disc is $I = \frac{mR^2}{2}$ and liner velocity of the point of the disc surface correlates with angular velocity according to $v = \omega R$, we obtain

$$W_k = \frac{mv^2}{2} + \frac{I\omega^2}{2} = \frac{m\omega^2 R^2}{2} + \frac{mR^2\omega^2}{2\cdot 2} =$$

$=0.75mR^2\omega^2 = 0.75 \cdot 1 \cdot 0.04 \cdot 0.25 = 7.5 \cdot 10^{-3}$ J.

Problem 59

The 25-kg disc of radius 0.3 m is at rest when the constant 15 N·m counterclockwise couple is applied. Determine the disc's angular velocity when it has rotated through 5 revolutions (a) by applying the Newton's 2 Law for rotation, and (b) by applying the principle of work and energy.

Solution

(a) Newton's 2 Law $M = I\varepsilon$ gives the angular acceleration of the disc as

$$\varepsilon = \frac{M}{I}.$$

The moment of inertia of the disc is $I = \frac{mR^2}{2}$, therefore, the angular acceleration is

$$\varepsilon = \frac{M}{I} = \frac{2M}{mR^2}.$$

From the kinematics of rotational motion for accelerated motion from the rest we have

$$\begin{cases} 2\pi N = \dfrac{\varepsilon t^2}{2}, \\ \omega = \varepsilon t. \end{cases}$$

Then $\omega = \sqrt{4\pi N\varepsilon} = \sqrt{4\pi N \cdot \dfrac{2M}{mR^2}} = \sqrt{\dfrac{4\pi \cdot M \cdot N}{mR^2}} = \sqrt{\dfrac{4\pi \cdot 15 \cdot 5}{25 \cdot 0.3^2}} = 20.5$ rad/s.

(b) Applying the law of conservation of energy, the work is

$$A = \int_0^{2\pi N} M d\varphi = M \cdot 2\pi N.$$

On the other hand, the work is equal to the change of rotational kinetic energy

$$A = \Delta E_k = E_{k2} - E_{k1} = \frac{I\omega^2}{2} - 0 = \frac{mR^2\omega^2}{4}.$$

Equating the right hand side of the equations for the work, we obtain

$$M \cdot 2\pi N = \frac{mR^2\omega^2}{4},$$

$$\omega = \sqrt{\frac{4\pi \cdot M \cdot N}{mR^2}} = \sqrt{\frac{4\pi \cdot 15 \cdot 5}{25 \cdot 0.3^2}} = 20.5 \text{ rad/s}.$$

Problem 60

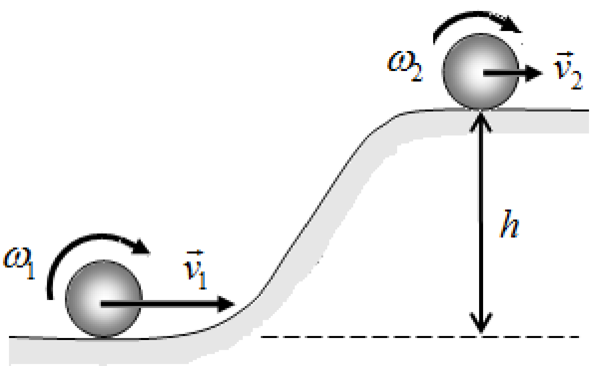
A bowling ball encounters a 0.8 m vertical rise on the way back to the ball rack, as the drawing illustrates. Ignore frictional losses and assume that the mass of the ball is distributed uniformly. The translational speed of the ball is 4 m/s at the rise. Find the translational speed at the top.

