NATIONAL TECHNICAL UNIVERSITY
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DEPARTMENT OF PHYSICS

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STUDY GUIDE

“WAVE OPTICS”

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OPTICS

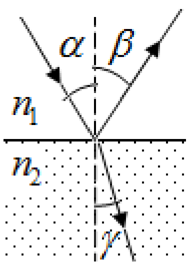
Optics is the part of physics that deals with the propagation of light through transparent media and its interaction with substance. Optical effects can be divided into three broad classes: those which can be explained without reference to the fact that light is fundamentally a wave or particle phenomenon, those which can be explained only on the basis that light is a wave phenomenon, and those which can be only explained on the basis that light is the flux of particles (photons). So, there are three *main parts of optics:* geometric optics; wave (physical) optics; and quantum optics.

Chapter 1. GEOMETRIC OPTICS

Geometric optics does not make any explicit assumption about the nature of light; it tends to suggest that light consists of a stream of massless particles. This is certainly what scientists, including, most notably, Isaac Newton, generally assumed up until about the year 1800. In geometric optics, light is treated as a set of *rays*, emanating from a source, which propagate through transparent media according to a set of four simple laws.

1. The law of *rectilinear propagation*, which states that light rays propagating through a homogeneous transparent medium, propagate in straight lines.

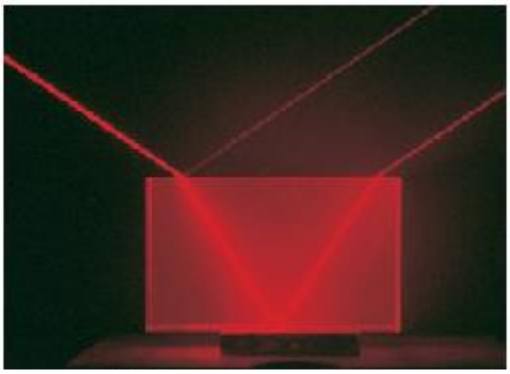
2. The law of *independent propagation* of light rays, which states that light rays at the point of intersection do not disturb each other (*superposition principle*);



3. The law of *reflection*, which governs the interaction of light rays with conducting surfaces. This law states that the incident ray, the reflected ray, and the normal to the surface of the mirror all lie in the same plane. Furthermore, the angle of reflection β is equal to the angle of incidence α. Both angles are measured with respect to the normal to the mirror.

4. The law of *refraction*, which is generally known as *Snell's law*, governs the behavior of light-rays as they propagate across a sharp interface between two transparent dielectric media. This law states that the incident ray, the refracted ray, and the normal at the point of incidence all lie in the same plane. Furthermore,

$$n_{21} = \frac{\sin\alpha}{\sin\gamma} = \frac{n_2}{n_1},$$



where α is the angle of incidence (between the incident ray and the normal to the interface), and γ is the angle of refraction (between the refracted ray and the normal to the interface). The quantities n_1 and n_2 are termed the *refractive indices* of media 1 and 2, respectively, n_{21} is *relative refractive index* of the 1st medium relative to the 2nd medium.

When light passes from one transparent medium to another, it's refracted because the speed of light is different in the two media. The *index of refraction (refractive index)*, n, of a medium is defined as the ratio of the speed of light in vacuum $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ and the speed of light in the medium $v = \frac{1}{\sqrt{\varepsilon\varepsilon_0\mu\mu_0}}$, i.e.,

$$n = \frac{c}{v} = \frac{\sqrt{\varepsilon\varepsilon_0\mu\mu_0}}{\sqrt{\varepsilon_0\mu_0}} = \sqrt{\varepsilon\mu} \approx \sqrt{\varepsilon},$$

where ε_0 and ε are is the electric permittivities of free space and the substance, μ_0 and μ are the magnetic permeabilities of free space and the substance, respectively.

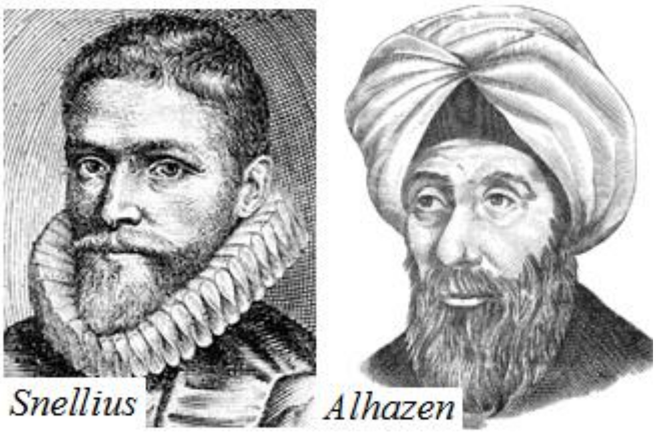
From this definition, we see that the index of refraction is a dimensionless number that is greater than or equal to one because v is always less than c. Further, $n = 1$ is for vacuum.

When light passes from one dielectric medium to another its velocity changes, but its frequency f remains unchanged (the color of the light doesn't change). Since, a speed $v = \lambda f$ for all waves, where λ is the wave-length, it follows that the wavelength of light must also change as it crosses an interface between two different media. When light propagates from medium with refractive index n_1 to medium with refractive index n_2 , the ratio of the wavelengths in the two media is given by

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1 f}{v_2 f} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = n_{21}.$$

If the first medium is vacuum, $\lambda_1 \equiv \lambda_0$ (in vacuum) and $\lambda_2 = \lambda$ (in substance),

$$n = \frac{\lambda_0}{\lambda}.$$



The law of refraction is called by the name of the Dutch naturalist, Astronomer and mathematician Willebord Snell (*Lat*. Snellius), (1580-1626), which he formulated in 1621. Earlier, this law and other laws of optics were investigated and described by the great Arab scholar Ibn al-Haytham (*Lat*. Alhazen, أبو علي، الحسن بن الحسن بن الهيثم) (965-1039), who was called “The Father of optics”.

Chapter 2. WAVE OPTICS

In wave (physical) optics, light is considered to propagate as electromagnetic (EM) wave. It is the form of radiant energy.

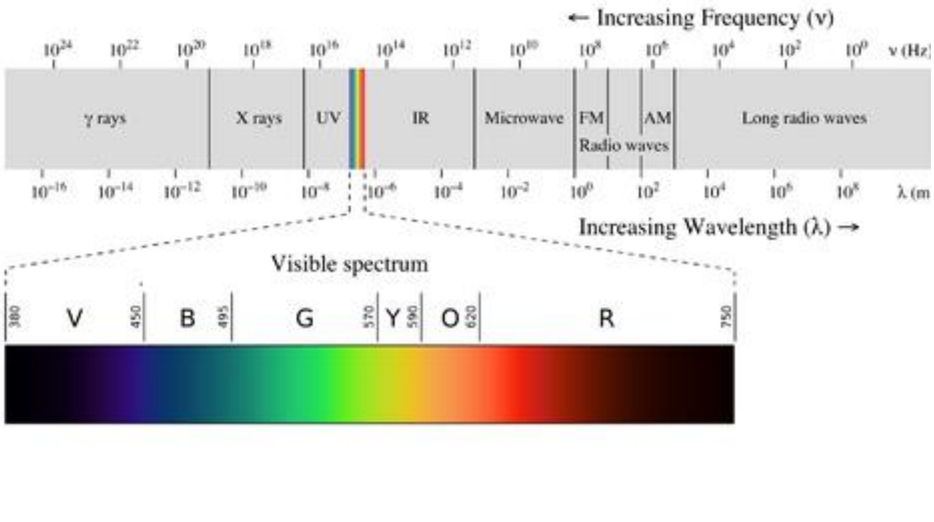
Visible light, the most familiar form of electromagnetic waves, may be defined as the part of the spectrum that is detected by the human eye. Light is produced by the rearrangement of electrons in atoms and molecules. The wavelengths of visible light are classified as colors ranging from violet ($\lambda = 4 \cdot 10^{-7}$ m) to red ($\lambda = 7.6 \cdot 10^{-7}$ m). The eye's sensitivity is a function of wavelength and is greatest at a wavelength of about $5.6 \cdot 10^{-7}$ m (yellow green).

Generally, physical optics studies other ranges of the EM waves.

Ultraviolet (UV) light covers wavelengths ranging from about $\lambda = 4 \cdot 10^{-7}$ m (400 nm) down to $\lambda = 6 \cdot 10^{-10}$ m (0.6 nm). An important source of ultraviolet light is the Sun.

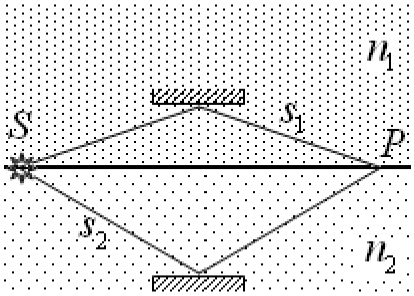
Infrared (IR) waves, produced by hot objects, have wavelength from $\lambda = 10^{-3}$ m to $\lambda = 7.6 \cdot 10^{-7}$ m.

X-rays are electromagnetic waves with wavelengths from about $\lambda = 10^{-8}$ m (10 nm) down to $\lambda = 10^{-13}$ m (10^4 nm). The most common source of X-rays is the acceleration of high-energy electrons bombarding a metal target.



1. INTERFERENCE

1.1. Coherence



The distance that the wave covered in vacuum is geometrical path length s. The optical path length (or optical distance) L is the product of the geometric length of the path light follows through the system, and the index of refraction of the medium through which it propagates.

$$L = n \cdot s\,.$$

$[s] = [L]$=m.

If two rays are propagating in the media with refractive indexes n_1 and n_2, and covered the optical distances L_1 and L_2, respectively, the optical path difference is

$$\Delta = L_1 - L_2 = n_1 s_1 - n_2 s_2\,.$$

The phase difference $\Delta\varphi$ of two waves that, after having been in phase initially, have traversed optical path lengths L_1 and L_2, respectively, is

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta\,.$$

Coherence is the matched progress of several oscillating or wave processes.

Coherent waves are monochromatic (having the equal frequencies) waves which are correlated to each other in phase. These phase relationships are maintained over long time. *Coherent sources* emit light waves of the same wavelengths or frequencies, which are always in phase with each other or have a constant phase difference, i.e., *coherent waves*. Two coherent waves can produce the phenomenon of interference.

Ordinary light is *incoherent* because it comes from independent atoms which emit on time scales of about 10^{-8} seconds.

1.2. Interference. Constructive and destructive interference

If two light waves passing through some common point P where each of them causes a displacement, according to the principle of superposition the resultant displacement is given by a vector sum of two displacements produced by each of the waves. If both displacements are along the same direction we can use the expression for calculation the amplitude of resultant oscillation $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\alpha$. Since intensity I of waves is proportional to A^2, $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\Delta\alpha$. And finally, assuming $I_1 = I_2 = I_0$, we obtain

$$I = 2I_0 + 2I_0\cos\Delta\alpha.$$

In dependence on the phase difference there are different cases are possible:

1. *Constructive* interference (maximum intensity)

$$\begin{cases} \cos\Delta\alpha = 1 \quad \Rightarrow \quad \Delta\alpha = \pm 2\pi k, \quad k = 0,1,2,... \\ \Delta\alpha = \dfrac{2\pi}{\lambda}\Delta = \pm 2\pi k \quad \Rightarrow \Delta = \pm k\lambda = \pm 2k\dfrac{\lambda}{2}, \end{cases} \qquad I = 4I_0$$

2. *Destructive* interference (minimum intensity)

$$\begin{cases} \cos\Delta\alpha = -1 \quad \Rightarrow \quad \Delta\alpha = \pm(2k+1)\pi, \quad k = 0,1,2,... \\ \Delta\alpha = \dfrac{2\pi}{\lambda}\Delta = \pm(2k+1)\pi \quad \Rightarrow \quad \Delta = \pm(2k+1)\dfrac{\lambda}{2} \end{cases} \qquad I = 0$$

At the points of maximum intensity the light waves emanating from each source are *in phase*. So the constructive interference occurs, resulting in a *light patch* on the screen. The general condition for constructive interference is that the path-length difference between the two waves be an *integer number* of wave-length, or *even* number of half-wave-lengths:

$$\Delta = k\lambda = 2k\frac{\lambda}{2},$$

where $k = 0, 1, 2,...$

At the points of minimum intensity waves are *out of phase,* so the destructive interference occurs, resulting in a *dark patch* on the screen. The general condition for destructive interference on the screen is that the difference in path-length between the two waves be a *half-integer* number of wavelengths, or *odd* number of half-wave-lengths:

$$\Delta = \left(k + \frac{1}{2}\right)\lambda = \left(2k+1\right)\frac{\lambda}{2},$$

where $k = 0, 1, 2, ...$

Therefore, *interference* is a phenomenon of *redistribution* of light on account the superposition of the coherent waves.

1.3. Coherent and incoherent sources of light. Huygens's Principle

The sources of light emitting waves having constant initial phase difference are called *coherent sources*.

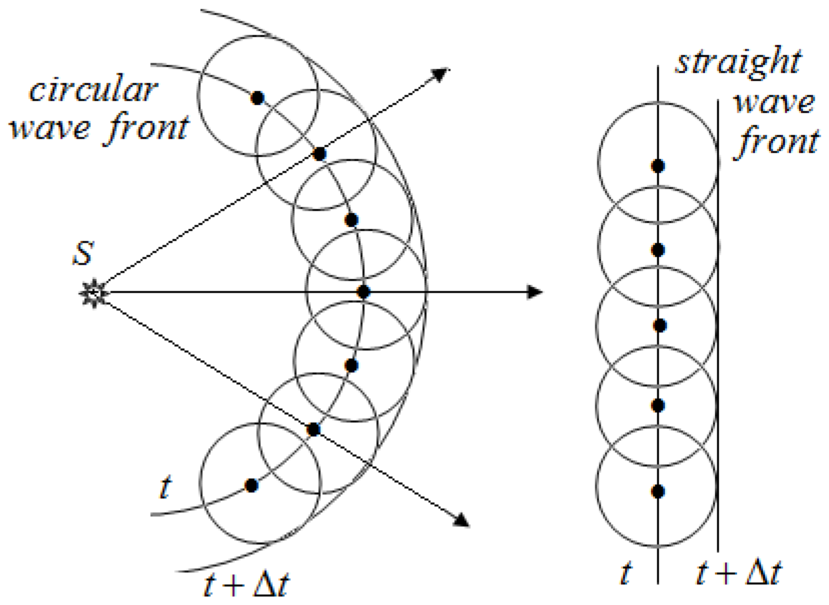
The sources of light emitting waves with random phase difference are called *incoherent sources*. For interference phenomenon, the sources must be coherent. So it is necessary to "prepare" light from readily available incoherent light sources - which typically emit individual, uncoordinated, short wave trains of fixed phase of no longer than 10^{-8} seconds - so that the light from such sources remains coherent over periods of time long enough to overlap and produce visible interference patterns. There are generally two ways to do this.

