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LECTURE NOTES

«ELECTRICITY»

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ELECTRICITY AND MAGNETISM

Chapter 1. ELECTRIC FIELDS IN FREE SPACE

I. ELECTRIC FIELD AND ELECTRIC CHARGES

1. Electric charge

Now it is known that four fundamental interactions underlie the basis of all the variety of natural phenomena: gravitational, strong and weak nuclear and electromagnetic interactions. Each type of interactions deals with a certain characteristic of a particle, for example, gravitational – with mass, and electromagnetic – with an electric charge.

Electric charge is the technical term to indicate that an object has been prepared so as to participate in electric forces. Then, an electric charge is a numerical rating of how strongly an object participates in electric forces.

The unit of charge is the Coulomb.

$$[q] = \text{Coulomb} = \text{C}.$$

Fundamental properties of an electric charge are the following.

1. There are two types of electric charges: *positive* and *negative*.

If a rod of ebonite is rubbed with fur, it gains the power to attract light bodies, such as pieces of paper. The discovery that rubbed amber could attract silk was mentioned by Thales (640-546 BC). «Electricity» comes from the Greek word for amber – electron (ηλεκτρον). In 1750 Benjamin Franklin (1706-1790), a pioneer of electrostatics, gave the name of “positive electricity” to the charge on a glass rod rubbed with silk, and “negative electricity” to that on an ebonite rod rubbed with fur. The experiment shows that two positive, or two negative charges repel each other, but a positive and negative charge attract each other.

2. Law of conservation of charge

An even more fundamental reason for using positive and negative signs for electric charge is the experiments showing that charge is conserved according to this definition: in any closed system, the total amount of charge is a constant. This is why we observe that rubbing initially uncharged substances together always results in gaining one substance a certain amount of one type of charge, while the other one acquires an equal amount of the other type of charge. Conservation of charge seems natural in the model in which matter is made up of positive and negative particles. If the charge on each particle is a fixed property of that type of particle, and if the particles themselves can be neither created nor destroyed, then conservation of charge is inevitable.

3. A charge is a *scalar* quantity. A magnitude of a charge is *independent* on the choice of a frame of reference.

4. Charge is a *quantized magnitude*. A tiny quantity of negative charge is carried by *electron* discovered by J.J.Thomson. Electrons are present in all atoms. As atoms are electrically neutral consequently the total negative charge on the electrons is equal to the positive charge on the nucleus. The nucleus contains tiny quantities of positive charge – *protons*. The magnitude of the charge of electron which is equal to the charge of proton is *elementary charge* $e = 1.6 \cdot 10^{-19} \text{ C}$.

2. Electric field. Coulomb's law. Electric field-strength

Between 1785 and 1787, the French physicist Charles Augustine de Coulomb (1736-1806) (Fig. 1.2, a) performed a series of experiments involving electric charges, and eventually established what is nowadays known as *Coulomb's law*. He used a torsion balance to perform his experiment (Fig. 1.2, b). Assuming that there are very small charged particles which can be treated as point, Coulomb's law says:

The force between two point charges (Fig. 1.1) is directly proportional to the product of the charges (q_1, q_2) and inversely proportional to the square of the

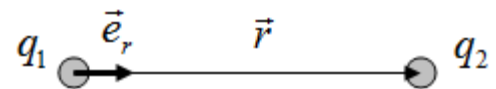


Figure 1.1

separation between their centers (r) and acts along the line joining the centers of these particles.

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_2}{r^2} \vec{e}_r = k \frac{q_1 \cdot q_2}{r^2} \vec{e}_r, \quad (1.1)$$

where $\epsilon_0 = 8,85 \cdot 10^{-12} \text{F} \cdot \text{m}^{-1}$ is the *electric permittivity* of free space, \vec{e}_r is the unit vector of \vec{r} , $k = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \text{N} \cdot \text{m}^2 \cdot \text{C}^{-2} = 9 \cdot 10^9 \text{m} \cdot \text{F}^{-1}$ is an *electrostatic constant* or the *force constant* (for vacuum and for air we use the same value of k).

Opposite charges will produce an *attractive* force while similar charges will produce a *repulsive* force.

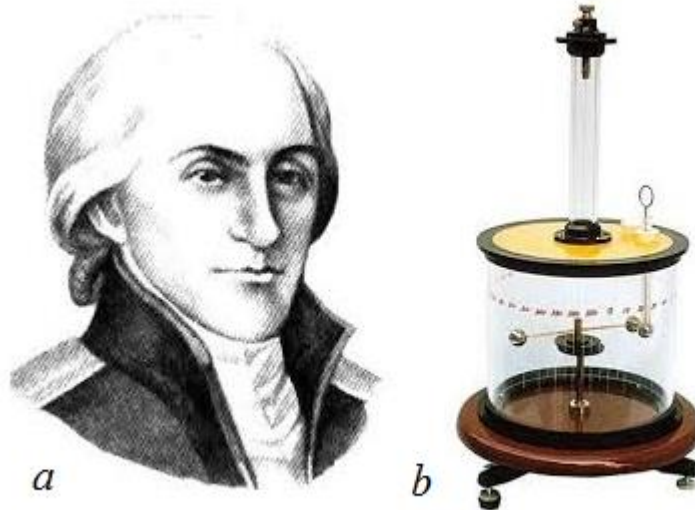


Figure 1.2 – Charles-Augustin de Coulomb (a) and torsion balance (b)

If the interaction of charges takes place in a medium other than vacuum/air the proportionality coefficient

$$k = \frac{1}{4\pi\epsilon\epsilon_0} \quad (1.2)$$

where ϵ is the *relative permittivity* of that medium (see Chapter 3). It is defined as the ratio of the force in vacuum to the force in any other medium between the same pair of charges separated by the same distance r . The relative permittivity ϵ is greater than 1 for any medium other than vacuum/air. This relative permittivity enters into the denominator in Coulomb's law defined by formula (1.1), therefore, the force

acting between the charges is always smaller in any medium other than vacuum or air.

The term "field" used in physics means the space in which some quantity is defined for each point of this space. If this is a vector quantity, we have a vector field. If the quantity is a scalar, we have a scalar field. In the chapter concerning electricity we will be defining the electric field. Such a field exists in the vicinity of any electric charge or charges. Thus, an **electric field** can be defined as a region where electric force is experienced. If we consider electric charges which do not move (or, in other words, stationary charges) we can call it more precisely – an **electrostatic field**.

The force exerted on a charged body in an electric field depends on the charge of the body and on the strength or intensity of the field. If we wish to explore the variation in strength of an electric field, we must place a **test charge** (q_{test}) which is small enough not to upset the field by its introduction at the point concerned.

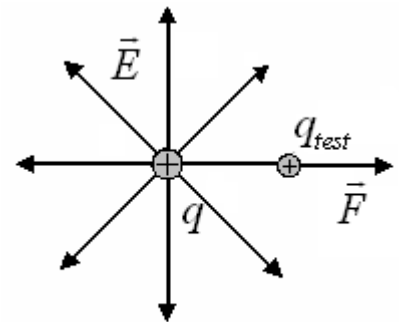


Figure 1.3

$$\frac{\vec{F}_1}{q_{test1}} = \frac{\vec{F}_2}{q_{test2}} = \dots = k \frac{q \cdot q_{test}}{r^2 q_{test}} \vec{e}_r = k \frac{q}{r^2} \vec{e}_r = \vec{E}. \quad (1.3)$$

The **electric field-strength** (or **electric field**, or **electric intensity**) \vec{E} at any point is defined as the force per unit positive charge which it exerts at that point. Its direction is that of the force exerted on a positive charge (Fig. 1.3).

$$[E] = \text{V} \cdot \text{m}^{-1}.$$

Therefore, the **electric field-strength due to a point charge** is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \vec{e}_r \quad (1.4)$$

From the definition of the field-strength it is possible to find the force exerted on any charge q at any point of field

$$\vec{F} = q \cdot \vec{E} \quad (1.5)$$

The **principle of superposition of fields**: electric field-strength of a system of point motionless charges is equal to the vector sum of field-strengths of these charges

which are created by them independently,

$$\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i^2} \vec{e}_{r_i} \quad (1.6)$$

The electric field can be represented graphically by **field lines** (Fig. 1.4). These lines are drawn in such a way that, at a given point, the tangent of the line has the direction of the electric field at that point. The density of lines is proportional to the magnitude of the electric field. Each field line starts on a positive point charge and ends on a negative point charge. Since the density of field lines is proportional to the strength of the electric field, the number of lines emerging from a positive charge must also be proportional to the charge.

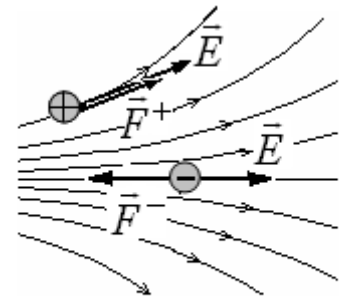


Figure 1.4

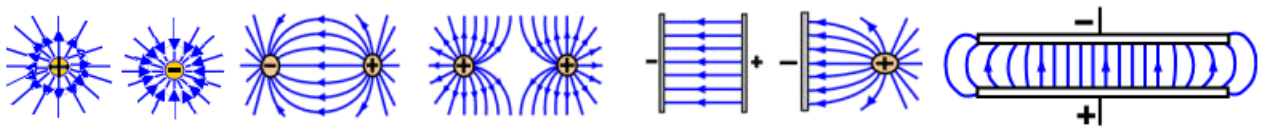


Figure 1.5

Fig. 1.5 shows the electric fields of different charged objects.

3. Distribution of charges

In most applications the charges are supposed to be non-discrete, i.e., distributed continuously over some region. The following three different distributions will be used in this case are

line charge (line density of charge) (λ): the charge per unit length;

surface charge (surface density of charge) (σ): the charge per unit area;

volume charge (volume density of charge) (ρ): the charge per unit volume

$$\lambda = \frac{dq}{dl}; \quad \sigma = \frac{dq}{dS}; \quad \rho = \frac{dq}{dV}, \quad (1.7)$$

where dq is a charge at length dl , on a surface dS , and in a volume dV , respectively.

