Contact interaction of a ball with a toroidal running track with a closely shaped power law profile

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Abstract. The influence of surface shape of contacting bodies on the distribution of contact pressure between them and the stress-strain state of these bodies is studied in the paper. The case study covers contact interaction between a ball piston and a running track of a radial hydrovolumetric transmission. The elastostatics problem is reduced to the contact of bodies of revolution: a sphere and a toroid. The profile of the axial section of the toroid is given in the form of a power function defined by two variable parameters. These parameters determine the distribution of contact pressure between these bodies, as well as its magnitude. The regularities of this influence have been established. The dependency of the maximum value of the contact pressure and the equivalent von Mises stresses on varied parameters are established. Constructed parametric models of contact interaction make it possible to perform multivariate calculations of the stress-strain state of contacting bodies. The analysis of the obtained results makes it possible to build a specialized research database. In turn, this database is the basis for the justification of project decisions when creating progressive structures, which include complex profile details. Accordingly, rational profiles of contacting bodies are determined according to the strength criterion.

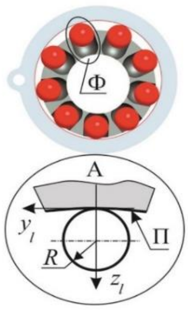
Keywords: Contact Interaction, Hydrovolumetric Transmission, Ball Pistons, Surface Shape, Contact Pressure, Stress-Strain State.

1 Introduction

The radial hydrovolumetric transmissions (HVT) can be designed to be compact, powerful and durable and thus are suitable as an integral part of vehicle powertrains. In particular, HVT GOP-900 shown in Fig. 1 is a promising solution offered by Kharkiv Morozov Machine Building Design Bureau [1]. The power drives of this type are considered for agricultural and military vehicles, since they provide continuously variable transition ratio. This is beneficial for the smoothness of operation for the former and agility for the latter. Wind power plans is another potential application of compact hydrostatic transmissions.

The initial development of the hydraulic machines of this design was hindered by the leakage of the hydraulic liquid in the gap between the ball piston and the cylinder walls [2]. This problem was successfully solved. Nowadays there is a renewed interest towards this design as it offers a solution combining unprecedented power density ratio with small dimensions and lightweight construction. New ways to improve the performance of the hydrostatic transition keep being discovered. For instance, tilted axis solution was recently suggested in order to reduce the lateral force between the ball pistons and the cylinder [3].

One of the bottlenecks of this design is the transmission of force between the ball pistons and the stator ring. The pistons perform translational motion along the rotor cylinders (zone Φ) either pumping hydraulic liquid in the hydraulic drive or force rotation of the rotor in the motor side of the transmission [2]. In both cases the active traction is transmitted via mechanical contact interaction between the ball piston and the running track (see Fig. 1). The two bodies have initial pointwise contact (see zone A in Fig. 1) in the unloaded state. This means that the contact zone is highly localized and the high contact pressure causes stress concentration under the working conditions. The magnitude of the contact pressure and the equivalent stresses in both bodies determine the strength of the HVT as a whole. That is, what is sought is such a profile of the axial section Π of the running track, which provides the lowest possible level of contact pressure and stresses.



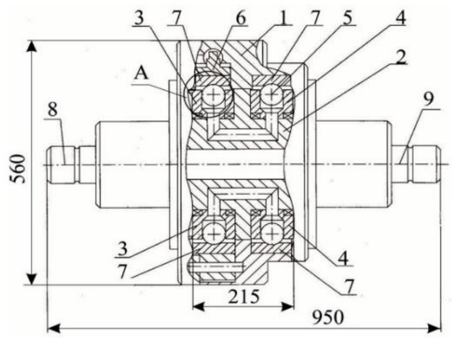


Fig. 1. Contact interaction of ball pistons with the stator ring Φ shown in the plane A of the running track profile Π: 1 – housing; 2 – block of pin distributors; 3 – pump cylinder block (rotor); 4 – cylinder block of the hydraulic motor (rotor); 5 – ball-piston; 6 – pump stator; 7 – a running track Π on the pump and the hydraulic motor; 8 and 9 – input and output shafts of HVT

The possible geometry of the running track is limited by the design of the transmission. Its depth cannot be increased beyond certain limit without part interference. The rational choice of the profile requires the analysis of multiple variants. The paper delivers appropriate model, methods and tools that can be used to obtain an engineering solution for an important component developed by domestic industry.