Solution

The bowling ball is rolling without slipping, therefore, its energy consists of the kinetic energy of translational motion (E_{kt}) and kinetic energy of rotational motion (E_{kr}):

$$E_1 = E_{p1} + E_{k1} = E_{p_1} + E_{kt1} + E_{kr1} = E_{p_1} + \frac{mv_1^2}{2} + \frac{I\omega_1^2}{2}.$$

The potential energy is at the bottom of the rock is zero. The moment of inertial of the solid sphere of the mass m and radius R is $I = \frac{2mR^2}{5}$, and the angular velocity (ω) relates to the linear velocity of the ball's center of mass (v) as $v = \omega R$. Then



$$E_{k1} = \frac{mv_1^2}{2} + \frac{\cancel{2}m \cdot \cancel{R}^2 \cdot v_1^2}{5 \cdot \cancel{2} \cdot \cancel{R}^2} = \frac{7mv_1^2}{10}.$$

At the top of the rack the ball's energy is given by

$$E_2 = E_{p2} + E_{k2} = E_{p2} + E_{kt2} + E_{kr2} = mgh + \frac{mv_2^2}{2} + \frac{I\omega_2^2}{2} = mgh + \frac{7mv_2^2}{10}.$$

At the absence of friction, $E_1 = E_2$, therefore,

$$\frac{7mv_1^2}{10} = mgh + \frac{7mv_2^2}{10}.$$

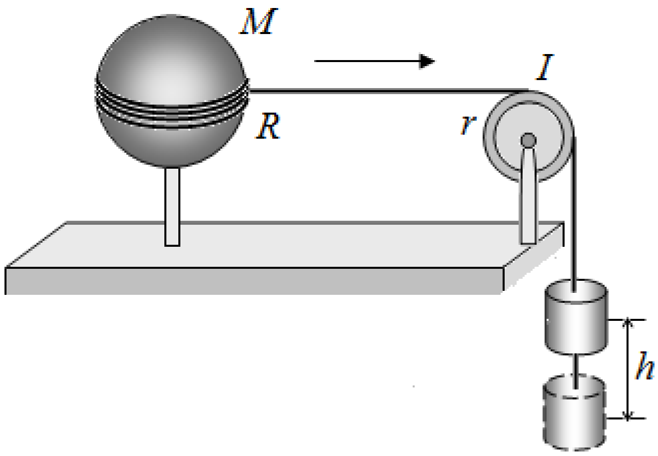
The translational velocity of the ball at the top is

$$v_2 = \sqrt{v_1^2 - \frac{10gh}{7}} = \sqrt{4^2 - \frac{10 \cdot 9.8 \cdot 0.8}{7}} = 2.2 \text{ m/s}.$$

Problem 61

A uniform spherical shell of mass M = 21 kg and radius R = 1.5 m rotates about a vertical axis on frictionless bearings. A massless cord passes around the equator of the shell, over a pulley of rotational inertia I = 1kg· m² and radius r = 0.6 m, and is attached to a small object of mass m = 7 kg. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object after it falls a distance h = 5 m from rest? Use energy considerations.

Solution



According to the law of conservation of energy, the potential energy of descending load transforms into the kinetic energy of rotating motion of the pulley and spherical shell, and translational motion of the load.

$$mgh = \frac{mv^2}{2} + \frac{I_1\omega_1^{\ 2}}{2} + \frac{I_2\omega_2^{\ 2}}{2},$$

where m and v are the mass and velocity of the load; $I_1 = \frac{2}{3}MR^2$ and ω_1 are the moment of inertia and the angular velocity of the spherical shell; I_2 and ω_2 are the moment of inertia and the angular velocity of the pulley.

The linear velocity of the load is equal to the linear velocity of the cord, which is connected to the angular velocities of the pulley and the shell

$$v = \omega_1 R = \omega_2 r.$$

$$mgh = \frac{mv^2}{2} + \frac{1}{2}\cdot\frac{2}{3}MR^2\left(\frac{v}{R}\right)^2 + \frac{1}{2}\cdot I_2\left(\frac{v}{r}\right)^2 = \frac{mv^2}{2} + \frac{Mv^2}{3} + \frac{I_2 v^2}{2r^2}.$$

The speed of the load may be found from

$$mgh = v^2\left(\frac{m}{2} + \frac{M}{3} + \frac{I_2}{2r^2}\right),$$

as following

$$v = \sqrt{\frac{mgh}{(m/2) + (M/3) + \left(I_2/2r^2\right)}}$$

$$v = \sqrt{\frac{7\cdot 9.8\cdot 5}{(7/2) + (21/3) + \left(1/2\cdot 0.6^2\right)}} = 5.37 \text{ m/s}.$$

