MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL TECHNICAL UNIVERSITY «KHARKIV POLYTECHNIC INSTITUTE»

GUIDELINES

of calculation and graphical performing the task on the topic «CALCULATION OF LINEAR DC ELECTRIC CIRCUITS»

for courses "Theoretical Basics of Electrical Engineering", "Theory of Electrical and Magnetic Circuits ", "Theory of Electric Circuits"

for students of specialties 141 "Electric Power, Electrical Engineering and Electromechanics", 151 "Automation and computer-integrated technologies", 171 "Electronics"

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INTRODUCTION

These guidelines contain the necessary theoretical basis, tasks and examples of calculations of linear DC electric circuits.

The task is on the topic «Linear DC electric circuits» of the first semester of students' study of the courses: «Theoretical Basics of Electrical Engineering», «Theory of Electrical and Magnetic Circuits», «Theory of Electric Circuits».

The first semester of students' study in these courses is significant for further study of both theoretical basics of electrical engineering and other specialized disciplines of electrical engineering, for which theoretical basics of electrical engineering are the basic discipline. During the first semester, students learn the basic laws and methods of calculation of DC and AC circuits (Ohm's and Kirchhoff's laws, mesh current method and nodal potentials methods, the equivalent generator method and the superposition method), which are the base for practical calculations of electric circuits for various purposes.

Self-study students' work by this task should be considered as the basis for successful learning in the next semesters.

At this task performance, students can use sources at the end of the guidelines, which contain the theory and examples of the considered problems solution.

Guidelines are for students of specialties 141, 151, 171 of full-time and distance forms of study.

1. SCOPE

These guidelines establish the requirements for performing calculation and graphical task at the courses «Theoretical Basics of Electrical Engineering», «Theory of Electrical and Magnetic Circuits», «Theory of Electric Circuits» study.

The content of the calculation and graphical task corresponds to the training program for students of 141, 151, 171 specialties.

2. PERFORMANCE PURPOSE

As a result of this task performance students must:

1) know the basic laws and methods of linear DC electric circuits calculations;

2) be able to:

a) apply the laws and methods of electric circuits calculation to solve specific tasks;

b) make rational choice of the optimal method of solution based on the task analysis.

3. PERFORMANCE ORDER

Calculation and graphical task is performed in the study of the topic's theoretical material.

4. REQUIREMENTS FOR THE TASK

4.1 Each section should contain:

1) the task (schematic and parameters values);

2) theoretical substantiation of the applied method;

3) calculations;

4) conclusions (comparison of the used methods).

4.2 Task should be written on A4 (297×210) mm paper sheets.

Left, right, bottom and top margins should be at least 20 mm.

4.3 Paper sheets of the work should be numbered in the upper right corner. There is no number on the title page, but it is included in the total numbering of the sheets.

4.4 Text of the work should be written on one side of a sheet in one of the following ways:

a) handwritten – clear, legible handwriting with letters and numbers not less than 2.5 mm high. Font density in the work should be the same;

b) printed – size 12–14 at 1.5 intervals.

5. GENERAL PROVISIONS

An electric circuit is a set of devices designed to transmit, distribute and transform electromagnetic energy, processes in which can be described by such concepts as electromotive force (EMF), current and voltage.

Such concepts as branch, node and loop are used in the analysis of electric circuits.

A branch is a section of an electric circuit between two nodes, formed by one element or several series-connected elements, through which the same current flows (fig. 5.1 a, b).

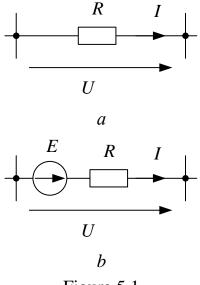


Figure 5.1

A node is a junction of three or more branches (fig. 5.2).

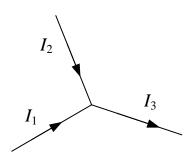
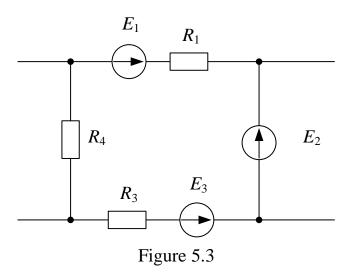


Figure 5.2

A loop is any closed path that runs across several branches (fig. 5.3).



Calculations of electric circuits are based on application of the basic laws of the theory of circuits: Ohm's, Joule-Lenz's and Kirchhoff's laws.

