NATIONAL TECHNICAL UNIVERSITY
“KHARKIV POLYTECHICAL INSTITUTE”

DEPARTMENT OF PHYSICS

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PROBLEM SOLVING GUIDE

“MAGNETISM”

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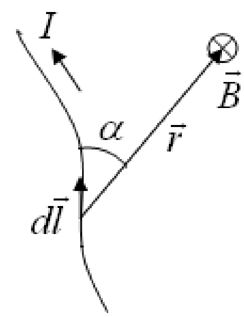
BASIC FORMULAS

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\left[d\vec{l}, \vec{r}\right]}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{I\left[d\vec{l}, \vec{e}_r\right]}{r^2}$$



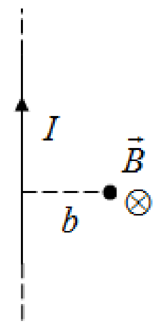
where $d\vec{l}$ is an infinitesimal length of the conductor carrying electric current I, $\vec{r}$ is the vector distance from the current to the field point, $\vec{e}_r$ is unit vector to specify the vector $\vec{r}$, $\mu_0 = 4\pi \cdot 10^{-7}$ (H/m) is the *magnetic permeability* of free space.

The magnitude of magnetic field vector is

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot dl \cdot \sin\alpha}{r^2}.$$

Magnetic field (magnetic induction) due to a *long straight wire* is

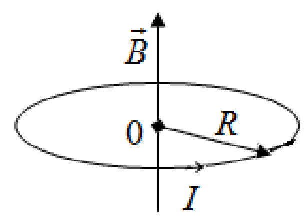
$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{b} = \frac{\mu_0}{2\pi} \cdot \frac{I}{b}.$$



The direction is given by a *right-hand rule*: point the thumb of your right hand in the direction of the current, and your fingers indicate the direction of the circular magnetic field lines around the wire.

Magnetic field (magnetic induction) in the center *of a circular loop:*

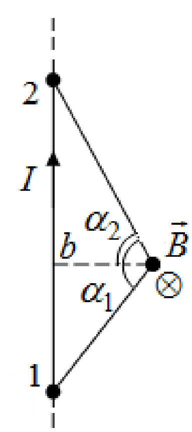
$$B_0 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{R}.$$



A right-hand rule for circular loop: curl the fingers of your right hand in the direction of the current flow, and your thumb points in the direction of the magnetic field inside the loop.

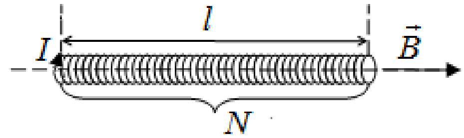
Magnetic field (magnetic induction) at a *distance b from a straight wire of finite length*

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \left(\sin\alpha_2 - \sin\alpha_1\right)$$



Magnetic field on the axis of a solenoid of

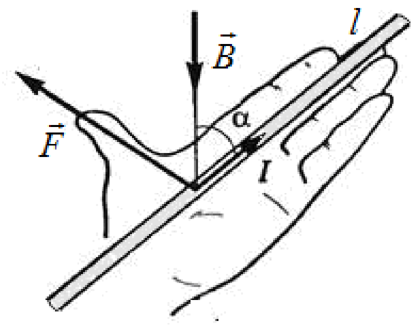
$$B = \mu_0 \frac{IN}{l} = \mu_0 nI$$



where N is the number of turns over the axial length l.

Ampere's Law: Magnetic force (*Ampere's force*) on a current-carrying wire $\left(d\vec{l}\right)$ in magnetic field $\vec{B}$ is

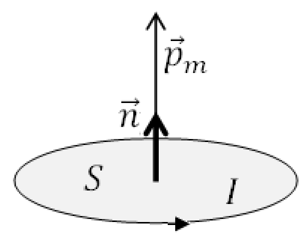
$$d\vec{F} = I\left[d\vec{l}, \vec{B}\right].$$



The magnitude of Ampere's force is

$$dF = Idl\sin\alpha$$

The direction of the force $\vec{F}$ may be visualized by the *left hand rule*: lines of magnetic field go into palm, fingers direction is direction of the current, and then the thumb points the direction of the force.



The force (per unit length) between two parallel current-carrying conductors (I_1 and I_2) separated by distance d

$$F_l = \mu_0 \frac{I_1 I_2}{2\pi d}$$

Magnetic dipole moment

$$\vec{p}_m = I \cdot S \cdot \vec{n},$$

where I is the current in the loop and S is its area $\vec{n}$ is a normal to a loop. The direction of the magnetic moment is perpendicular to the current loop according to the right-hand rule.

The torque $\vec{M}$ on a loop of the area S with current I in the magnetic field $\vec{B}$ and its magnitude

$$\vec{M} = \left[\vec{p}_m, \vec{B}\right],$$

$$M = p_m B\sin\alpha = ISB\sin\alpha.$$

The *Lorentz force* is acting on a particle of a charge q moving at the velocity $\vec{v}$:

$$\vec{F} = q\left[\vec{v}, \vec{B}\right],$$

and its magnitude is

$$F = qvB\sin\alpha.$$

The equation for magnetic flux: Φ is the magnetic flux, $\vec{B}$ is the magnetic field strength, α is the angle between $\vec{B}$ and a normal to the plane of the loop $\vec{n}$; S is the area of the loop; $\vec{S} = S \cdot \vec{n}$.

$$\Phi = \left(\vec{B}, \vec{S}\right) = BS\cos\alpha .$$

The elementary work done for displacement of the current-carrying conductor in the magnetic field

$$dA = I \cdot d\Phi$$

This is the equation for induced emf. ε is the induced emf, $d\Phi$ is the change in magnetic flux, and dt is the change in time.

$$\varepsilon = -\frac{d\Phi}{dt}$$

The equation for the induced emf ε in a loop (or conductor) of width ***l*** that is moving into or out of a magnetic field $\vec{B}$ at a velocity $\vec{v}$.

$$\varepsilon = Blv .$$

A self-induced emf is proportional to the time rate of change of the current

$$\varepsilon_s = -L\frac{dI}{dt},$$

where L is the inductance (depends on the geometry of the coil and other physical characteristics.

The inductance of a uniformly wound solenoid having N turns, the length l and the cross-section area S is

$$L = \mu_0 \frac{N^2}{l^2} Sl = \mu_0 n^2 V ,$$

where $n = N/l$ is the number of turns per unit length, V is the interior volume of the solenoid.

Constants	
Speed of light (EM waves) in vacuum or air	$c = 3 \cdot 10^{8}$ m/s
Electric constant (vacuum permittivity)	$\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m
$k = 1/4\pi\varepsilon_0$	$9 \cdot 10^{9}$ N·m^2·C^{-2}H/m
Magnetic constant (vacuum permeability)	$\mu_0 = 4\pi \cdot 10^{-7} = 1.2566 \cdot 10^{-6}$
$\frac{\mu_0}{4\pi}$	10^{-7} H/m
Elementary charge (proton, electron)	$e = 1.6 \cdot 10^{-19}$ C
α -particle charge	$q_\alpha = 2e = 3.2 \cdot 10^{-19}$ C
Mass of electron	$m_e = 9.1 \cdot 10^{-31}$ kg
Mass of proton	$m_p = 1.66 \cdot 10^{-27}$ kg
Mass of α -particle	$m_\alpha \approx 4m_p = 6.64 \cdot 10^{-27}$ kg
Electron-Volt	1eV=$1.6 \cdot 10^{-19}$ J

Properties of substances		
Substance	*Density, kg/m^3*	*Resistivity, $n\Omega \cdot m$*
Aluminum	$2,6 \cdot 10^{3}$	25
Iron	$7,9 \cdot 10^{3}$	87
Copper	$8,6 \cdot 10^{3}$	17

Problem 1

Two long parallel wires carry the currents I_1 *= 20 A and* I_2 *=30 A separated by a distance d = AB =10 cm. Find the magnetic field in the points:*

1). M_1 (M_1A = 2 cm); 2). M_2 (AM_2 = 4 cm); 3). M_3 (BM_3 = 3 cm); 4). M_4 (AM_4= 6 cm, BM_4 = 8 cm); 5). M_5 (AM_5 = BM_5 = 10 cm); 6). Find the point M_6 on AB line, where the magnetic field is zero.

Solution

According to the superposition principle, the magnetic field intensity at point M_1 is the vector sum of the magnetic fields at this point created by both currents independently:

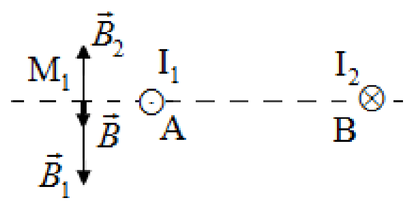
$$\vec{B} = \vec{B}_1 + \vec{B}_2.$$

The magnetic fields $\vec{B}_1$ and $\vec{B}_2$ we'll find using Biot-Savart Law. The magnitude of field $\vec{B}$ generated by a long, thin current is

$$B = \frac{\mu_0 I}{2\pi b}.$$

Plugging in I_1, we have $B_1 = \dfrac{\mu_0 I_1}{2\pi \cdot AM_1}$, and plugging I_2, we obtain

$$B_2 = \frac{\mu_0 I_2}{2\pi \cdot BM_1} = \frac{\mu_0 I_2}{2\pi \cdot \left(AM_1 + AB\right)}.$$



According to Bio-Savart Law

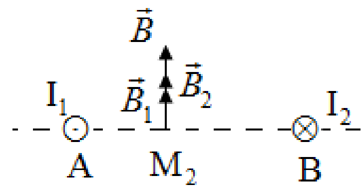
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^3}\left[d\vec{l}, \vec{r}\right],$$

the field due to a wire is perpendicular to a radial vector from the wire $\vec{r}$ and to the infinitesimal wire segment $d\vec{l}$. Since both vectors ($\vec{r}$ and $d\vec{l}$) are in the figure plane, the magnetic field $\vec{B}_1$ and $\vec{B}_2$ are directed perpendicularly to the line AB. The directions of each vector are given by the right hand rule: $\vec{B}_1$ is directed downwards and $\vec{B}_2$ - upwards. The magnitude of the magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$ at the point M_1 is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{AM_1} - \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{(AM_1 + AB)} =$$

$$= \frac{\mu_0}{4\pi} \cdot 2 \cdot \left(\frac{20}{2 \cdot 10^{-2}} - \frac{30}{12 \cdot 10^{-2}} \right) = 10^{-7} \cdot 2 \cdot (1000 - 250) = 1.5 \cdot 10^{-4} \text{ T.}$$