Methods of producing coherent sources:

- by *division of the wave front*: in this method the wave front which is the locus of points of same phase is divided into two (or the large amount) parts. The example is Young' double slit method;

- by *division of amplitude*: in this method the amplitude of a wave is divided into two parts by successive reflections.

Interference and further, diffraction, are explained by wave theory first proposed by a Dutch physicist and mathematician Christian Huygens (1629-1695). The assumptions of Huygens theory are: a source sends waves in all possible directions. The locus of points of medium oscillating in same phase is called a *wave front*. From the point source, the wave front is spherical: while for a line source the wave front is cylindrical. The distant wave front is plane.




Christian Huygens

Huygens's Principle: Each point of a wave front acts as a source of secondary waves (spherical wavelets). These wavelets travel with the velocity of light in the medium. A surface tangent to the wavelets at given instant constitutes the new position of the wave front is called the envelope of the wavelets.

Each point on the wave front acts as a point source that emits secondary.

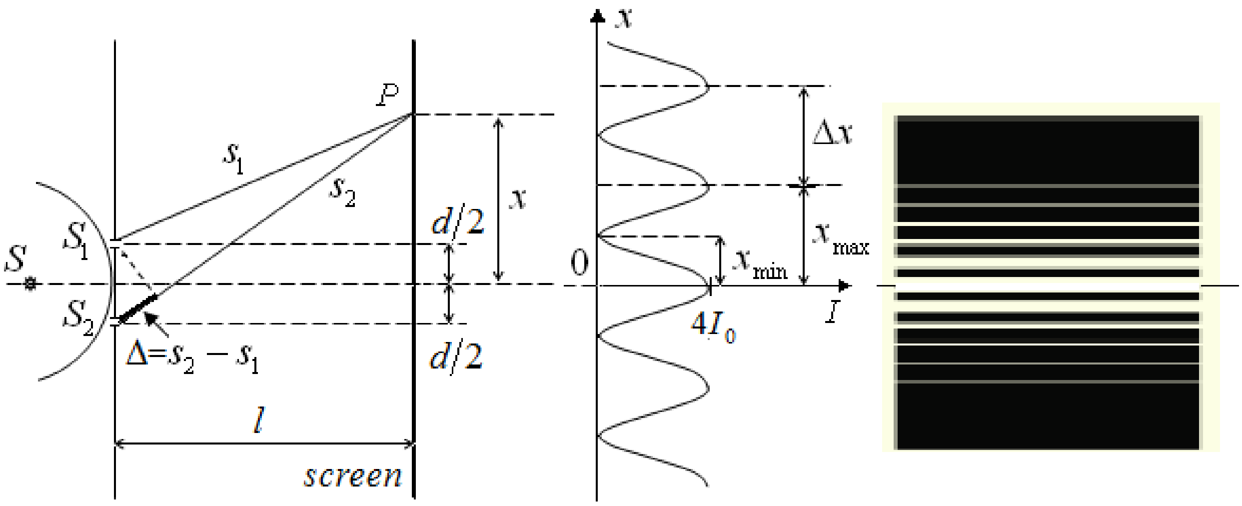
1.4. Double-slit interference. Young's experiment

In 1801 English scientist and physician Thomas Young (1773 – 1829) conducted an experiment to understand to the nature of light. Young thought that light acted like a wave and with his double slit experiment, he could see how light waves would interact if they intersected. The monochromatic point source *S* is a source of light whose spherical wave front (circular in the drawing) falls on two slits to create secondary sources S_1 and S_2 according to Huygens's Principle. They


Thomas Young

act as coherent sources. Spherical waves radiating out from the two secondary sources S_1 and S_2 maintain a fixed phase relationship with each other as they spread out and overlap on the screen, to produce a series of alternate bright and dark regions. The production of the bright points takes place when the crest of one wave meets the crest of the other wave or the trough of one wave meets the trough of the other wave, the waves are in phase, and these points appear bright. This type of interference is said to be *constructive interference*.

At points where the crest of one wave meets the trough of the other wave, the waves are in opposite phase, and these points appear dark. This type of interference is said to be *destructive interference*. The alternate regions of bright and dark are referred to as interference fringes which are shown at the right side of the figure.



The coordinates of the bright and dark fringes (interference maxima and minima) are, respectively

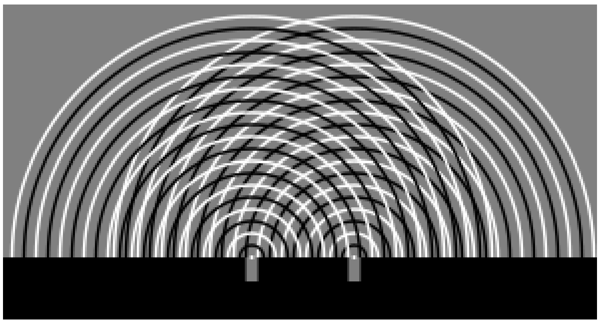
$$x_{\max} = \pm 2k \cdot \frac{\lambda_0 \cdot l}{2nd} = \pm 2k \cdot \frac{\lambda \cdot l}{2d},$$

$$x_{\min} = \pm(2k+1) \cdot \frac{\lambda_0 \cdot l}{2nd} = \pm(2k+1) \cdot \frac{\lambda \cdot l}{2d},$$

where $k = 0, 1, 2, ...$ is the number of interference maximum and minimum; λ_0 and λ are the wavelength in the free space and in medium; l is the distance from the slits to the screen; n is the refractive index, and d is the separation between the slits.

The distance between two bright (dark) fringes is

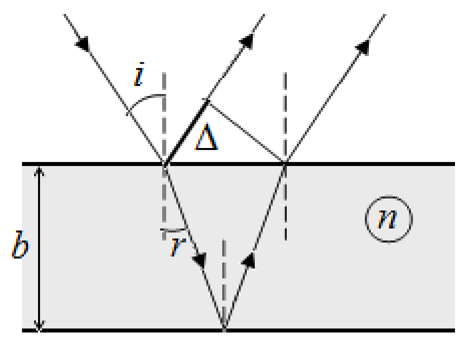
$$\Delta x = x_{\max 1} - x_{\max 0} = \frac{\lambda_0 \cdot l}{n \cdot d} = \frac{\lambda \cdot l}{d}.$$



1.5. Thin film interference

Double slit interference, described in the previous paragraph, is rarely observed in nature. And there is another type of interference that is quite frequently observed, namely thin film interference. Swirling colors on an oil slick, colors on a soap bubble, the purple tinge on an expensive camera lens, coloration of the wings of certain moths and butterflies are all examples of thin film interference.

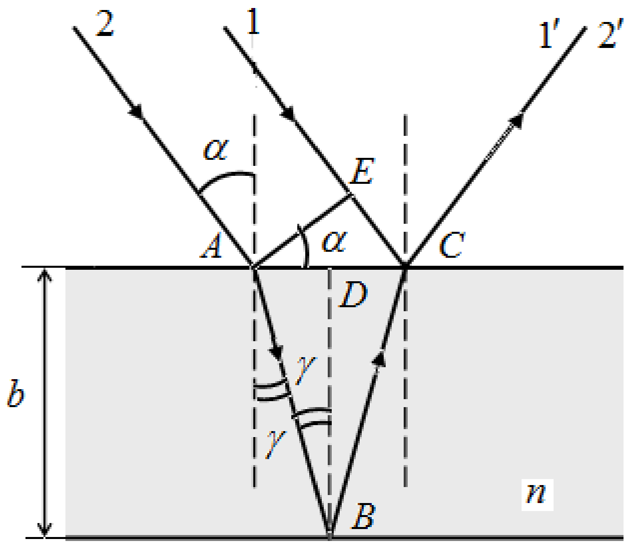
Light wave interference results when two waves are traveling through a medium and meet up at the same location. When a light wave reaches the boundary between two media, a portion of the wave reflects off the boundary and a portion is transmitted across the boundary. The reflected portion of the wave remains in the original medium. The transmitted portion of the wave enters the new medium and continues traveling through it until it reaches a subsequent boundary. If the new medium is a thin film, then the transmitted wave does not travel far before it reaches a new boundary and undergoes the usual reflection and transmission behavior. Thus, there are two



waves that emerge from the film: one wave that is reflected off the top of the film and the other wave that reflects off the bottom of the film.

(a) *Film of uniform thickness*

Let us determine this optical path length difference for thin film interference.



The first beam was propagated in the air $(n_0 = 1)$, so its optical path length is equal to the geometric path length $EC = s_1$. The second beam was propagated in a substance with a refractive index n, so the optical path length is $n \cdot (AB + BC) = ns_2$.

Thus, the optical path length difference, that is equal to the difference of the optical path lengths, is $\Delta = ns_2 - n_0 s_1 = ns_2 - s_1$.

From ΔADB: $AB = \dfrac{DB}{\cos\gamma} = \dfrac{b}{\cos\gamma}$, $\quad s_2 = 2 \cdot AB = \dfrac{2b}{\cos\gamma}$,

$$AD = BD \cdot \tan\gamma = b \cdot \tan\gamma, \quad AC = 2AD = 2b \cdot \tan\gamma.$$

From ΔAEC: $s_1 = EC = AC \sin\alpha = 2b \cdot \tan\gamma \cdot \sin\alpha$,

$$s_1 = 2b \cdot \tan\gamma \cdot \sin\alpha; \quad s_2 = \frac{2b}{\cos\gamma}.$$

Then the optical path length difference is

$$\Delta = \frac{2bn}{\cos\gamma} - 2b \cdot \tan\gamma \cdot \sin\alpha = 2b \cdot \frac{(n - \sin\gamma \cdot \sin\alpha)n}{(\cos\gamma)n} = 2b \cdot \frac{n^2 - n \cdot \sin\alpha \cdot \sin\gamma}{n \cdot \cos\gamma}.$$

From the reflection law $\dfrac{\sin\alpha}{\sin\gamma} = n_{21} = \dfrac{n}{n_0} = n$, $\quad n \cdot \sin\gamma = \sin\alpha$.

Since $\cos\gamma = \sqrt{1 - \sin^2\gamma}$, $n \cdot \cos\gamma = n\sqrt{1 - \sin^2\gamma} = \sqrt{n^2 - n^2 \sin^2\gamma} = \sqrt{n^2 - \sin^2\alpha}$,

then $\Delta = 2b \cdot \dfrac{n^2 - (n \cdot \sin\gamma)\sin\alpha}{n \cdot \cos\gamma} = 2b \cdot \dfrac{n^2 - \sin^2\alpha}{\sqrt{n^2 - \sin^2\alpha}} = 2b\sqrt{n^2 - \sin^2\alpha}$.

When light incident from a less dense medium (smaller n) reflects off the boundary with a denser (larger n) medium a $180°$ phase change on reflection occurs. Substitution of this phase shift in the relationship between the optical phase difference and the optical path length difference $\Delta\varphi = \frac{2\pi}{\lambda} \cdot \Delta$ gives the *additional path length difference*

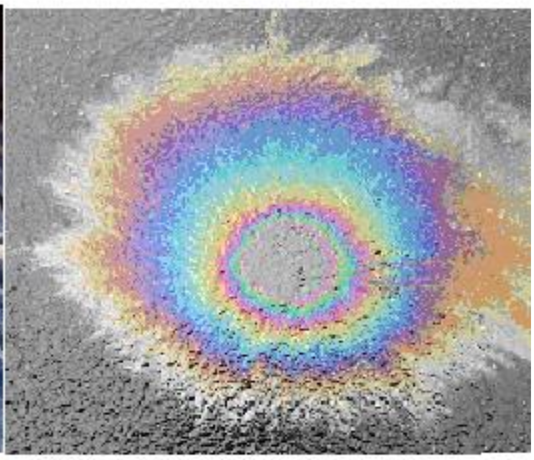
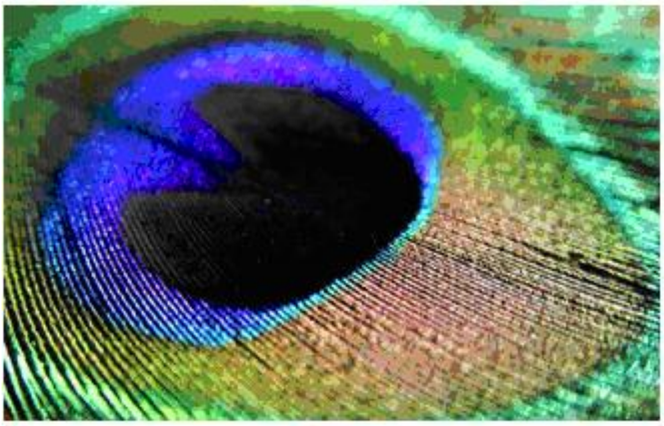
$$\Delta' = \frac{\Delta\varphi \cdot \lambda}{2\pi} = \frac{\pi \cdot \lambda}{2\pi} = \frac{\lambda}{2}\pi .$$

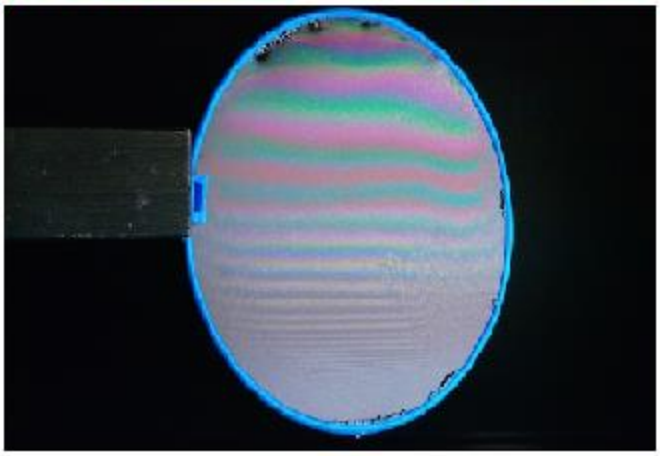
Incorporating this phase change leads to the expression for optical path length difference

$$\Delta = 2b\sqrt{n^2 - \sin^2\alpha} \pm \frac{\lambda}{2} ,$$

where b is the thickness of the film, n is the refractive index of the thin film substance, α is the angle of incidence, λ is the wavelength of the incident light.