Ohm's Law

Ohm's Law for a circuit section that does not contain an EMF (fig. 5.1, a) establishes a relationship between current (I) and voltage (U) in this section:

$$I = \frac{U}{R}.$$
(5.1)

According to Ohm's law, for a circuit section, that contains an EMF (fig. 5.1, b), voltage drop over a resistor of the circuit section is equal to algebraic sum of the electromotive force (*E*) in this section and voltage applied to this section:

$$RI = E + U . (5.2)$$

Current is equal to a ratio of a sum of EMF in the section and applied voltage to resistance (R) of the circuit section:

$$I = \frac{E+U}{R}.$$
(5.3)

Important: if current, EMF or voltage directions in a branch coincide, these quantities in (5.2), (5.3) should be written with "plus" sign and, if not, with "minus" sign.

Kirchhoff's current law (KCL)

Algebraic sum of all currents that converge into a node of a branched electric circuit is equal to zero:

$$\sum_{k=1}^{n} I_k = 0, (5.4)$$

where n is the number of branches connected to the node.

Sign rule: currents directed to a node are negative, currents directed away from a node are positive.

Example:

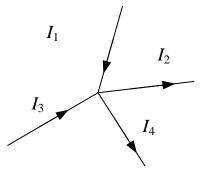


Figure 5.4

 $-I_1 + I_2 - I_3 + I_4 = 0.$

Kirchhoff's voltage law (KVL)

Algebraic sum of voltage drops in any closed loop of electric circuit is equal to algebraic sum of EMFs in this loop:

$$\sum_{k=1}^{m} R_k I_k = \sum_{k=1}^{l} E_k , \qquad (5.5)$$

where m is the number of resistors in the loop; l is the number of EMF sources in the loop.

Sign rule: EMFs that match the loop direction are taken positive; EMFs that do not match the loop direction are taken negative. Similarly for voltage drops: if current over circuit resistor coincides with the loop direction, voltage drop over it is taken positive, and if not – negative. Direction of a loop and directions of currents in branches are chosen arbitrarily.

Example:

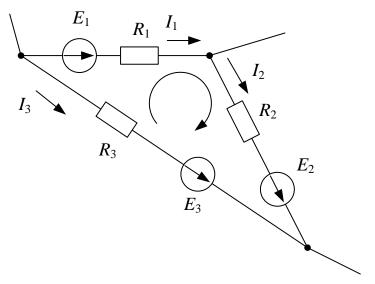


Figure 5.5

$$R_1I_1 + R_2I_2 - R_3I_3 = E_1 + E_2 - E_3.$$

A loop that differs from other loops by at least one branch is called an *independent loop*.

Important: if there is a current source in a circuit, then do not choose an independent loop through this source, because the internal resistance of the ideal current source is considered to be infinitely large, and voltage drop over such a source depends on the circuit to which it is connected. Therefore, equations by Kirchhoff's voltage law for loops that include branches with current sources do not write. Current sources are considered only in the equations written by Kirchhoff's current law.

Any electric circuit can be calculated (currents in any branch can be determined), applying Kirchhoff's laws. The total number of equations written by Kirchhoff's current and voltage laws is equal to the number of branches **b** (this number does not include branches with current sources).

KCL = n - 1 (where n is the number of nodes of the circuit) equations are written by Kirchhoff's current law for nodes of an electric circuit.

KVL = b - KCL equations are written by Kirchhoff's voltage law for circuit independent loops.

Power balance

Power balance is equality of power generated in an electric circuit by all energy sources (EMFs and current sources) to a sum of power of all circuit consumers:

$$\sum P_{gen} = \sum P_{cons} \,. \tag{5.6}$$

Power generated by an EMF source is determined as follows:

$$P_E = E \cdot I \,, \tag{5.7}$$

where *E* is EMF;

I is current through the branch with EMF.

If direction of current flowing through EMF source coincides with the direction of the EMF, then a product $E \cdot I$ is included in the power balance equation with a positive sign (fig. 5.6, *a*) and, if not, with a negative sign – energy consumer mode (fig. 5.6, *b*).



Figure 5.6

Power generated by a current source is determined as follows:

$$P_J = J \cdot U \,, \tag{5.8}$$

where *J* is current source;

U is voltage drop between the nodes to which this source is connected, it is directed from the positive pole of the source to the negative one (fig. 5.7).