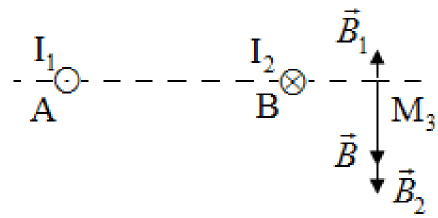
2) The similar reasoning for the point M_2 leads to the result that the magnitude of the magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$ is



$$B = B_1 + B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{AM_2} + \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{(AB - AM_2)} =$$

$$= \frac{\mu_0}{4\pi} \cdot 2 \cdot \left(\frac{20}{4 \cdot 10^{-2}} + \frac{30}{6 \cdot 10^{-2}} \right) = 10^{-7} \cdot 2 \cdot (500 + 500) = 2 \cdot 10^{-4} \text{ T.}$$

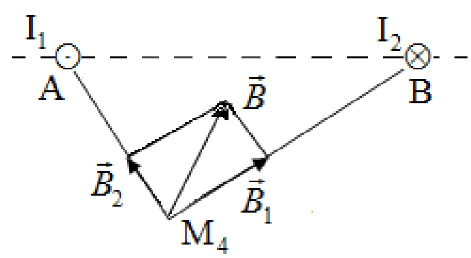
3) For the point M_3 the magnitude of the magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$ is



$$B = B_2 - B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{BM_3} - \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{(AB + BM_3)} =$$

$$= \frac{\mu_0}{4\pi} \cdot 2 \cdot \left(\frac{30}{3 \cdot 10^{-2}} - \frac{20}{13 \cdot 10^{-2}} \right) = 10^{-7} \cdot 2 \cdot (1000 + 154) = 2{,}3 \cdot 10^{-4} \text{ T.}$$

4) If the separation between the currents AB = 10 cm, the distances AM_4 = 6 cm and BM_4 = 8 cm, then their ratio is AB: BM_4: AM_4 = 10:8:6 = 5:4:3, therefore, the triangle ABM_4 is so called Egyptian right triangle with an angle $\angle AM_4B = 90^0$. In this case



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{AM_4}, \; B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{BM_4}.$$

The magnitude of the magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$ at the point M_4 is

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi} \cdot \frac{2I_1}{AM_4} \right)^2 + \left(\frac{\mu_0}{4\pi} \cdot \frac{2I_2}{BM_4} \right)^2} =$$

$$= \frac{2\mu_0}{4\pi} \sqrt{\left(\frac{I_1}{AM_4} \right)^2 + \left(\frac{I_2}{BM_4} \right)^2} = 2 \cdot 10^{-7} \sqrt{\left(\frac{20}{6 \cdot 10^{-2}} \right)^2 + \left(\frac{30}{8 \cdot 10^{-2}} \right)^2} = 1 \cdot 10^{-4} \text{ T.}$$

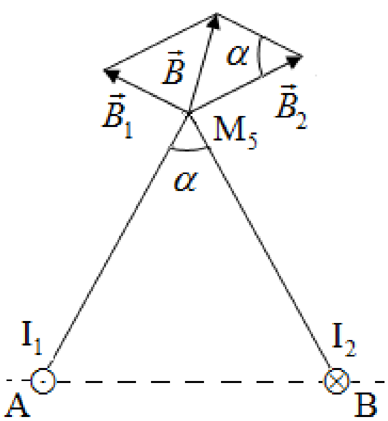
5) If the point M_5 is at a distance 10 cm from each current we obtain equilateral triangle, and the magnitude of the magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$ at the point M_5 is

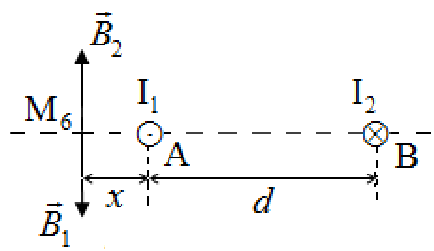
$$B = \sqrt{B_1^2 + B_2^2 - 2B_1B_2\cos\alpha} =$$

$$= \sqrt{\left(\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{AM_4}\right)^2 + \left(\frac{\mu_0}{4\pi}\cdot\frac{2I_2}{BM_4}\right)^2 - 2\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{AM_4}\frac{\mu_0}{4\pi}\cdot\frac{2I_2}{BM_4}\cos\alpha} =$$

$$= \frac{2\mu_0}{4\pi}\sqrt{\left(\frac{I_1}{AM_4}\right)^2 + \left(\frac{I_2}{BM_4}\right)^2 - 2\frac{I_1}{AM_4}\frac{I_2}{BM_4}\cos\alpha} =$$

$$= 2\cdot 10^{-7}\sqrt{\left(\frac{20}{10\cdot 10^{-2}}\right)^2 + \left(\frac{30}{10\cdot 10^{-2}}\right)^2 - 2\frac{20}{10\cdot 10^{-2}}\frac{30}{10\cdot 10^{-2}}\cdot\frac{1}{2}} = 5{,}3\cdot 10^{-5}\ \text{T}.$$





6. Basing on the examined 1-3 cases we can make a conclusion that the point, where the magnetic field is zero, can't be between the points A and B as the vectors $\vec{B}_1$ and $\vec{B}_2$ are directed in the same direction. The "zero-point" may be to the left from the current I_1 or to the right from the current I_2. But taking into account the magnitudes of the currents $(I_1 < I_2)$, the point M_6 has to be to the left from the point A.

Let $AM_6 = x$. Since $\vec{B} = \vec{B}_1 + \vec{B}_2$ and $B_1 = B_2$, then

$$\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{AM_6} = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{BM_6}$$

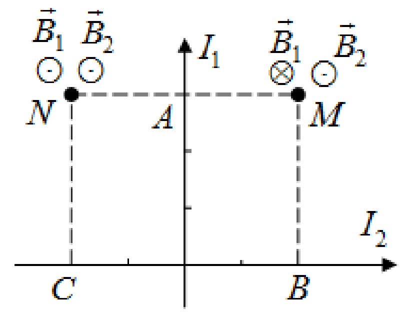
and $\dfrac{I_1}{x} = \dfrac{I_2}{d+x}$.

$$x = \frac{I_1 d}{I_2 - I_1} = \frac{20\cdot 10\cdot 10^{-2}}{30-20} = 0{,}2\,\text{m}.$$

Problem 2

Two infinitely long rectilinear conductors are arranged perpendicular to each other in one plane as shown in Figure. Calculate the magnetic field at points M and N if I_1 = 3A and I_2 = 4 A. The distances AM = AN = 2 cm and BM = CN = 4 cm.

Solution



Magnetic inductions at each of the points according to the Principle of Superposition are

$$\vec{B}_M = \vec{B}_{M1} + \vec{B}_{M2},$$

$$\vec{B}_N = \vec{B}_{N1} + \vec{B}_{N2}.$$

The magnetic fields $\vec{B}_1$ and $\vec{B}_2$ created by the currents I_1 and I_2 at points M and N can be found by means of Bio-Savart Law: the directions of vectors are determined by $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\left[d\vec{l}, \vec{r}\right]}{r^3}$, and the magnitudes of magnetic fields are determined by $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$. For finding $\vec{B}$ directions use the following rule: a wire with electric current going through it produces a magnetic field going in circles around it. To find the direction of the magnetic field, point your thumb in the direction of the current. Then, curl your fingers around the wire. The direction your fingers curl tells you the direction that the magnetic field is pointing

Depending on the direction of the vectors $\vec{B}_1$ and $\vec{B}_2$ (see the figure) the resultant field at the points M and N are:

1). At the point M:

$$B_M = B_{M1} - B_{M2} = \frac{\mu_0}{4\pi}\left(\frac{2I_1}{AM} - \frac{2I_2}{BM}\right) = 10^7\left(\frac{2 \cdot 3}{2 \cdot 10^{-2}} - \frac{2 \cdot 4}{4 \cdot 10^{-2}}\right) = 8 \text{ A/m}$$

2). At the point N:

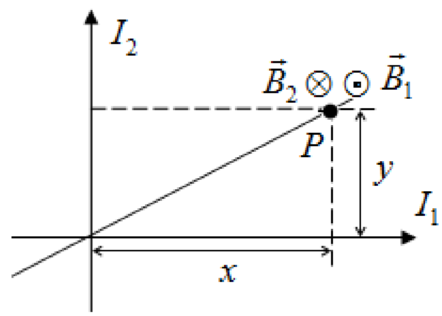
$$B_N = B_{N1} + B_{N2} \frac{\mu_0}{4\pi}\left(\frac{2I_1}{AN} + \frac{2I_2}{BN}\right) = 10^7\left(\frac{2 \cdot 3}{2 \cdot 10^{-2}} - \frac{2 \cdot 4}{4 \cdot 10^{-2}}\right) = 39.8 \text{ A/m}.$$

Problem 3

The mutually perpendicular conductors carrying the currents I_1 and I_2 are shown in Figure. Assuming the conductors to be infinitely long, find the locus of points at which the magnetic field is zero.

Solution

Let y be the distance from a certain point on the conductor carrying current I_1 to a point P where the net field is zero and x is the distance from the point P to a certain point on the conductor carrying current I_2. Since the net field at the point P is zero, $\vec{B} = \vec{B}_1 + \vec{B}_2 = 0$, and each magnetic field induction is equal to $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{b}$, $B_1 = B_2$. Hence,



$$\frac{\mu_0}{4\pi}\frac{2I_2}{x} = \frac{\mu_0}{4\pi}\frac{2I_1}{y}$$

or $\quad y = \frac{I_1}{I_2} x$.

This is the equation of a straight line passing through the origin. Therefore, the locus of points where the net field is zero is a straight line passing through the origin, with gradient equal to I_1/I_2.

Problem 4

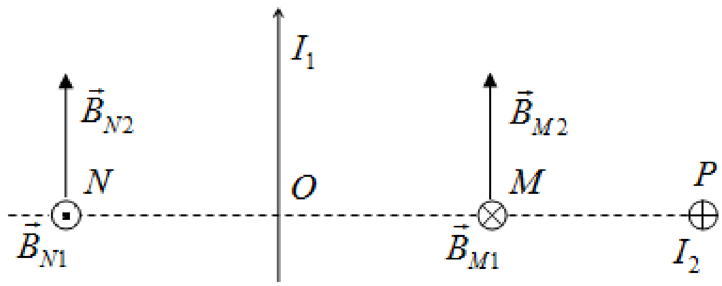
Two current-carrying wires are perpendicular to each other. The current $I_1 = 20$ A in the first wire flows vertically upward and the current I_2=40 A in the other flows horizontally into the page. The horizontal wire is one meter to the right of the vertical wire. Find the magnetic fields at points M and N if OM = ON = 0.5 m?

Solution

From Biot-Savart law, the magnetic field generated by a long, thin current-carrying wire is

$$B = \frac{\mu_0 I}{2\pi b},$$

where b the distance is separated the wire and the point where the magnetic field is determined.