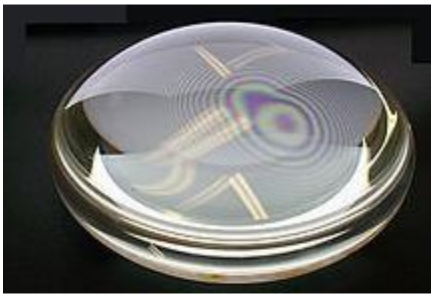
The theory of the interference due to a parallel film can now enable us to understand as to why films appear colored. The incident light is white and falls on a parallel-sided film. The incident light will split up by reflection at the top and bottom of the film. The split rays are in a position to interfere and interference of these rays is responsible for the colors. The bright or dark appearance of the reflected light depends upon the refractive index, the thickness of the film and the incidence angle. At a particular point of the film and for a particular position of the eye, the interfering rays of only certain wavelengths will have a path difference satisfying the conditions of bright fringe. Hence only such wavelengths (colors) will be present there. Other wavelengths will be present with diminished intensity. The colors for which the condition of minima is satisfied are absent. We know that the condition for maxima and minima in transmitted light are opposite to that of reflected light. Hence, the colors that are absent in reflected light would be present in transmitted light. The colors observed in transmitted and reflected light are complimentary.

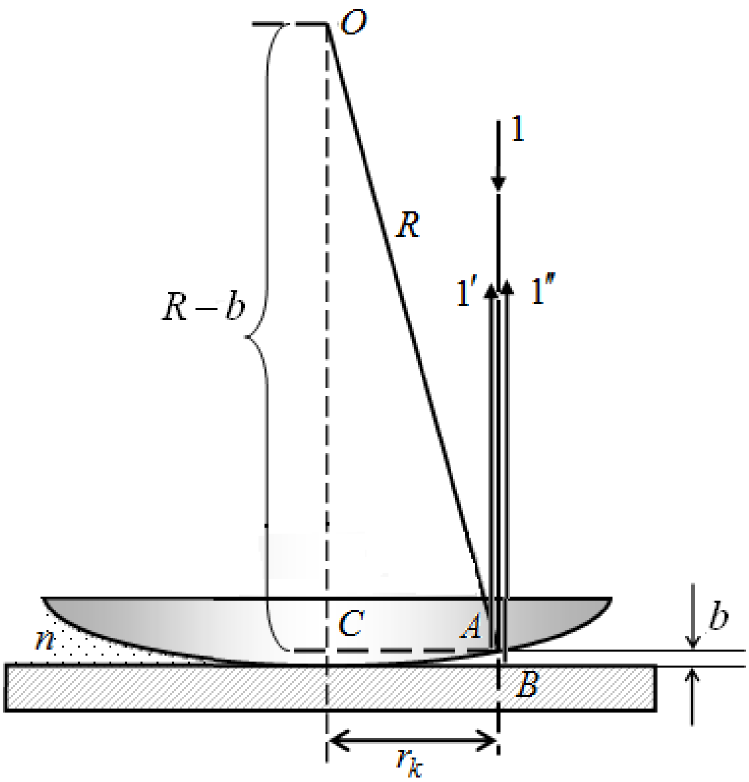




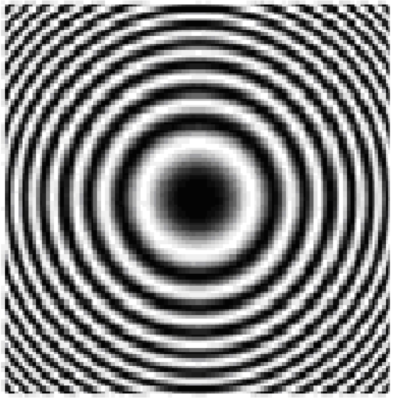



(b) *Newton's rings*

Another method for observing the interference is to place a plano-convex lens with long focal length on the top of a flat glass surface. With this arrangement, the air film in the gap between the glass surfaces varies in thickness from zero at the point of contact to a certain value b. When the air film is illuminated by monochromatic light normally, alternate bright and dark concentric circular rings are formed with dark spot at the centre. These circular fringes, discovered by Newton, are called *Newton's rings*. Generally, the substance with refractive index n may be between the lens and the glass surface. The interference is due to the combination between the beams reflected from the plate at point A and from the lower surface at the point B, respectively. Depending on the path length difference at different points separated from the center O by distance r the bright or dark rings can be observed.







The incident beam (1) is partially reflected at the point A (beam $1'$), partially passes the gap and is reflected at the point B (beam $1''$). If we observe Newton's rings in reflected light, then the optical path difference is twice the width of the gap (b) multiplied by the refractive index (n) of the medium in the gap, i.e. $\Delta = 2bn$. Taking into account the additional path length difference $\Delta' = \frac{\lambda}{2}$ due to reflection at point *A*, finally, the path length difference is $\Delta = 2bn + \frac{\lambda}{2}$.

From ΔCOA

$$R^2 = (R-b)^2 + r^2 = R^2 - 2Rb + b^2 + r^2.$$

Neglecting b^2 due to $b \ll R$ and $b \ll r$, we obtain $b = \frac{r^2}{2R}$.

The bright ring radii can be determined using the condition for constructive interference $\Delta = 2k \cdot \frac{\lambda}{2}$:

$$2bn + \frac{\lambda}{2} = 2k\frac{\lambda}{2}, \qquad 2bn = \frac{\cancel{2}nr^2}{\cancel{2}R} = (2k-1)\frac{\lambda}{2}.$$

As a result, the radii of *bright rings* are

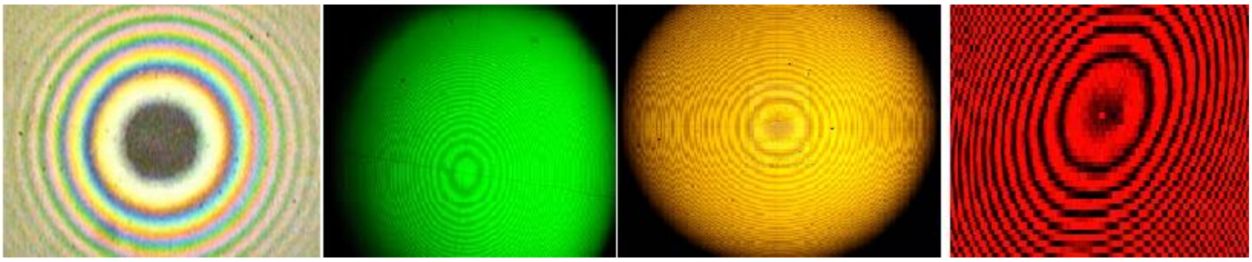
$$r_k = \sqrt{\frac{(2k-1)R\lambda}{2n}}, \quad k = 1, 2, \ldots;$$

Using the condition for destructive interference $\Delta = (2k+1) \cdot \frac{\lambda}{2}$, the radii of *dark rings* may be calculated

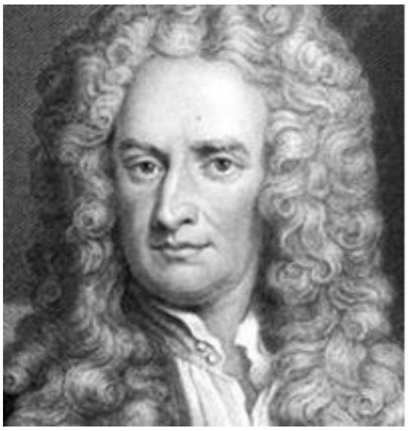
$$2bn + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2}, \qquad 2bn = \frac{\cancel{2}nr^2}{\cancel{2}R} = \cancel{2}k\frac{\lambda}{\cancel{2}}$$

$$r_k = \sqrt{\frac{kR\lambda}{n}}, \qquad k = 0, 1, 2, \ldots,$$

where k is the number of ring; R is the radius of lens curvature; λ is the light wavelength; and n is the refractive index of the substance in the gap between the lens and the glass surface.

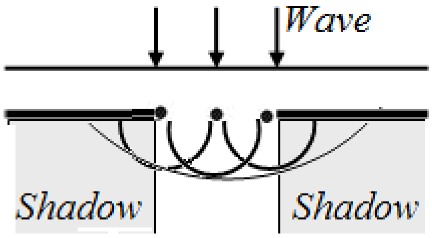


This interference patterns were firstly observed by the great English physicist and mathematician Isaac Newton (1643-1727) in 1717. When illuminated with monochromatic light, he found that the radii of rings increases with increasing wavelength, and when illuminated with white light, the light rings have an iridescent color, while the color changes with increasing distance from the center from violet to red. Newton could not give a sufficient explanation of the observed phenomenon. The nature of Newton's rings was explained by T. Young.



2. DIFFRACTION

Diffraction is a set of phenomena observed during the propagation of light in a medium with sharp inhomogeneities (near the boundaries of opaque or transparent bodies, small holes, etc.) and associated with deviations from the laws of geometric optics.



The common between interference and diffraction is the redistribution of the light flux at the superposition of coherent waves.

The difference is that interference comes from a finite number of discrete sources, and diffraction comes from continuously located sources.

As a result of diffraction, light waves envelop obstacles that are of the order of light wavelength and penetrate the region of geometrical shadow.

If a plane wave normally falls on a hole in opaque screen, each point of the wave front in this hole according to the Huygens principle is a source of secondary waves. The envelope of these waves, which determines the position of the wave front at the next instant of time, enters the region of the geometric shadow.

2.1 Fresnel diffraction

Diffraction phenomenon can be classified under two groups: Fresnel diffraction and Fraunhofer diffraction. In the Fresnel diffraction, the light source and the screen are at the finite distances from the obstacle producing diffraction. In this case the wave front undergoing diffraction is either spherical or cylindrical. In the Fraunhofer diffraction, the source and the screen are at the infinite distances from the obstacle producing diffraction, hence, the wave front is plane. The diffracted parallel beams are brought to focus with the help of a convex lens.

Augustin-Jean Fresnel (1788 -1827), a French civil engineer and physicist, essentially developed Huygens principle introducing several innovative provisions into it:

Augustin – Jean Fresnel

- all secondary sources of the wave front emanating from the same source are coherent;

- equal areas of the wave surface radiate equal intensities;

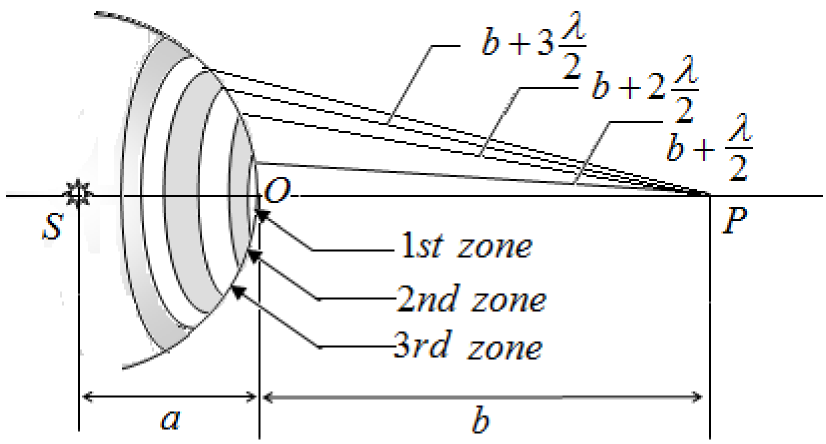
- each section of the wave surface radiates independently (the superposition principle for secondary sources);

- each element of the wave surface radiates predominantly in the direction of the external normal to it while the amplitude of the secondary spherical wave is proportional to the area of the element. The resulting oscillation at the observation point is determined by the formula

$$E = \int_S K(\varphi) \cdot \frac{A}{r} \cdot \cos(\omega t - kr + \alpha)\, dS,$$

where E is the light vector magnitude, A is the amplitude of the light oscillation, r is the distance from the element to the observation point, k is the wave number, $K(\varphi)$ is the coefficient depending on the angle φ between the normal to dS and the direction from dS to the observation point.

Calculation using the above formula is quite complex hence Fresnel proposed a method of quantitative calculations of the diffraction pattern for cases characterized by a certain symmetry. This simple method is known as *Fresnel half-period zone method*.



Fresnel assumed that a wave front that started from a source P can be divided into large number of strips which are known as *Fresnel's half period zones (HPZ).* If the distance between the points O and P is b then the distances to the edges of these zones from the point P have to be $b+\frac{\lambda}{2}$, $b+2\frac{\lambda}{2}$, …, $b+k\frac{\lambda}{2}$. The resultant effect at any point on the screen is due to the combined effect of all the secondary waves from various zones. Since the path difference between the wavelets originating from two consecutive HPZ's and reaching the point P is $\frac{\lambda}{2}$ the phase difference is π.

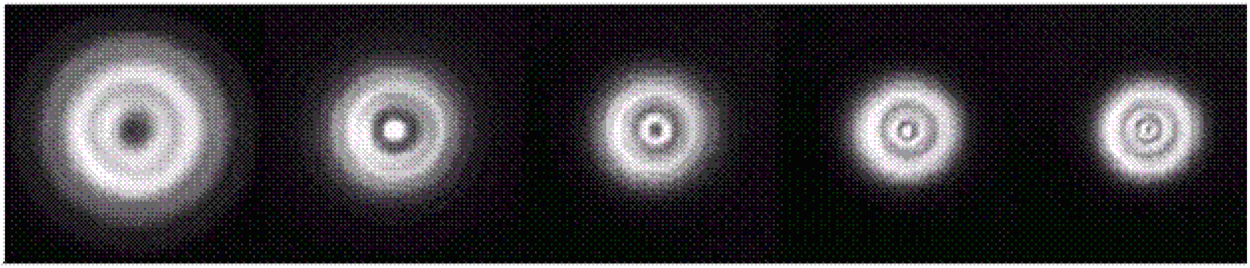
Therefore, waves that arrive at P from two contiguous zones damp one another, whereas the action of zones separated by one zone is added.

$$A = A_1 - A_2 + A_3 - A_4 + ... = \frac{A_1}{2} + \left(\frac{A_1}{2} - A_2 + \frac{A_3}{2} \right) + \left(\frac{A_3}{2} - A_4 + \frac{A_5}{2} \right) + ...$$

If the areas of zones are approximately equal $S_1 \approx S_2 \approx S_3 \approx$, the amplitudes are approximately too: $A_1 \approx A_2 \approx A_3 \approx ...$ As a result, if

1. the first HPZ is open, $A = A_1$.
2. two HPZ are open, $A = 0$.
3. three HPZ are open, $A = A_1$.
4. for large number of HPZ, the amplitude of light at point P due to whole wave front is half the amplitude due to the 1st HPZ.: $A = \frac{A_1}{2}$.