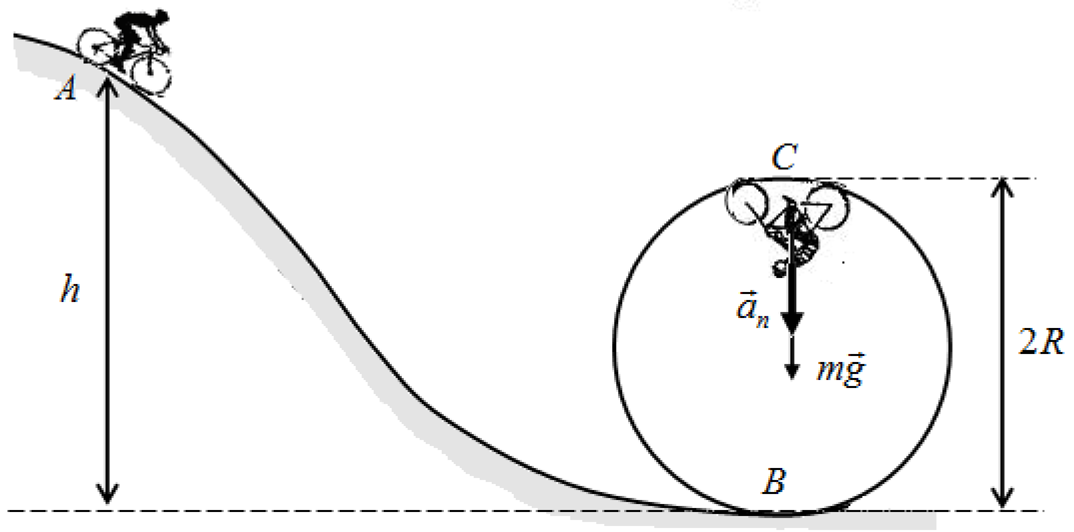
Problem 62

From what height above the bottom of the loop must the cyclist in the figure start in order to just make it around the loop of radius R *=3 m. The mass of the cyclist with the bicycle is* m*=75 kg, the mass of each wheel is* m_0 *= 1.5 kg. Assume that the wheels are hoops with the moment of inertia* $I = m_0 r^2$, *and there is no friction.*

Solution

For the cyclist to just make him around the loop, his speed at the highest point of the loop must be such that force of gravity on him is sufficient to provide the force needed to keep him in a circular path. For this to be the case,

$$ma_n = \frac{mv^2}{R} = mg,$$



or the velocity at point C must be given by

$$v = \sqrt{gR}$$

According the law of conservation of energy for this isolated system potential energy of the cyclist at his initial point A of motion ($W_p = mgh$) is equal to

$$W_p = W_{k1} + 2W_{k2} + W_{p1},$$

where $W_{k1} = \dfrac{mv^2}{2}$ is the kinetic energy of the cyclist with the cycle at point B,

The kinetic energy of rotating wheel is

$$W_{k2} = \frac{I\omega^2}{2} = \frac{m_0 r^2 v^2}{2 \cdot r^2} = \frac{m_0 v^2}{2},$$

and the potential energy of the cyclist at point C is

$$W_p = 2mgR.$$

Hence,

$$mgh = \frac{mv^2}{2} + 2 \cdot \frac{m_0 v^2}{2} + 2mRg \, .$$

Taking into account that the velocity is $v = \sqrt{gR}$, let’s write the conservation law:

$$mgh = \frac{mgR}{2} + 2 \cdot \frac{m_0 gR}{2} + 2mgR \, .$$

The cyclist has to start from the height

$$h = \frac{R}{2} + \frac{m_0}{m} gR + 2R = \frac{3}{2} + \frac{1.5 \cdot 3}{75} + 2 \cdot 3 = 7.56 \, \text{m}.$$

Problem 63

The wheel is a solid disc of mass M =3 kg and radius 40 cm. The suspended block has mass m=0.6 kg. If suspended block starts from rest and descends to a position 1 m lower, what is its speed when it is at this position?