Therefore, the magnetic fields created by the first and the second wires at the point M and M are

$$B_{M1} = \frac{\mu_0 I_1}{2\pi \cdot OM} = \frac{4\pi \cdot 10^{-7} \cdot 20}{2\pi \cdot 0.5} = 8 \cdot 10^{-6} \text{ T};$$

$$B_{M2} = \frac{\mu_0 I_1}{2\pi \cdot PM} == \frac{4\pi \cdot 10^{-7} \cdot 40}{2\pi \cdot 0.5} = 16 \cdot 10^{-6} \text{ T};$$

$$B_{N1} = \frac{\mu_0 I_1}{2\pi \cdot ON} = \frac{4\pi \cdot 10^{-7} \cdot 20}{2\pi \cdot 0.5} = 8 \cdot 10^{-6} \text{ T};$$

$$B_{N2} = \frac{\mu_0 I_2}{2\pi \cdot PN} = \frac{4\pi \cdot 10^{-7} \cdot 40}{2\pi \cdot 1.5} = 5.33 \cdot 10^{-6} \text{ T}.$$

According to the principle of superposition

$$\vec{B}_M = \vec{B}_{M1} + \vec{B}_{M2},$$

$$\vec{B}_N = \vec{B}_{N1} + \vec{B}_{N2}.$$

Determination of the directions of the magnetic fields with the help of the Right-hand rule gives that at point M: $\vec{B}_{M1}$ - into the page, $\vec{B}_{M2}$ - upwards; $\vec{B}_{N1}$ - out of page, $\vec{B}_{N2}$ - upwards. Then the magnitudes of magnetic at the points M and N are

$$B_M = \sqrt{B_{M1}^2 + B_{M2}^2} = \sqrt{\left(8 \cdot 10^{-6}\right)^2 + \left(16 \cdot 10^{-6}\right)^2} = 17.9 \cdot 10^{-6} \text{T},$$

$$B_N = \sqrt{B_{N1}^2 + B_{N2}^2} = \sqrt{\left(8 \cdot 10^{-6}\right)^2 + \left(5.33 \cdot 10^{-6}\right)^2} = 9.6 \cdot 10^{-6} \text{T}..$$

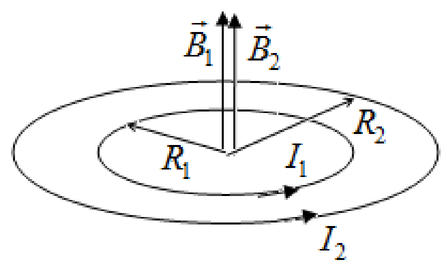
Problem 5

Two concentric circles of radius 2 cm and 4 cm respectively carry currents of 2A and 4A in a clockwise direction. What is the net magnetic field at the center? If the direction of current in the inner circle is reversed, what is net magnetic field at the center?

Solution

According to Bio-Savart Law the magnetic field due to the circular loop is directed normal to the plane of the loop and its magnitude is equal to $B = \frac{\mu_0 I}{2R}$.

Then the magnitudes of the magnetic fields of the currents I_1 and I_2 are



$$B_1 = \frac{\mu_0 I_1}{2R_1} = \frac{4\pi \cdot 10^{-7} \cdot 2}{2 \cdot 0.02} = 6.28 \cdot 10^{-5}\,\text{T},$$

$$B_2 = \frac{\mu_0 I_2}{2R_2} = \frac{4\pi \cdot 10^{-7} \cdot 4}{2 \cdot 0.04} = 6.28 \cdot 10^{-5}\,\text{T}.$$

Both vectors $\vec{B}_1$ and $\vec{B}_2$ are in the same direction, the resultant field B is also the same:

$$B = B_1 + B_2 = 6.28 \cdot 10^{-5} + 6.28 \cdot 10^{-5} = 1.256 \cdot 10^{-4}\,\text{T}.$$

If the direction of the current in the inner circle is reversed, the direction of $\vec{B}_1$ reverses. Now $\vec{B}_1$ and $\vec{B}_2$ are equal in magnitude and opposite in direction. Hence the net magnetic field is zero.

Problem 6

A current I =50 A flows through a straight wire AB of finite length. Find the magnetic field B at distance r =0.2 m from the wire, the ends of the wire making inner angles β_1=30^0 and β_2=45^0 with P.

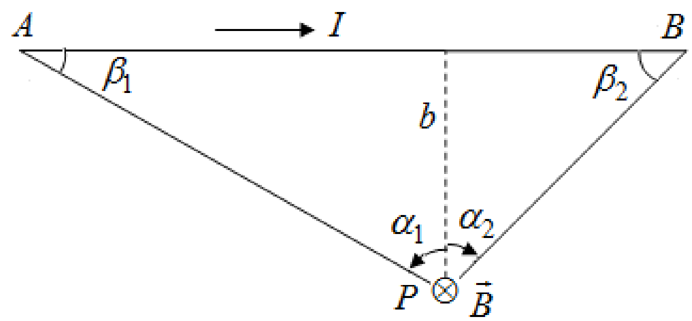
Solution

Each element of AB creates at P the magnetic field dB which may be found from Biot–Savart law and is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot \cos\alpha \cdot d\alpha}{b}.$$

Since the direction of the contribution dB at point P for all such elements is identical, i.e. at right angles to the plane of paper, the resultant field is obtained by integrating dB from A to B

$$B = \int_{\alpha_1}^{\alpha_2} dB = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \int_{\alpha_1}^{\alpha_2} \cos\alpha \cdot d\alpha .$$



The given angles β_1=30^0 and β_2=45^0 give us $\alpha_1 = 60^0 = \frac{\pi}{3}$ and $\alpha_2 = 45^0 = \frac{\pi}{4}$. Then

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \int_{-\frac{\pi}{3}}^{+\frac{\pi}{4}} \cos\alpha \cdot d\alpha = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \sin\alpha \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{4}} = \frac{\mu_0}{4\pi} \cdot \frac{I}{b}\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right) =$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \cdot 1.57 = \frac{4 \cdot 10^{-7} \cdot 50 \cdot 1.57}{0.2} = 1.57 \cdot 10^{-4} \text{ T}.$$

Problem 7

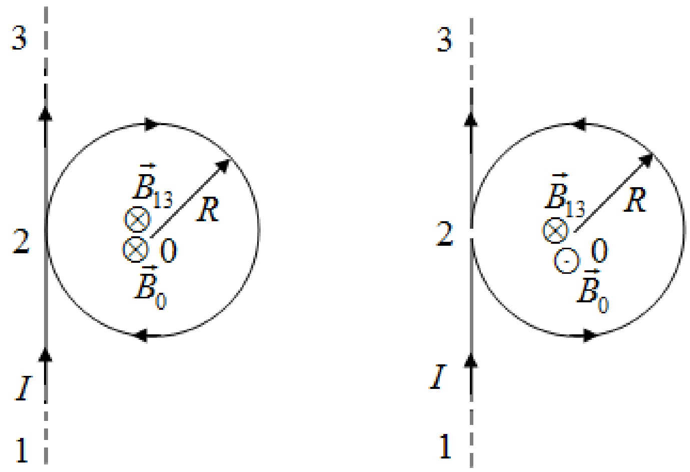
A conductor consists of a circular loop of radius R and two straight long sections, as shown in Figure. The wire lies in the plane of the paper and carries a current I. Determine the magnitude and direction of the magnetic field at the center of the loop.

Solution

The magnetic field is the superposition of the fields created by long wire and a circular current loop. It is necessary to pay attention that the conductor consists of two straight semi-infinite sections. Hence, the field at point O is equal to

$\vec{B} = \vec{B}_{12} + \vec{B}_{23} + \vec{B}_0$.

According to Bio-Savart Law and right hand rule the magnetic inductions $\vec{B}_{12}$ and $\vec{B}_{23}$ point in the same direction, into the page. As for circular loop, depending



on the direction of the current in it, the magnetic field induction $\vec{B}_0$ may be into or out of page.

The magnitudes of the magnetic fields of semi-infinite sections we can find using the following expression:

$$B = \int_{\alpha_1}^{\alpha_2} dB = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \int_{\alpha_1}^{\alpha_2} \cos\alpha \cdot d\alpha .$$

$$B_{13} = B_{12} + B_{23} = \frac{\mu_0}{4\pi} \cdot \frac{I}{R} \int_{-\frac{\pi}{2}}^{0} \cos\alpha \cdot d\alpha + \frac{\mu_0}{4\pi} \cdot \frac{I}{R} \int_{0}^{\frac{\pi}{2}} \cos\alpha \cdot d\alpha =$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{R} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\alpha \cdot d\alpha = \frac{\mu_0}{4\pi} \cdot \frac{2I}{R}.$$

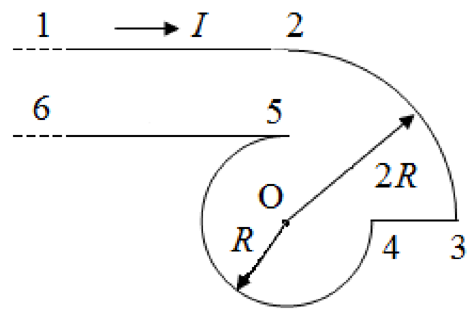
So, two semi-infinite sectors of wire create the magnetic field the same as the infinitely long straight wire.

The field at the point O is

$$B = B_{13} \pm B_0 = \frac{\mu_0 I}{2\pi R} \pm \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2R}\left(\frac{1}{\pi} \pm 1\right).$$

Problem 8

A wire is formed into the shape showed in the Figure. A current $I = 20$ A flows in the circuit, and $R = 0.2$ m. Determine the magnitude and direction of the magnetic field at the center O.



Solution

The Superposition Principle states that net magnetic field produced at any point by a system of sources is equal to the vector sum of all individual fields, produced by each source at this point. Therefore, the magnetic field at the point O is

$$\vec{B} = \vec{B}_{12} + \vec{B}_{23} + \vec{B}_{34} + \vec{B}_{45} + \vec{B}_{56}.$$

Let us determine the magnitudes of magnetic fields of all curve and linear segments of circuit.