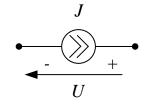


Figure 5.7

Consumed power is determined as a sum of power of all resistors in an electric circuit:

$$P_{cons} = \sum_{k=1}^{n} R_k I_k^2 , \qquad (5.9)$$

where n is the number of branches containing resistors.

Equation $P = RI^2$ expresses Joule – Lenz law and is the power that determines amount of energy released in a resistor in the form of heat per time unit.

6. THE CONTENT OF THE TASK

6.1 At disconnected *MN* branch:

1. Write a system of equations by Kirchhoff's laws (solution of this system is not necessary).

2. Determine currents through all circuit branches by the mesh currents method and the nodal potentials method. Choose the most appropriate method for the scheme and justify this choice.

Check correctness of the task solution by substituting the found currents into the system of equations, written by Kirchhoff's laws.

3. Write a power balance and check that it is satisfied. Estimate an error.

4. Using the equivalent generator method, determine currents in two branches, which numbers are indicated in Table 2.

Note. The currents included in the open-circuit voltage equation can be determined by superposition method or by equivalent circuit transformations.

6.2 At MN branch connection, determine potential of node A.

Tasks are given in Attachment A. The variant number coincides with the number in the group journal.

7. EXAMPLES OF DC ELECTRIC CIRCUITS CALCULATION

It is necessary to calculate electric circuit (determine the branches currents), shown in fig. 7.1

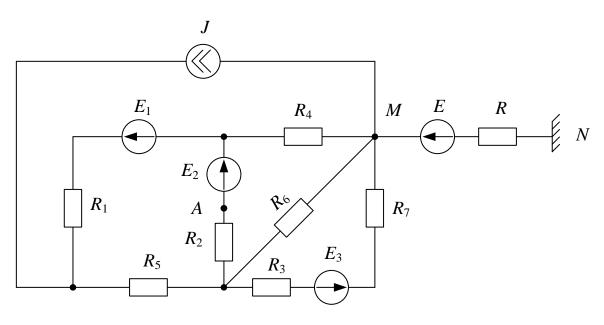


Figure 7.1

Circuit parameters have the following values:

J = 10 A; E = 15 V; $E_1 = 30$ V; $E_2 = 12$ V; $E_3 = 16$ V; R = 3 Ohm; $R_1 = 1$ Ohm;

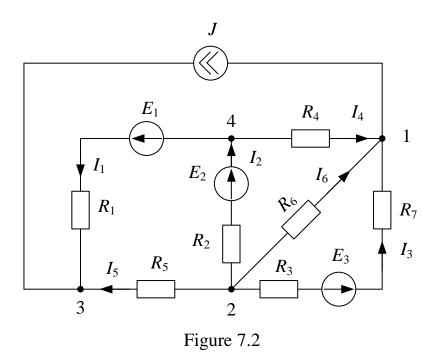
 $R_2 = 4$ Ohm; $R_3 = 1$ Ohm; $R_4 = 2$ Ohm; $R_5 = 4$ Ohm; $R_6 = 4$ Ohm; $R_7 = 3$ Ohm.

7.1. Writing systems of equations by Kirchhoff's laws

In the circuit with *MN* branch disconnected, it is necessary to determine the number of nodes and branches.

The number of nodes is 4. Nodes should be provided by sequence numbers. Directions of currents should be indicated in each branch. In a branch containing EMF, it is convenient to choose the same current direction as EMF direction, in branches that do not contain EMF, currents directions are selected arbitrarily. It is not necessary to indicate current direction in a branch with current source.

The diagram shows 6 currents (fig. 7.2).



Thus, the diagram shown in fig. 7.2 contains 6 branches without current sources and 4 nodes (b = 6, n = 4).

Equations for three nodes (KCL = n - 1 = 4 - 1 = 3) can be written by Kirchhoff's current law, and equations for three loops (KVL = b - KCL = 6 - 3 = 3), in which loop directions are chosen arbitrarily (indicated in the diagram by circular arrows) can be written by Kirchhoff's voltage law (fig. 7.3).

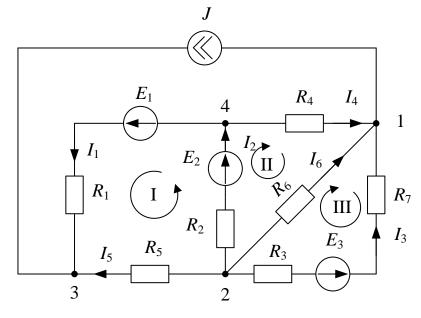


Figure 7.3

Equations for 1-st, 2-nd and 3-d nodes can be written by Kirchhoff's current law, and equations for I-st, II-nd and III-d loops can be written by Kirchhoff's voltage law.