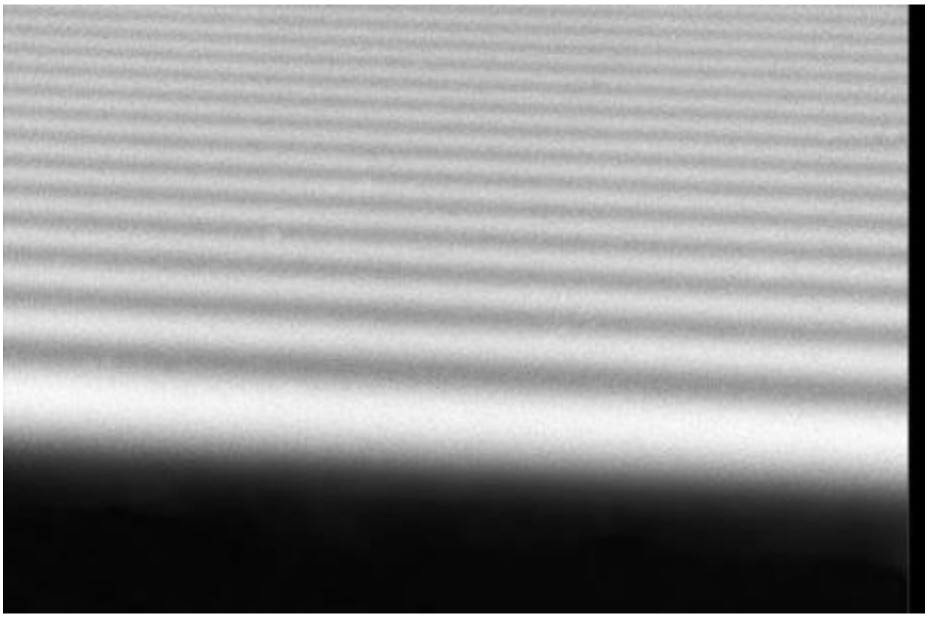
(a) Fresnel diffraction by *circular aperture* demonstrates the dark and bright points in the center of diffraction pattern when respectively even or odd numbers of Fresnel zones are open.

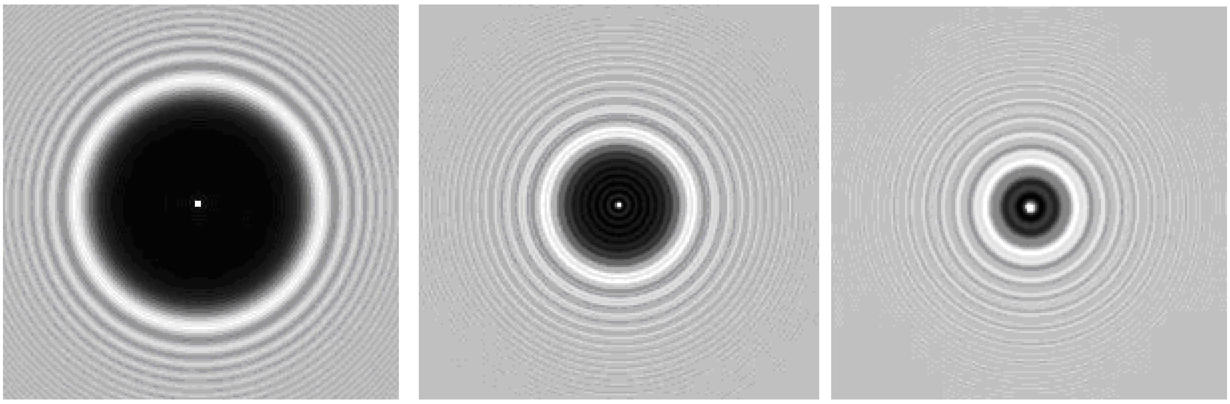



$k = 2$ $\quad$ $k = 3$ $\quad$ $k = 4$ $\quad$ $k = 5$ $\quad$ $k = 6$

Fresnel diffraction due to a circular aperture, k is a number of open zones

(b) Fresnel diffraction due to the knife-edge and the opaque disc





Fresnel diffraction due to circular disks of different diameters

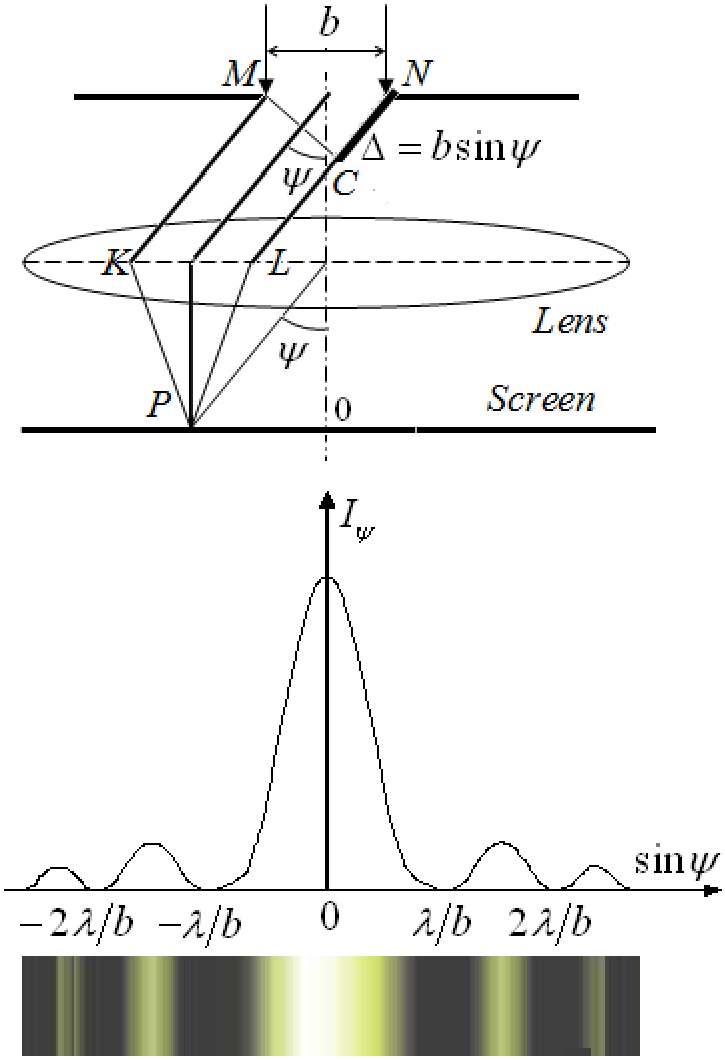
2.2. Fraunhofer diffraction

Joseph von Fraunhofer

Fraunhofer diffraction of light waves takes place when the diffraction pattern is viewed at a long distance from the diffracting object, and also when it is viewed at the focal plane of an imaging lens. This technique names after Joseph von Fraunhofer (1787

– 1826), German physicist who first studied the dark lines of the Sun's spectrum and used extensively the diffraction grating, a device that disperses light more effectively than a prism does.

In Fraunhofer diffraction lines joining the source and the screen are subparallel. If a lens is placed in front of aperture Fraunhofer diffraction pattern can be seen in the focal plane of the lens.



Fraunhofer diffraction at a single slit gives alternately dark and bright fringes observed at the viewing plane. The Fraunhofer diffraction pattern has maximum intensity at the central location which is determined by the rectilinear propagation law of light, and a series of peaks of decreasing intensity and width on each side symmetrically

In single slit diffraction, instead of point source assumption for slit we use a fine slit to illustrate diffraction effects. Break down the slit for many point sources. Hence light from one portion interferes with another portion and the pattern at the point of observation depends on angle ψ .

A phase difference for waves travelling from edges of the slit is

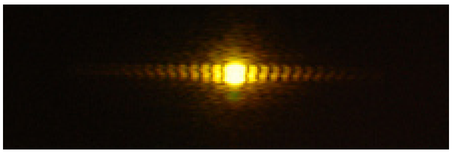
$\Delta = b\sin\psi$.

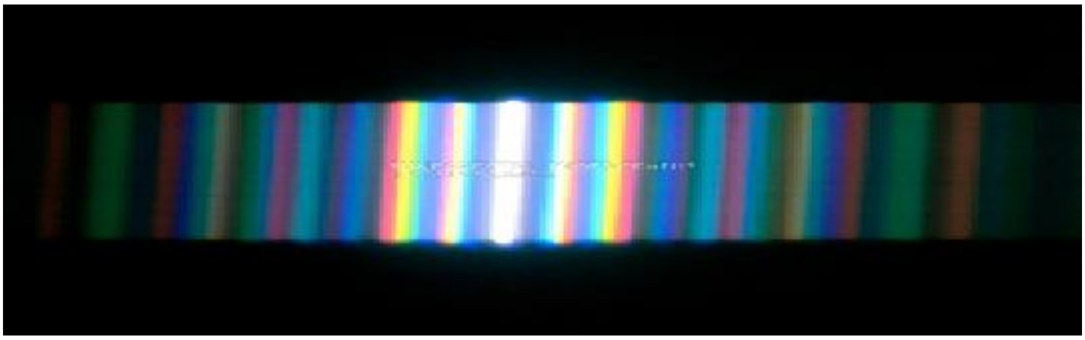
The resulting amplitude at the observation point P is zero or non-zero depending on the number of Fresnel zones located on the width of the slit:

$$b\sin\psi = \pm 2k\frac{\lambda}{2}, \qquad k = 1, 2, \ldots \qquad \text{- } \textit{diffraction minimum}$$

$$b\sin\psi = \pm(2k+1)\frac{\lambda}{2}, \qquad k = 0, 1, 2, \ldots \qquad \text{- } \textit{diffraction maximum}$$

The Fraunhofer diffraction pattern for a long narrow slit in an opaque screen consists of a set of light and dark parallel fringes.





2.3. Diffraction grating

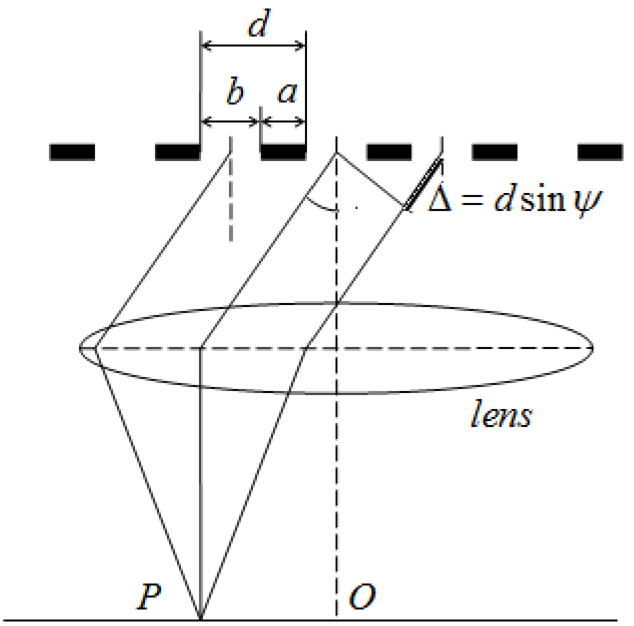
If b is a slit width and a is an opaque gap width, $d = a + b$ is *a grating constant.*

Each slit acts as a source and all sources are in phase. At some arbitrary direction away from horizontal, the beams must travel different path lengths to arrive at the screen. If the path-length difference is equal to one wavelength or an integer number of wavelengths, bright line (constructive interference) is observed at screen. Therefore the *condition for maxima at* an angle ψ is

$$d\sin\psi = \pm k\lambda, \qquad k = 0,\ 1,\ 2\ \ldots$$

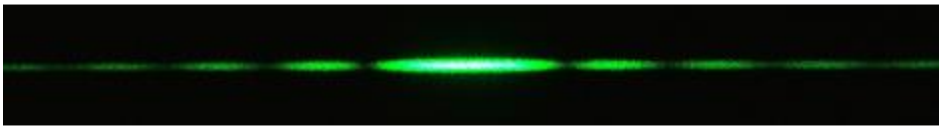
The *condition for minima* is the same as for single slit diffraction:

$$b\sin\psi = \pm 2m\frac{\lambda}{2}, \quad m = 1, 2, 3 \ldots$$



Diffraction grating is one of them, which allows a beam of light to be resolved into different colors. Diffraction grating consists usually of thousands of narrow, closely spaced parallel slits (or grooves).

If we illuminate diffraction grating with monochromatic light of laser source, then the beam is splited into several divergent beams in accordance with parameters of grating. Particularly the angles between the beams will be inversely proportional to the period of grating. Investigating the interference pattern on the remote screen we can make a conclusion about parameters of grating. And, on the contrary, knowing the parameters of grating we can easily calculate the wavelength of light. In the figure below you can see the result of the diffraction of light at a single slit.



If the light is not monochromatic, then the angle of diffraction depends upon the wavelength. In this case, the radiation spectrums of different orders will appear instead of the beams. If the diffraction grating is placed in front of objective glass of photo-camera and a photo of the candle flame is taken. If the central image coincides with the one without the grating, then the images in higher orders are dissolved in spectrum similar to rainbow. This property of diffraction is used for investigation of the spectrums of the optical radiation.

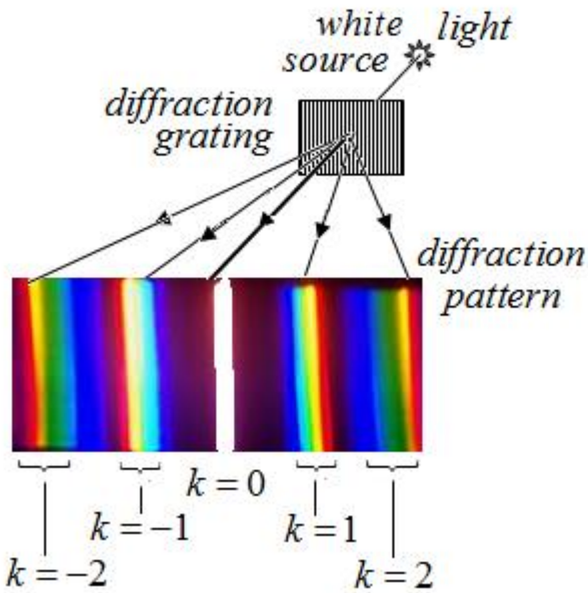
Diffraction gratings can be used to split light into its constituent wavelengths (colors). In general, it gives better wavelength separation than does a prism, although the output light intensity is usually much smaller.

By shining a light beam into a grating whose spacing d is known, and measuring the angle ψ where the light is imaged, one can measure the wavelength λ. This is the manner in which the atomic spectra of various elements were first measured.