A linear segment 1-2 is a semi-infinite segment which magnetic field may be determines as

$$B_{12} = \frac{\mu_0}{4\pi} \cdot \frac{I}{2R} \int_{-\frac{\pi}{2}}^{0} \cos\alpha \, d\alpha = \frac{\mu_0 \cdot I}{4\pi \cdot 2R}\left(\sin 0 - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{\mu_0 \cdot I}{4\pi \cdot 2R}.$$

A curvilinear segment 2-3 is a quarter of the circular loop which magnetic field with provision for the radius of the arc is $2R$ may be presented as

$$B_{23} = \frac{1}{4}\mu_0 \frac{I}{2(2R)} = \mu_0 \frac{I}{16R}.$$

A producing the line 3-4 passes through the point O, therefore,

$$\vec{B}_{34} = 0.$$

A curvilinear segment 4-5 is three fourth of the circular loop of radius R, then the magnetic field created by this segment is

$$B_{45} = \frac{3}{4} \cdot \mu_0 \frac{I}{2R} = \mu_0 \frac{3I}{8R}.$$

Finally, the rectilinear semi-infinite segment 5-6 gives

$$B_{56} = \frac{\mu_0}{4\pi} \cdot \frac{I}{R} \int_{-\frac{\pi}{2}}^{0} \cos\alpha \, d\alpha = \frac{\mu_0 \cdot I}{4\pi \cdot R}\left(\sin 0 - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{\mu_0 \cdot I}{4\pi \cdot R}.$$

The directions of the all vectors $\vec{B}$ may be determined according to Biot-Savart Law (they all normal to the paper) using the right hand rule. As a result, $\vec{B}_{12}$, $\vec{B}_{23}$ and $\vec{B}_{45}$ are directed into the paper, and $\vec{B}_{56}$ - out of the paper.

$$\vec{B} = \overset{\otimes}{\vec{B}_{12}} + \overset{\otimes}{\vec{B}_{23}} + \cancel{\vec{B}_{34}} + \overset{\otimes}{\vec{B}_{45}} + \overset{\odot}{\vec{B}_{56}}.$$

The magnitude of the magnetic field at the point O is

$$B = \mu_0 \frac{I}{4\pi \cdot 2R} + \mu_0 \frac{I}{16R} + \mu_0 \frac{3I}{8R} - \mu_0 \frac{I}{4\pi \cdot R} = \frac{\mu_0 I}{4R}\left(\frac{1}{2\pi} + \frac{1}{4} + \frac{3}{2} - \frac{1}{\pi}\right) \approx 5 \cdot 10^{-5}$$

T.

Problem 9

A wire of 0.2 m long carrying a current of 10 A is at right angles to a magnetic field. The force on the wire is 0.1 N. 1). What is the strength of the magnetic field? 2). What would its strength be if the wire is at an angle of 30^0 to the field?

Solution

The magnetic force acting on the current-carrying wire in the magnetic field $\vec{B}$ is

$$\vec{F} = I\left[\vec{l}, \vec{B}\right],$$

where $\vec{l}$ is a length vector with magnitude l and directed along the direction of the electric current.

The magnitude of this force is

$$F = IlB\sin\alpha.$$

Magnetic field strength $\vec{H}$ is related with the magnetic field induction $\vec{B}$ by

$$\vec{B} = \mu\mu_0\vec{H},$$

where μ is the magnetic permeability of substance (for vacuum $\mu = 1$), $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic constant (or magnetic permeability of free space).

1). The magnetic field strength when the wire is at right angle to a magnetic field is

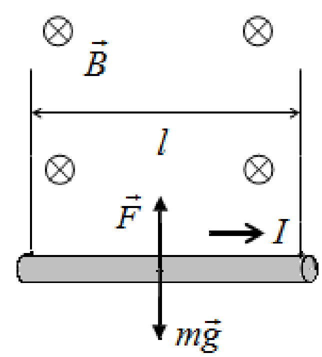
$$H = \frac{B}{\mu_0} = \frac{F}{\mu_0 Il \sin\alpha} = \frac{0.1}{4\pi \cdot 10^{-7} \cdot 10 \cdot 0.2} = 3.98 \cdot 10^4 \text{ A/m.}$$

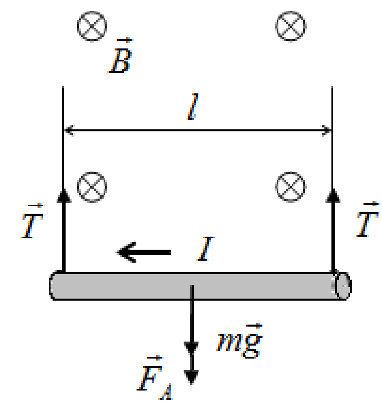
2). If the wire is at an angle of 30^0 to the field the magnetic field strength is

$$H = \frac{B}{\mu_0} = \frac{F}{\mu_0 Il \sin\alpha} = \frac{0.1}{4\pi \cdot 10^{-7} \cdot 10 \cdot 0.2 \cdot \sin 30^\circ} = 7.96 \cdot 10^4 \text{ A/m.}$$

Problem 10

A conductor of the length l=0.2 m suspended by two flexible wires as in the figure below has a mass per unit length m_0 = 0.05 kg/m. The magnetic field is B = 2.1 T and is directed into the page. 1). What current must exist in the conductor for the tension in the supporting wires to be zero? 2). What will be the tension in each wire if the direction of the current is reversed, keeping the magnetic field same as before and neglecting the mass of the wires?





Solution

1). In order that the tension in the wires be zero, the magnetic force $\vec{F} = I\left[\vec{l}, \vec{B}\right]$ acting on the conductor must exactly cancel the downward gravitational force $m\vec{g}$, i.e.,

$$m\vec{g} + \vec{F} = 0.$$

The magnitude of magnetic force is $F = IlB\sin\alpha$. Taking into account that $\sin\alpha = 1$ and $mg = m_0 l\,\mathrm{g}$, we get

$$m_0 lg = IlB.$$

Finally, from $m_0 g = IB$, the current in the wire is

$$I = \frac{m_0 g}{B} = \frac{0.05 \cdot 9.8}{2.1} = 0.23 \text{ A.}$$

The direction of the current may be determined using the left hand rule: if you orient your left hand so that the outstretched fingers point in the direction of the current and the magnetic field lines enter into your palm, the extended thumb points in the direction of the magnetic force on the conductor. Placing the hand in such a way, we can find that the current flows to the right.

2). Keeping the magnetic field same as in the first part, if the direction of current is reversed, then the force due to the magnetic field and gravity act downwards. Then from

$$m\vec{g} + 2\vec{T} + \vec{F}_A = 0$$

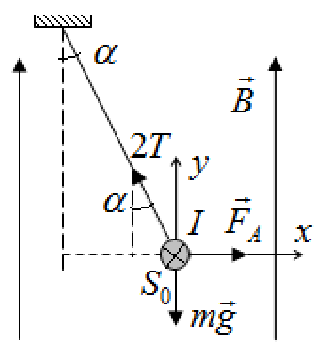
we obtain

$$T = 0.5(mg + F_A) = 0.5l(m_0 g + IB) = 0.5 \cdot 0.2(0.05 \cdot 9.8 + 0.23 \cdot 2.1) \approx 0.1\text{N}.$$

Problem 11

An aluminum conductor having a length of 20 cm and cross-section area S_0= 70 mm^2 is suspended on two thin conductors. If the conductor carries a current of 11.7 A it deflects from the vertical in homogenous magnetic field B= 5 mT by an angle α. Find this angle.

Solution



First of all we have to realize which forces are acting on the current-carrying conductor. Since the conductor is placed in a magnetic field it is a magnetic force (Ampere's force) $\vec{F}_A$. Its direction given by the left-hand rule points to the right. Next, there is the gravity $m\vec{g}$ acting on the conductor downwards. The sum of these forces gives the final force acting on the conductor (see picture) which compensates the forces of the two thin wires ($2\vec{T}$) on which the conductor is hanged.

$$m\vec{g} + 2\vec{T} + \vec{F}_A = 0.$$

The projections of this equation on x- and y-axes are

$$\begin{cases} -2T\sin\alpha + F_A = 0, \\ 2T\cos\alpha - mg = 0. \end{cases}$$

Ampere's force is $F_A = IlB\sin\alpha = IlB$ (as $\sin\alpha = \sin 90^0 = 1$), and mass of the conductor is

$m = \rho_{Al} V = \rho_{Al} S_0 l$.

Dividing the first equation of the system by the second equation, we obtain

$$\frac{\cancel{2}\cancel{T}\sin\alpha}{\cancel{2}\cancel{T}\cos\alpha} = \frac{F_A}{mg},$$

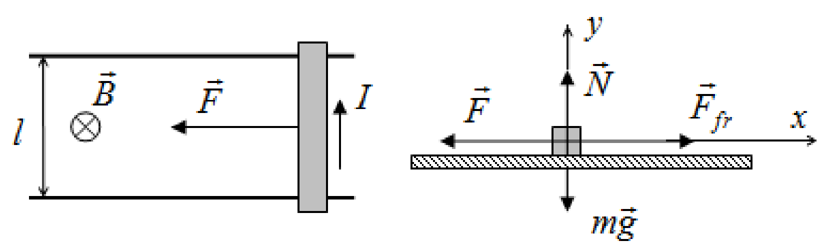
$$\tan\alpha = \frac{F_A}{mg} = \frac{I\cancel{l}B}{\rho_{Al} S_0 \cancel{l} g} = \frac{IB}{\rho_{Al} S_0 g} = \frac{11.7 \cdot 5 \cdot 10^{-3}}{2.7 \cdot 10^3 \cdot 70 \cdot 10^{-6} \cdot 9.8} = 0.032,$$

$\alpha = \arctan\ 0.032 = 1.8^0$.

Problem 12

A copper rod is mounted on two parallel rails separated by a distance l =15 cm, which are located in the magnetic field directed into the page (see the Figure). The rod is at a right angle to the rails. The coefficient of friction between the rod and the rail is k = 0.2. To move the rod uniformly it is necessary to apply the voltage U = 4.55·10^{-4}V across the rod. Find the induction of the magnetic field.

Solution



The rod with the current I moves in magnetic field $\vec{B}$ due to the action of the Ampere's force $\vec{F} = I\left[\vec{l}, \vec{B}\right]$.

The equation of its uniform motion is

$m\vec{g} + \vec{F} + \vec{N} + \vec{F}_{fr} = 0$,

and the projections of this equation on x and y – axes are

$$\begin{aligned} x &: \begin{cases} F_{fr} - F = 0, \\ N - mg = 0, \end{cases} \\ y &: \end{aligned}$$

In these equations the friction force is

$F_{fr} = kN$,

where k is the coefficient of friction and N is the normal force: $N = mg$.

The magnitude of Ampere's force is $F = Ilb$ as $\sin\alpha = 1$.

According to Ohm's Law, the current is $I = U/R$, where the resistance is $R = \rho\dfrac{l}{S_0}$ and ρ is the resistivity of copper; l is its length and S_0 is a cross-section area.

The mass of the rod is $m = DV = DS_0 l$, where D is the density of the rod substance.