Thus, the system of Kirchhoff's equations is follows:

$$\begin{cases}
J - I_4 - I_6 - I_3 = 0; \\
I_5 + I_2 + I_6 + I_3 = 0; \\
- J - I_1 - I_5 = 0; \\
R_1 I_1 - R_5 I_5 + R_2 I_2 = E_1 + E_2; \\
R_2 I_2 + R_4 I_4 - R_6 I_6 = E_2; \\
R_6 I_6 - (R_7 + R_3) I_3 = -E_3.
\end{cases}$$
(7.1)

7.2 Mesh current method

The mesh current method is based on Kirchhoff's voltage law. It means that currents in branches can be expressed through mesh currents, each of which circulates in its own mesh. The number of equations for determining the mesh currents is equal to the number of independent loops (KVL).

To determine currents in branches by mesh current method, it is necessary:

1) Determine the number of equations in the system of equations. It coincides with the number of independent loops and is determined by the formula:

$$KVL = b - (n - 1) = 6 - (4 - 1) = 3.$$

2) Indicate in the schematic loops directions (they are chosen arbitrarily and will be used as the directions of mesh currents in these loops).

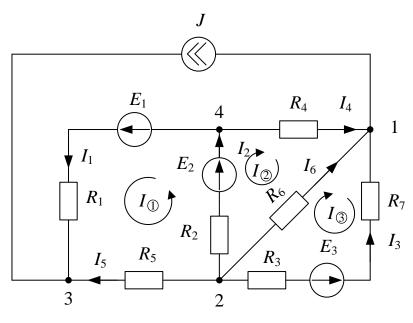


Figure 7.4

3) Write a system of equations of KVL order. For circuits in which KVL = 3, it has the form:

$$\begin{cases} I_{(1)}R_{11} + I_{(2)}R_{12} + I_{(3)}R_{13} = E_{(1)}; \\ I_{(1)}R_{21} + I_{(2)}R_{22} + I_{(3)}R_{23} = E_{(2)}; \\ I_{(1)}R_{31} + I_{(2)}R_{32} + I_{(3)}R_{33} = E_{(3)}. \end{cases}$$
(7.2)

In this system:

 $I_{(1)}, I_{(2)}, I_{(3)}$ are mesh currents; R_{11}, R_{22}, R_{33} are self (or total) mesh resistances of I-st, II-nd and III-d loops; $R_{12}, R_{13}, R_{21}, R_{23}, R_{31}, R_{32}$ are mutual (or adjacent) mesh resistances;

 $E_{(1)}, E_{(2)}, E_{(3)}$ are EMFs of I-st, II-nd and III-d loops.

4) To determine the loops' EMFs, it is necessary to draw an equivalent circuit in which current source J should be replaced by additional EMFs. To do this, one arbitrary closed loop for current source J is selected. It is indicated by a dotted arrow in the diagram (fig. 7.5). Action of current source is taken into account by introduction of additional EMFs, which should be placed beside each resistor in the branches through which this loop passes. Each such EMF should have direction opposite to direction of current J in the corresponding

branch, and their values are equal to product of current J and corresponding resistance R.

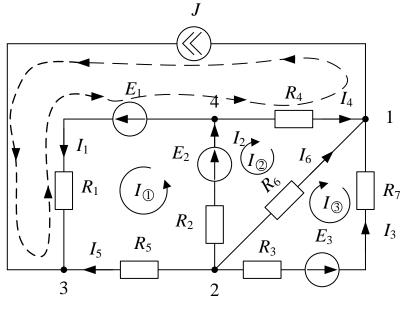


Figure 7.5

These rules are based on the compensation theorem for resistances through which contour current J goes (fig. 7.6).

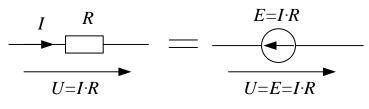


Figure 7.6

The branch with current source should then be removed from the equivalent circuit. As a result, an equivalent circuit will have only those loops in which unknown mesh currents are flowing.

Positive mesh currents' directions are selected arbitrarily and are indicated in the diagram.