II. GAUSS' LAW

1. Electric flux

We have already shown how electric fields can be described by means of electric field lines. Suppose that the magnitude of field-strength is equal (not only proportional) to the number of lines per unit area. In this case the number of lines per unit area is $E \cdot dS \cdot \cos \alpha$. This scalar quantity is an elementary *electric flux* (*flux of the vector \vec{E}*) through the area

$$d\Phi = (\vec{E}, d\vec{S}) = \vec{E} \cdot d\vec{S} = E \cdot dS \cdot \cos \alpha, \quad (1.8)$$

where $d\vec{S} = dS \cdot \vec{n}$ is the infinitesimal area, whose normal \vec{n} makes an angle α with the direction of the electric field. \vec{E} is a vector quantity (Fig. 1.5). Hence, it is useful to represent the area dS by the vector $d\vec{S}$ as well. The length of this vector is the size of the area, while its orientation is perpendicular to the area. It is in the direction of the normal \vec{n} . The normal to the surface can point into two different directions. For a closed surface, by convention, the normal points outward. So, by our definition of $d\vec{S}$ we can write the flux as the dot product of \vec{E} and $d\vec{S}$.

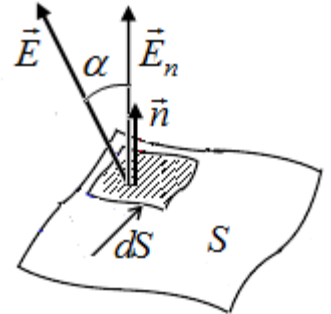


Figure 1.6

The *total electric flux* through any surface \vec{S} can be evaluated by integrating the differential flux over the surface S :

$$\Phi = \int_S d\Phi = \int_S \vec{E} \cdot d\vec{S}. \quad (1.9)$$

$$[\Phi] = \text{V} \cdot \text{m}.$$

The flux through a given surface can be positive or negative, since the cosine can be positive or negative. If the field lines are parallel to the normal vector \vec{n} , the flux is *positive*. If the field lines are anti-parallel to the normal vector \vec{n} , the flux is *negative*.

2. Gauss' law

The relationship between the electric field flux through the closed surface and the charge in the volume that is surrounded by this surface was formulated by the German mathematician and astronomer Johann Carl Friedrich Gauss (1777-1855) in 1831 (Fig. 1.7).



Figure 1.7

Gauss' law (theorem). The total flux of the electric field (of the vector \vec{E}) through any closed surface whatever its shape depends only on the total charge enclosed by the surface

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{inside}}{\epsilon_0}. \quad (1.10)$$

In electrostatics, electric field lines start on a positive charge and end on a negative charge. If there are no charges inside a closed surface, there are no sources or sinks of field lines inside the surface. All the field lines that enter through the surface into the volume enclosed by the surface must also emerge from the surface. The net flux through the surface is zero.

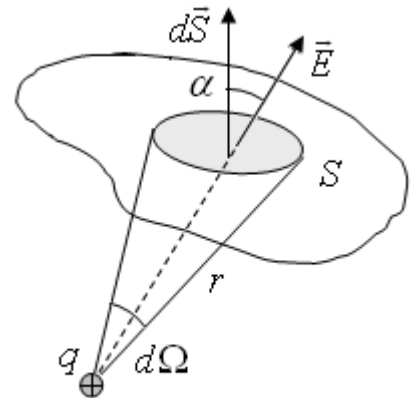


Figure 1.8

To prove the theorem we shall consider a field of one point charge. Enclose this charge by an arbitrary closed surface S (the **Gaussian surface**) and calculate the flux of the field-strength vector \vec{E} through an elementary area $d\vec{S}$.

$$d\Phi = \vec{E} \cdot d\vec{S} = E dS \cos \alpha = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot dS \cdot \cos \alpha = \frac{q}{4\pi\epsilon_0} \cdot \frac{dS \cdot \cos \alpha}{r^2} = \frac{q}{4\pi\epsilon_0} d\Omega,$$

where $d\Omega$ is a solid angle subtended by an elementary surface dS with the top at the point of the disposition of a charge (Fig. 1.8).

Let's integrate over the surface S :

$$\Phi = \oint_S d\Phi = \oint_S \frac{q}{4\pi\epsilon_0} \cdot d\Omega = \frac{q}{4\pi\epsilon_0} \oint_S d\Omega = \frac{q}{4\pi\epsilon_0} \cdot 4\pi = \frac{q}{\epsilon_0}.$$

We'll get the similar result if a system of point charges is situated inside the enclosed surface. Then according to the principle of superposition we have

$$\begin{aligned} \oint_S \vec{E} d\vec{S} &= \oint_S (\vec{E}_1 + \vec{E}_2 + \dots) d\vec{S} = \oint_S \vec{E}_1 d\vec{S} + \oint_S \vec{E}_2 d\vec{S} + \dots = \Phi_1 + \Phi_2 + \dots = \\ &= \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \dots = \frac{(q_1 + q_2 + \dots)}{\epsilon_0} = \frac{q}{\epsilon_0}. \end{aligned}$$

3. Application of Gauss' law

Gauss' law by itself can be used for finding the electric field of a point charge at rest, and the principle of superposition can be used for finding the electric field of an arbitrary charge distribution.

1. A field of uniform charged infinite sheet (charged plane)

Consider a charged plane with surface charge density σ (Fig. 1.9). The field-strength vector \vec{E} is perpendicular to the surface and at symmetric points is identical in magnitude and opposite in direction. As Gaussian surface we can choose a simple right circular cylinder with faces parallel to the plane of charge. The field lines are parallel to the sides of the cylinder, so the sides do not contribute to the flux. The flux both through the top surface and through the

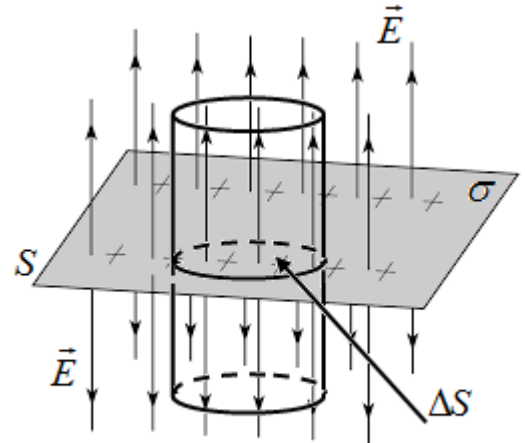


Figure 1.9

bottom surface of cylinder is $E \cdot \Delta S$, where ΔS is the area of the top (the bottom). The charge inside the cylinder is $\sigma \cdot \Delta S$. Then according to Gauss' theorem, $2E \cdot \Delta S = \frac{\sigma \cdot \Delta S}{\epsilon_0}$ and $E = \frac{\sigma}{2\epsilon_0}$. The projection of the vector \vec{E} to a normal to the charged plane E_n is

$$E_n = \frac{\sigma}{2\epsilon_0}. \quad (1.11)$$

The field of an infinite planar charge distribution is uniform. It does not decrease with distance.

If $\sigma > 0$, then $E_n > 0$, and the vector \vec{E} is directed off the charged plane; if $\sigma < 0$, therefore, $E_n < 0$, and the vector \vec{E} is directed to the charged plane. The fact that field-strength does not depend on distance from the plane means that the field is uniform ($\vec{E} = \text{const}$). This result is valid for an infinite plane as the influence of its end was not taken into account. Moreover, it is approximately valid for the central part of the finite charged plane.

2. A field of two parallel charged planes

The most important application of the above result is the superposition of the fields from two planar charge distributions which are separated by some distance.

Let two parallel planes be oppositely charged with surface charge densities $+\sigma$ and $-\sigma$ (Fig. 1.10). Considering the field of two planes as the superposition of fields of every charged plane, we can estimate that the field between the planes is

$$E = \frac{\sigma}{\epsilon_0}. \quad (1.12)$$

The fields add to yield a uniform field between the planes, but they precisely cancel outside the planes to give zero net fields outside. Between the planes the field points from the positive plane towards the negative one. This is the common configuration of a parallel plate capacitor.

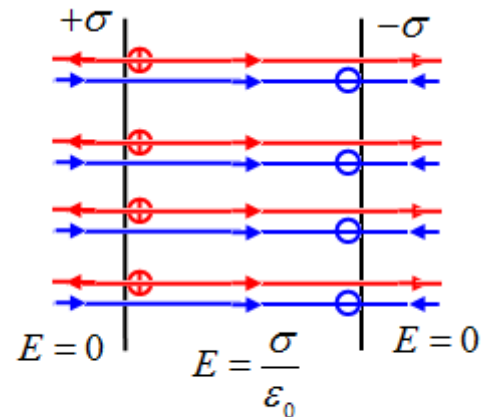


Figure 1.10

3. A field of the infinite round cylinder

Let a cylinder of radius a be charged with the linear charge density λ (Fig. 1.11).

The field in this case is of radial character: the vector \vec{E} at each point is

perpendicular to the axes of the cylinder and the magnitude of the vector \vec{E} depends only on the distance from the axis of the cylinder. We choose the cylinder as Gaussian surface. Its axis coincides with the axis of the charged cylinder. The flux through its end faces is equal to zero and through its lateral surface is $E_r 2\pi r h$, where E_r is the projection of the vector \vec{E} to the position vector \vec{r} whose direction coincides with the normal to the lateral surface of the cylinder of radius r and height h . Then according to Gauss' theorem, when $r > a$, we obtain

$$E_r \cdot 2\pi r h = \frac{\lambda h}{\epsilon_0} .$$

Then

$$E_r = \frac{\lambda}{2\pi\epsilon_0 r} . \quad (1.13)$$

If $\lambda > 0$, then $E_r > 0$, i.e., the vector of field-strength is directed off the charged cylinder.

When $r < a$, there are no charges inside the enclosed surface, therefore, $E = 0$ there .

4. A field of a spherical surface

The flux through a spherical surface of radius r , concentric with the spherical surface carrying a charge q on its surface is

$$E_r = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} . \quad (1.14)$$

This is the same result as that for a point charge. This means that outside a charged sphere, the field behaves as if all the charges on the sphere were centrally concentrated (Fig. 1.12).

There are no charges inside a closed surface ($r < a$), therefore, $E = 0$ there.