Combining all expressions in $F_{fr} - F = 0$, we obtain

$$kN - IlB = 0,$$

$$kDS_0 lg - \frac{US_0}{\rho \cancel{l}}\cancel{l}B = 0.$$

The induction of the magnetic field is

$$B = \frac{kD\cancel{S_0}lg\rho}{U\cancel{S_0}}.$$

Substituting the given and reference data (for copper: the density $D = 8.93\cdot 10^3$ kg/m^3 and the resistivity $\rho = 1.7\cdot 10^{-8}$), we obtain

$$B = \frac{kDlg\rho}{U} = \frac{0.2\cdot 8.93\cdot 10^3\cdot 0.15\cdot 9.8\cdot 1.7\cdot 10^{-8}}{4.55\cdot 10^{-4}} = 9.81\cdot 10^{-2}\text{ T}.$$

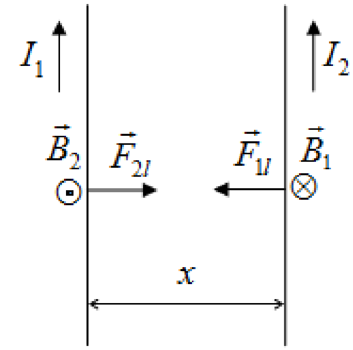
Problem 13

Two long parallel rectilinear conductors are 10 cm apart. The currents I_1 *= 20 A and* I_2 *=40 A flow through the conductors in the same direction. What is the magnitude of the magnetic field* B_1 *created by* I_1 *at the location of* I_2*? What is the force per unit length exerted by* I_1 *on* I_2*? What is the magnitude of the magnetic field* B_2 *created by* I_2 *at the location of* I_1*? What is the force per unit length exerted by* I_2 *on* I_1*? What work is required (per unit length of the conductors) to move them apart a distance of 30 cm.*

Solution

The magnetic induction B of the field, generated by a long, thin current flowing in the wire at the distance x x from the wire, is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{x}.$$



Plugging in I_1, we have the induction magnetic field created by I_1 at the location of I_2

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{x}.$$

This magnetic field depends on the distance from I_1, but because the wires are parallel, the B field from I_1 is constant along I_2. We can use the right hand rule to determine that $\vec{B}_1$ is perpendicular to both I_1 and x.

The similar result for

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{x}.$$

The force $\vec{F}$ on a current carrying wire of the length l in a uniform magnetic field $\vec{B}$ is

$$\vec{F} = I\left[\vec{l}, \vec{B}\right].$$

Taking into account that the angle between $\vec{l}$ and $\vec{B}$ is 90^0, and $\sin\alpha = 1$, the force per unit length is

$$F_{1l} = \frac{F_1}{l} = \frac{I_1 \cdot l \cdot B_2 \sin\alpha}{l} = I_1 \cdot B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 \cdot I_2}{x},$$

$$F_{2l} = \frac{F_1}{l} = \frac{I_2 \cdot l \cdot B_1 \sin\alpha}{l} = I_2 \cdot B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 \cdot I_2}{x}.$$

$$F_l = F_{1l} = F_{2l} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 \cdot I_2}{x} = \frac{10^{-7} \cdot 2 \cdot 20 \cdot 40}{0.1} = 1.6 \cdot 10^{-4}\,\text{N/m}.$$

Work done is

$$dA_l = \left(\vec{F}_l, d\vec{x}\right) = F_l \cdot dx \cdot \cos\alpha = -F_l \cdot dx = -\frac{\mu_0}{4\pi} \cdot \frac{2I_1 \cdot I_2}{x} dx.$$

$$A_l = \int_{x_1}^{x_2} F_l \cdot dx = \frac{\mu_0}{4\pi} \cdot 2I_1I_2 \int_{x_1}^{x_2} \frac{dx}{x} = \frac{\mu_0}{4\pi} \cdot 2I_1I_2 \cdot \ln\frac{x_2}{x_1} =$$

$$= 10^{-7} \cdot 2 \cdot 20 \cdot 40 \cdot \ln\frac{30}{10} = 1.758 \cdot 10^{-4}\ \text{J/m}.$$

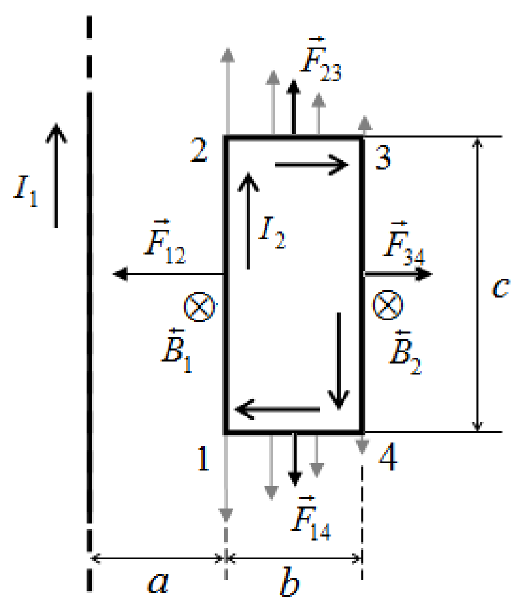
Problem 14

In Figure, the current in the long, straight infinite wire is $I_1 = 5$ A and the wire lies in the plane of the rectangular loop, which carries I_2=10 A. The dimensions are a= 0.15 m, b = 0.1 m, and c = 0.5 m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

Solution

Even though there are forces in opposite directions on the loop, we must remember that the magnetic field is stronger near the wire $\vec{B}_1$ than it is farther away $\vec{B}_2$:

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I_1}{2\pi(a+b)}$$



By symmetry the forces exerted on sides 23 and 14 (the horizontal segments of length b) are equal and opposite, and therefore cancel. The magnetic field in the plane of the loop is directed into the page to the right of I_1. By the right-hand rule, $\vec{F}_{12} = I_2\left[\vec{l}_{12}, \vec{B}_1\right]$ is directed toward the left for side 12 of the loop and a smaller force $\vec{F}_{34} = I_2\left[\vec{l}_{34}, \vec{B}_2\right]$ is directed toward the right for side 34. Therefore, we should expect the net force to be to the left. The magnitudes of these forces allow for the fact that magnetic field and the conductor are mutually perpendicular to each other ($\sin\alpha = \sin 90^0 = 1$), are

$$F_{12} = I_2 l_{12} B_1 \sin\alpha = \frac{\mu_0 I_1 I_2 c}{2\pi a},$$

$$F_{34} = I_2 l_{34} B_2 \sin\alpha = \frac{\mu_0 I_1 I_2 c}{2\pi(a+b)}.$$

The net force is $\vec{F} = \vec{F}_{12} + \vec{F}_{34}$, and its magnitude directed to the left is

$$F = F_{12} - F_{34} = \frac{\mu_0 I_1 I_2 c}{2\pi a} - \frac{\mu_0 I_1 I_2 c}{2\pi(a+b)} = \frac{\mu_0 I_1 I_2 c}{2\pi}\left(\frac{1}{a} - \frac{1}{a+b}\right) = \frac{\mu_0 I_1 I_2 cb}{2\pi a(a+b)}.$$

Substituting the given data, we obtain

$$F = \frac{4\pi \cdot 10^{-7} \cdot 5 \cdot 10 \cdot 0.5 \cdot 0.1}{2\pi \cdot 0.15 \cdot (0.15 + 0.1)} = 1.33 \cdot 10^{-5}\text{N}.$$

Problem 15

A coil of the turn area $S = 200$ cm^2 is mounted in a uniform magnetic field $B = 8 \cdot 10^{-4}$ T. There is a current of 20 A in the coil, which has 25 turns. When the plane of the coil makes an angle of 60^0 with direction of the field, what is the torque tending to rotate the coil?

Solution

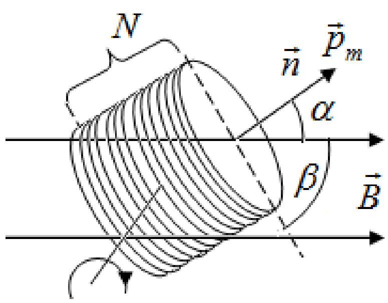
The torque on the circuit in the field of the magnetic induction $\vec{B}$ is

$$\vec{M} = \left[\vec{p}_m, \vec{B}\right],$$

where $\vec{p}_m$ is the magnetic moment of the circuit directed along the normal $\vec{n}$ to the turn plane. The magnitude of the magnetic moment is

$$p_m = ISN,$$

where N is the number of the turns in the circuit, I is the current in the circuit, and S is the area it encloses.



The direction of $\vec{p}_m$ is given by the right hand rule: wrap the fingers of your right hand around the circuit in the direction of the current, and the direction in which your thumb points then be the direction of $\vec{p}_m$.

The magnitude of the torque is

$$M = p_m B \sin\alpha = ISNB \sin\alpha,$$

where α is the angle between the normal to the plane of turn and the direction of the magnetic field $\vec{B}$. The given angle is the angle that the plane of the coil makes with direction of the field. Then the angle that we need is $\alpha = 90^0 - \beta = 90^0 - 60^0 = 30^0$.

Inserting all given data, we find

$M = ISNB\sin\alpha = 20 \cdot 0.02 \cdot 25 \cdot 8 \cdot 10^{-4} \cdot \sin 30^0 = 4 \cdot 10^{-3}$ N·m.

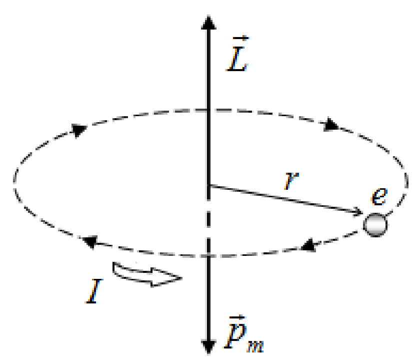
Problem 16

Find the ratio of the magnetic moment and orbital angular momentum for the electron orbiting in the atom about the nucleus.

Solution

In the classical model we assume that an electron moves with constant speed v in a circular orbit of radius r about the nucleus. Because the electron travels a distance of $2\pi r$ (the circumference of the circle) in a time interval *T*, its orbital speed is $v = 2\pi r/T$. The current I associated with this orbiting electron is its charge e divided by *T*. Using $T = 2\pi/\omega$ and $\omega = v/r$, we have

$$I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}.$$



The magnetic moment associated with this current loop is $\vec{p}_m = I\vec{S} = IS\vec{n}$, where $S = \pi r^2$ is the area enclosed by the orbit, and $\vec{n}$ is the unit normal to the loop. Therefore,

$$p_m = IS = \frac{ev}{2\pi r} \cdot \pi r^2 = \frac{evr}{2}.$$

Since the magnitude of the angular momentum of the electron is $L = mvr$, the ratio of the magnetic and orbital angular momentum is

$$\frac{p_m}{L} = \frac{e\cancel{vr}}{2m\cancel{vr}} = \frac{e}{2m}.$$

This is so called gyromagnetic (or magnetic) ratio which magnitude for electron is

$$\frac{p_m}{L}=\frac{e}{2m}=\frac{1.6\cdot10^{-19}}{2\cdot9.1\cdot10^{-31}}\approx8.8\cdot10^{10}\text{ C/kg.}$$

Its important to note that vectors $\vec{p}_m$ and $\vec{L}$ point in opposite directions because the electron is negatively charged particle.