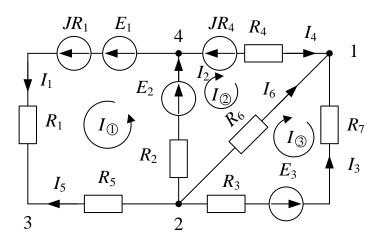


Figure 7.7 – Equivalent circuit in which current source action is compensated by additional EMFs and current source is removed

5) Calculate self and adjacent mesh resistances, as well as loop EMFs using the equivalent schematic.

 R_{11}, R_{22}, R_{33} are self (or total) mesh resistances. They are equal to a sum of the resistances included in a mesh (always positive):

$$R_{11} = R_1 + R_5 + R_2 = 1 + 4 + 4 = 9$$
 Ohm;
 $R_{22} = R_2 + R_4 + R_6 = 4 + 2 + 4 = 10$ Ohm;
 $R_{33} = R_6 + R_7 + R_3 = 4 + 3 + 1 = 8$ Ohm.

 $R_{12}, R_{13}, R_{21}, R_{23}, R_{31}, R_{32}$ are adjacent (mutual) mesh resistances, i.e. resistances of the branches common to two meshes. These resistances can be positive or negative. They are positive if mesh currents flow through a common branch in one direction; they are negative if mesh currents flow through a common branch in opposite directions. If two loops do not have common branches, their mutual mesh resistance is zero.

$$R_{12} = R_{21} = R_2 = 4$$
 Ohm; $R_{13} = R_{31} = 0$ Ohm; $R_{23} = R_{32} = -R_6 = -4$ Ohm.

 $E_{(1)}, E_{(2)}, E_{(3)}$ are loop EMFs. The loop EMF is equal to algebraic sum of EMFs of all sources in a considered mesh, taking into account additional

EMFs if they are included, to regard influence of the removed from the equivalent circuit current sources. EMF is written with a positive sign if its direction coincides with the mesh current direction, and with a negative sing if not:

$$E_{(1)} = E_2 + E_1 + R_1 J = 12 + 30 + 10 = 52 \text{ V};$$

$$E_{(2)} = E_2 - R_4 J = 12 - 20 = -8 \text{ V};$$

$$E_{(3)} = -E_3 = -16 \text{ V}.$$

6) The found parameters should be written in the system of equations (7.2):

$$\begin{cases} 9I_{(1)} + 4I_{(2)} = 52; \\ 4I_{(1)} + 10I_{(2)} - 4I_{(3)} = -8; \\ -4I_{(2)} + 8I_{(3)} = -16. \end{cases}$$

Solving this system of equations, we get the mesh currents values:

 $I_{(1)} = 8,57$ A, $I_{(2)} = -6,29$ A, $I_{(3)} = -5,14$ A.

7) Determine current of each branch according to the initial scheme (fig. 7.5) using the calculated mesh currents and the source current *J* flowing through the selected loop. If there are several mesh currents in a branch, then their algebraic sum will be that branch current. <u>Important:</u> calculation of currents in the branches through which source current *J* is looped, *J* value should be included in algebraic sum of the mesh currents flowing through the considered branch with regard of its real direction.

<u>The rule of signs</u>: mesh currents and current of current source, which direction coincides with the branch current direction are taken with a positive sign, and the currents which direction is opposite to the branch current direction are taken with a negative sign.

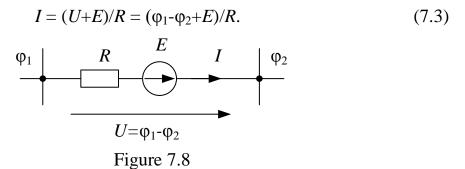
$$I_1 = I_{(1)} - J = 8,57 - 10 = -1,43$$
 A;
 $I_2 = I_{(1)} + I_{(2)} = 8,57 - 6,29 = 2,28$ A;
 $I_3 = -I_{(3)} = 5,14$ A;

$$I_4 = I_{(2)} + J = -6,29 + 10 = 3,71$$
 A;
 $I_5 = -I_{(1)} = -8,57$ A;
 $I_6 = I_{(3)} - I_{(2)} = -5,14 + 6,29 = 1,15$ A.

7.3 Nodal potentials method

The nodal potential method is based on Kirchhoff's current law.