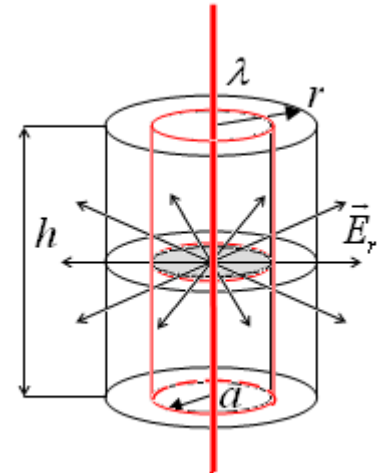


Figure 1.11

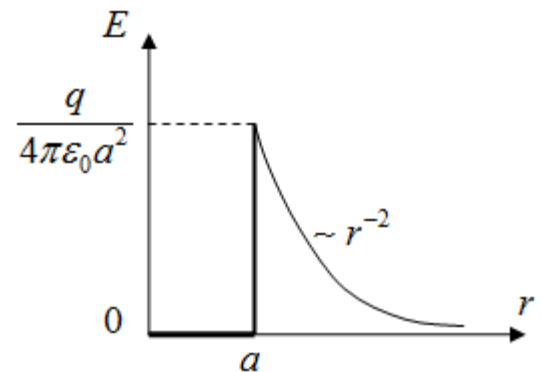


Figure 1.12

5. A field of a uniform charged sphere

Imagine a sphere of radius a with charge q uniformly distributed inside. The symmetry of the charge distribution requires a spherically symmetric electric field. The field must either point radially inward (toward the center) or outward (from the center of the sphere). For a spherical symmetric charge distribution, the magnitude of E can therefore depend only on the radial coordinate r and on the charge q . To

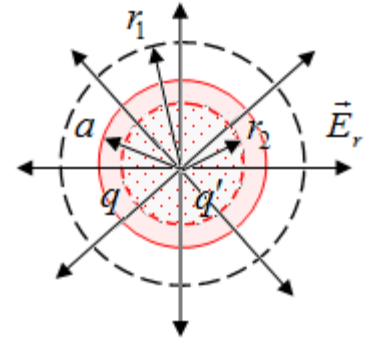


Figure 1.13

determine E as a function of r , we use Gauss' law. We draw a spherical Gaussian surface of radius r centered in the center of the spherical charge distribution. The radius r of the surface can be larger ($r = r_1$) or smaller ($r = r_2$) than the radius a (Fig. 1.13).

Let r be greater than a , so Gaussian surface encloses the entire charge distribution. The electric field is radial, the vector \vec{E} is normal to any surface element dS . Thus, from Gauss' law

$$\Phi = \oiint \vec{E} d\vec{S} = \oiint E dS = E \cdot 4\pi r^2 = q_{inside} / \epsilon_0 = q / \epsilon_0.$$

Therefore, we have for $r > a$

$$\vec{E}_r = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \vec{e}_r. \quad (1.15)$$

The field outside a spherically symmetric charge distribution looks like the field of a point charge. If q is positive, the field points outward, and if q is negative, it points inward.

The charge q_{inside} can be written as the charge density ($\rho = q/V$) times the volume of the charged sphere ($V = 4\pi R^3/3$). Therefore, we can write down

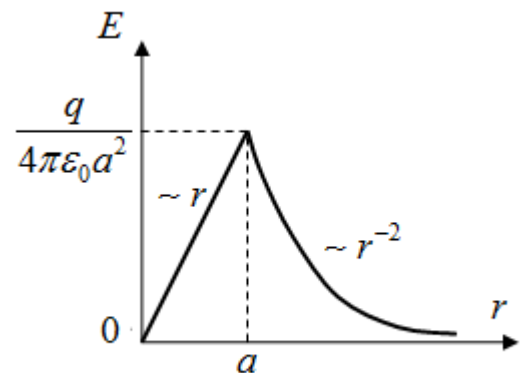


Figure 1.14

$$\vec{E} = \frac{\rho a^3}{3\epsilon_0 r^2} \vec{e}_r. \quad (1.1)$$

Let r be smaller than a , hence the Gaussian surface encloses only a part of the charge distribution. The charge inside the sphere enclosed by the Gaussian spherical surface of radius r ($r \leq a$) is $q' = q(r/a)^3$, because the charges are proportional to volumes where they are, and the volumes are proportional to cubes of radii. Then according to Gauss' theorem

$$E_r \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \cdot q \cdot \left(\frac{r}{a}\right)^3$$

and for $r \leq a$

$$E_r = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^3} \cdot r. \quad (1.17)$$

The field inside the charge distribution increases linearly with r . Its direction is outward for a positive distribution, and inward for a negative distribution (Fig. 1.14).

III. CIRCULATION OF THE VECTOR \vec{E} . POTENTIAL

1. The circulation theorem for the vector \vec{E}

The electrostatic field is a stationary field of the central forces, therefore, it is a conservative field, i.e., the work of the forces of this field does not depend on the path and depends only on a position of the initial and final points of motion.

If the unit positive charge is taken as a test charge, the elementary work of forces at the

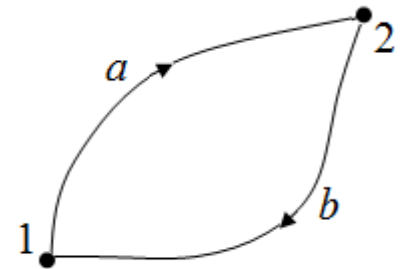


Figure 1.15

displacement $d\vec{l}$ is $\vec{E}d\vec{l}$, and the total work done on path 1-2 is $\int_1^2 \vec{E}d\vec{l}$. If such a linear

integral is taken around a closed path it is a **circulation of the vector \vec{E}** .

The **Circulation Theorem** states that the circulation of the vector \vec{E} in any electrostatic field is equal to zero, i.e.,

$$\oint \vec{E} d\vec{l} = 0. \quad (1.18)$$

To prove this theorem let us divide a closed path into two sections 1a2 and 2b1 (Fig. 1.15). As the linear integral $\int \vec{E} d\vec{l}$ does not depend on the form of the path,

$$\int_{12}^a \vec{E} d\vec{l} = \int_{12}^b \vec{E} d\vec{l}. \text{ But } \int_{12}^b \vec{E} d\vec{l} = -\int_{21}^b \vec{E} d\vec{l}. \text{ Therefore, } \int_{21}^b \vec{E} d\vec{l} = -\int_{12}^b \vec{E} d\vec{l}. \text{ Hence,}$$

$$\int_{12}^a \vec{E} d\vec{l} + \int_{21}^b \vec{E} d\vec{l} = \int_{12}^a \vec{E} d\vec{l} - \int_{12}^b \vec{E} d\vec{l} = 0.$$

The field possessing the property $\oint \vec{E} d\vec{l} = 0$ is a **potential field**; therefore, an electrostatic field is a potential field.

A conclusion of this theorem is the fact that electric field lines cannot close on themselves; they diverge from a positive charge and converge on a negative charge. Moreover, the number of electric-strength lines per any unit area has to be identical for any uniform field.

2. Potential. Potential due to a point charge and a system of point charges

Besides force description of electric fields (by means of electric field-strength) there is another type of description. Let us consider it.

The fact that a linear integral $\int_1^2 \vec{E} d\vec{l}$ representing the work done in taking a unit positive charge from point 1 to point 2 does not depend on the path form allows to state that there is some scalar function of coordinates whose loss is

$$\varphi_1 - \varphi_2 = \int_1^2 \vec{E} d\vec{l}. \quad (1.19)$$

Put a test charge q_{test} in the electric field of a charge q and estimate the ratio of potential energy of a test charge to its magnitude.

$$\frac{W_1}{q_{test1}} = \frac{W_2}{q_{test2}} = \dots = k \frac{q \cdot q_{test}}{r \cdot q_{test}} = k \frac{q}{r} = \varphi. \quad (1.20)$$

This scalar quantity describing the given point of an electric field is a **potential** of

an electric field. Potential is an energy characteristic of an electric field and it is equal to the potential energy of a unit positive charge at the given point of this field.

The **potential due to a point charge** is equal to

$$\varphi = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}. \quad (1.21)$$

$$[\varphi] = \text{Volt} = \text{V}.$$

As any definite integral, $\int_1^2 \vec{E}d\vec{l}$ is taken up to a constant whose value is defined arbitrarily. For example, for an electrostatic field it is accepted that the potential at infinity is equal to zero.

According to the principle of superposition, a **potential due to a system of point charges** is

$$\varphi = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} \quad (1.22)$$

So if we wish to find the potential at any point, we merely calculate the potential due to each charge, and add the potential algebraically, since potentials are scalars.

3. Potential gradient and field-strength. Work of an electric field

From $\varphi_1 - \varphi_2 = \int_1^2 \vec{E}d\vec{l}$ it follows that $(-d\varphi) = \vec{E}d\vec{l}$. And in the case of displacement of a charge along axis x , $E_x = -\frac{d\varphi}{dx}$. Similarly, for displacements along other coordinate axes $E_y = -\frac{d\varphi}{dy}$ and $E_z = -\frac{d\varphi}{dz}$. Then

$$\vec{E} = -\left(\frac{\partial\varphi}{\partial x} \vec{i} + \frac{\partial\varphi}{\partial y} \vec{j} + \frac{\partial\varphi}{\partial z} \vec{k} \right) = -\text{grad}\varphi. \quad (1.23)$$

Graphical representation of the electric field in terms of potential uses the concept of **equipotential surfaces**, i.e., surfaces over which the potential is constant. Equipotential surfaces can be drawn throughout any space in which there is an electric

field. An equipotential surface has the property that, along any direction lying in the surface, there is no electric field; for there is no potential gradient. Equipotential surfaces are, therefore, always at the *right angle* to force lines (Fig. 1.16).