If λ_1 and λ_2 are two nearly equal wavelengths between which the spectrometer can hardly distinguish, the *resolving power* (*resolvance*) of the grating R is defined to be

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda},$$

where $\lambda = \dfrac{\lambda_1 + \lambda_2}{2}$.

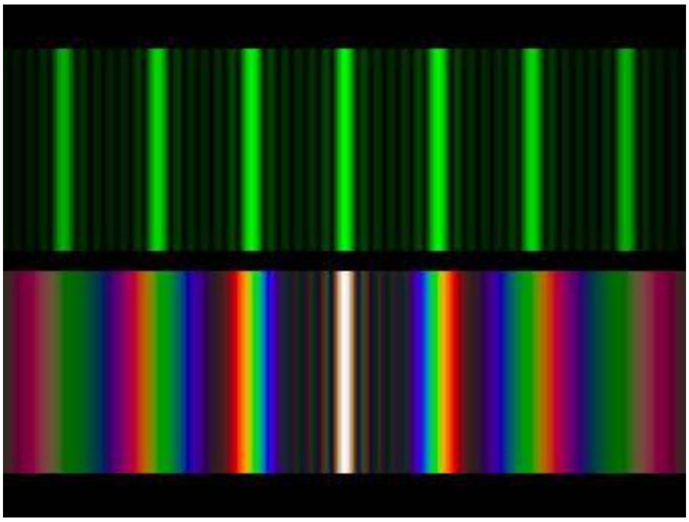


Resolvance is the measure of ability to resolve the different wavelength components. If N lines are illuminated on the grating it can be shown that the resolvance is

$$R = kN = kN_0 l = k\frac{l}{d},$$

where k is an order of a grating spectrum (maxima); l is the length of a diffraction grating; N_0 is a number of lines per unit length.

Lattices for UV-radiation have N_0 = 500 – 2500 mm^{-1}, for a visible light - N_0 = 300 - 1500 mm^{-1}, for IR-radiation - N_0 = 1 - 300 mm^{-1}.



PROBLEMS

Problem 1

Light of wavelength λ_0 = 600 nm is traveling from the air to the glass ($n = 1.5$). Find the change in the velocity, the frequency, the wavelength and the color of light.

Solution

The frequency is unchanged characteristic of light: it isn't changed at different transitions from one media to another. The frequency of given wave is

$$f = \frac{c}{\lambda} = \frac{3 \cdot 10^{8}}{6 \cdot 10^{-7}} = 5 \cdot 10^{14}\,\text{Hz}.$$

The wavelength in the glass is

$$\lambda = \frac{\lambda_0}{n} = \frac{6 \cdot 10^{-7}}{1.5} = 4 \cdot 10^{-7}\,\text{m}.$$

The velocity of the wave is given by

$v = \lambda f = 4 \cdot 10^{-7} \cdot 5 \cdot 10^{14} = 2 \cdot 10^{8}$ m/s.

The color of the light isn't changed because it is determined by the frequency of light which is constant.

Problem 2

Find the path length d_1 in the water (n_1 =1.33) that contains the number of the wavelengths which is equal to the number of the wavelengths located on the distance d_2 = 5 mm in the glass (n_2 =1.5).

Solution

The absolute refractive index is the ration of the wavelengths in the vacuum (air) λ and in the substance. Therefore, the wavelength in the glass is

$$\lambda_2 = \frac{\lambda}{n_2}.$$

The number of the wavelengths is

$$k = \frac{d_2}{\lambda_2} = \frac{d_2 n_2}{\lambda}.$$

The wavelength of the light in the water is $\lambda_1 = \dfrac{\lambda}{n_1}$.

Then the desired distance is

$$d_1 = k\lambda_1 = \frac{d_2 n_2}{\lambda} \cdot \frac{\lambda}{n_1} = \frac{d_2 n_2}{n_1} = \frac{5 \cdot 10^{-3} \cdot 1.5}{1.33} = 5.64 \cdot 10^{-3} \text{ m.}$$

Problem 3

The coherent rays with the wavelength λ = 404 nm hit the screen at the point P. What is observed at this point if the path length difference is (a) 17.17 μm: (b) 12.12 μm?

Solution

The condition for the constructive $\Delta = \pm 2k\frac{\lambda}{2}$ and destructive $\Delta = (2k+1)\frac{\lambda}{2}$ interference may be written in the general form as

$$\Delta = \pm m\frac{\lambda}{2},$$

where $m = 2k$ is an even number for constructive interference, and $m = 2k + \frac{1}{2}$ is an odd number for destructive interference.

(1) Magnitude of m for the first case is

$$m = \frac{2\Delta}{\lambda} = \frac{2\cdot 17.17\cdot 10^{-6}}{404\cdot 10^{-9}} = 85.$$

The result is an odd integer; therefore, there is the interference minimum at this point.

(b) For the second case

$$m = \frac{2\Delta}{\lambda} = \frac{2\cdot 12.12\cdot 10^{-6}}{404\cdot 10^{-9}} = 60.$$

There is the interference maximum at this point since the result is an even integer.

Problem 4

In an interference pattern from two slits, the seventh-order bright fringe is 32.1 mm from the zeroth-order bright fringe. The double slit is 5 meters away from the screen, and the two slits are 0.691 mm apart. Calculate the wavelength of the light.

Solution

The coordinate of the k-maximum is

$$x_{\max} = \pm 2k \cdot \frac{\lambda \cdot l}{2d} = \pm \frac{k \cdot \lambda \cdot l}{d}.$$

From this equation, the wavelength is

$$\lambda = \frac{d \cdot x_7}{k \cdot l} = \frac{0.691 \cdot 10^{-3} \cdot 32.1 \cdot 10^{-3}}{7 \cdot 5} = 6.34 \cdot 10^{-7} \text{ m}.$$

Problem 5

The separation between the slits in Young's experiment is d = 1.5 mm, the distance to the screen l = 2 m. Determine the distance between the interference fringes on the screen if the wavelength is λ = 670 nm.

Solution

The width of the interference fringe is

$$\Delta x = \frac{\lambda \cdot l}{n \cdot d}.$$

Taking into account that the experiment is carried out in the air, $n = 1$. Then the width is equal to

$$\Delta x = \frac{\lambda \cdot l}{d} = \frac{670 \cdot 10^{-9} \cdot 2}{1.5 \cdot 10^{-3}} = 8.9 \cdot 10^{-4} \text{ m}.$$

$$d = \frac{\lambda_{film}}{4} = \frac{4.074 \cdot 10^{-7}}{4} = 1.019 \cdot 10^{-7} \text{ m.}$$

Problem 6

In the double-slit experiment, the distance between the slits is $d = 0.15\,mm$*, 'slit-screen' separation is* $l = 120\,cm$*, the wavelength is* $\lambda = 600\,nm$*, and position of the point P is* $x = 1.44$ *cm. What is the path length difference* Δ *for the rays from two slits arriving at point P? Does point P correspond to a maximum, a minimum, or an intermediate condition?*

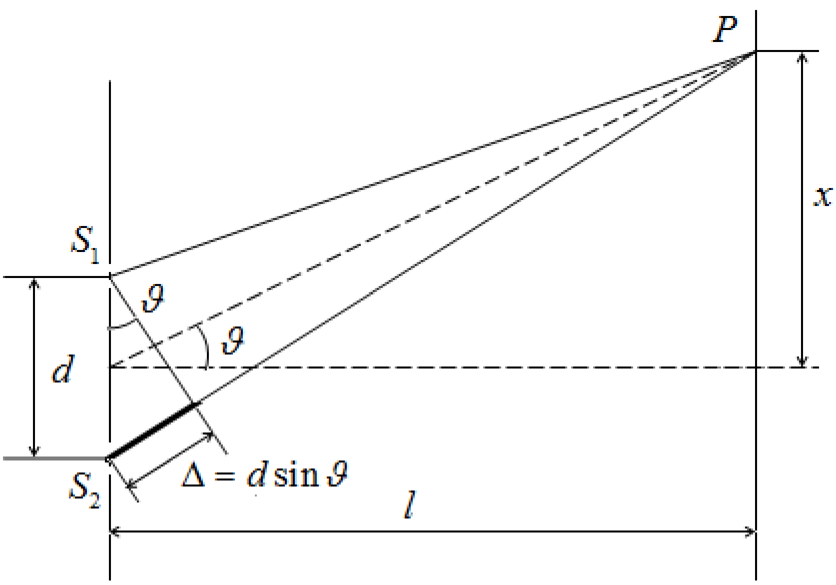
Solution

The path length difference is given by $\Delta = d\sin\vartheta$. When $l \gg x$, the angle ϑ is small and we can make the approximation $\sin\vartheta = \tan\vartheta = \frac{x}{l}$. Thus,

$$\Delta = d\sin\vartheta \approx d\tan\vartheta = \frac{d \cdot x}{l} = \frac{0.15 \cdot 10^{-3} \cdot 1.44 \cdot 10^{-2}}{1.2} = 1.8 \cdot 10^{-6} \text{m.}$$

The ratio of the path difference and the wavelength is

$$\frac{\Delta}{\lambda} = \frac{1.8 \cdot 10^{-6}}{600 \cdot 10^{-9}} = 3.$$



Since the path difference is an integer multiple of the wavelength $\Delta = 3\lambda$ or , in other words, it is equal to the even number of half-wavelengths $\Delta = 2k\dfrac{\lambda}{2}$, the intensity at the point P is the maximum.

Problem 7

In Young's experiment, monochromatic light falling on two slits 20 μm apart produces the fifth-order fringe at a 7.8° angle. What is the wavelength of the light used?

Solution

As was shown in the previous problem, the path difference can be expressed as $\Delta = d\sin\vartheta$. Taking into account that the "slits-screen" separation is much larger than the coordinates of the interference maxima ($l \gg x$), the small-angle approximation may be used: $\sin\vartheta = \tan\vartheta = \dfrac{x}{l}$. Therefore, $\Delta = d\tan\vartheta$.

For the fifth-order maximum $\Delta = 2k\dfrac{\lambda}{2} = 2\cdot 5\cdot\dfrac{\lambda}{2} = 5\lambda$. As a result,

$d\tan\vartheta = 5\lambda$.

Solving for λ, we obtain

$$\lambda = \frac{d\tan\vartheta}{5} = \frac{2\cdot 10^{-5}\cdot\tan 7.8°}{5} = 5.5\cdot 10^{-7}\ \text{m}.$$

The resulting wavelength corresponds to the wavelength of green light .

Problem 8

Natural light beam passes through two slits 0.58 mm apart in Young's experiment. How far apart are the second-order fringes for red (λ_R = 720 nm) and green (λ_G =530 nm) light beams on a screen 1.0 m away?

Solution

For constructive interference, the path difference is a multiple of the wavelength

$$\Delta = d\sin\vartheta = 2k\frac{\lambda}{2} = k\lambda \text{ , where } k = 0, 1, 2, ...,$$

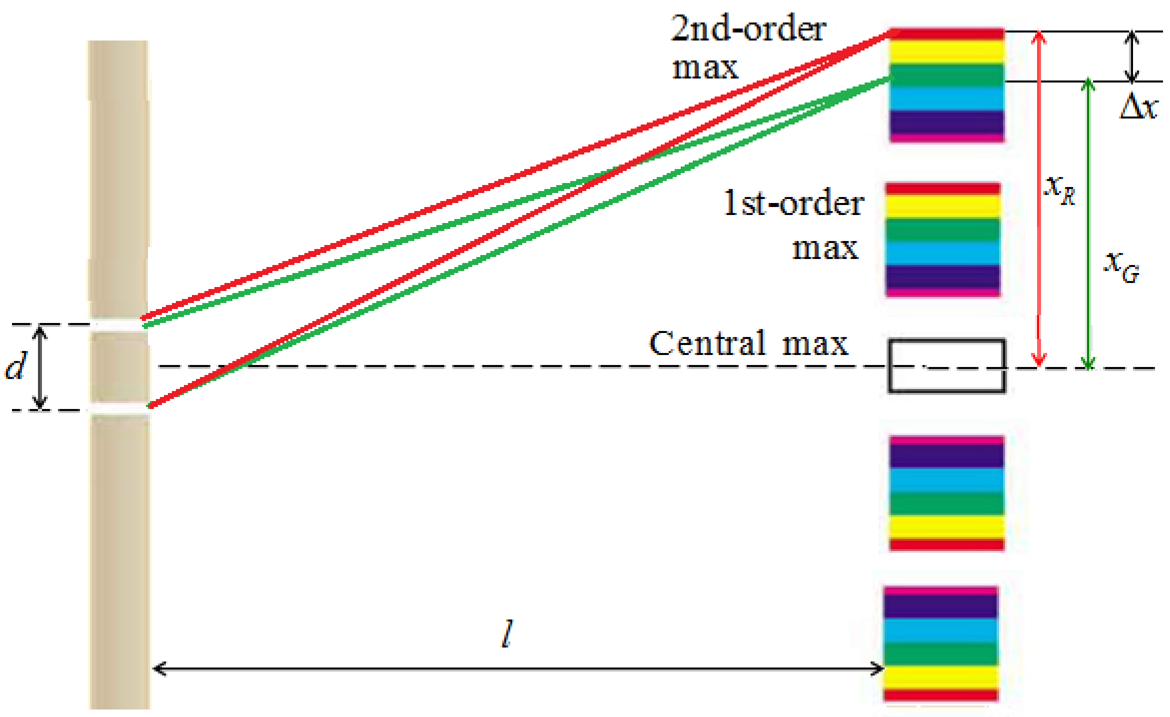
The location of the fringe the screen is $x = l\tan\vartheta$. But for small angles $\sin\vartheta = \tan\vartheta$, and

$$x = l\tan\vartheta \approx l\sin\vartheta = l\frac{k\lambda}{d}$$

For two given wavelengths, the coordinates of the second order maxima are

$$x_R = \frac{kl\lambda_R}{d} = \frac{2\cdot 1\cdot 720\cdot 10^{-9}}{0.58\cdot 10^{-3}} = 2.48\cdot 10^{-3} \text{ m},$$

$$x_G = \frac{kl\lambda_G}{d} = \frac{2\cdot 1\cdot 530\cdot 10^{-9}}{0.58\cdot 10^{-3}} = 1.83\cdot 10^{-3} \text{ m}.$$



Thus, two second-order fringes related to given wavelengths are separated by the distance

$$\Delta x = x_R - x_G = 2.48\cdot 10^{-3} - 1.83\cdot 10^{-3} = 6.5\cdot 10^{-4}\text{m}.$$

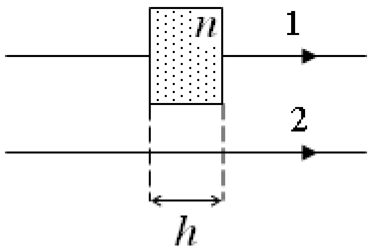
Problem 9

In the Young's experiment, thin glass plate is located on the path of one of the rays. As a result, the central bright fringe is displaced to the position which was the location of the fifth bright fringe. The beam is normal to the plate surface. The refractive index of glass is n =1.5. The wavelength λ =600 nm. Find the thickness of the glass plate.