2 Literature Review

Contact interaction of bodies along the surfaces of complex shape is found in many structures. The shape of the surfaces is often very close to each other. In particular, this applies to elements of technological systems, engines, suspension systems, hydraulic transmissions, mechanical drives, gears, etc. The contact between such conjugated bodies is realized by close or nearly coincident surfaces. At the same time, the use of traditional Hertz-type models [4] or other closed-form solutions [5] is limited due to the closeness of the dimensions of the contact area to the dimensions of the bodies themselves. On the other hand, the contact model with coincident (congruent) surfaces cannot be used, because small changes in the distribution of the initial gap can in this case lead to significant changes in contact areas and contact pressure. Such sensitivity is typical for nonlinear problems including the contact problems. If we apply an approach based on the use of correction coefficients that take into account the influence of the unevenness of the distribution of the initial gap between the bodies, then, despite the quantitative errors, the qualitative pictures and trends that are inherent in the distribution of contact pressure and the stress-strain state (SSS) of the interacting bodies will inevitably be distorted. It is also valid to note that the direct numerical formulation of the problem using the finite element method (FEM) [4] from the very beginning introduces an error due to the fact that the geometric shape of the contacting surfaces of the bodies is approximated by finite elements. The error introduced by this approximation is comparable to the true disturbance of their geometric shape. At the same time, the needs of practice require the implementation of adequate, operational and reliable analysis of the contact interaction of bodies along surfaces of close shape. No universal models and methods that would account for the disturbances in the shape of the contacting surfaces of various origins of (errors in manufacturing, assembly, displacement during operation, wear, etc.) for the contact interaction are currently available.

The existing formulations of contact problems [6] are evaluated with regard to their application for the problems outlined in the current study. In particular, three aspects come into consideration. The first aspect concerns general variational formulations, which have significant potential for taking into account a variety of factors. It is precisely this property that is necessary for the class of problems considered in this paper. For example, the apparatus of the theory of variational inequalities [7] enables the reduction of the contact problem to the problem of minimizing some energy functional [8]. This formulation can be easily augmented to account for new factors and physical features by mere introduction of additional additive terms to this functional [9]. For the formulations based on Kalker's variational principle [10] the minimized functional is the complementary energy. This leads to effective numerical treatment of problems with contact [11]. Discrete pressure elements are developed to three dimensional contact problems [12] that also include friction. The formulation of the normal contact problem can be expanded to include variation in the body geometry or the properties of contact layers [13]. The second aspect concerns specifically new physical factors that should be taken into account in the case of contact interaction of closely shaped bodies. This is, for example, the question of the influence of the shape of the contacting surfaces [14], adhesion [15], roughness [16] and other factors on the contact interaction.

The microasperities model is applied to identify the impact of the random geometry properties on the adhesion between surfaces[17]. Periodically textured bodies are studied with respect to their relative slip in contact [18]. The scaling of the contact stiffness with the roughness is established in [19]. The effect of fine scale and coarse scale roughness on adhesion is studied in [20]. The problem of adhesive contact of a rigid body with a thin elastic coating is solved in [21]. The instabilities in adhesive contact of rough bodies and the value of the pull-off force are evaluated in [22]. Instability of the contact area in adhesive contact due to the shear force is characterized in [23]. Axisymmetric thermoelastic contact is analyzed in [24]. A multilayered model [25] and Pasternak foundation model [26] are offered for the contact problems of functionally graded bodies. The effect of the Poisson's ratio on the adhesive pull-off force is studied for a stiff sphere with non-symmetric perturbation of the contact geometry in [27]. Viscoelastic dissipation for a repeated indentation of a paraboloid is derived in [28]. Pull-off dynamics is studied for adhesive contact of viscoelastic solids in [29]. Thermomechanical slip and the tangential contact tractions between a half-space and a sphere are analyzed in [30].