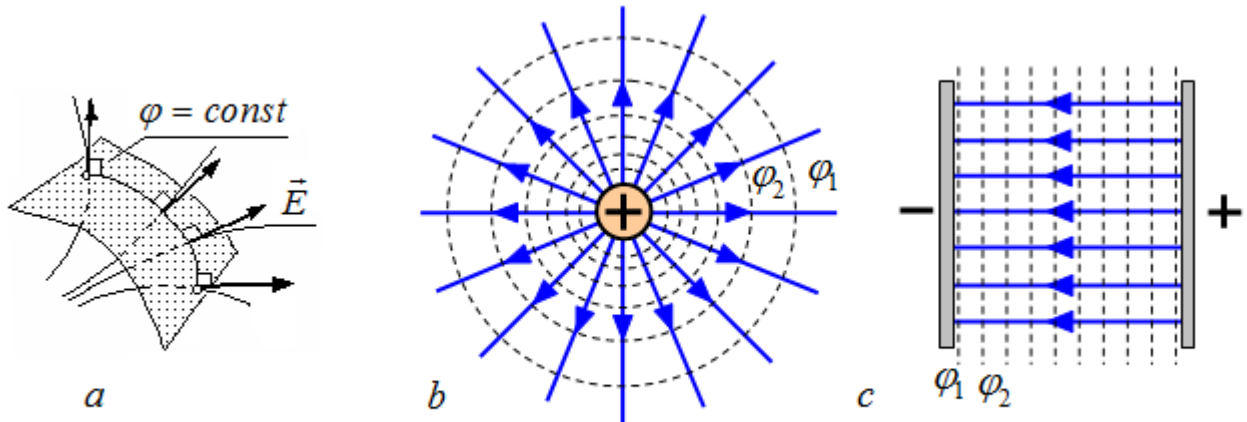


Figure 1.16 – The equipotential surfaces

When we have plotted the equipotentials, they turn out to be more useful than the lines of force. A line of force diagram appeals to the imagination, and helps us to see what would happen to a charge in the field. But it tells us little about the strength of the field, at the best, if it is more carefully drawn, we can only say that the field is strongest where the lines are closest. But equipotentials can be labeled with the value of potential they represented; and from their spacing we can find the actual value of the potential gradient, and hence the field-strength.

If we know the potential difference between two points in the field it is possible to calculate the work done in taking the charge from one point to another. For a unit

positive charge the work done is $\int_1^2 \vec{E} d\vec{l}$, therefore, the work for any charge is

$$A_{12} = q \int_1^2 \vec{E} d\vec{l} = q(\phi_1 - \phi_2) \quad (1.24)$$

An easy way of calculating the work done is the essential advantage of a potential. One more advantage is in the fact that in many cases of calculating field-strength it is easier firstly to calculate potential and then its gradient rather than calculate the field-strength directly. Really, to calculate the potential it is necessary to take one integral and to calculate the field-strength – three integrals (as it is a vector quantity).

4. Comparison between electrostatic and gravitational fields

Comparing these electrostatic and gravitational forces, it is clear that the charge in electrostatics plays a similar role to that of the mass in gravity. A major difference is that while the gravitational force is always attractive, the electrostatic force can be either attractive or repulsive.

	<i>Electric field</i>	<i>Gravitational field</i>
Force formula	$\vec{F} = k \frac{q_1 \cdot q_2}{r^2} \vec{e}_r$	$\vec{F} = \gamma \frac{m_1 \cdot m_2}{r^2} \cdot \vec{e}_r$
Force direction	attractive/repulsive	attractive
Field strength	$E = F/q$ (N/C)	$E = F/m$ (N/kg)
Strength outside the isolated sphere	$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \vec{e}_r$	$\vec{g} = \gamma \frac{m}{r^2} \cdot \vec{e}_r$
Potential outside the isolated sphere	$\varphi = \pm \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$	$\varphi = \gamma \frac{m}{r}$

IV. ELECTRIC DIPOLE

1. A field of an electric dipole

An *electric dipole* is a system consisting of a positive charge and a negative charge having equal magnitudes (Fig. 1.17).

Let's calculate a *potential* of a dipole field.

$$\begin{aligned} \varphi &= \varphi_+ + \varphi_- = \frac{1}{4\pi\epsilon_0} \cdot \left(\frac{q}{r_+} - \frac{q}{r_-} \right) = \frac{1}{4\pi\epsilon_0} \cdot q \cdot \left(\frac{r_- - r_+}{r_+ \cdot r_-} \right) = \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q \cdot p \cdot \cos \vartheta}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cdot \cos \vartheta}{r^2}. \end{aligned} \quad (1.25)$$

Vector

$$\vec{p} = q \cdot \vec{l} \quad (1.25)$$

is called a **dipole moment** (the vector \vec{l} directed from a *negative* charge to a *positive* charge).

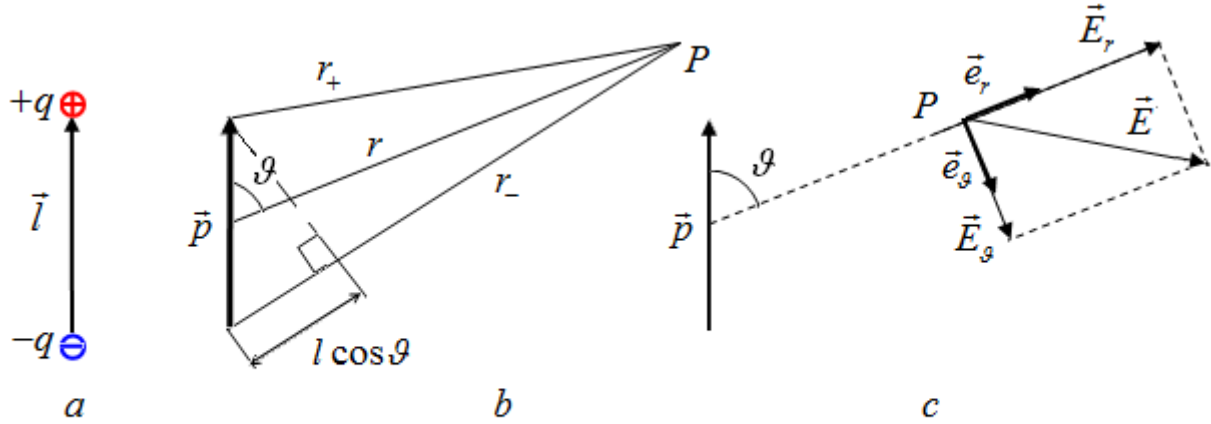


Figure 1.17 – *a* – electric dipole; *b* – distances to point P; *c* – electric field of dipole

The **electric field strength** of a dipole may be calculated taking into account its relationship with potential. Therefore, we'll determine the projections of \vec{E} on two mutually perpendicular directions along the unit vectors \vec{e}_r and \vec{e}_g .

$$E_r = -\frac{\partial \varphi}{\partial r} = -\frac{\partial}{\partial r} \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{p \cdot \cos \vartheta}{r^2} \right) = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p \cdot \cos \vartheta}{r^3}, \quad (1.27)$$

$$E_g = -\frac{\partial \varphi}{r \cdot \partial \vartheta} = -\frac{1}{r} \cdot \frac{\partial}{\partial \vartheta} \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{p \cdot \cos \vartheta}{r^2} \right) = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cdot \sin \vartheta}{r^3}, \quad (1.28)$$

$$E = \sqrt{E_r^2 + E_g^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \cdot \sqrt{1 + 3\cos^2 \vartheta}. \quad (1.29)$$

2. The electric dipole in an external electrostatic field

In an external non-uniform electrostatic field ($\vec{E} \neq const$) dipole is subjected to the force

$$\vec{F} = p \frac{\partial \vec{E}_{ext}}{\partial l}. \quad (1.2)$$

The analysis of this formula shows that

1) $\frac{\partial \vec{E}}{\partial l}$ – the directional derivative of the vector \vec{E} in a uniform field – is equal to

zero, therefore, the net force acting on the dipole is $\vec{F} = 0$ since $\vec{F}_+ = -\vec{F}_-$ (Fig. 1.18).

2) When the angle α formed by \vec{p} and \vec{E} vectors is $\alpha < 90^\circ$, the dipole is involved into a field, and when $\alpha > 90^\circ$, the dipole is pushed out from a field.

Moreover, the **torque** on a dipole placed into the external electrostatic field is

$$\vec{M} = [\vec{p}, \vec{E}] \quad (1.31)$$

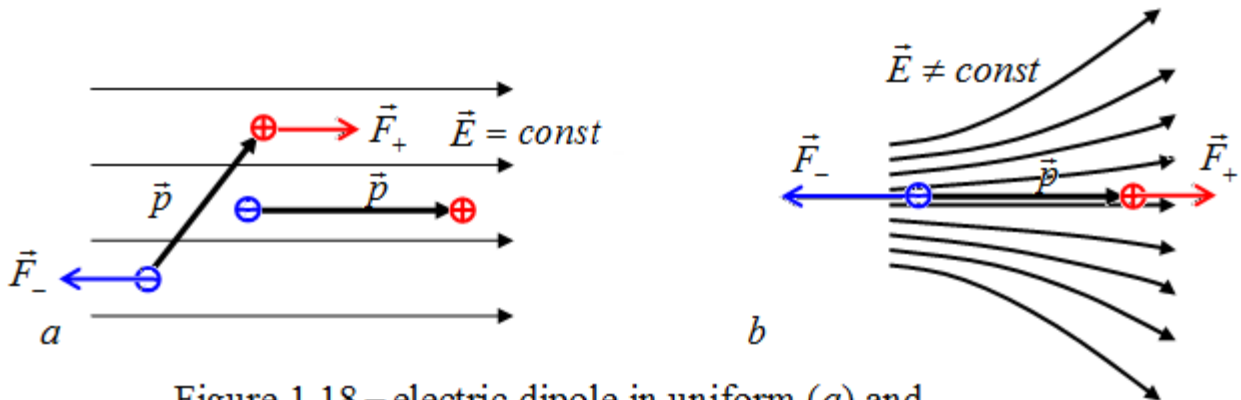


Figure 1.18 – electric dipole in uniform (a) and non-uniform (b) electric field

This torque aspires to turn the dipole to establish a dipole moment along the vector of external electric field \vec{E} .

Thus, in a non-uniform external field \vec{E} the electric dipole behaves as follows: under the action of a torque the dipole aspires to be established along the field and under the action of net force – to move in the direction where the field-strength magnitude is greater. These movements take place simultaneously.