Solution

The bright fringe is produced as a result of constructive interference

$\Delta = \pm 2k\frac{\lambda}{2} = \pm k\lambda$, where k = 0, 1, 2, …



For the fifth bright line

$\Delta_5 = k\lambda = 5 \cdot \lambda$.

The glass plate gives the addition path length difference

$\Delta = n \cdot h - h = (n-1)h$,

which determines the fifth fringe production. Therefore, from $\Delta_5 = \Delta$, we obtain $5\lambda = (n-1)h$, and

$$h = \frac{5\lambda}{n-1} = \frac{5 \cdot 600 \cdot 10^{-9}}{1.5-1} = 6 \cdot 10^{-6} \text{ m.}$$

Problem 10

White light falling on the soap (n = 1.33) film makes an angle 30° with the film surface. Find the minimum thickness of the film when the reflected light is yellow (λ = 600 nm).

Solution

If the incident beam makes an angle $30°$ with the film surface, the incidence angle will be $\alpha = 60°$ since the angle of incidence is the angle that the incident beam makes with the normal to the surface.

The path length difference at the interference in the thin films is $\Delta = 2b\sqrt{n^2 - \sin^2\alpha} \pm \frac{\lambda}{2}$.

Since the reflected beams are yellow, the constructive interference takes place; therefore, the path length difference is equal to the even number of the half-wavelengths:

$$2b\sqrt{n^2 - \sin^2\alpha} - \frac{\lambda}{2} = 2k\frac{\lambda}{2},$$

$$b = \frac{(2k+1)\lambda}{4\sqrt{n^2 - \sin^2\alpha}}.$$

The minimum thickness is at $k = 0$ $(\sin 60^0 = 0.866)$

$$b = \frac{\lambda}{4\sqrt{n^2 - \sin^2\alpha}} = \frac{600\cdot 10^{-9}}{4\sqrt{1.33^2 - 0.866^2}} = 1.48\cdot 10^{-7}\ \text{m}.$$

Problem 11

White light at normal incidence illuminates the surface of the glass (n *=1.5) film of thickness* b *= 0.4 μm. Find the wavelength in visible range (400 – 700 nm) for which the constructive interference of reflected light is realized.*

Solution

The path length difference at normal incidence with consideration of the additional path length difference connected with the reflection ($\Delta' = \pm\lambda/2$) is $\Delta = 2bn \pm \frac{\lambda}{2}$. The condition for the constructive interference is $\Delta = \pm 2k\frac{\lambda}{2}$. Then,

$$2bn - \frac{\lambda}{2} = 2k\frac{\lambda}{2},$$

$$2bn = (2k+1)\frac{\lambda}{2},$$

$\lambda = \frac{4bn}{2k+1}$, where $k = 0,\ 1,\ 2,...$

Calculation gives:

$k = 0$ $\quad\lambda = 4bn = 4 \cdot 0,4 \cdot 10^{-6} \cdot 1.5 = 2.4 \cdot 10^{-6}$ м,

$k = 1$ $\quad\lambda = \frac{4bn}{3} = \frac{2.4 \cdot 10^{-6}}{3} = 0.8 \cdot 10^{-6}$ м,

$k = 2$ $\quad\lambda = \frac{4bn}{5} = \frac{2.4 \cdot 10^{-6}}{5} = 0.48 \cdot 10^{-6}$ м,

$k = 3$ $\quad\lambda = \frac{4bn}{7} = \frac{2.4 \cdot 10^{-6}}{7} = 0.34 \cdot 10^{-6}$ м.

It is obvious that only $\lambda = 0.48 \cdot 10^{-6}$ m meets the requirements of the problem.

Problem 12

Monochromatic light with a frequency of $7.5 \cdot 10^{14}$ Hz is traveling through the air when it reaches a thin film ($n = 1.45$). The incidence angle is 40^0. Determine the minimum thickness of the film that will result in de constructive interference of the reflected light.

Solution

The wavelength of the incident light is $\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{7.5 \cdot 10^{14}} = 4 \cdot 10^{-7}$ m.

The path length difference at the interference in thin films is

$$\Delta = 2b\sqrt{n^2 - \sin^2\alpha} \pm \frac{\lambda}{2}.$$

The condition for destructive interference is $\Delta = (2k+1)\frac{\lambda}{2}$. Therefore

$$\Delta = 2b\sqrt{n^2 - \sin^2\alpha} + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2},$$

$$2b\sqrt{n^2 - \sin^2\alpha} = k\lambda,$$

$$b = \frac{k\lambda}{\sqrt{n^2 - \sin^2\alpha}}$$

The minimum thickness is at $k = 1$,

$$b_{\min} = \frac{\lambda}{\sqrt{n^2 - \sin^2\alpha}} = \frac{4\cdot 10^{-7}}{\sqrt{1.45^2 - \sin^2 40^0}} = 3.08\cdot 10^{-7}\,\text{m}.$$

Problem 13

White light is incident on a soap film (n = 1.3) in air. The reflected light looks bluish because the red light (λ = 670 nm) is absent in the reflection. What is the minimum thickness of the soap film?

Solution

At the normal incidence of the light on the thin soap film the path length difference is $\Delta = 2bn + \frac{\lambda}{2}$. This path length difference has to be multiple of the odd number of the half-wavelength:

$$2bn + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2}.$$

It gives $2bn = k\lambda$ and $b = \frac{k\lambda}{2n}$.

The minimum thickness for destructive interference realization foe the red light is at $k = 1$:

$$b_{\min} = \frac{\lambda}{2n} = \frac{670\cdot 10^{-9}}{2\cdot 1.3} = 258\cdot 10^{-9}\,\text{m}.$$

Problem 14

A picture frame manufacturer wishes to design picture frames that provide minimal glare from the glass cover. In order to achieve this, a thin plastic film (n = 1.35) is placed on the glass surface (n_1 = 1.52). If light reflected in the middle of the visible spectrum with a wavelength of 550 nm is to be minimized, what film thickness is required?

Solution

The reflection from the "air-film" interface will be 180° out of phase with the incident light. The reflection from the "film-glass" interface will also be 180° out of phase with the incident light. (Both reflections occur at a less dense to more dense boundary.) Therefore, two reflected rays would be in phase if the path difference were zero. To produce destructive interference between these two rays, and hence minimize the glare, the path difference must be $\lambda/2$. To achieve this, the film thickness must be $\lambda/2$, where λ is the wavelength of the light in the film. According to the definition, the refractive index of the plastic film is $n = \frac{\lambda_{air}}{\lambda_{film}}$. The wavelength in the film is

$$\lambda_{film} = \frac{\lambda_{air}}{n} = \frac{550 \cdot 10^{-9}}{1.35} = 4.074 \cdot 10^{-7}\,\text{m}.$$

The thickness of the film has to be $d = \frac{\lambda}{4} = \frac{4.074 \cdot 10^{-7}}{4} = 1.0185 \cdot 10^{-7}$ m.

Problem 15

In Newton's rings apparatus, the radii of the k*-th and (k + 20)-th dark rings are found to be 0.162 and 0.368 cm, respectively, when light of wavelength 546 nm is used. Calculate the radius of curvature, R, of the lower surface of the lens.*

Solution

The radius of the k-th dark ring is given by

$$r_k = \sqrt{kR\lambda}\ .$$

The radius of (k + 20)-th dark ring is

$$r_{k+20} = \sqrt{(k+20)R\lambda}\ .$$

Squaring above equations, subtracting and solving for R, the radius of curvature of the lower lens is

$$R = \frac{r_{k+20}^2 - r_k^2}{20\cdot\lambda} = \frac{\left(0.368\cdot10^{-2}\right)^2 - \left(0.162\cdot10^{-2}\right)^2}{20\cdot546\cdot10^{-9}} = 1\,\mathrm{m}.$$

Problem 16

The radius of the 10th dark ring in Newton's rings apparatus changes from 60 to 50 mm when a liquid is introduced between the lens and the plate. Calculate the refraction index of the liquid.

Solution

The radius of the dark ring when the air is in gaping is given by

$$r_k = \sqrt{kR\lambda}\ .$$

When the gap is filled by liquid, the radius is

$$r_k' = \sqrt{\frac{kR\lambda}{n}}\ .$$

$$\frac{r_k}{r_k'} = \sqrt{\frac{kR\lambda n}{kR\lambda}} = \sqrt{n}\ .$$

The refractive index is equal to

$$n = \left(\frac{r_k}{r_k'}\right)^2 = \left(\frac{60}{50}\right)^2 = 1.44\,.$$

Problem 17

The Newton's rings apparatus is illuminated by the monochromatic light which is normal to the upper surface of the plane-convex lens. The radii of the adjacent dark rings are 4 mm and 4.38 mm. The radius of curvature of the lens is R *=6.4 m. Find the numbers of the rings and the wavelength of the light.*

Solution

Let the numbers of the rings be k and $k+1$. Then their radii are

$$\begin{cases} \sqrt{kR\lambda} = r_k = 4\cdot 10^{-3}, \\ \sqrt{(k+1)R\lambda} = r_{k+1} = 4.38\cdot 10^{-3}. \end{cases}$$

$$\sqrt{\frac{k}{k+1}} = \frac{4\cdot 10^{-3}}{4,38\cdot 10^{-3}} = 0,913,$$

$$k = (k+1)\cdot 0,834.$$

$$k = 5, \qquad k+1 = 6.$$

The wavelength from the expression $r_k = \sqrt{kR\lambda}$ is

$$\lambda = \frac{r_k^2}{k\cdot R} = \frac{16\cdot 10^{-6}}{5\cdot 6.4} = 5\cdot 10^{-7}\ \text{m}.$$

Problem 18

The Newton's rings apparatus is illuminated by the monochromatic light which is normal to the upper surface of the plane-convex lens. The distance between the fifth and twenty fifth bright rings is 4.8 mm. Find the distance between the fourth and sixteenth rings.

Solution

Using the formulas for the radii of the bright $r_k = \sqrt{\dfrac{(2k-1)R\lambda}{2}}$ and dark $r_k = \sqrt{kR\lambda}$ rings, the given distance is expressed as

$$r_{25} - r_5 = \sqrt{\frac{49R\lambda}{2}} - \sqrt{\frac{9R\lambda}{2}} = \sqrt{R\lambda}\left(\frac{7-3}{\sqrt{2}}\right) = 2.83\sqrt{R\lambda}\,.$$

The sought distance is

$$r_{16} - r_4 = \sqrt{16R\lambda} - \sqrt{4R\lambda} = 2\sqrt{R\lambda}\,.$$

The ration of these two expressions is

$$\frac{r_{25} - r_5}{r_{16} - r_4} = \frac{2.83\sqrt{R\lambda}}{2\sqrt{R\lambda}}\,.$$

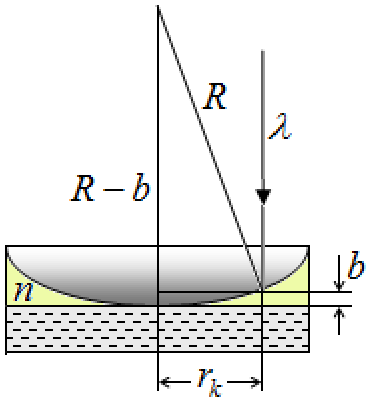
Then the distance between the fourth and sixteenth rings is

$$r_{16} - r_4 = (r_{25} - r_5)\cdot 0.707 = 4.8\cdot 10^{-3}\cdot 0.707 = 3.4\cdot 10^{-3}\ \text{m.}$$

Problem 19

The Newton's ring apparatus is illuminated by the monochromatic light (λ = 500nm). The gaping between the lens and the glass plate is filled with water. Find the thickness of the water layer at the locus of the forth bright ring.

Solution



From the drawing of the Newton's ring apparatus,

$$R^2 = (R-b)^2 + r_k^2,$$

$$R^2 = R^2 - 2Rb + b^2 + r_k^2\,.$$

Since $b \ll R$, we neglect b^2, and obtain $b = \dfrac{r_k^2}{2R}$.

The radius of k-th bright ring is

$$r_k = \sqrt{\frac{(2k-1)R\lambda}{2n}}.$$

$$b = \frac{(2k-1)\cancel{R}\lambda}{4n\cancel{R}} = \frac{(2k-1)\lambda}{4n}.$$

Substituting the given data: λ = 500 nm, $n = 1.33$ and $k = 4$, we obtain

$$b = \frac{(2\cdot 4-1)\cdot 500\cdot 10^{-9}}{4\cdot 1.33} = 6.58\cdot 10^{-7}\,\text{m}.$$

DIFFRACTION

Problem 1

A slit of width d is illuminated by the light of wavelength 550 nm. Find the slit width b when

(a) the first maximum falls at an angle of diffraction $30°$*;*

(b) the first minimum falls at an angle of diffraction $30°$.

Solution

(a) The condition of the diffraction maximum is $b\sin\psi = k\lambda$. The slit width is

$$b = \frac{k\lambda}{\sin\psi}.$$

Substituting the given data $k = 1$ $\lambda = 550$ nm = $5.5\cdot 10^{-7}$ m, and $\sin 30° = 0.5$ gives

$$b = \frac{1\cdot 5.5\cdot 10^{-7}}{0.5} = 1.1\cdot 10^{-6}\ \text{m} = 1.1\ \mu\text{m}.$$

(b) The condition of the diffraction minimum is $b\sin\psi = (2k+1)\dfrac{\lambda}{2}$. The slit width is

$$b = \frac{(2k+1)\lambda}{2 \cdot \sin\psi}.$$

After substituting the given data $k = 0$ $\lambda = 550\,\text{nm}$ = $5.5 \cdot 10^{-7}\,\text{m}$, and $\sin 30° = 0.5$ we obtain

$$b = \frac{1 \cdot 5.5 \cdot 10^{-7}}{2 \cdot 0.5} = 5.5 \cdot 10^{-7}\ \text{m} = 0.55\ \mu\text{m}.$$

Problem 2

Light of the wavelength λ = 589 nm falls on the slit of width b = 2.25 μm. Find the angular deviations for the diffraction minimum. Determine the number of minima that this slit gives. How many minima does this slit give?