Problem 17

An electron in the ground state of hydrogen atom is revolving in anti-clockwise direction in a circular orbit of radius $R = 5.3\cdot10^{-11}$ *m at the velocity* $v = 2.19\cdot10^{6}$ *m/s. 1) Find the magnetic moment of electron* p_m. *2) The atom is placed in a uniform magnetic field (B = 2 T) such that the plane normal of the electron orbit makes an angle of* 30^0 *with the magnetic induction* $\vec{B}$. *Find the torque experienced by the orbiting electron.*

Solution

Current constituted as a result of the motion of electron is equal to

$$I=-\frac{e}{T}=-\frac{ev}{2\pi R},$$

where $e=1.6\cdot10^{-19}$ C is the elementary charge, and v is a linear velocity of its motion along the circular path.

Magnetic moment is

$$\vec{p}_m = IS\vec{n},$$

where $S=\pi R^2$ is the area of the circle, and $\vec{n}$ is the normal to the plane of the circle.

The magnitude of the magnetic moment is

$$p_m = IS = \frac{ev\cancel{\pi} R^{\cancel{2}}}{2\cancel{\pi}\cancel{R}}=\frac{evR}{2}=\frac{1.6\cdot10^{-19}\cdot2.19\cdot10^{6}\cdot5.3\cdot10^{-11}}{2}=9.3\cdot10^{-24}\ \text{A·m}^2.$$

The torgue experienced by the orbiting electron is $\vec{M}=\left[\vec{p}_m,\vec{B}\right]$, and its magnitude is

$$M = p_m B\sin\alpha,$$

where α is the angle between the normal to the plane of the orbit and the magnetic induction vector $\vec{B}$.

Therefore,

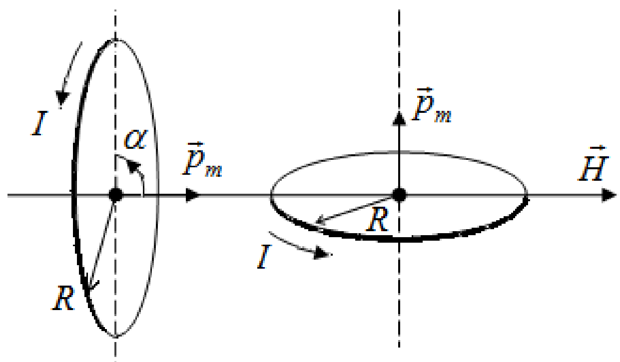
$M = p_m B \sin 30^0 = 9.3 \cdot 10^{-24} \cdot 2 \cdot 0.5 = 9.3 \cdot 10^{-24}$ N·m.

Problem 18

The circular loop of the radius R = 2 cm is located perpendicularly to the direction of the uniform magnetic field of the strength H = 150 A/m. The current in the loop is I = 2 A. Find the work which is required to turn the loop by the angle α = 90^0 about the axis coinciding with the diameter of the loop. Suppose the current is unchanged during the turn.

Solution

The problem may be solved by two methods.



1. The torgue acting on the loop is

$\vec{M} = \left[\vec{p}_m, \vec{B} \right]$,

where $\vec{p}_m = I\vec{S} = IS\vec{n}$ is the magnetic moment of the loop, S is an area of the loop, $\vec{n}$ unit normal to the loop.

The magnetic field induction $\vec{B}$ related to the magnetic field strength $\vec{H}$ according to $\vec{B} = \mu_0 \vec{H}$, where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic constant (or vacuum permeability).

The magnitude of the torque is $M = p_m B \sin\alpha$, where α is the angle between $\vec{p}_m$ and $\vec{B}$.

In the initial position when the vectors $\vec{p}_m$ and $\vec{B}$ are of the same direction, and the torque $M = 0$ and the loop is in the equilibrium state. For turning the loop from this state the external force has to do the work.

The elementary work of the external force is

$dA = M\,d\alpha = p_m B\sin\alpha\,d\alpha = ISB\sin\alpha\,d\alpha = I\pi R^2 B\sin\alpha\,d\alpha$.

The work don at the turn by the finitesmal angle is

$$A = \int_0^{90^0} I\pi R^2 B\sin\alpha d\alpha = I\pi R^2 B\int_0^{90^0}\sin\alpha d\alpha = I\pi R^2 B\left(-\cos\alpha\right)\Bigg|_0^{90^0} = I\pi R^2 B =$$

$$= I\pi R^2\mu_0 H = 2\pi\cdot 4\cdot 10^{-4}\cdot 4\pi\cdot 10^{-7}\cdot 150\cdot 10^3 = 3{,}16\cdot 10^{-5}\text{ J.}$$

2. The work may be determined by another method using the expression for the work of magnetic field $dA = Id\Phi$, where $\Phi = BS\cos\alpha$ is the magnetic flux, α is the angle between $\vec{B}$ (or $\vec{H}$) and the normal to the loop.

In the initial position the magnetic flux $\Phi_1 = BS$ is maximum, and in the final position $\Phi_2 = 0$. Then

$$dA = Id\Phi = I(\Phi_2 - \Phi_1) = I\left(0 - BS\right) = -IBS = -I\pi R^2\mu_0 H = -3{,}16\cdot 10^{-5}\text{ J.}$$

We have obtained the same value of the work but with the opposite sign because in the first case we found the work of external force (it is positive), and in the second case we found the work of magnetic field. This work is negative as the external force done this work.

Problem 19

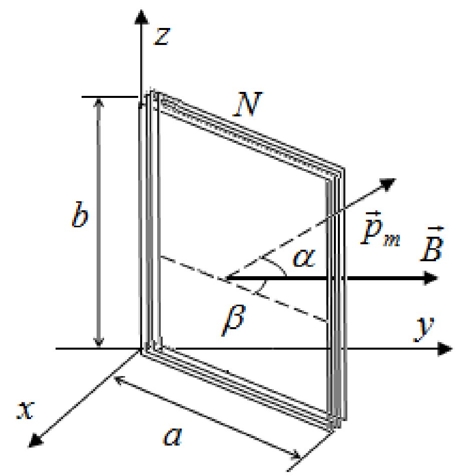
The rectangular loop (a = 0.5 m, b=0.6 m) consists of N = 75 turns and carries a current of I = 5 A. A magnetic field B = 2 T is directed along the +y axis. The loop is free to rotate about the z-axis. Determine the magnitude of the net torque exerted on the loop. State whether the β *= 30° angle will increase or decrease.*

Solution

The torque on the loop is given by $\vec{M} = \left[\vec{p}_m, \vec{B}\right]$, where $\vec{p}_m = NIS\vec{n}$ is the magnetic moment of the loop.

The magnitude of the torque is $M = p_m B \sin\alpha$, where $p_m = NIS = NIab$, and the angle α between the normal to the plane of the loop and the magnetic field is

$\alpha = 90^0 - \beta = 90^0 - 30^0 = 60^0$.



The magnitude of the net torque exerted on the loop is

$M = NIabB\sin\alpha$.

$M = 75 \cdot 5 \cdot 0.5 \cdot 0.6 \cdot 2 \cdot 0.866 \approx 195$ N.

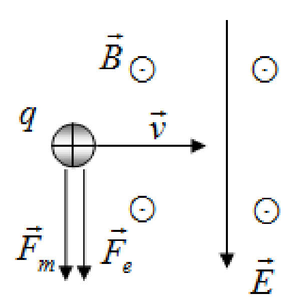
When a current-carrying loop is placed in a magnetic field, the loop tends to rotate such that its normal becomes aligned with the magnetic field. The normal to the loop makes an angle of 60° with respect to the magnetic field. Since this angle decreases as the loop rotates, and the angle β increases.

Problem 20

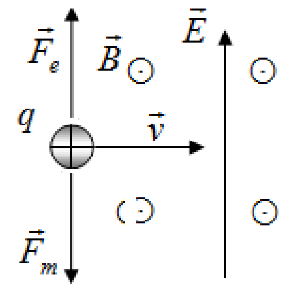
A positively charged particle entering a 0.6-T magnetic field (directed out of the paper). The particle has a speed of 300 m/s and moves perpendicular to the magnetic field. Just as the particle enters the magnetic field, an electric field is turned on. What must be the magnitude and direction of the electric field such that the net force on the particle is twice the magnetic force? Determine the conditions under which the particle moves in the same direction as it moved before entering into the fields.

Solution

According to the expression $\vec{F} = q\left[\vec{v}, \vec{B}\right]$, the magnetic force on the positively charged particle is toward the bottom of the page in the drawing in the text. If the presence of the electric field is to double the magnitude of the net force on the charge, the electric force and, consequently, the electric field must also be directed towards the bottom of the page.



Furthermore, if the magnitude of the net force on the particle is twice the magnetic force, the electric force $F_e = qE$ must be equal in magnitude to the magnetic force $F_m = qvB\sin\alpha$ (with $\sin\alpha = 1$).



$qE = qvB\sin\alpha$.

Then, solving for E

$E = vB = 300 \cdot 0.6 = 180$ V/m.

If the particle4 is moving without changing its direction in the magnetic and electric fields, it means that the net force on this particle is zero. This is possible when electric force is equal to the magnetic force in the magnitude and opposite in direction, i.e., when the electric field is directed upwards to the top of the page.

Problem 21

*A proton (charge +e, mass m_p) is accelerated through a potential difference $\Delta\varphi$=100 V. The particle enters a uniform magnetic field $\vec{B}$ **(**B= 2T) with a velocity in a direction perpendicular to $\vec{B}$.*

1) Describe the motion of the proton.

2) Determine the values of the radius of the circular path, the orbital period for the motion, cyclotron's frequency and the angular momentum of the particle.

Solution

The speed of the particles can be found from the kinetic energy resulting from the change in electric potential given. An electric field changes the speed of charged particle according to

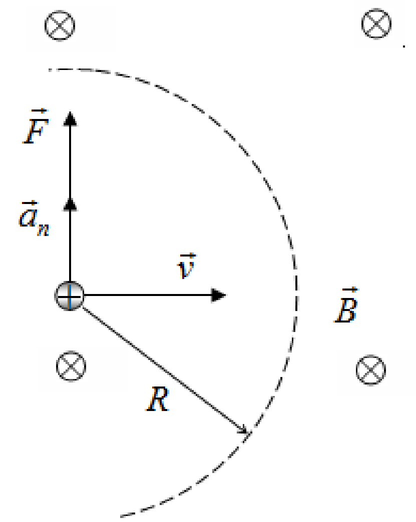
$\Delta W_k = W_{k2} - W_{k1} = A_e$.