In an external electrostatic field the dipole has the **potential energy**. If we assume that l is very small and $dl \sim l$, we obtain

$$W = q_+ \varphi_+ + q_- \varphi_- = q(\varphi_+ - \varphi_-) = q \cdot \frac{\partial \varphi}{\partial l} \cdot l = q \cdot l \cdot (-E),$$

$$W = -\vec{p} \cdot \vec{E} \quad (1.32)$$

Chapter 2. CONDUCTORS IN ELECTRIC FIELD

I. THE FIELD IN SUBSTANCE

1. Conductors, isolators, semi-conductors

All materials are divided into three categories depending on how easily they allow the charges (i.e., electrons) to flow along them. These are:

- conductors (metals, for example),
- semi-conductors (silicon is a good example), and
- insulators (rubber, wood, plastic, for example).

Most materials are either conductors or insulators. The difference between them is that in conductors, the outermost electrons in the atoms are so loosely bound to their atoms that they're free to travel around. In insulators, on the other hand, the electrons are much more tightly bound to the atoms, and are not free to flow. Semi-conductors are a very useful intermediate class, not as conductive as metals but considerably more conductive than insulators. By adding certain impurities to semi-conductors in the appropriate concentrations the conductivity can be well-controlled.

There are three ways that objects can be given a net charge. These are:

1. ***Charging by friction*** – this is useful for charging insulators. If you rub one material with another (say, a plastic ruler with a piece of paper), electrons have a tendency to be transferred from one material to the other. For example, rubbing glass with silk or saran wrap generally leaves the glass with a positive charge; rubbing polyvinylchloride rod with fur generally gives the rod a negative charge.

2. ***Charging by conduction*** – is useful for charging metals and other conductors. If a charged object touches a conductor, some charge will be transferred between the object and the conductor, charging the conductor with the same sign as the charge on the object.

3. ***Charging by induction*** – is also useful for charging metals and other conductors. Again, a charged object is used, but this time it is only brought close to the conductor, and does not touch it. If the conductor is connected to ground (ground is basically anything neutral that can give up electrons to, or take electrons from, an

object), electrons will either flow to it or away from it. When the ground connection is removed, the conductor will have a charge opposite in sign to that of the charged object.

An example of induction by means of a negatively charged object and an initially-uncharged conductor (for example, a metal ball on a plastic handle):

1. Bring the negatively-charged object close to, but not touching, the conductor. Electrons on the conductor will be repelled from the area nearest the charged object.
2. Connect the conductor to ground. The electrons on the conductor want to get as far away from the negatively-charged object as possible, so some of them flow to ground.
3. Remove the ground connection. This leaves the conductor with a deficit of electrons.
4. Remove the charged object. The conductor is now positively charged.

2. Micro- and macro-field

The true electric field in any substance (micro-field) varies rather sharply in space and in time. It is different in various points of atoms and in gaps between them. Detailed description of this field is solving incredibly complex problem. Moreover, it is absolutely unnecessary for solving macroscopic problems. For many purposes more simple and average description is suitable (macro-field).

This averaging is carried out through so-called physically infinitesimal volume, i.e. the volume containing a great amount of atoms but having sizes many times smaller than the distances on which the macro-field varies noticeably. Thus, a *field in substance* is

$$\vec{E} = \vec{E}_{macro} = \langle \vec{E}_{micro} \rangle \quad (2.1)$$

3. Influence of substance on a field

When any substance is carried into the electric field the displacement of positive and negative charges (nucleus and electrons) takes place that leads to partial division

of these charges. Uncompensated charges of opposite signs appear in different places of the substance. This phenomenon is *electric induction* and these charges are *induced charges*. These induced charges create additional electric field which simultaneously with the initial (external) fields forms the total (net) field.

Knowing an external field and the distribution of the induced charges it is possible to neglect the role of substance taken into account with help of induced charges.

4. A field inside and outside a conductor

When placing a conductor into an electric field, the displacement of charges takes place immediately, and as a result the field inside the conductor becomes equal to zero (Fig. 2.1). Excessive charges are in a thin superficial layer. The absence of a field inside a conductor means that the potential in a conductor is the same in all points, i.e., any conductor in an electrostatic field represents the equipotential area and its surface is an equipotential surface. Hence, the vector of field-strength is directed along the normal to the surface as if the vector is directed along the tangent, the charges move on surfaces and their equilibrium is impossible.

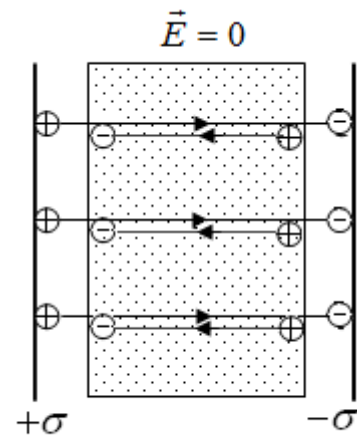


Figure 2.1

If a part of the substance is removed from the conductor the field in it is not changed. The excess charges are still on an external surface and the field inside the cavity is equal to zero. This property is the basis of *electrostatic protection*, i.e., screening, for example, of measuring instruments from the influence of external electric fields.

II. CAPACITANCE. CAPACITORS

1. Capacitance of an isolated conductor

Consider a conductor removed from other charges. The potential of the conductor

is proportional to its charge. Hence, the ratio q/φ does not depend on the magnitude of the charge. For any isolated conductor this ratio has its specific magnitude which is called the **capacitance** of a solitary conductor

$$C = \frac{q}{\varphi}. \quad (2.2)$$

The capacitance depends on the sizes and the form of a conductor.

$$[C] = \frac{C}{V} = \text{Farad} = F.$$

2. Capacitors. Capacitance. Energy of a charged capacitor

A system of conductors which has capacitance much greater than the solitary conductor capacitance and is independent on surrounding bodies is called a **capacitor**.

The simplest capacitor (**parallel plate capacitor**) consists of two plane conductors (plates) located at a small distance from each other. The main characteristic of the capacitor is capacitance.

The **capacitance of a capacitor** is the ratio of the charge on either plate to the potential difference between the plates (voltage),

$$C = \frac{q}{U}. \quad (2.3)$$

Now we can find out what factors influence the capacitance

$$\left. \begin{array}{l} E = \frac{\sigma}{\varepsilon_0} = \frac{q}{S \cdot \varepsilon_0} \\ U = E \cdot d = \frac{q \cdot d}{S \cdot \varepsilon_0} \end{array} \right\} C = \frac{q}{U} = \frac{q \cdot S \cdot \varepsilon_0}{q \cdot d} = \frac{\varepsilon_0 \cdot S}{d}. \quad (2.4)$$

If a dielectric is used between the plates of the capacitor the capacitance is

$$C = \frac{\varepsilon_0 \cdot \varepsilon \cdot S}{d}, \quad (2.5)$$

where ε is a dielectric permittivity.

A charged capacitor is a store of **electric energy**:

$$W = \frac{qU}{2} = \frac{CU^2}{2} = \frac{q^2}{2C}.$$

Chapter 3. ELECTRIC FIELD IN DIELECTRICS

I. POLARIZATION OF DIELECTRICS

1. Dielectrics

<i>Type of molecules</i>	<i>Polar molecules</i>	<i>Non-polar molecules</i>
Characteristic of the molecule	The centers of positive and negative charges are displaced relatively each other	The centers of positive and negative charges coincide
Dipole moment	Proper dipole moment is present	Proper dipole moment is absent
Examples	HCl, H ₂ O	H ₂ , N ₂ , O ₂
Influence of external field	The field turns the dipole along the field-strength lines	The field displaces the centers of positive and negative charges in the opposite directions and molecule acquires an induced dipole moment
Polarization	<i>Orientation polarization</i>	<i>Electronic polarization</i>
Type of dipole	Rigid dipole	Elastic dipole

Dielectrics (insulators) are the substances which practically do not conduct electric current (10^{15} - 10^{25} times worse than conductors). It means that in dielectrics there are no charges capable to move by significant distances creating a current.

Placing even neutral dielectric into external electric field essentially modifies both the field and the dielectric.

To understand the causes it is necessary to consider the features of dielectrics

structure. Dielectric consists either of neutral molecules (polar, non-polar) or of ions (ionic crystals).

Ionic crystals (for example, NaCl) consist of two sublattices inserted into each other in sites of which the charged ions are located. In an external field the positive sublattice is displaced along the field and the negative sublattice is displaced in the opposite direction. It is so-called **ionic polarization**.

Polarization of a dielectric is its transition into the state when its total dipole moment becomes nonzero. The mechanism of polarization is related to a certain structure of a dielectric.

2. Volume and surface bound charges

As a result of polarization uncompensated charges appear on the surface and, generally, in the volume of a dielectric.

Consider a plate of a neutral inhomogeneous dielectric where the density increases with the growth of x (Fig. 3.1).

Without an external field the densities of positive and negative charges are equal to each other at any point of dielectric since the dielectric is neutral, but the volume densities of charge increase with growth of x because of heterogeneity of the dielectric. The distributions of positive and negative charges are similar.

The external field displaces the positive charges inside the atoms or molecules along the field and negative charges - in the opposite direction. Uncompensated charges appear on the surface and in the volume of the dielectric. These uncompensated charges appeared as a result of polarization of the dielectric are called **polarization charges** (or **bound charges**). The last term underlines that the freedom of transition of such charges is limited: they can be displaced only inside electrically neutral molecules.

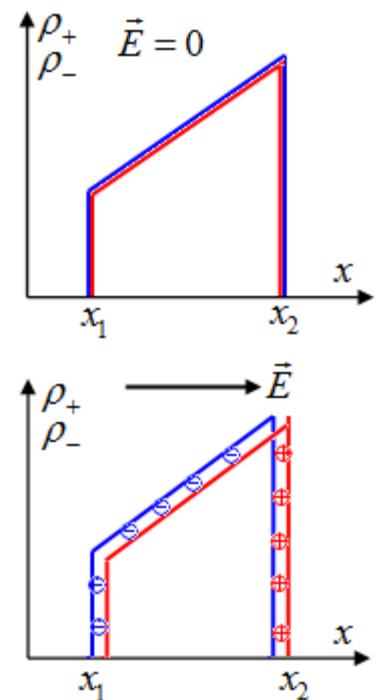


Figure 3.1

We'll mark the bound charges with a prime (for example, q' or ρ'). The charges which are not part of molecules of a dielectric are *extraneous charges*. These charges can be both inside and outside dielectric.