Solution

The condition for diffraction minimum is $b\sin\psi = \pm 2k\frac{\lambda}{2}$, so the number of maximum is

$$k = \pm\frac{b\sin\psi}{\lambda}.$$

Since $\sin\psi \le 1$,

$$k_{\max} \le \frac{b}{\lambda} = \frac{2.25 \cdot 10^{-6}}{589 \cdot 10^{-9}} = 3.83.$$

As k is the number of the diffraction minimum, it has to be integer, therefore, $k_{\max}$ = 3. Note! We didn't round the obtained result 3.83 according to the mathematical rules. If we round it to 4, we would obtain $\sin\psi$ greater than 1. But it is nonsense. Therefore, we omitted the fractional part of 3.83 and obtained the result k = 3.

The angles that are related to the diffraction minima may be found according to

$$\sin\psi = \frac{k\lambda}{b}.$$

$$k=1 \qquad \sin\psi_1=\frac{1\cdot 589\cdot 10^{-9}}{2\cdot 10^{-6}}=0.295, \quad \psi_1=17.1^\circ;$$

$$k=2 \qquad \sin\psi_2=\frac{2\cdot 589\cdot 10^{-9}}{2\cdot 10^{-6}}=0.59, \quad \psi_2=36.2^\circ;$$

$$k=3 \qquad \sin\psi_3=\frac{3\cdot 589\cdot 10^{-9}}{2\cdot 10^{-6}}=0.885, \quad \psi_3=62.3^\circ.$$

This slit gives six diffraction minima; three minima on the each side of the central maximum, i.e., $K=2k_{\max}=2\cdot 3=6$.

Problem 3

The monochromatic light of wavelength 600 nm illuminates the slit of width $b=$ 30 μm. There is the convex lens behind the slit and the observation screen in its focal plane. What is observed on the screen at the diffraction angles that are (a) 4.59° and (b) 6.32°?

Solution

The conditions for diffraction maximum $b\sin\psi=\pm(2k+1)\dfrac{\lambda}{2}$ and minimum $b\sin\psi=\pm 2k\dfrac{\lambda}{2}$ may be written in the general form as

$$b\sin\psi=\pm m\frac{\lambda}{2},$$

where m is an odd number for maximum and an even number for minimum, relatively.

Then,

$$m=\frac{2b\sin\psi}{\lambda}.$$

(a) For the first given diffraction angle,

$$m_1=\frac{2b\cdot\sin\psi_1}{\lambda}=\frac{2\cdot 30\cdot 10^{-6}\sin 4.59^\circ}{600\cdot 10^{-9}}=8.$$

The result is an even integer, therefore, $m_1 = 2k = 8$, $k = 4$. Thus, there is the 4th diffraction minimum.

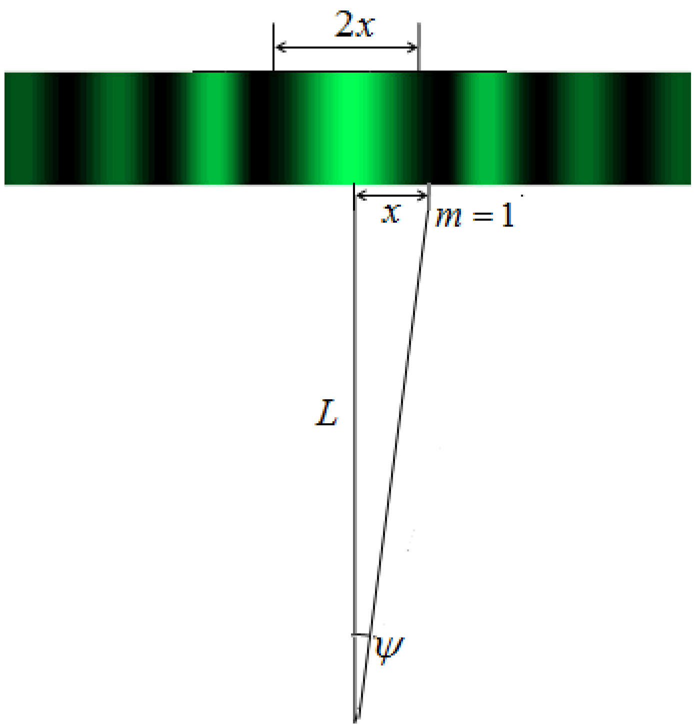
(b) For the second diffraction angle,

$$m_2 = \frac{2b\sin\psi_2}{\lambda} = \frac{2\cdot 30\cdot 10^{-6}\sin 6.32°}{600\cdot 10^{-9}} = 11.$$

The result is an odd integer, therefore, $m_2 = 2k+1 = 11$, $k = 5$. This is the 5th diffraction maximum.

Problem 4

Light of wavelength 500 nm falls from a distant source on a slit 0.5 mm wide. Find the distance between the dark bands on either side of the central band of diffraction pattern (the central maximum width) observed on screen placed 2 m from the slit.



Solution

As it is seen from the figure, the desired distance $2x$ may be calculated from the relationship

$$\tan\psi = \frac{x}{L}.$$

$$x = L\cdot\tan\psi.$$

Using the condition for diffraction minimum $b\sin\psi = m\lambda$, we can determine the sine of the diffraction angle, and, therefore, the angle ψ. It should be noted that in the most cases, the angle of diffraction is so small that $\sin\psi \approx \tan\psi$, especially, for the maxima and minima of the first orders. So,

$$\tan\psi = \sin\psi = \frac{m\lambda}{b},$$

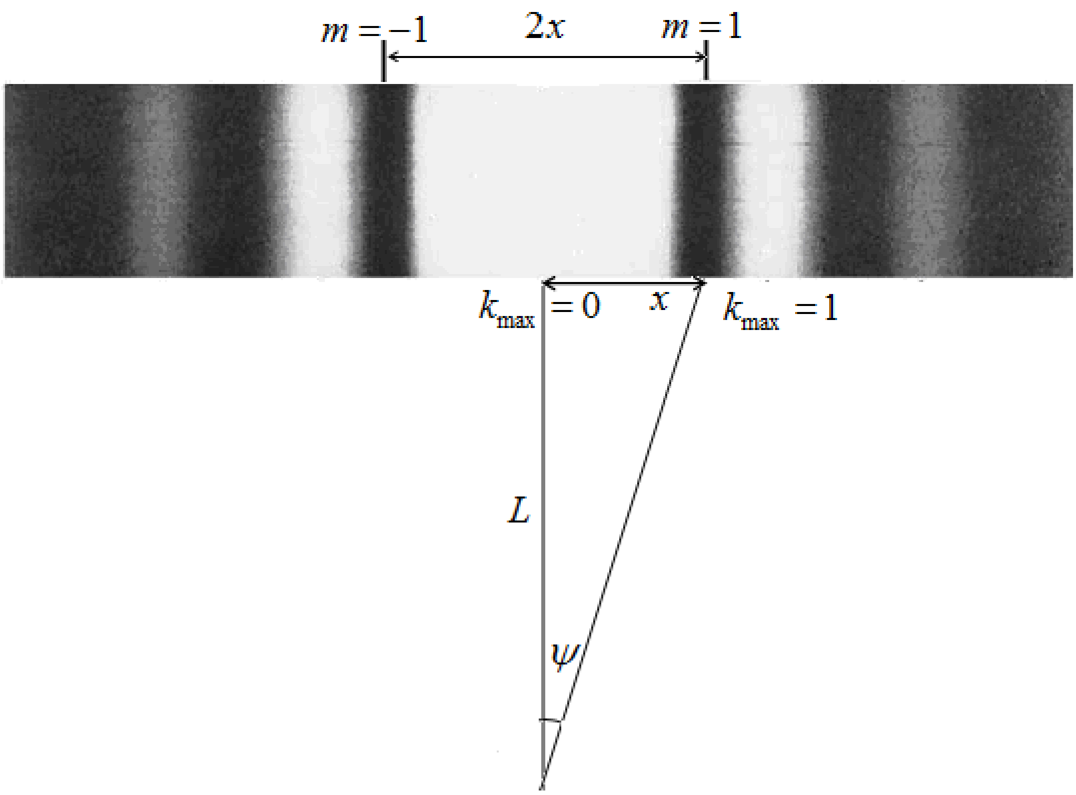
$$2x = 2L \cdot \tan\psi = 2L \cdot \frac{m\lambda}{b} = \frac{2 \cdot 2 \cdot 1 \cdot 500 \cdot 10^{-9}}{0.5 \cdot 10^{-3}} = 4 \cdot 10^{-3}\,\text{m} = 4\text{ mm}.$$

Problem 5

A diffraction pattern using blue light with a wavelength of λ = 438 nm creates a central maximum that is $2x$ = 6.50 cm wide. If the slit width used is b =17.5 µm, how far away is the screen?

Solution

The width of the maximum is equal to the separation between the adjacent minima, therefore, in this case, the distance between the first minima on the both



sides of the central maximum.

The condition for diffraction minimum is $b\sin\psi = m\lambda$.

The sine of the first angle of diffraction is

$$\sin\psi = \frac{m\lambda}{b} = \frac{\lambda}{b} = \frac{438\cdot 10^{-9}}{17.5\cdot 10^{-6}} = 0.025\,.$$

$$\psi = \arcsin 0.025 = 1.43°.$$

On the other side, the tangent of the diffraction angle can be expressed using the "slit-screen" separation L and the distance from the center of the diffraction pattern to the first order minimum x. As a result, the required distance is

$$L = \frac{x}{\tan\psi} = \frac{0.065}{2\cdot\tan 1.43°} = 1.3\text{m}.$$

Problem 6

A coherent beam of light from a hydrogen discharge tube falls normally on a diffraction grating of 8000 lines per centimeter. Calculate the angular deviation of each line in the first-order spectrum. Do any lines of the second-order spectrum overlap the first-order spectrum? (For the hydrogen discharge tube, λ_{red} = 656.3 nm; $\lambda_{blue-green}$ = 486.1 nm, λ_{blue} = 434.0 nm, λ_{violet} = 410.1 nm.)

Solution

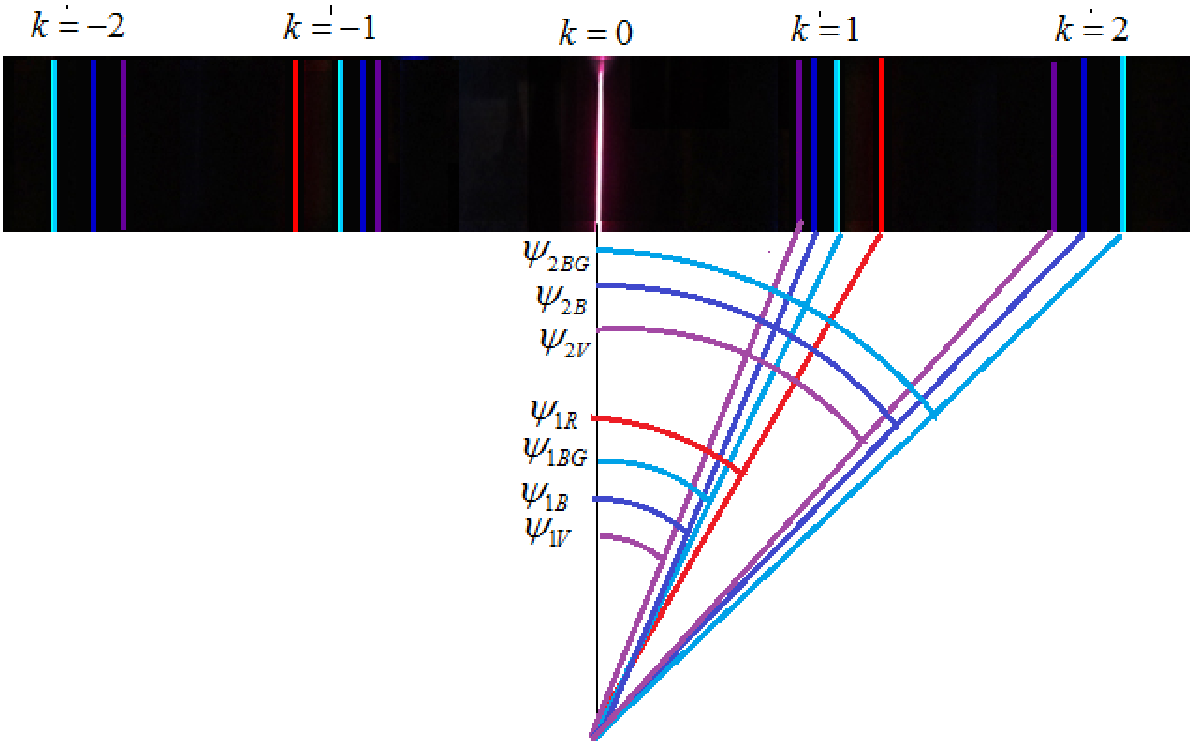
Since the grating has $N_0 = 8000$ lines per centimeter, the grating spacing is given by:

$$d = \frac{1}{N_0} = \frac{1}{8000}\text{cm} = 1.25\cdot 10^{6}\,\text{m}.$$

We can calculate the deviation of each component in turn by using the diffraction grating equation for bright fringes:

$d\sin\psi = k\lambda$, where $k = 0, 1, 2, ...$

It is necessary to calculate the angles of the first order fringes and the second order fringes for the given lights and to compare them to determine whether they have overlap. The general idea is to compare the biggest angle $\psi_{\max 1}$ of the first-order fringes and the smallest angle $\psi_{\min 2}$ of second-order fringes. If $\psi_{\max 1} > \psi_{\min 2}$, the overlap happens. Otherwise, no overlap occurs.



(1) The first-order maximum:

Violet line: $$\sin\psi_{1V} = \frac{k\lambda}{d} = \frac{1\cdot 410\cdot 10^{-9}}{1.25\cdot 10^{-6}} = 0.328$$

$\psi_{1V} = \arcsin 0.328 = 19.14°$.

Blue line: $$\sin\psi_{1B} = \frac{k\lambda}{d} = \frac{1\cdot 434\cdot 10^{-9}}{1.25\cdot 10^{-6}} = 0.347,$$

$\psi_{1B} = \arcsin 0.347 = 20.31°$

Blue-green line: $$\sin\psi_{1BG} = \frac{k\lambda}{d} = \frac{1\cdot 486.1\cdot 10^{-9}}{1.25\cdot 10^{-6}} = 0.389,$$

$\psi_{1BG} = \arcsin 0.389 = 22.88°$

Red line: $\sin\psi_{1R} = \dfrac{k\lambda}{d} = \dfrac{1\cdot 656.3\cdot 10^{-9}}{1.25\cdot 10^{-6}} = 0.525$

$\psi_{1R} = \arcsin 0.525 = 31.67°$

(b) The second-order maximum

Violet line: $\sin\psi_{2V} = \dfrac{k\lambda}{d} = \dfrac{2\cdot 410\cdot 10^{-9}}{1.25\cdot 10^{-6}} = 0.656$

$\psi_{2V} = \arcsin 0.656 = 41°$.