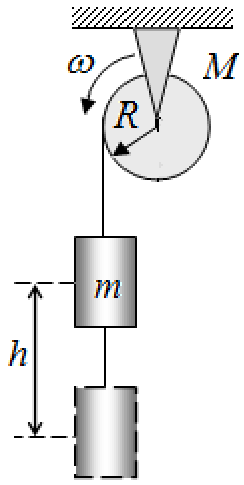
Solution

The work done on the system of the block and the wheel is due to the gravitational force $m\vec{g}$ acting on the hanging block. From the law of conservation of energy, the change in potential energy of the block

$$\Delta E_p = E_{p1} - E_{p2} = mg\Delta h$$

must be equal to the change in kinetic energy of the system

$$\Delta E_k = E_{k2} - E_{k1} = \Delta E_{kt} + \Delta E_{kr} = \left(E_{kt2} - E_{kt1}\right) + \left(E_{kr2} - E_{kr1}\right),$$



where ΔE_t is the change in translational kinetic energy of the block, and ΔE_r is the change in rotational kinetic energy of the wheel.

The system begins from rest ($E_{kt1} = E_{kr1} = 0$), so we can write

$$mg\Delta h = E_{kt2} + E_{kr2} = \frac{mv^2}{2} + \frac{I\omega^2}{2},$$

where v is the speed of the block in its final position. It is also the speed of the string at this instant as well as the speed of a point on the rim of the wheel at this instant. Therefore, $\omega = v/R$. In addition, because the wheel is a solid disc, its moment of inertia is $I = \frac{MR^2}{2}$. Consequently,

$$mg\Delta h = \frac{mv^2}{2} + \frac{M\cancel{R^2}v^2}{2 \cdot 2 \cdot \cancel{R^2}} = \frac{mv^2}{2} + \frac{Mv^2}{4} = \frac{v^2(2m+M)}{4},$$

Solving for v, we find that

$$v = \sqrt{\frac{4mg \cdot \Delta h}{2m+M}} = \sqrt{\frac{4 \cdot 0.6 \cdot 9.8 \cdot 1}{2 \cdot 0.6 + 3}} = 2.37 \text{ m/s}.$$

Problem 64

Energy of 500 J is spent in increasing the speed of fly–wheel from 60 rev/min to 360 rev/min. Find the moment of inertia of the wheel.

Solution

The speed of rotation was changed from $n_1 = 30$ rev/min = 0.5 rev/s to $n_2 = 360$ rev/min = 6 rev/s, consequently, the kinetic energy of the wheel was changed. According to the Work–Energy Theorem, the work is a quantitative measure of changes in energy. Hence the work is equal to the increment of the kinetic energy of the wheel:

$$A = \Delta W_k = W_{k2} - W_{k1}.$$

Taking into account that $\omega = 2\pi n$

$$A = \Delta W_k = W_{k2} - W_{k1} = \frac{I\omega_2^2}{2} - \frac{I\omega_1^2}{2} = \frac{4 \cdot I \cdot \pi^2}{2}\left(n_2^2 - n_1^2\right).$$

Substituting given data, we obtain

$$I = \frac{A}{2 \cdot \pi^2 \cdot \left(n_2^2 - n_1^2\right)} = \frac{500}{2\pi^2\left(36 - 0.25\right)} = 4.4\,\text{kg}\cdot\text{m}^2.$$

Problem 65

Find the linear velocities and accelerations of centers of sphere, disc and hoop that roll down an inclined plane without slipping. The incline of height $h = 1$ m makes an angle of 30^0 to the horizontal. The initial velocity of all objects $v_0 = 0$. Compare calculated velocities and accelerations with the velocity and acceleration of the box, which slides from this incline without friction.

Solution

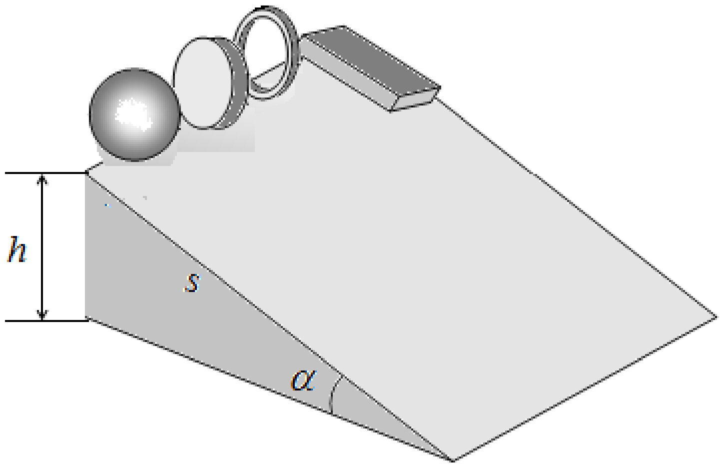
Apply the conservation of $W_p = W_k$.