During polarization of a dielectric the volume and surface bound charges may generally appear in it. Hence, the field in a dielectric is the sum of a field of extraneous charges and a field of bound charges $\vec{E} = \vec{E}_0 + \vec{E}'$.

II. POLARIZATION \vec{P} AND ITS PROPERTIES

1. Polarization \vec{P}

It is naturally to characterize the process of polarization of a dielectric by means of dipole moment of unit volume. If the external field is non-uniform or the dielectric is inhomogeneous the degree of polarization appears to be non-uniform in different points of a dielectric. To describe the polarization at a given point we'll select a physically infinitesimal volume containing this point and find the vector sum of dipole moments of molecules in this volume. The vector \vec{P} that is equal to dipole moment of unit volume of a dielectric is *polarization* of a dielectric.

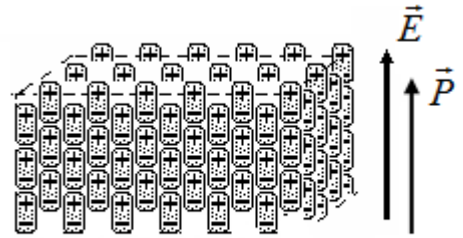


Figure 3.2

$$\vec{P} = \frac{1}{\Delta V} \sum_i \vec{p}_i. \quad (3.1)$$

If the volume ΔV contains ΔN molecules, then

$$\vec{P} = \frac{1}{\Delta V} \sum_i \vec{p}_i \cdot \frac{\Delta N}{\Delta N} = \frac{\Delta N}{\Delta V} \cdot \frac{\sum_i \vec{p}_i}{\Delta N} = n \cdot \langle \vec{p} \rangle, \quad (3.2)$$

where n is the *molecule concentration*; $\langle \vec{p} \rangle$ is an *average dipole moment* of a molecule.

$$[P] = C/m^2.$$

2. Relationship between the vectors \vec{P} and \vec{E}

For the majority of isotropic dielectrics the dependence of polarization on field-strength is linear, i.e.,

$$\vec{P} = \kappa \epsilon_0 \vec{E}, \quad (3.3)$$

where κ is the unitless positive magnitude describing the properties of the dielectric called as *dielectric susceptibility*. The usual values of κ for the large majority of dielectrics are several units or tens.

But there is a whole class of dielectrics (ferroelectrics) with nonlinear dependence between these quantities.

3. Ferroelectrics

Dielectrics with nonlinear dependence $P(E)$ which is determined by the previous history of a dielectric, i.e., preceding magnitudes of the field-strength, are *ferroelectrics*. The examples of these dielectrics are $\text{NaKC}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$, BaTiO_3 , KH_2PO_4 .

Their properties are:

1. High magnitudes of dielectric susceptibility (~ several thousands);
2. The dielectric susceptibility depends on field-strength;
3. Dependence $P(E)$ looks like a loop called as a *hysteresis loop*.

The explanation of the properties and the behavior of ferroelectrics is following. Owing to interaction of the particles in a ferroelectric crystal the dipole moments are spontaneously established in a parallel. Areas of spontaneous polarization (*ferroelectric domains*) appear. Without a field the total dipole moment of a crystal is equal to zero. In external field there is a growth of the domains oriented along the field and turning of dipole moments of domains. The dipole moment of a crystal

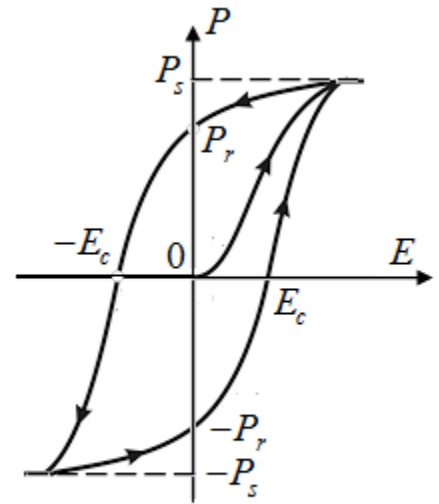


Figure 3.3

becomes nonzero.

Ferroelectrics have the temperature at which they lose their ferroelectric properties and become the usual dielectrics. This temperature is *Curie's temperature*. For example, Seignette's salt $\text{NaKC}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$ is ferroelectric in the temperature interval from 255K up to 297K.

4. Piezoelectric effect

The *piezoelectric effect* (from Greek *piezo* [$\pi\epsilon\zeta\omega$], which means 'squeeze' or 'press') is the effect in which energy is converted between mechanical and electrical forms. It was discovered in the 1880 by French scientists Jacques and Pierre Curie. Specifically, when a pressure is applied to a polarized crystal, the resulting mechanical deformation results in an electric charge. Piezoelectric microphones serve as a good example of this phenomenon. Microphones turn an acoustical pressure into a voltage. On the contrary, when an electric field is applied to a piezoelectric crystal, the crystal undergoes a mechanical deformation which can in turn create an acoustical pressure (*backward piezoelectric effect*, 1881, Gabriel Lippmann). An example of this can be seen in piezoelectric speakers. (These are the cause of those annoying system beeps that are too common in today's computers).

5. Properties of the vector \vec{P} field

1. For the vector \vec{P} field *Gauss' law* is

$$\oint \vec{P} d\vec{S} = -q', \quad (3.4)$$

i.e., the flux of the vector \vec{P} through any closed surface whatever its shape is equal to taken with the opposite sign the excess bound charge of a dielectric in the volume enclosed by the surface.

2. The volume density of the excess bound charges inside a dielectric is equal to zero under simultaneous realization of two conditions: a) the dielectric should be homogeneous; b) there are no extraneous charges inside it. Therefore, if a homogeneous isotropic dielectric is placed into an electric field the bound charges

appear only on its surface.

3. The field of the vector \vec{P} depends on all charges (both bound and extraneous). The bound charges determine only the flux of the vector \vec{P} through a closed surface but not the field of the vector \vec{P} .

III. THE VECTOR \vec{D}

1. Gauss' law for the vector \vec{D} field

Gauss' law for the vector \vec{E} is

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{inernal}}{\epsilon_0}. \quad (3.5)$$

Since the sources of a field \vec{E} are all charges (bound and extraneous),

$$\oint_S \epsilon_0 \cdot \vec{E} \cdot d\vec{S} = q + q' = q - \oint_S \vec{P} \cdot d\vec{S};$$

$$\oint_S (\epsilon_0 \vec{E} + \vec{P}) d\vec{S} = q.$$

Put

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}, \quad (3.6)$$

then

$$\oint_S \vec{D} d\vec{S} = q. \quad (3.7)$$

This is **Gauss' law for the vector \vec{D}** : the flux of the vector \vec{D} through any closed surface whatever its shape is equal to total extraneous charge in the volume enclosed by the surface.

The vector \vec{D} (*electric induction* or *electric displacement*) is an auxiliary vector without any deep physical sense.

2. Relationship between the vectors \vec{D} and \vec{E} . Dielectric permittivity

For isotropic dielectrics polarization $\vec{P} = \kappa\epsilon_0\vec{E}$, hence,

$$\vec{D} = \epsilon_0\vec{E} = \vec{P} = \epsilon_0\vec{E} + \kappa\epsilon_0\vec{E} = (\kappa + 1)\epsilon_0\vec{E} = \epsilon\epsilon_0\vec{E}. \quad (3.8)$$

The dimensionless quantity

$$\epsilon = \kappa + 1 \quad (3.9)$$

is *dielectric permittivity* of the substance. It is the basic characteristic of a dielectric. For all dielectrics $\epsilon > 1$; for vacuum $\epsilon = 1$. Magnitude of dielectric permittivity depends on the nature of the dielectric: $\epsilon \approx 1$ (gases); $\epsilon \sim$ several thousands units (ceramics); $\epsilon = 81$ (distilled water).

Dielectric permittivity shows how a dielectric *decreases* the force of interaction between charges, in other words, permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium.

Chapter 4. DIRECT ELECTRIC CURRENT

I. OHM'S LAW

1. Current. Current density

Suppose a potential difference is established between two points, and some charges are released. These charges will be acted on by the electric force and start to move. If we measure the amount of charge that flows through a given point in a certain time interval, we can define the *electric current* (i.e., the charge flowing through that point per unit time) via

$$I = \frac{dq}{dt} \quad (4.1)$$

The unit of current is

$$[I] = \frac{\text{Coulomb}}{\text{second}} = \text{Ampere} = \text{A}.$$

Physical currents in electric circuits consist of electrons in the wire moving from lower to higher electric potential. Since electrons by convention have negative charge, the net effect is equivalent to having a positive current flow from higher to lower electric potential. *Conventional current direction* is the direction which is opposite to electron direction.

Electric current may consist not only of electrons, but of other types of charged particles, for example, *ions*. The word "ion" is ancient Greek for "going" and was introduced by Michael Faraday (1791-1867) to designate those electrically charged particles that migrate to one or another pole when an electric field is set up in a solution. The ions may be positive and negative.

Without an electric field the *carriers of current* (electrons, ions) move chaotically. In electric fields the directed motion at a velocity \vec{u} (*drift velocity*) is superimposed on a random motion of carriers.

As an electric current can be distributed non-uniformly over the surface through which it flows we introduce the vector of a *current density* \vec{j} .

The magnitude of the vector \vec{j} is equal to a ratio of current dI through the unit area dS_{\perp} located at the given point perpendicularly to the current direction

$$j = \frac{dI}{dS_{\perp}}. \quad (4.2)$$

The direction of \vec{j} coincides with the direction of the velocity \vec{u} of the positive carriers (or it is opposite to the direction of the negative carriers).

If there are negative and positive carriers, the current density is

$$\vec{j} = \rho_+ \vec{u}_+ + \rho_- \vec{u}_-, \quad (4.3)$$

where ρ_+ and ρ_- are the electric densities of the positive and the negative carriers, \vec{u}_+ , \vec{u}_- are their velocities.