Blue line: $\sin\psi_{2B} = \dfrac{k\lambda}{d} = \dfrac{2\cdot 434\cdot 10^{-9}}{1.25\cdot 10^{-6}} = 0.694$,

$\psi_{2B} = \arcsin 0.694 = 87.9°$.

Blue-green line: $\sin\psi_{1BG} = \dfrac{k\lambda}{d} = \dfrac{2\cdot 486.1\cdot 10^{-9}}{1.25\cdot 10^{-6}} = 0.778$,

$\psi_{2BG} = \arcsin 0.778 = 51.1°$.

Red line gives $\sin\psi_1 = \dfrac{k\lambda}{d} = \dfrac{2\cdot 656.3\cdot 10^{-9}}{1.25\cdot 10^{-6}} > 1$. This is the impossible result. So the red line is absent in the second-order spectrum produced by this diffraction grating.

To determine whether the second-order spectrum for violet line overlaps the first-order spectrum, we need to compare the angles corresponding to red line in the 1st order spectrum and the violet line in the 2nd order spectrum, namely, $\psi_{2V} = 41°$ and $\psi_{1R} = 31.67°$. Since $31.67° < 41°$, overlapping of the spectra is not observed.

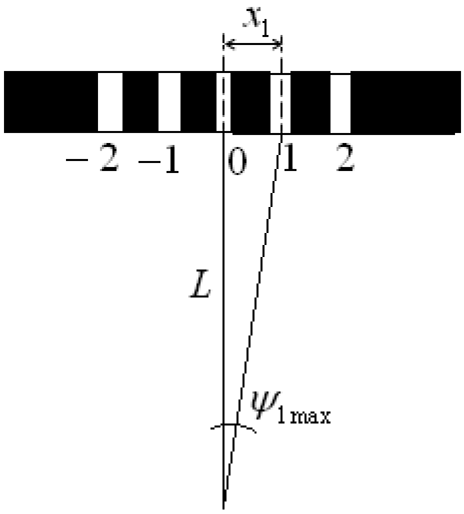
Problem 7

The monochromic light with the wavelength λ =600 nm falls on the diffraction grating. The distance between the zeroth and the first maxima on the screen separated by distance L = 2.5 m from the grating, is Δx =4 cm. Find: (a) the grating constant; (b) maximum diffraction angle; (c) the total number of maxima which this grating gives.

Solution

The condition for diffraction maximum is $d\sin\psi = \pm k\lambda$, $k = 0, 1, 2, \dots$. For the first $(k = 1)$ maximum:

$d\sin\psi_1 = \lambda$.



On other hand, $\tan\psi_1 = \dfrac{x_1}{L}$,

$$\psi_1 = \arctan\frac{x_1}{L} = \arctan\frac{4\cdot 10^{-2}}{2.5} = \arctan 0.016.$$

$\psi_1 = 0.9°$.

The grating constant is

$$d = \frac{\lambda}{\sin\psi_1} = 3.75\cdot 10^{-5}\text{m}.$$

The highest order of diffraction spectrum can be calculated, assuming that the maximum magnitude of $\sin\psi = 1$, Then,

$$k_{\max} = \frac{d\cdot(\sin\psi)_{\max}}{\lambda} = \frac{d}{\lambda} = \frac{3.75\cdot 10^{-5}}{600\cdot 10^{-9}} = 62.5.$$

The resulting value has to be rounded to an integer, since k is the number of the diffraction spectrum. It should be remembered that the value of the sine should not be greater than 1, therefore, we remove the fractional portion. As a result, the highest number of diffraction spectrum is $k_{\max} = 62$.The diffraction angle for this spectrum can be calculated as

$$\sin\psi_{\max} = \frac{k_{\max}\cdot\lambda}{d} = \frac{62\cdot 600\cdot 10^{-9}}{3.75\cdot 10^{-5}} = 0.992,$$

$\psi_{\max} = \arcsin 0.992 = 82.75°$

The total number of diffraction maxima is 62 at each side and one central maximum:

$K = 2k_{\max} + 1 = 2\cdot 62 + 1 = 125.$

Problem 8

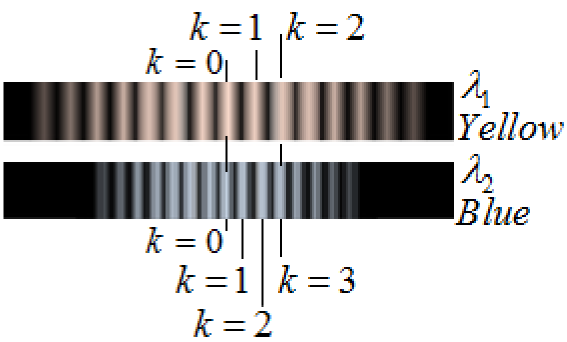
A parallel beam of white light is shines normally on a diffraction grating with 6500 lines per 1 cm. Assuming the wavelengths of yellow and blue light in air are λ_1 = 600 nm and λ_2 = 400 nm, respectively, show that there is overlapping between yellow and blue spectra. Find the orders of the overlapping spectra.

Solution

The conditions for diffraction maxima for the yellow and blue lights in the case of their overlapping are

$$\begin{cases} d \cdot \sin\psi = k_1\lambda_1, \\ d \cdot \sin\psi = k_2\lambda_2. \end{cases}$$

Since the left sides of the expressions are the same, $k_1\lambda_1 = k_2\lambda_2$. The resulting equation contains two unknown quantities and can not be solved from a formal point of view. But, if we recall that the k_1 and k_2 are the numbers of the diffraction spectra, and hence the integers, then we obtain



$$\frac{k_1}{k_2} = \frac{\lambda_2}{\lambda_1} = \frac{400}{600} = \frac{2}{3}.$$

The obtained result indicates that the yellow line in the 2nd-order diffraction spectrum is overlapped by the blue line of the 3rd-order spectrum

Problem 9

Diffraction grating with 100 lines per 1 mm is at the distance 2 m from the screen. It is illuminated by white light that strikes normally to the grating. Find the *width of the first order diffraction maximum if the visible light refer to the wavelength range between λ_V = 380 nm (violet) and λ_R =760 nm (red).*

Solution

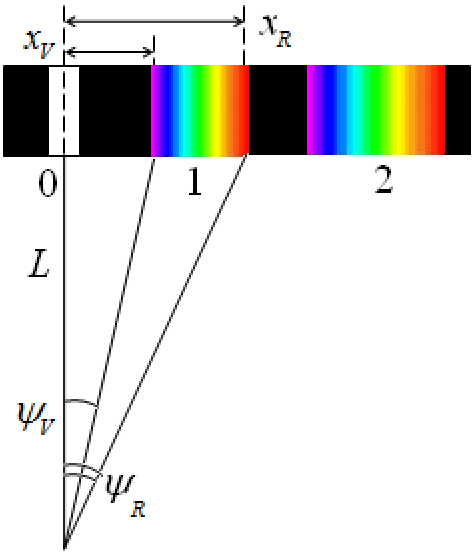
If the number of lines per unit length N_0 is 100 lines per 1 mm (or 100000 lines per 1 meter), the grating spacing can be found as

$$d = \frac{1}{N_0} = \frac{1}{10^5} = 10^{-5}\,\text{m}.$$

The positions of the diffraction maxima (excluding the central maximum) according to $d\sin\psi = \pm k\lambda$ depend on the wavelength λ. That is why the diffraction grating is the spectroscopic instrument.

The conditions for diffraction maxima for two wavelengths and the equations based on the geometric relationship of the experimental parameters are

$$\begin{cases} d\cdot\sin\psi_V = 1\cdot\lambda_V, \\ d\cdot\sin\psi_R = 1\cdot\lambda_R, \\ x_V = L\cdot\tan\psi_V, \\ x_R = L\cdot\tan\psi_R. \end{cases}$$



Two diffraction angles expressed from the first two equations are

$$\sin\psi_V = \frac{\lambda_V}{d} = \frac{380\cdot10^{-9}}{10^{-5}} = 0.038,$$

$$\sin\psi_R = \frac{\lambda_R}{d} = \frac{760\cdot10^{-9}}{10^{-5}} = 0.076.$$

Since the angles for the first orders of diffraction are very small, $\tan\psi_V \approx \sin\psi_V$ and $\tan\psi_R \approx \sin\psi_R$. Therefore,

$$x_V = L\cdot\tan\psi_V \approx L\cdot\sin\psi_V,$$

$$x_R = L\cdot\tan\psi_R \approx L\sin\psi_r.$$

Finally, the sought width of the first maximum is

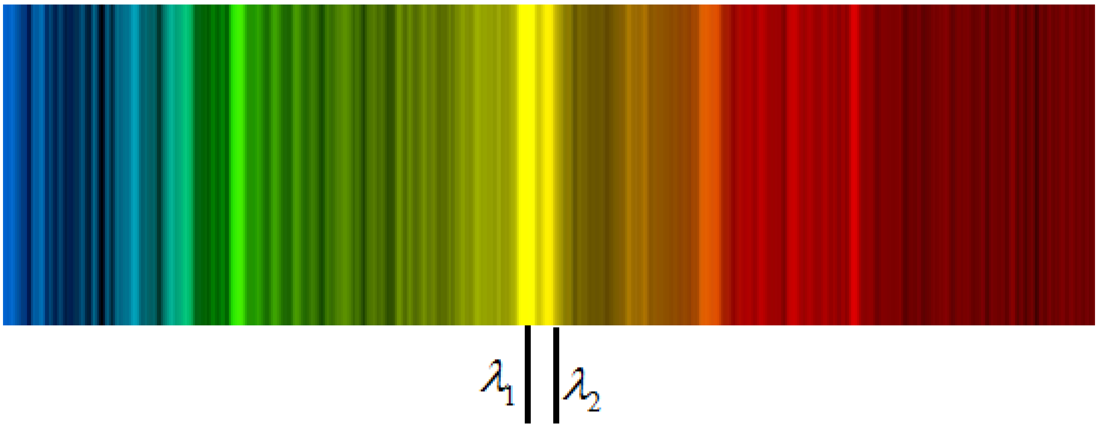
$$\Delta x = x_R - x_V = L\cdot\tan\psi_R - L\cdot\tan\psi_V \approx L(\sin\psi_R - \sin\psi_V),$$

$$\Delta x = (0.076 - 0.036) = 0.076 \text{ m.}$$

Problem 10

Find the resolvance of the diffraction grating if it must have to resolve the components of Sodium doublet, i.e. two closed lines of Na with the wavelengths λ_1 *= 589.0 nm and* λ_2 *= 589.6 nm. Calculate the number of lines of this grating that it can resolve these lines in the third order spectrum.*

Solution



The resolving power (resolvance) of diffraction grating is a dimensionless measure of ability to separate adjacent spectral lines, or the difference in wavelength between two lines (λ_1 and λ_2) of equal intensity that can be distinguished in a given order of their spectrum λ.

$$R = \frac{\lambda}{\Delta\lambda}.$$

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{589 + 589.6}{2} = 589.3 \text{ nm,}$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = 589.6 - 589 = 0.6 \text{ nm.}$$

Resolvance is

$$R = \frac{\lambda}{\Delta\lambda} = \frac{589{,}3}{0{,}6} = 982.$$

$$R = kN,$$

where k is the number of the diffraction spectrum in which the lines are observed separately, and N is the total number of lines of the grating.

Substituting $k = 3$ and $R = 982$ gives the sought number of lines

$$N = \frac{R}{k} = \frac{982}{3} = 328 \text{ lines.}$$

Problem 11

Diffraction pattern is obtained by means of the diffraction grating of the length l = 0.5 cm with N_0 =100 mm^{-1}. Find the order of the spectrum in which two lines with the wavelengths λ_1 = 578nm and λ_2 = 580 nm can be clearly resolved.

Solution

The resolvance of the diffraction grating is

$$R = \frac{\lambda}{\Delta\lambda} = kN_0 l \, .$$

The average wavelength is equal to

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{578 + 580}{2} = 579 \text{ nm.}$$

The wavelength difference is

$$\Delta\lambda = \lambda_2 - \lambda_1 = 580 - 578 = 0.2 \text{ nm.}$$

The order of the spectrum is determined as

$$k = \frac{\lambda}{\Delta\lambda \cdot N_0 \cdot l} = \frac{579 \cdot 10^{-9}}{0.2 \cdot 10^{-9} \cdot 10^5 \cdot 0.005} = 5.8 \approx 6 \, .$$

Note! We obtained the result as a broken number. The number of spectrum has to be integer; therefore, it is necessary to round the obtained result. Remember that we have to round it to the greater integer regardless the mathematical rule: 5.2 and 5.8 are to round to 6, because the lines couldn’t be resolved in the 5th order spectrum. It is possible in the spectra of $k \geq 6$ as the width of the spectrum increases with its number.

Now it is necessary to check if this grating allows obtaining the 6th spectrum. Since $k = \dfrac{d \cdot \sin\psi}{\lambda}$, the largest spectrum number can be calculated assuming $(\sin\psi)_{\max} = 1$ and substituting $\lambda = \lambda_2 > \lambda_1$.

$$k_{\max} = \frac{d \cdot (\sin\psi)_{\max}}{\lambda_2} = \frac{1}{N_0 \lambda_2} = \frac{1}{10^5 \cdot 580 \cdot 10^{-9}} = 17.24 \approx 17 \; .$$

The maximum spectrum number is $k_{\max} = 17$, therefore, it is possible to observe the given lines in the spectra of $k \geq 6$.

Problem 12

A diffraction grating 4 cm wide produces a deviation of 30 degrees in the second order spectrum with light of wavelength 660 nm. What is the total number of lines on the grating?

Solution

The total number of lines depends on the number of lines per unit length N_0 and the width of the diffraction grating as $N = N_0 \cdot l$.

In turn, the grating spacing is $d = \dfrac{1}{N_0}$.

From the condition for diffraction maximum $d \sin\psi = k\lambda$, the grating spacing is

$$d = \frac{2\lambda}{\sin\psi} = \frac{2 \cdot 660 \cdot 10^{-9}}{\sin 30°} = 2.64 \cdot 10^{-6} \text{ m.}$$

The total number of lines is

$$N = N_0 \cdot l = \frac{l}{d} = \frac{4 \cdot 10^{-2}}{2.64 \cdot 10^{-6}} = 15152 \text{ lines.}$$