Initially, the each object possesses only gravitational potential energy. When it reaches the bottom of the ramp, this potential energy has been converted to the translational and rotational kinetic energy according to the law of conservation of mechanical energy. Taking into account that the moments of inertia of the objects mentioned above are $I_{sp} = \frac{2}{5}mR^2$, $I_{disc} = \frac{1}{2}mR^2$, $I_{hoop} = mR^2$, and the angular speed of rolling objects is related to the liner speed according to $\omega = \frac{v}{R}$, we obtain

$$W_p = W_k = \begin{cases} W_{k_{sp}} = \dfrac{mv^2}{2} + \dfrac{\cancel{2} m \cancel{R^2} v^2}{5 \cdot \cancel{2} \cdot \cancel{R^2}} = 0,7mv^2 & \Rightarrow \quad v = \sqrt{\dfrac{gh}{0,7}} = 3,74\,\text{m/s}, \\ W_{k_{disc}} = \dfrac{mv^2}{2} + \dfrac{m \cancel{R^2} v^2}{2 \cdot 2 \cdot \cancel{R^2}} = 0,75mv^2 & \Rightarrow \quad v = \sqrt{\dfrac{gh}{0,75}} = 3,61\,\text{m/s}, \\ W_{k_{hoop}} = \dfrac{mv^2}{2} + \dfrac{m \cancel{R^2} v^2}{2 \cdot \cancel{R^2}} = mv^2 & \Rightarrow \quad v = \sqrt{gh} = 3,13\,\text{m/s}, \\ W_{k_{box}} = \dfrac{mv^2}{2} & \Rightarrow \quad v = \sqrt{2gh} = 4,43\,\text{m/s}. \end{cases}$$

Decelerated motion of all objects is described by the following kinematic equations

$$\begin{cases} s = v_0 t + \dfrac{at^2}{2}, \\ v = v_0 + at, \end{cases}$$



Then the acceleration of the objects is

$$a = \frac{v^2 - v_0^2}{2s}.$$

The initial velocities $v_0 = 0$, the final velocities we have found above, the travelled distance is $s = h/\sin\alpha$, where h is the height of incline. The accelerations of the objects are

$$a = \frac{v^2 \cdot \sin\alpha}{2h} = \begin{cases} a_{sp} = \dfrac{g \cdot \sin\alpha}{0,7 \cdot 2} = \dfrac{9,8 \cdot 0,5}{1,4} = 3,5\,\text{m/s}^2, \\ a_{disc} = \dfrac{g \cdot \sin\alpha}{0,75 \cdot 2} = \dfrac{9,8 \cdot 0,5}{1,5} = 3,27\,\text{m/s}^2, \\ a_{hoop} = \dfrac{g \cdot \sin\alpha}{2} = \dfrac{9,8 \cdot 0,5}{2} = 2,45\,\text{m/s}^2, \\ a_{box} = \dfrac{\cancel{2} \cdot g \cdot \sin\alpha}{\cancel{2}} = 9,8 \cdot 0,5 = 4,9\,\text{m/s}^2. \end{cases}$$

Problem 66

The wheel during the time t = 60 s of decelerated motion diminish the frequency of rotation from n_1=5 rev/s to n_2=3 rev/s. Find the amount of revolutions N, that were made for this time period, an angular acceleration ε of the wheel, braking torque M and the work of braking force A. The wheel is the hoop of mass m= 1 kg and radius R = 0.2 m.