The problem of substantiating adequate models of the SSS taking into account the contact interaction on close or almost coincident (congruent) surfaces has not been fully resolved. The third aspect is the approximation of the given distributions and sought fields when solving contact problems by numerical methods. First of all, we are talking about methods of finite [31] and boundary elements [32]. Here it is worth noting that it is the variational formulations of contact problems that lead to the construction of adequate and correct numerical models.

In summary, it can be noted that various models and methods of solving the problems of contact interaction of close-shaped bodies have been developed so far. At the same time, there is a problem of creating numerical solutions of these problems based on them. The approach described in [13] look as promising solution to the outlined issues. They meet the listed above requirements such as: 1) variational formulation; 2) incorporation of various physical factors; 3) numerical implementation in the form of finite and boundary elements models.

Furthermore all these aspects can be combined with a parametric description of the models of the studied objects. This creates advantages when solving this class of problems. That is why this approach was adapted in this paper to the analysis of the elements of radial HVT providing parametric solution of the problem of contact interaction of bodies of complex shape.

3 Research Methodology

A variational approach is applied to the analysis of contact interaction of bodies of complex shape. In the general form the elastostatic equilibrium is determined by the mininimizations problem. It consists in the minimization of functionals

$$J_1\,(u,p,f)\ \rightarrow \min \text{ on } K_1\colon l(u)\geq 0 \ \text{ or } \ J_2\,(q,p,f)\ \rightarrow \min \text{ on } K_2\colon q\geq 0. \qquad (1)$$

Here J_1, J_2 are the functionals of total energy or total complementary energy correspondingly; u is the displacement filed defined in the domains $\Omega_1,\ \Omega_2,$ occupied by the deformable solids in contact; q is the contact pressure distribution defined on the matching surfaces $S_1,\ S_2$, over which the contact may occur; p is the array of parameters controlling various features of the problem (geometry, physical properties, etc.) that present the subject of interest; f are the external loading; $K_1,\ K_2$ are the convex sets containing either displacements or the contact pressure constrained by the inequality conditions of the unilateral normal contact; l is the linear algebraic operator mapping the potential contact surfaces through which the impenetration conditions of contact are defined. Functionals $J_1,\ J_2$ can be augmented with additional terms that introduce new factors to the system. The quadratic terms need to be added in order to account for the local deformations of the intermediate surface layers (for example, roughness, films or coatings)

$$J_1 := J_1 + \frac{1}{2}\int cw^2 dS;\ J_2 := J_2 + \frac{1}{2}\int \lambda q^2 dS, \tag{2}$$

with c, λ being the contact stiffness and compliance accordingly, while $w = w(u)$ expressing the relative penetration of the two bodies or else the compression of the contact layer. The integration is performed over the entire surface of possible contact. Note, that this formulation allows for the positive penetration unlike the case of normal contact of smooth bodies without intermediate layer. Thus the corresponding inequality constraint is relaxed. The contact pressure though is still limited to nonnegative values only. The following linear constitutive relations for the contact pressure and penetration come out of the quadratic energy terms (2)

$$q = c \cdot w,\ \ w = \lambda \cdot q. \tag{3}$$

The approximation of the unknown fields and discretization of the minimization problem is performed by MFBE. In the linear elastic case a quadratic programming problem with linear inequality constraints is obtained. It can be solved by various optimization procedures. The resulting solution may be represented as

$$u = u(\mathbf{r}, p);\quad q = q(\boldsymbol{\rho}, p) \tag{4}$$

where $\mathbf{r}, \boldsymbol{\rho}$ are the coordinates vectors of discretization points in the solid domains $\Omega_1,\ \Omega_2$, or the contact surfaces $\mathrm{S}_1,\ \mathrm{S}_2$. The expressions (4) explicitly determine the parametric dependence of the sought-for values on the parameters p. The effect of parameter variation on the contact pressure distribution and the SSS of the modeled system can be analyzed. Furthermore, the optimal parameter values that deliver desired design objectives with regard to certain criteria (for example, strength) can be ultimately determined from these simulation data.