Using the generalized Ohm's law, it is possible to express currents in branches by the potential difference of the branch nodes:



All currents in branches can be determined by calculating the nodal potentials, which reduces the number of equations compared to the system of equations by Kirchhoff's laws. Potential of one node can be assumed to be zero, as only the potential difference affects the current. Thus, the system of equations relatively the nodal potentials has the order of KCL = n - 1 (where n is the number of nodes in a circuit):

$$\begin{cases}
G_{11}\phi_{1} + G_{12}\phi_{2} + G_{13}\phi_{3} + \dots + G_{1n}\phi_{n} = J_{(1)}; \\
G_{21}\phi_{1} + G_{22}\phi_{2} + G_{23}\phi_{3} + \dots + G_{2n}\phi_{n} = J_{(2)}; \\
G_{31}\phi_{1} + G_{32}\phi_{2} + G_{33}\phi_{3} + \dots + G_{3n}\phi_{n} = J_{(3)}; \\
\dots \\
G_{n1}\phi_{1} + G_{n2}\phi_{2} + G_{n3}\phi_{3} + \dots + G_{nn}\phi_{n} = J_{(n)};
\end{cases}$$
(7.4)

where $\phi_1, \phi_2, \phi_3, ..., \phi_n$ are unknown nodal potentials of nodes 1, 2, 3,..., *n*.

To calculate the currents by nodal potentials method, it is necessary to perform the following actions:

1) Assign random numbers to the schematic nodes (1, 2, 3, 4).

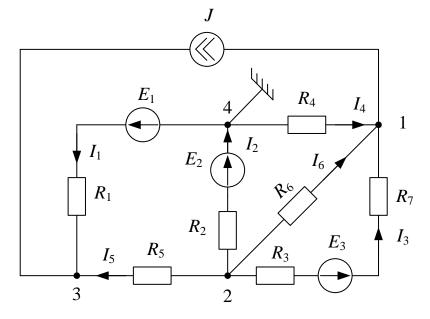


Figure 7.9

Important: The node which potential is expedient to assume equal to 0 should be assigned the largest number. This node is called the base. As a base node, it is advisable to choose a node to which the maximum number of branches is connected to simplify the solution process. It will be shown below how to choose a base node in circuits containing an ideal EMF source. In the considered problem, node number 4 is selected as the base node: $\varphi_4 = 0$.

2) Write a system of equations of the form (7.4) of KCL order. At KCL=3 the system has three equations, each with three terms:

$$\begin{cases} G_{11}\phi_1 + G_{12}\phi_2 + G_{13}\phi_3 = J_{(1)}; \\ G_{21}\phi_1 + G_{22}\phi_2 + G_{23}\phi_3 = J_{(2)}; \\ G_{31}\phi_1 + G_{32}\phi_2 + G_{33}\phi_3 = J_{(3)}, \end{cases}$$
(7.5)

where G_{11}, G_{22}, G_{33} are self-conductance of nodes 1, 2, 3; $G_{12}, G_{13}, G_{21}, G_{23}, G_{31}, G_{32}$ are mutual conductances between nodes 1 and 2, 1 and 3, 2 and 3;

 $J_{(1)}, J_{(2)}, J_{(3)}$ are nodal current sources (or nodal currents) of nodes 1, 2, 3.

3) Determine the nodal parameters (self and mutual conductance of nodes and nodal current sources).

Self conductance is always positive and is equal to sum of conductances of the branches connected to this node:

$$G_{11} = \frac{1}{R_4} + \frac{1}{R_6} + \frac{1}{R_3 + R_7} = \frac{1}{2} + \frac{1}{4} + \frac{1}{1+3} = 1 \text{ S};$$

$$G_{22} = \frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_6} + \frac{1}{R_3 + R_7} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{1+3} = 1 \text{ S};$$

$$G_{33} = \frac{1}{R_1} + \frac{1}{R_5} = \frac{1}{1} + \frac{1}{4} = 1,25 \text{ S}.$$

Mutual conductance is always negative and is equal to sum of conductances of the branches that directly connect considered two nodes; if two nodes are not directly connected by any branches, their mutual conductance is equal to 0:

$$G_{12} = G_{21} = -\left(\frac{1}{R_6} + \frac{1}{R_3 + R_7}\right) = -\left(\frac{1}{4} + \frac{1}{1+3}\right) = -0.5 \text{ S};$$

$$G_{13} = G_{31} = 0;$$

$$G_{23} = G_{32} = -\frac{1}{R_5} = -\frac{1}{4} = -0.25 \text{ S}.$$

To determine nodal current sources, it is necessary to perform equivalent replacement of EMF sources by current sources. The currents of such sources are determined by the formula

$$J_{k} = \frac{E_{k}}{R_{k}}.$$

Figure 7.10

The nodal current source of a node is equal to algebraic sum of the currents of current sources and the additional current sources formed by EMF connected to the corresponding node.