Electrons ($\rho_- < 0, u_+ = 0$) are the carriers in conductors, therefore, the current density in them is

$$\vec{j} = \rho_- \vec{u}_-.$$

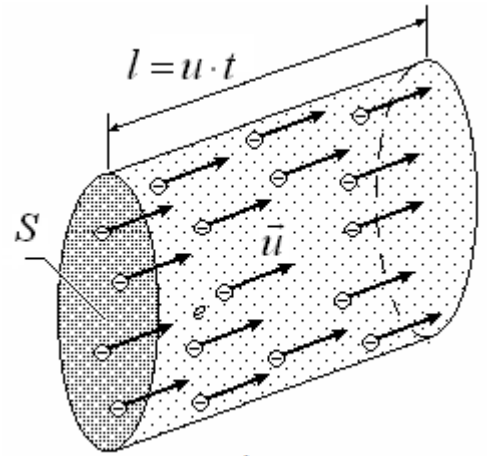


Figure 4.1

The current through the surface S is

$$I = \int_S \vec{j} \cdot d\vec{S}, \quad (4.4)$$

where $d\vec{S} = dS \cdot \vec{n}$, and \vec{n} is the positive normal to the surface dS .

Let's find the relationship between a current density and a speed of directed motion of electrons.

The speed of electrons can be written as $u = l/t$, where l is the distance for time t (Fig. 4.1). The time can be found by using the definition of current,

$$I = \frac{dq}{dt} \Rightarrow \int_0^{eN} dq = I \int_0^t dt \Rightarrow N \cdot e = I \cdot t \Rightarrow t = \frac{N \cdot e}{I};$$

Now the speed

$$u = \frac{l \cdot I}{N \cdot e} = \frac{l \cdot S}{N} \cdot \frac{I}{S} \cdot \frac{1}{e}.$$

Define the *free electron density* $n = \frac{N}{V}$ and then obtain the speed of electrons (*drift speed*)

$$u = \frac{j}{ne}. \quad (4.5)$$

2. Ohm's law

The relationship between the potential difference and current is given by the law that was discovered by German physicist and mathematician Georg Simon Ohm (1789-1854) in 1827 (Fig. 4.2).



Figure 4.2

Ohm's law in integral form: the voltage $U = \varphi_1 - \varphi_2$ and current I are proportional

$$U = I \cdot R. \quad (4.6)$$

The constant of proportionality R is called the *resistance*.

The unit of resistance is Volt/Ampère, or Ohm.

$$[R] = \text{V/A} = \text{Ohm} = \Omega.$$

Voltage can be thought of as the pressure pushing charges along a conductor, while the electric resistance of a conductor is a measure of how difficult it is to push the charges along.

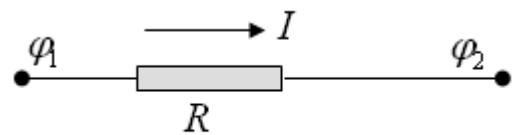


Figure 4.3

Using the flow analogy, electric resistance is similar to friction. For water flowing through a pipe, a long narrow pipe provides more resistance to the flow than does a short fat pipe. The same applies for flowing currents: long thin wires provide more resistance than short thick wires do.

The *resistance of a conductor* R depends on its length, cross-sectional area, material and temperature, and also on the configuration of current in a conductor. For the uniform cylindrical conductor (a wire) resistance is

$$R = \rho \frac{l}{S}, \quad (4.7)$$

where ρ is *resistivity of material* (ρ is dependent on the microscopic properties of the conducting material), l and S are the length and the cross-sectional area of a wire, relatively.

At the atomic level, currents are pictured as the flow of the outer electrons of atoms through the material. Resistance then results from collisions of electrons with other electrons and with atoms. From this we would expect that raising the temperature of a material would increase the resistance, as the added heat energy would cause the electrons move faster and hence collide more often.

$$\rho = \rho_0(1 + \alpha t^\circ), \quad (4.8)$$

where ρ_0 is the resistivity at 0°C ; α is a *temperature coefficient of resistance*, and t° is the temperature in Celcius degrees.

The materials for which the relationship between the voltage and the resistance is in accordance with Ohm's law are *ohmic conductors* (or *linear conductors*); the materials with more complicated relationship are *non-ohmic conductors*.

Let's find Ohm's law in the differential form. Taking into account that $I = j \cdot dS$; $U = E \cdot dl$, and $R = \rho \frac{dl}{dS}$, substitute these expressions into the formula $I = \frac{U}{R}$, and receive

$$j = \frac{1}{\rho} E. \quad (4.9)$$

Then Ohm's law in differential form is

$$\vec{j} = \frac{1}{\rho} \vec{E} = \sigma \vec{E}, \quad (4.10)$$

where $\sigma = \rho^{-1}$ is an *electric conductivity*.

$$[\sigma] = \Omega^{-1} \cdot \text{m}^{-1} = \text{Siemens} \cdot \text{m}^{-1} = \text{S} \cdot \text{m}^{-1}.$$

3. Fundamental Ohm's law

If all forces acting on carriers of current are reduced only to electrostatic force, the positive carriers move from higher to lower electric potential and negative carriers – in the opposite direction. This process would lead to the equalization of potential and as a result to the stoppage of a current. Hence, the forces of another origin (not electrostatic) have to act in a direct-current circuit. They can be stipulated by a chemical and physical heterogeneity of a conductor (for example, the forces originating from the junction of heterogeneous conductors – in galvanic cells, or conductors with different temperature – in thermocouples) and some other reasons. These are so-called *extraneous forces*.

If \vec{E}^* is a field-strength of extraneous forces (the magnitude of \vec{E}^* is equal to the extraneous force acting on a unit positive charge) then *fundamental Ohm's law* in the differential form is

$$\vec{j} = \sigma(\vec{E} + \vec{E}^*). \quad (4.11)$$

4. Ohm's law for the open and closed circuits. Electromotive force (emf)

Let's consider an electric current in a wire. The current direction coincides with the axis of a wire and $\vec{j} = const$ at any point of the wire cross-section S (Fig. 4.4).

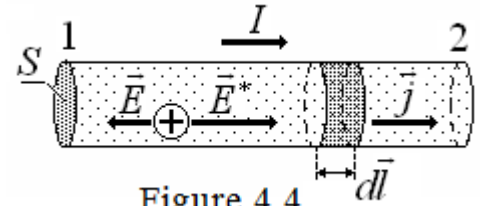


Figure 4.4

Divide the expression $\vec{j} = \sigma(\vec{E} + \vec{E}^*)$ by σ , make a dot product of it and infinitesimal length $d\vec{l}$ and then take an integral over the length of the wire from the point 1 up to the point 2.

$$\int_1^2 \frac{\vec{j}}{\sigma} d\vec{l} = \int_1^2 \vec{E} \cdot d\vec{l} + \int_1^2 \vec{E}^* \cdot d\vec{l}.$$

The first component is the potential drop on the section 1-2 of the wire:

$$\int_1^2 \frac{\vec{j}}{\sigma} d\vec{l} = \int_1^2 \rho \cdot \vec{j} \cdot d\vec{l} = \int_1^2 \rho \cdot j_l \cdot dl = \int_1^2 \rho \cdot \frac{I}{S} \cdot dl = I \int_1^2 \rho \cdot \frac{dl}{S} = IR_{12}. \quad (4.12)$$

The second component is the potential difference (p.d.) of the points 1 and 2:

$$\int_1^2 \vec{E} \cdot d\vec{l} = \varphi_1 - \varphi_2. \quad (4.13)$$

The third component is

$$\int_1^2 \vec{E}^* \cdot d\vec{l} = E_{12}, \quad (4.14)$$

where E_{12} is the electromotive force (emf) operating on the section 12.

$[E] = \text{Volt} = \text{V}$.

Electromotive force is the physical value equal to the work of the extraneous forces produced in moving a positive unit charge around the closed circuit.

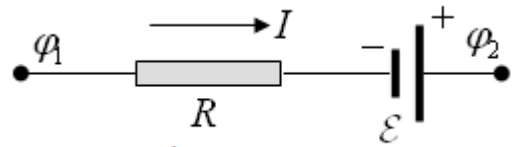


Figure 4.5

If emf promotes the motion of positive carriers of a current in the chosen direction it is positive, otherwise, it is negative.

Ohm's law for the open circuit is

$$IR = \varphi_1 - \varphi_2 + E_{12}. \quad (4.15)$$

For a complete circuit the points 1 and 2 coincide, hence, $\varphi_1 = \varphi_2$, The total resistance of the circuit consists of the external (load) resistance R and source resistance $r < R_{total} = R + r$.

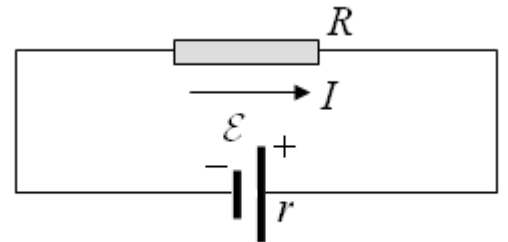


Figure 4.6

In this case, the **Ohm's law for the complete (closed) circuit** takes on form

$$IR_{total} = E. \quad (4.16)$$

II. COMPLICATED CIRCUITS. KIRCHHOFF'S RULES

Ohm's law is applicable only for simple circuits. For complicated circuits, Kirchhoff's laws can be used to find current or voltage. There are two generalized laws: Kirchhoff's current law and Kirchhoff's voltage law. They were formulated by the German scientist Gustav Robert Kirchhoff (1824-1887) in 1845 (Fig. 4.7).

The first Kirchhoff's rule (**Current Law**) refers to any junction (node) of complicated circuit. A **junction** is the point in the circuit where three or more conductors are joined together. Another name of the first rule is **the junction theorem**. This theorem is explained by the law of conservation of charge.

The first Kirchhoff's rule:

The algebraic sum of the currents at any junction of circuit is zero:

$$\sum_k I_k = 0. \quad (4.17)$$

A current is reckoned to be positive if it flows towards the node and negative if it flows away from it. It should not really matter which you choose to be the positive or negative current, as long as you stay consistent.