The proposed methodology was applied to the analysis of contact interaction of the ball piston and the running track of the HVT. The bodies have the shape of a ball and a toroid, correspondingly. Both of them have surfaces of revolution as shown in Fig. 2 although the axes of revolution are not the same. The toroidal surface of the running track on the stator ring may have an arbitrary profile. The particular shape of the profile Π defined by a power function is chosen for the present analysis:

$$z_l = h \cdot \left(\frac{|y_l|}{R}\right)^n. \tag{5}$$

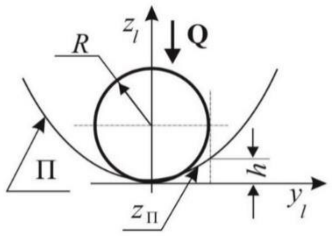


Fig. 2. Contact of a ball with a toroid

The expression (5) contains two parameters $p_1 \equiv h = \alpha R$, $p_2 \equiv n$. Note that the transverse dimension y_l and the coefficient h are both scaled relative to the ball piston radius R. The reference configuration of the profile is set with the dimensionless parameter $\alpha_0 = 0.5$ for a quadratic function (5) so that $p_1^0 = R/2, p_2^0 = 2$. This profile nearly follows the circular shape of the ball piston which means that the gap between the bodies is extremely small along y-direction. Still the numerical analysis is capable to capture this feature. The solution of the contact problem provides the maximal value of the contact pressure $q_{\max} = \max q(\alpha, n, Q)$ and the maximal value of the equivalent von-Mises stress $\sigma_{\max} = \max \sigma(\alpha, n, Q)$ for any given normal force Q. These results can be processed in the dimensionless form with the output given by the variables $\gamma_q = q_{\max}(\alpha, n, Q)/q_{\max}(1/2,\ 2,\ Q)$ and $\gamma_\sigma = \sigma_{\max}(\alpha, n, Q)/\sigma_{\max}(1/2,\ 2,\ Q)$. They represent relative reduction/growth of the two considered stress factors that can be used to sort out infeasible design points.

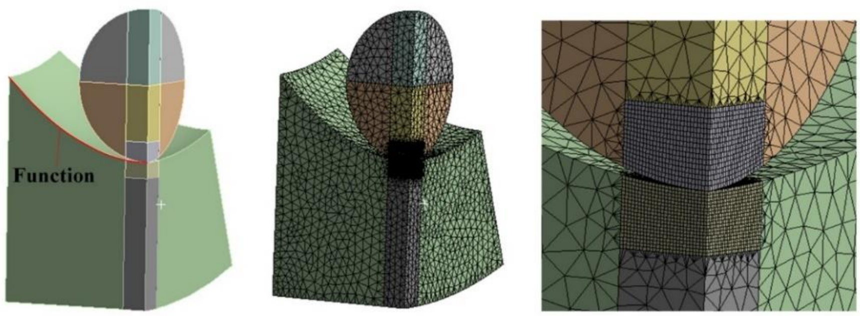


Fig. 3. 1/4 cut geometric and finite-element models of the ball and toroidal ring in contact

4 Results and Discussion

The geometric and finite-element models of the studied system is shown in Fig. 3. A quadrant cut out of the two bodies is only included to the analysis due to the symmetries of the problem.

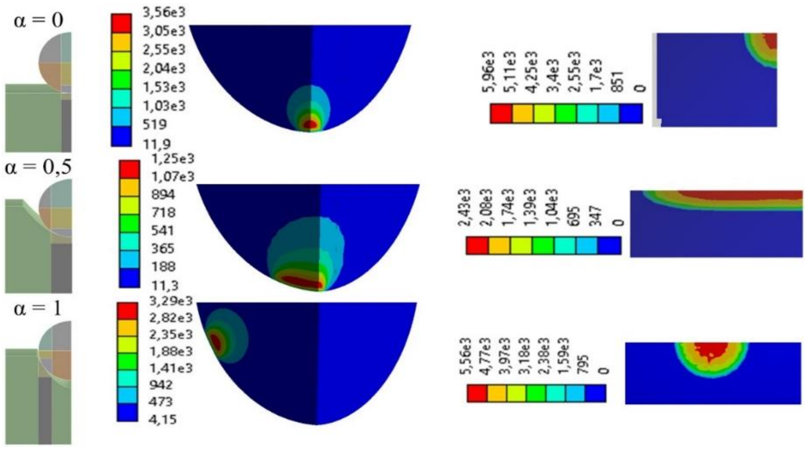


Fig. 4. Contact pressure distributions (left) and equivalent von Mises stress fields (right) with variable α in case of quadratic profile with n = 2