Since the initial velocity of the proton $v_0 = 0$, its initial kinetic energy $W_{k1} = \frac{mv_0^2}{2} = 0$. The final kinetic energy is $W_{k2} = \frac{mv^2}{2}$, and the work of accelerating electric field is $A = q\Delta\varphi = e\Delta\varphi$. Then

$$\frac{mv^2}{2} = e\Delta\varphi .$$

The velocity of the proton when it enters the magnetic field is

$$v=\sqrt{\frac{2e\Delta\varphi}{m}}.$$



The force on the proton in a magnetic field is given by $\vec{F}_m = q\left[\vec{v},\vec{B}\right]$, where $\vec{v}$ is a velocity of the proton and $\vec{B}$ is the magnetic field induction. This force is always perpendicular to the direction of motion of the particle, and will therefore only change the direction of motion, and not the magnitude of the velocity. If the charged particle is moving in a uniform magnetic field, with strength $\vec{B}$, that is perpendicular to the velocity v, then the magnitude of the magnetic force is given by

$$F_m = qvB = evB,$$

and its direction is perpendicular to $\vec{v}$. As a result of this force, the particle will carry out uniform circular motion. The radius of the circle is determined by the requirement that the strength magnetic force is equal to the centripetal force.

The normal (centripetal) acceleration is perpendicular to the velocity and has a magnitude $a_n = \frac{v^2}{R}$, where R is the radius of the circle. Applying Newton's second law to the proton motion $m\vec{a}_n = \vec{F}_m$, we obtain

$$\frac{mv^2}{R} = evB,$$

$$R = \frac{mv}{eB}.$$

The orbital period for the proton's motion is

$$T = \frac{2\pi R}{v} = \frac{2\pi m\cancel{v}}{\cancel{v}eB} = \frac{2\pi m}{eB}.$$

Cyclotron's frequency is inverse proportional to the orbital period

$$f = \frac{1}{T} = \frac{eB}{2\pi m}.$$

An angular momentum of the proton is

$$L = mvR = \frac{(mv)^2}{eB}.$$

Substituting the given data, we obtain

$$v = \sqrt{\frac{2e\Delta\varphi}{m}} = \sqrt{\frac{2\cdot 1.6\cdot 10^{-19}\cdot 100}{1.66\cdot 10^{-27}}} = 1.4\cdot 10^{5}\,\text{m/s}.$$

$$R = \frac{1.66\cdot 10^{-27}\cdot 1.4\cdot 10^{5}}{1.6\cdot 10^{-19}\cdot 2} = 7.26\cdot 10^{-4}\,\text{m}.$$

$$T = \frac{\cancel{2}\pi\cdot 1.66\cdot 10^{-27}}{1.6\cdot 10^{-19}\cdot \cancel{2}} = 5.18\cdot 10^{-9}\,\text{s}.$$

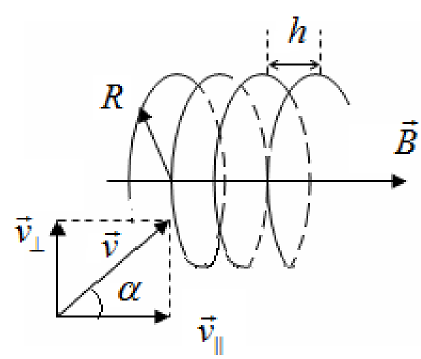
$$f = \frac{1}{5.18\cdot 10^{-9}} = 1.93\cdot 10^{8}\,\text{Hz}.$$

$$L = \frac{\left(1.66\cdot 10^{-27}\cdot 1.4\cdot 10^{5}\right)}{1.6\cdot 10^{-19}\cdot 2} = 1.7\cdot 10^{-25}\,\text{kg·m}^2\text{·s}^{-1}.$$

Problem 22

An electron after accelerated motion through a potential difference U =500 V enters a uniform magnetic field $\vec{B}$ (B = 0.05 T) with a velocity $\vec{v}$ that makes an angle α = 60^0 with the magnetic field. Find the trajectory of the electron and its parameters.

Solution



The speed of the electron is obtained from the work-energy theorem:

$$eU = \frac{mv^2}{2}.$$

Hence

$$v = \sqrt{\frac{2eU}{m}} = \sqrt{\frac{2\cdot 1.6\cdot 10^{-19}\cdot 500}{9.1\cdot 10^{-31}}} = 1.33\cdot 10^{7}\,\text{m/s}.$$

The electron is acted on by the magnetic Lorentz force: $\vec{F} = e\left[\vec{v},\vec{B}\right]$. The cross product is perpendicular to both $\vec{v}$ and $\vec{B}$, so the force will not change the magnitude of the velocity, and therefore the magnitude of the force vector also stays constant: $F = evB\sin\alpha$.

The velocity of the electron may be decompose into two components

$$\begin{cases} v_{\perp} = v\sin\alpha, \\ v_{\parallel} = v\cos\alpha. \end{cases}$$

Since the force has no component parallel to the $\vec{B}$

$$F_{\parallel} = evB\sin\left(\vec{v}_{\parallel}\,^\wedge\vec{B}\right) = 0,$$

the motion of the electron in the direction of the field lines is uniform at $v_{\parallel}=$ const.

The motion at $\vec{v}_{\perp}$ in a plane perpendicular to the field lines is along the circular path. The radius of its orbit can be determined by using Newton's second law.

$$ma_n = m\frac{v_{\perp}^2}{R} = ev_{\perp}B\sin\left(\vec{v}_{\perp}\,^\wedge\vec{B}\right) = ev_{\perp}B,$$

$$R = \frac{mv_{\perp}}{eB} = \frac{mv\sin\alpha}{eB} = \frac{9.1\cdot 10^{-31}\cdot 1.33\cdot 10^{7}\cdot 0.866}{1.6\cdot 10^{-19}\cdot 0.05} = 1.31\cdot 10^{-3}\,\text{m}.$$

Thus the trajectory of the electron is a helix with its axis parallel to the field lines. In a reference frame moving along the electron at a parallel speed of $v_{\parallel} = v\cos\alpha$, the electron is moving in a circular orbit at a tangential speed of $v_{\perp} = v\sin\alpha$, and its orbital period is

$$T = \frac{2\pi R}{v_{\perp}} = \frac{2\pi m}{eB} = \frac{2\pi\cdot 9.1\cdot 10^{-31}}{1.6\cdot 10^{-19}\cdot 0.05} = 7.14\cdot 10^{-10}\,\text{s},$$

independently of the initial direction of the velocity of the electron.

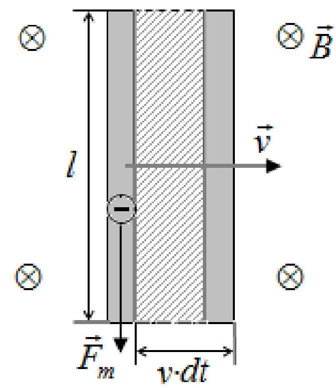
Travelling at a speed of $v_{\parallel} = v\cos\alpha$, the electron covers the distance h in a time period T:

$$h = v_{\parallel}T = \frac{2\pi mv\cos\alpha}{eB} = \frac{2\pi\cdot 9.1\cdot 10^{-31}\cdot 1.33\cdot 10^{7}\cdot 0.5}{1.6\cdot 10^{-19}\cdot 0.05} = 4.75\cdot 10^{-5}\,\text{m}.$$

Problem 23

A conducting rod 4 meters in length is placed in a magnetic field at right angles to the direction of $\vec{B}$ *which has a magnitude of 0.3 Tesla. If the rod is moved with a speed of 3 m/s in a direction perpendicular to its length and perpendicular to* $\vec{B}$, *what emf is induced across the ends of the rod? Indicate the direction of the induced emf.*

Solution



We first determine the direction of the induced emf in the rod. We can do this by using the magnetic component of Lorentz force $\vec{F}_m = q\left[\vec{v}, \vec{B}\right]$. The force acting on the negative charges in the rod moving to the right is directed downward, and the bottom of the rod will become the negative end of the emf.

The emf produced is given by:

$$\mathrm{E} = -\frac{d\Phi}{dt} = -\frac{d\left(\vec{B}, \vec{S}\right)}{dt} = -\frac{d\left(B \cdot S \cdot \cos\alpha\right)}{dt} = -B\frac{dS}{dt} = -B\frac{lvdt}{dt} = -Blv.$$

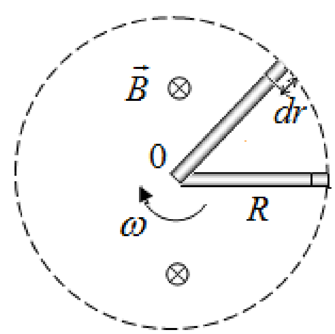
Substituting the given data, we obtain the magnitude of the emf:

$\mathrm{E} = 0.3 \cdot 4 \cdot 3 = 3.6\,\mathrm{V}.$

Problem 24

A rod of length R = 50 cm rotates at an angular velocity ω = 50 rad/s in a uniform magnetic field of induction B = 2 mT. Determine the emf E *developed across the ends of the rod.*

Solution



Consider an element of length dr situated a distance r from the end of the rod (see Figure) If the linear velocity of the rod is at right angles to the field $\vec{B}$ by which the element dr is moving, the emf developed across the element is given by

$$d\mathrm{E} = B \cdot v \cdot dr = B \cdot \omega \cdot r \cdot dr,$$

$$\mathrm{E} = \int d\mathrm{E} = B\omega \int_0^R r \cdot dr = \frac{B\omega R^2}{2}.$$

The same result may be obtained by the following reasoning.