Solution

Decelerated motion of the wheel is described by the following kinematic equations

$$\begin{cases} 2\pi N = 2\pi n_1 t - \dfrac{\varepsilon t^2}{2}, \\ 2\pi n_2 = 2\pi n_1 - \varepsilon t. \end{cases}$$

An angular acceleration is

$$\varepsilon = \frac{2\pi\left(n_1 - n_2\right)}{t} = \frac{2\pi\left(5-3\right)}{60} = 0.21 \text{ rad/s}^2.$$

The amount of revolutions is

$$N = n_1 t - \frac{\varepsilon t^2}{4\pi} = 5\cdot 60 - \frac{0.21\cdot 60^2}{4\pi} = 240.$$

The moment of inertia of the hoop respectively the centre of mass is

$$I = mR^2 = 1\cdot 0.2^2 = 0.04 \text{ kg·m}^2.$$

Using the Second Newton's Law for rotation, the braking torque may be found as

$$M = I\varepsilon = 0{,}04\cdot 0{,}21 = 8{,}4\cdot 10^{-3}\,\text{N·m}.$$

The work of braking force is determined by the increment of the kinetic energy of the rotating wheel

$$A = W_{k2} - W_{k1} = \frac{I\omega_2^2}{2} - \frac{I\omega_1^2}{2} = \frac{4\pi^2 I}{2}\left(n_2^2 - n_1^2\right) =$$

$$= 2\cdot I\pi^2\left(n_2^2 - n_1^2\right) = 2\cdot 0.04\cdot \pi^2\left(3^2 - 5^2\right) = -12.63 \text{ J}.$$

Problem 67

An ice skater begins a spin by rotating at an angular velocity of 2.2 rad/s with both arms and one leg outstretched. At that time her moment of inertia is 0.52 kg·m². She then brings her arms up over her head and her legs together, reducing her moment of inertia by 0.21 kg·m². At what angular velocity will she then spin?

Solution

Because there are no acting external torques (any friction is ignored here), angular momentum is conserved and we can write that

$$I_1\omega_1 = I_2\omega_2.$$

In this case the skater's moment of inertia has decreased and so her angular velocity will increase.

We find

$$\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{0.52 \cdot 2.2}{0.21} = 5.4 \text{ rad/s}.$$

Problem 68

A 50 kg student is spinning on the merry-go-round that has a mass of 100 kg and a radius of 2 m. She walks from the edge of merry-go-round towards the center. If the angular speed of the merry-go-round is initially 2 rad/s^2, what is its angular speed when the student reaches a point 0.5 m from center?

Solution

Because there is no external torques, the angular momentum of the system (merry-go-round plus student) is conserved.

$$L = L',$$

$$L_m + L_s = L'_m + L'_s.$$

Determine the moments of inertia. Treat the merry-go-round as a solid disc, and treat the student as a point mass.

$$I_m = I'_m = \frac{MR^2}{2}, \quad I_s = mR^2, \quad I'_s = mr^2.$$

$$I_m\omega_1 + I_s\omega_1 = I'_m\omega_2 + I'_s\omega_2.$$

$$\omega_1\left(\frac{MR^2}{2}+mR^2\right)=\omega_2\left(\frac{MR^2}{2}+mr^2\right)$$

Substitute the values into the equations and solve

$$\omega_2=\omega_1\frac{MR^2+2mR^2}{MR^2+2mr^2}=\omega_1\frac{M+2m}{M+2m\left(\frac{r}{R}\right)^2}=2\cdot\frac{100+2\cdot 50}{100+2\cdot 50\left(\frac{0.5}{2}\right)^2}=3.76\,\text{rad/s.}$$

Problem 69

A merry-go-round of radius $R = 6$ m with nearly frictionless bearings and a moment of inertia $I = 3000$ kg·m² is turning at $n_1 = 3$ rpm when the motor is turned off. If there were 10 children of $m = 25$ kg average mass initially out at the edge of the carousel and they all move into the center and huddle $r = 1$ m from the axis of rotation, find the angular velocity of the carousel. If then the brakes are applied, find the torque required to stop the carousel in 15 s.