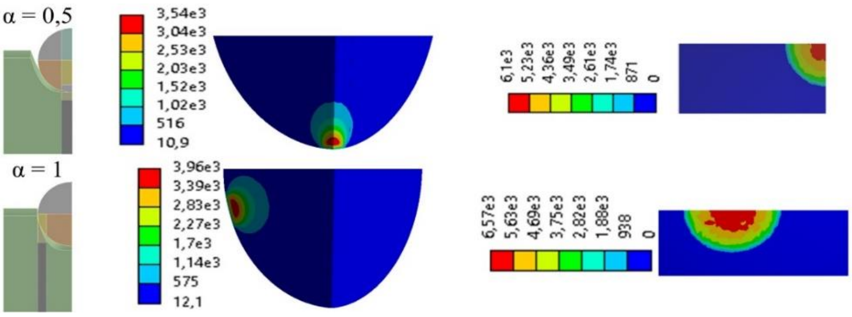


Fig. 5. Contact pressure distributions (left) and equivalent von Mises stress fields (right) with variable α in case of profile with the power exponent n = 4

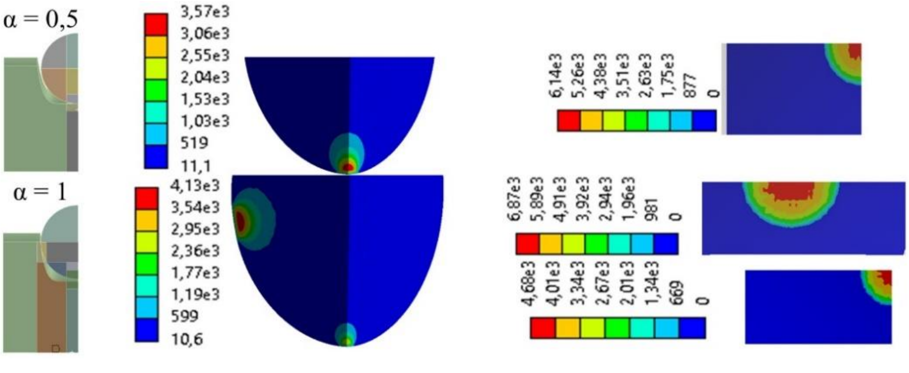
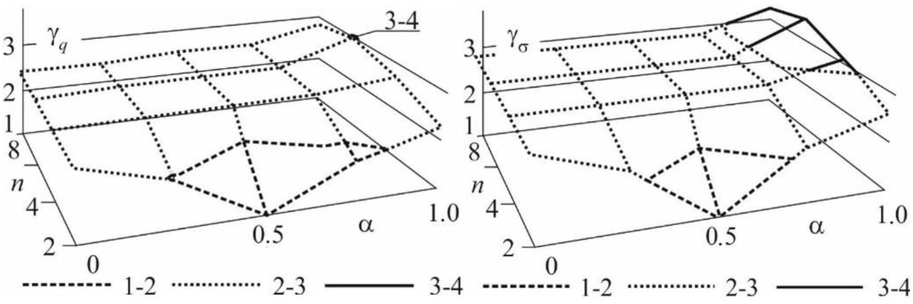


Fig. 6. Contact pressure distributions (left) and equivalent von Mises stress fields (right) with variable α in case of profile with the power exponent n = 8