The emf according to the Faraday's law of induction is

$$\mathrm{E} = -\frac{d\Phi}{dt}.$$

Since the change of the magnetic flux $d\Phi$ takes place for the time of one revolution, and this the period of rotation is equal to $T = \dfrac{2\pi R}{v}$, the emf is

$$\left|\mathrm{E}\right| = \frac{B \cdot \Delta S}{T} = \frac{B\cancel{\pi} R^{\cancel{2}} v}{2\cancel{\pi}\cancel{R}} = \frac{BvR}{2} = \frac{B\omega R^2}{2}.$$

On substituting the values of B, R and ω, we have

$$\mathrm{E} = \frac{2 \cdot 10^{-3} \cdot 50 \cdot (0.5)^2}{2} = 1.25 \cdot 10^{-2}\ \mathrm{V}.$$

Problem 25

A coil of 1000 turns and 12 cm radius flips 180° about an axis that points northward. The coil has a resistance of 4.8 Ω*. The vertical component of Earth's magnetic field is 46 µT. Find the total charge that flows when the coils flips.*

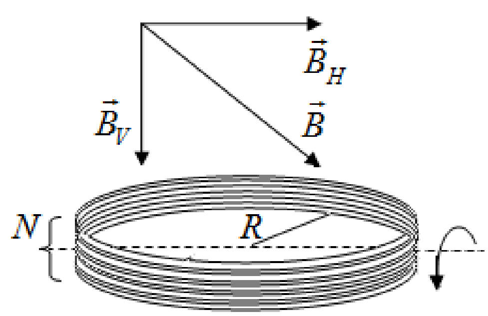
Solution

Notice that the horizontal component of the Earth's field contributes no flux. All the flux through the loop is due to the vertical component. Faraday's Law requires,

$$\mathrm{E} = -N\frac{d\Phi}{dt}.$$

The charge that flows is related to Ohm's Rule,

$$\mathrm{E} = IR = \frac{dq}{dt}.$$



Setting the voltage equal,

$$\frac{dq}{dt}R = -N\frac{d\Phi}{dt},$$

$$\int_0^q dq = -\frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi,$$

$$q = -\frac{N}{R}(\Phi_2 - \Phi_1) = \frac{N}{R}(\Phi_1 - \Phi_2).$$

The initial flux is just the product of the vertical component of the field and the area ($\cos\alpha = 1$):

$$\Phi_1 = \left(\vec{B}_V, \vec{S}\right) = B_V S\cos\alpha = B_V \pi r^2.$$

The final flux in just the opposite of the initial flux ($\cos\alpha = -1$):

$$\Phi_2 = \left(\vec{B}_V, \vec{S}\right) = B_V S\cos\alpha = -B_V \pi r^2.$$

$$q = \frac{N}{R}(\Phi_1 - \Phi_2) = \frac{N}{R}\left(B_V \pi r^2 - \left(-B_V \pi r^2\right)\right) = \frac{2NB_V \pi r^2}{R} =$$

$$= \frac{2 \cdot 1000 \cdot 46 \cdot 10^{-6} \cdot \pi \cdot (0.12)^2}{4.8} = 8.67 \cdot 10^{-4} \text{ C}.$$

Problem 26

A coil consists of N = 300 turns of wire. Each turn is a square of side a = 20 cm and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in t = 1s, what is the magnitude of the induced emf in the coil while the field is changing? What is the magnitude of induced current in the coil if its total resistance is $R = 4\,\Omega$?

Solution

The area of one turn of the coil is $S = a^2$.

The magnetic flux Φ_1 through the coil at $t_1 = 0$ is zero because $B = 0$ at that time. At $t_2 = 1$ s, the magnetic flux through one turn is $\Phi_2 = BS\cos\alpha = BS$, as $\cos\alpha$ due to the perpendicularity of the magnetic flux and the normal to the turn plane. The magnetic flux change is $\Delta\Phi = \Phi_2 - \Phi_1 = \Delta B \cdot S$. Therefore, the magnitude of the induced emf is

$$\mathrm{E} = N\frac{\Delta\Phi}{\Delta t} = N\frac{\Delta B \cdot S}{\Delta t} = N\frac{\Delta B \cdot a^2}{\Delta t} = \frac{300 \cdot 0.5 \cdot (0.2)^2}{1} = 6\,\text{V}.$$

The current in the coil

$$I = \frac{\mathrm{E}}{R} = \frac{6}{4} = 1.5\,\text{A}.$$

Problem 27

A 200-turn circular coil has a diameter of D =4 cm, a resistance of R = 80 Ω, and the two ends of the coil are connected together. The plane of the coil is perpendicular to a uniform magnetic field of magnitude B = 2 T. The direction of the field is reversed. Find the total charge that passes through a cross section of the wire. If the reversal takes Δt =0.2 s, find the average current and the average emf during the reversal.

Solution

An average current is equal to the total charge passing through the coil for the time Δt. Because the induced current is proportional to the induced emf and the induced emf, in turn, is given by Faraday's law, we can express q as a function of the number of turns of the coil, the magnetic field, the resistance of the coil, and the area of the coil. Knowing the reversal time, we can find the average current from its definition and the average emf from Ohm's law.

Faraday's law gives

$$\mathrm{E} = -N\frac{d\Phi}{dt} = -N\frac{d\left(BS\cos\alpha\right)}{dt} = -NS\frac{dB}{dt},$$

as $\cos\alpha = \cos 90^0 = 1$ due to perpendicularity of the magnetic field and coil's plane.

On the other hand, according to Ohm's law

$$\mathrm{E} = IR = \frac{dq}{dt}R.$$

Combining these equations, we obtain

$$\frac{dq}{dt}R = -NS\frac{dB}{dt}.$$

Since the charge passes during the time interval when the magnetic field is changing

$$dq = -\frac{NS}{R}dB,$$

$$q = -\frac{NS}{R}\int_{B}^{-B} dB = \frac{N\pi D^2 \cdot 2B}{4R} = \frac{N\pi D^2 B}{2R} = \frac{200\pi \cdot 16 \cdot 10^{-4} \cdot 2}{2 \cdot 80} = 2.53 \cdot 10^{-2}\ \mathrm{C}.$$

An average current is

$$I = \frac{q}{\Delta t} = \frac{2.53 \cdot 10^{-2}}{0.2} = 0.126 \text{ A.}$$

An average emf in the coil is

$$E = IR = 0.126 \cdot 80 = 10.11 \text{ V.}$$

Problem 28

The coil consists of N =20 turns of wire, each of area S = 0.1 m^2, and the total resistance of the wire is R = 10 Ω. The loop rotates in a magnetic field of B = 1 T at a constant frequency of n =50 Hz. Find the maximum induced emf $\mathrm{E}_{\max}$. *What is the maximum induced current* $I_{\max}$ *when the output terminals are connected to a low-resistance conductor?*

Solution

$$\mathrm{E} = -N\frac{d\Phi}{dt} = -N\frac{d\left(BS\cos\alpha\right)}{dt} = NBS\sin\alpha \cdot \frac{d\alpha}{dt} = NBS\omega\sin\alpha = \mathrm{E}_{\max}\sin\alpha .$$

Taking into account that $\omega = 2\pi n$,

$$\mathrm{E}_{\max} = NBS2\pi n = 20 \cdot 1 \cdot 0.1 \cdot 2 \cdot \pi \cdot 50 = 628\,\text{V.}$$

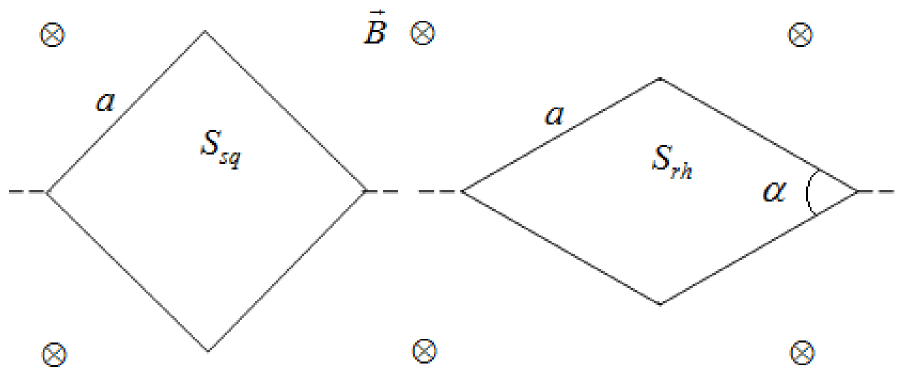
The maximum induced current

$$I_{\max} = \frac{\mathrm{E}_{\max}}{R} = \frac{628}{10} = 62.8\,\text{A.}$$

Problem 29

A square loop with length a = 0.5 m on each side is placed in a uniform magnetic field B = 2 T pointing into the page. During a time interval Δt =1ms the loop is pulled from its two edges and turned into a rhombus with acute angle as shown in the Figure. Assuming that the loop is made of iron wire with cross-section area S_0 = 1 mm^2, find the charge that passes through the loop and an average induced current in it.

Solution



Using Faraday's law, we have

$$\mathrm{E} = -\frac{d\Phi}{dt} = -\frac{d\left(\vec{B},\vec{S}\right)}{dt} = \frac{d\left(BS\cos\alpha\right)}{dt} = -B\frac{dS}{dt},$$

$$E = IR = \frac{dq}{dt}R,$$

$$\frac{dq}{dt}R = -B\frac{dS}{dt},$$

$$q = -\frac{B}{R}\int_{S_1}^{S_2} dS = -\frac{B\left(S_2 - S_1\right)}{R} = \frac{B\left(S_1 - S_2\right)}{R}.$$

Since the initial and the final areas of the loop are $S_{sq} = a^2$ and $S_{rh} = a^2 \sin 2\alpha = 0.866a^2$, respectively; the resistance of the loop is $R = \rho\frac{l}{S_0}$, where $\rho = 0{,}087 \cdot 10^{-6}$ Ω·m, $l = 4a$, the charge that passed through the loop is

$$q = \frac{B\left(S_1 - S_2\right)}{R} = \frac{B\left(a^2 - a^2\sin 2\alpha\right)S_0}{\rho l} = \frac{Ba^{\cancel{2}}S_0\left(1-\sin 2\alpha\right)}{4\cancel{a}\rho} =$$

$$= \frac{2\cdot 0.5\cdot 10^{-6}\left(1 - 0.866\right)}{4\cdot 0.087\cdot 10^{-6}} = 0.385 \text{ C}.$$

The average induced current is

$$I = \frac{q}{\Delta t} = \frac{0.385}{10^{-3}} = 385\,\text{A}.$$

Problem 30

Calculate (a) the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25 cm and its cross-sectional area is 4.00 cm^2; and (b) the self-induced emf in the solenoid if the current it carries is decreasing at the rate of 50 A/s.

Solution

The inductance of the solenoid is

$$L = \mu_0 \frac{N^2}{l^2} Sl = \mu_0 \frac{N^2}{l} S = \frac{4\pi \cdot 10^{-7} \cdot 300^2 \cdot 4 \cdot 10^{-4}}{0.25} = 1.81 \cdot 10^{-4}\,\text{H} = 0.181 \text{ mH}.$$

Using the expression for the self-induced emf and given that $\frac{dI}{dt} = -50$ A/s, we obtain

$$\varepsilon_s = -L\frac{dI}{dt} = -1.81 \cdot 10^{-4} \cdot (-50) = 9.05 \cdot 10^{-3}\,\text{V} = 9.05 \text{ mV}.$$