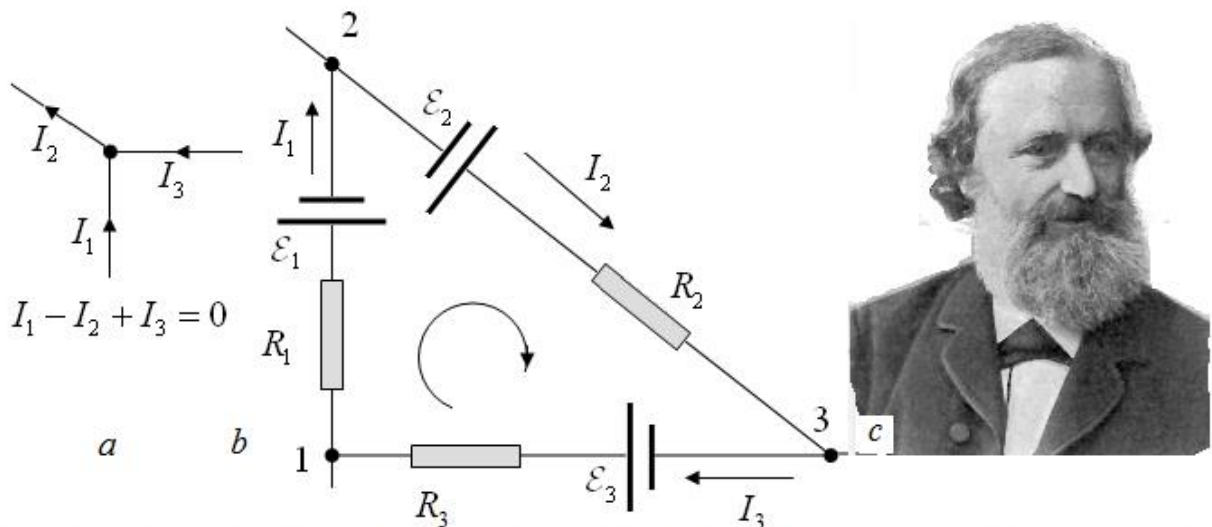


Figure 4.7 – The 1st (a) and 2nd (b) Kirchhoff's Rules; Gustav Robert Kirchhoff (c)

The first Kirchhoff's rule gives a set of equations. If m is the number of junctions in a complicated circuit, the amount of the independent equations is equal to $(m - 1)$.

The second Kirchhoff's rule (**Voltage Law**) connects the emf and **potential drops** (or **potential differences**) (p.d.) in a complete circuit. It refers to any closed loop. A

loop in a circuit is any closed conducting path. Another name of the second rule is *the Loop Theorem*. This theorem is explained by the law of conservation of energy and the fact that the electric force is conservative.

The second Kirchhoff's rule: The algebraic sum of the emf is equal to the algebraic sum of all p.d. in any closed loop of circuit *or* the total voltage around a closed loop must be zero.

$$\sum_i I_i R_i = \sum_k E_k. \quad (4.18)$$

To write down the equations according to the second rule we have to choose the direction of moving around the loop. The p.d. is positive if it is in the same direction as the net emf.

The amount of the independent equations according to the second rule is $[p - (m - 1)]$, where p is the amount of branches in the network. The *branch* (the *arm*) of a network is the part of circuit between two junctions.

III. THE ELECTRIC CURRENT EFFECTS

1. Heating effect of a current. Joule - Lenz's law

The conduction of electricity in metals is due to free electrons as they move through the metal. On their way they collide frequently with atoms. At each collision they lose some of their kinetic energy, and give it to the atoms which they strike. Thus, as the current flows through the wire, it increases the kinetic energy of vibration of the metal atoms: it generates heat in the wire. The electric resistance of the metal is due, we say, to its atom obstructing the drift of the electrons past them: it is analogous to mechanical friction. As the current flows through the wire, the energy lost by electrons per second is the electric power supplied by the battery which maintains the current. That power comes from the chemical energy liberated by these actions within the battery.

Joule-Lenz's law (or **Joule's First Law**) is a law that defines the amount of heat Q liberated in the conductor when an electric current I passes through it. The heat is

proportional to the conductor's resistance R , to the square of the current I^2 and the time t for which the current flows:

$$Q = I^2 \cdot R \cdot t = UI t = \frac{U^2}{R} t. \quad (4.19)$$



Figure 4.8 – James P. Joule (a) and Heinrich E.C. Lenz (b)

This law was established in 1841 by English physicist James Prescott Joule (1818-1889) and independently discovered and confirmed in 1842 by experiment of Russian physicist of Baltic German ethnicity Heinrich Friedrich Emil Lenz (1804-1865) (Fig. 4.8). The heating effect of conductors carrying currents is known as ***Joule heating***.

2. Chemical effect of a current. Faraday's laws of electrolysis

The passage of an electric current through a conducting solution causes chemical reactions. This is known as the ***chemical effect*** of electric current. Some of the chemical effects of electric current are the following: formation of bubbles of a gas on the electrodes, deposition of metal on electrodes, and changes the colour of solution. The process of decomposition of a chemical compound in a solution when an electric current passes through it is called ***electrolysis***. The solution that conducts electricity due to the presence of ions is called an ***electrolyte***.

Electrolysis takes place in so-called ***ion conductors (ionic solutions)*** where the electric current is due to the movement of ions and there are associated chemical changes. In a typical experiment of electrolysis, two metal rods are immersed in an

ionic solution and connected to the two ends of a battery. The dissociated ions appear at the two plates. The English scientist Michael Faraday (1791-1867) was the first to carry out quantitative investigations and summarize them in the now two famous laws of electrolysis (1832-1834) (Fig. 4.9, a). He introduced in the scientific terminology and popularized the words ‘anode’, ‘cathode’, ‘electrode’, and ‘ion’

Faraday's laws of electrolysis

I. The mass of an element deposited or liberated during electrolysis is proportional to the quantity of charge q that passes through the electrolyte.

$$m = k \cdot q = k \cdot I \cdot t . \quad (4.20)$$

where k is the ***electrochemical equivalent*** of an element (k is the mass of element deposited or liberated by 1 Coulomb of electric charge).

II. Electrochemical equivalents of elements are proportional to their chemical equivalents

$$\frac{k_1}{k_2} = \frac{x_1}{x_2} , \quad (4.21)$$

where $x = \frac{A}{n}$ is a ***chemical equivalent***; A is atomic mass of element; n is its valence.

$$\frac{x_1}{k_1} = \frac{x_2}{k_2} = F . \quad (4.22)$$

$F = e \cdot N_A = 96485.33 \approx 96500 \text{ C} \cdot \text{mol}^{-1}$ is ***Faraday's constant for electrolysis*** (the electric charge carried by one mole of electrons (or singly-ionized ions). It is equal to the product of the Avogadro number (constant) $N_A = 6.022 \cdot 10^{23} \text{ mol}^{-1}$, and the absolute value of the charge on an electron $e = 1.6 \cdot 10^{-19} \text{ C}$).

Mole (mol) is the fundamental SI unit of substance, defined as the amount of substance that contains as many elementary units (atoms, molecules, ions, etc.) as there are atoms in 0.012 kg of Carbon ^{12}C .

Two Faraday's laws may be incorporated into one *general law of electrolysis*.

$$m = k \cdot q = \frac{1}{F} \cdot \frac{A}{n} \cdot q = \frac{1}{F} \cdot \frac{A}{n} \cdot I \cdot t. \quad (4.23)$$

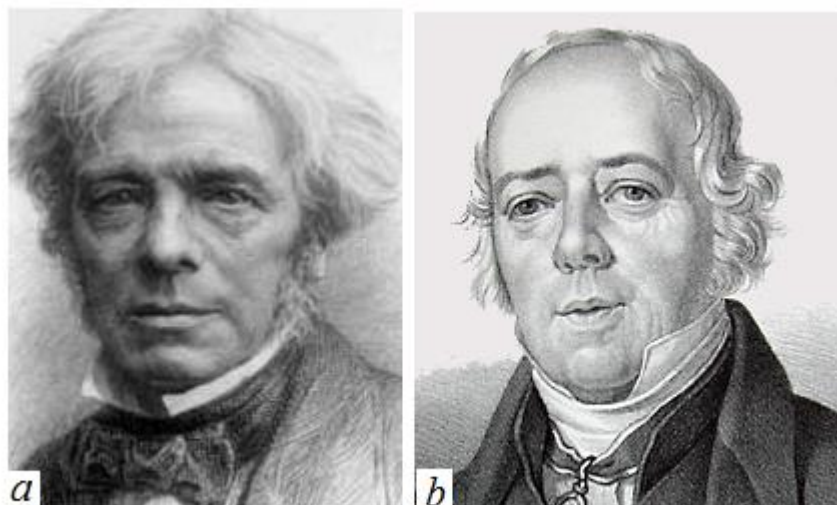


Figure 4.9 – Michael Faraday (a) and Hans Christian Oersted (b)

3. Magnetic effect of a current. Oersted's experiment

Danish physicist Hans Christian Oersted (1777-1851) (Fig. 4.9, b) was a professor of science at Copenhagen University. In 1820 he arranged a science demonstration to friends and students in his home. He planned to demonstrate the heating of a wire by an electric current, and also to carry out demonstrations of magnetism, for which he provided a compass needle mounted on a wooden stand. While performing his electric demonstration, Oersted noted to his surprise that every time the electric current was switched on, the compass needle moved. In the months that followed he worked hard trying to make sense out of the new phenomenon. In the end he published his findings without any explanation. French scientist Andre-Marie Ampère (1775-1836) read Oersted's report, and in the space of a week had repeated the observations and developed a mathematical theory describing how the magnetic fields depends on the current.

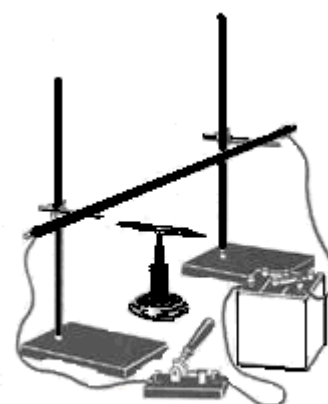


Figure 4.10

We'll consider the nature of this phenomenon in the next chapters.