Solution

Before the brakes are applied there are no external torques acting on the carousel (friction is absent in the bearings) so that we know angular momentum is conserved. Therefore, we can first write expressions for the initial and final angular momentum and then equate them to solve for the final rotational velocity. We have

$$L_1=I_1\omega_1$$

Initial moment of inertia I_1 consists of the moment of inertia of the carrousel I and the moments of inertia of the children at the edge:

$$I_1=I+N\cdot I_0=I+N\cdot mR^2=3000+10\cdot 25\cdot 6^2=12000\ \text{kg·m}^2,$$

treating the children as point masses at the edge of the carousel.

The angular velocity

$$\omega_1=2\pi n_1=\frac{2\pi\cdot 3}{60}=\frac{\pi}{10}=0.314\,\text{rad/s.}$$

The initial angular moment is

$L_1 = I_1 \cdot \omega_1 = 12000 \cdot 0.314 = 3768$ kg·m²/s.

Final moment of inertia I_2 consists of the moment of inertia of the carrousel I and the moments of inertia of the children at the distance r from the axis of rotation:

$I_2 = I + N \cdot I_0 = I + N \cdot mr^2 = 3000 + 10 \cdot 25 \cdot 1^2 = 3250$ kg·m².

Using conservation of angular momentum, we then can write

$L_1 = L_2 = I_2 \omega_2$.

So that the final angular velocity is

$$\omega_2 = \frac{L_1}{I_2} = \frac{3768}{3250} = 1.16 \text{ rad/s.}$$

From kinematics the angular velocity for decelerated motion depends on time

$\omega = \omega_0 - \varepsilon t$,

where ω_0 and ω are the initial and final angular speeds, respectively. For this problem, the braking begins at the speed $\omega_0 = \omega_2$ and ends when $\omega = 0$. Hence

$0 = \omega_2 - \varepsilon t$,

$$\varepsilon = \frac{\omega_2}{t} = \frac{1.16}{15} = 0.077 \text{ rad/s}^2.$$

Using Newton’s 2 Law for rotation and substituting the values we obtain

$M = I_2 \varepsilon = 3250 \cdot 0.077 = 250.25$ N·m.

Problem 70

A fly-wheel begins to rotate with an angular acceleration ε = 0.4 rad/s^2 and after t_1=10 s has the kinetic energy W_k=80 J. Find the angular momentum of the fly–wheel after t_2=30 s since the beginning of the rotation.

Solution

After t_1=10 seconds of rotation ($\omega_0 = 0$) an angular velocity of the fly–wheel is

$\omega_1 = \omega_0 + \varepsilon t_1 = \varepsilon t_1 = 0.4 \cdot 10 = 4$ rad/s.

Since kinetic energy for rotating body is

$W_k = \dfrac{I\omega_1^2}{2}$, and moment of inertia of the fly–wheel is

$I = \dfrac{2W_k}{\omega_1^2} = \dfrac{2 \cdot 80}{16} = 10$ kg·m^2.

After t_2 =20 seconds of rotation an angular velocity is

$\omega_2 = \varepsilon t_2 = 0.4 \cdot 30 = 12$ rad/s.

The angular momentum on this instant of time is equal to

$L = I\omega_2 = 10 \cdot 12 = 120$ kg·m^2·s^{-1}.

Problem 71

The hoop of radius R = 1 m is hooked and may oscillate in vertical plane. Find the period of hoop's oscillations.

Solution

The hoop in this problem is the compound pendulum, hence its period of oscillations is

$$T = 2\pi\sqrt{\frac{I}{mgx}},$$

where m is the hoop mass, I is the moment of inertia respectively the pivot point, and x is the distance between the pivot point and the centre of mass.

The moment of inertia may be determined using the parallel axes theorem (Huygens–Steiner theorem): Moment of inertia of a rigid body about any axis is sum of the body's moment of inertia about the parallel axis passing through the object's centre of mass and the product of the mass and the perpendicular distance between the two axes. Hence,

$$I = I_0 + mx^2 = mR^2 + mR^2 = 2mR^2 .$$

The distance $x = R$.The period of oscillations is

$$T = 2\pi\sqrt{\frac{2\cancel{m}R^{\cancel{2}}}{\cancel{m}g\cancel{R}}} = \sqrt{\frac{2R}{g}} = \sqrt{\frac{2\cdot 0,5}{9,8}} = 0,32\,\text{s}.$$