Several basic design parameters are preset to the fixed values such as the radius of the ball piston R = 0.03175 m, minimum diameter of the profile d = 0.3195 m, normal force Q = 100 kN. The materials of the stator ring and the ball pistons are special structural steels with nearly identical elastic properties defined by the elasticity modulus E = 210 GPa and the Poisson's coefficient ν = 0.3. The dimensionless parameters of the profile geometry are varied in the span $\alpha \in [0;\ 1]$, $n \in [2;\ 8]$.
The obtained distributions of the contact pressure and the equivalent von Mises stresses are shown in Fig. 4-6. The processed output in terms of the dimensionless stress factors $\gamma_q(\alpha, n)$ та $\gamma_\sigma(\alpha, n)$ is given in Fig. 7.



a *b*

Fig. 7. Relative maximal contact pressure $\gamma_q(\alpha, n)$ (*a*) and von Mises stress $\gamma_\sigma(\alpha, n)$ (*b*)

Several observations can be made based on the obtained results. The shape and size of the contact area change with the variation of toroidal profile of the running track. The gap between the stator ring and the ball piston grows slower in the transverse direction compared to the piston rolling direction. For small $1 >> \alpha \rightarrow 0$ and the power function (5) close enough to quadratic $n \rightarrow 2$ the contact area is oval. The contact pressure distribution has smooth and convex dome profile with the maximum located in the center point where both bodies touch initially. As α and n grow this picture transforms. The contact area gradually attains the more rectangular form with rounded corners or even further gets a dumbbell shape. Meanwhile the maximum of the contact pressure shifts from the center closer to the opposite edges of the elongated contact spot. For certain geometric parameters and load levels the contact spot may even loose connectivity and split into two distant areas. The maximal contact pressure and the dimensionless factor $\gamma_q(\alpha, n)$ shown in Fig. 7a vary significantly depending on the running track profile. The amplification of the contact pressure is two-fold or higher in the examined range of parameters. As concerns the equivalent stress magnitude and the corresponding stress factor $\gamma_\sigma(\alpha, n)$ given in Fig. 7b, they correlate well with the concentration of the contact pressure.

The obtained results can be used to identify rational profile parameters for the given criteria and restrictions. The apparent optimum at $\alpha = 0.5, n = 2$ displays the lowest maximal contact pressure and von Mises stress values due to the large contact area shown in Fig. 4. However this profile of the running track is unfeasible for the considered design of the axial hydrovolumetric machine. It requires to cut the groove which is too deep, which will cause the interference between the rotor block and the inner wall of the stator. The feasible domain of the design parameters does not cover the entire parameter space shown in Fig. 7. The reduction of stress levels that delivers the desired strength of the load bearing parts is limited by the finite dimensions of the hydrovolumetric drive and in particular the sizing of the running track profile. The advised strategy would consist in combining several means of stress reduction besides the optimal choice of the running track profile such as the adjustment of the material elastic properties and the compliance of the contact layer [31] to obtain the further improvement of the design.

5 Conclusion

The developed models, methods and analysis tools for contact interaction and the SSS of contacting complex shaped structural elements have demonstrated their utility and effectiveness. The parametric approach is based on the variable analysis of design solutions according to the strength criterion. This makes it possible to validate progressive technical solutions of these structures.

The effect of the surface shape on the contact interaction of the bodies and their stress-strain state is studied for the most critical components of the radial HVT that are the stator ring and the ball pistons. Characteristic regularities of changes in the contact area, contact pressure distributions, and equivalent von Mises stresses are obtained for the studied nonlinear mechanical system. The possibility of transition from a connected to a multi-split contact area was established when the power of the gap function is increased, for example, from 2 to 4. It was also found that by changing the shape of the contacting surfaces, it is possible to control the levels of contact pressure and von Mises stresses. Compared to the reference case, these levels can increase by 2-3 times or more.

The advantage of the proposed approach is the combination in a single model of the analytical description of the shape of the contacting surfaces, on the one hand, and the numerical modeling of the SSS of the contacting bodies, on the other hand. The developed analysis methodology can be applied to other machine components that transmit high loads through the contact interaction along complex shape working surfaces. The obtained stress factors are instrumental in the search for the progressive design solutions that satisfy the imposed strength criteria.

Acknowledgments

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