The rule of sings: if a source is directed to a node, then the term has a positive sign, if a source is directed from the node it has a negative sign.

$$J_{1} = -J + E_{3}G_{3} = -J + \frac{E_{3}}{R_{3} + R_{7}} = -10 + \frac{16}{1 + 3} = -6 \text{ A};$$

$$J_{2} = -E_{2}G_{2} - E_{3}G_{3} = -\frac{E_{2}}{R_{2}} - \frac{E_{3}}{R_{3} + R_{7}} = -\frac{12}{4} - \frac{16}{1 + 3} = -7\text{ A};$$

$$J_{3} = J + G_{1}E_{1} = J + \frac{E_{1}}{R_{1}} = 10 + \frac{30}{1} = 40 \text{ A}.$$

5) The obtained nodal parameters should be written into the system of equations (7.5):

$$\begin{cases} \phi_1 - 0.5\phi_2 &= -6; \\ -0.5\phi_1 + \phi_2 - 0.25\phi_3 &= -7; \\ -0.25\phi_2 + 1.25\phi_3 &= 40. \end{cases}$$

The solution of the system is the nodal potentials values:

$$\varphi_1 = -7,43 \text{ V}, \ \varphi_2 = -2,86 \text{ V}, \ \varphi_3 = 31,43 \text{ V}.$$

Voltage drop over each branch can be calculated as a difference of the potentials of nodes to which the branch is connected:

$$U_{mn} = \varphi_m - \varphi_n. \tag{7.6}$$

6) Calculate currents of branches using generalized Ohm's law (5.3) and relation (7.6):

$$I = \frac{E+U}{R}.$$
(7.7)

$$I_1 = \frac{E_1 + \varphi_4 - \varphi_3}{R_1} = \frac{30 - 31,43}{1} = -1,43 \text{ A};$$
$$I_2 = \frac{E_2 + \varphi_2 - \varphi_4}{R_2} = \frac{12 - 2,86}{4} = 2,28 \text{ A};$$

$$I_3 = \frac{E_3 + \varphi_2 - \varphi_1}{R_3 + R_7} = \frac{16 - 2,86 + 7,43}{1 + 3} = 5,14 \text{ A};$$

$$I_4 = \frac{\varphi_4 - \varphi_1}{R_4} = \frac{7,43}{2} = 3,72$$
 A;

$$I_5 = \frac{\varphi_2 - \varphi_3}{R_5} = \frac{-2,86 - 31,43}{4} = -8,57$$
 A;

$$I_6 = \frac{\varphi_2 - \varphi_1}{R_6} = \frac{-2,86 + 7,43}{4} = 1,14$$
 A.

Peculiarities of calculation of a circuit by nodal potentials method at presence of an ideal EMF

A branch containing only an ideal EMF has no resistance. It is not possible to calculate self and mutual conductances as well as nodal currents for the nodes to which such a branch is connected.

1) A node to which a negative clamp of an ideal EMF is connected is assigned as the base node having the largest number (N = n). Its potential is assigned zero:

 $\varphi_N = 0$.

2) Then potential of a node to which a positive clamp is connected will be equal to EMF voltage. It should be provided with previous number (N-1) (see fig. 7.11). The potential of this node is equal to:

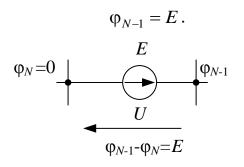


Figure 7.11

Thus, it is not necessary to write an equation for (N-1)-th node. The system for nodal potentials calculation will have one equation less: (N-2), but the number of terms in each equation will not change and will be equal to (N-1).

3) As terms in the left-hand sides of equations (7.5), which contain the products of φ_{N-1} and the mutual conductance between (*N*-1)-th node and other nodes are known, they should be transferred to the right-hand side of the corresponding equations.

4) The nodal parameters are calculated according to the rules described above.

5) As a result of the system of equation calculation, the values of the nodal potentials are got.

6) Currents of all branches except current of the branch containing ideal EMF are calculated using Ohm's law (7.7). Current of the branch with ideal EMF shoyuld be calculated by Kirchhoff's current law.

7.4 Power balance

Determine the generated power:

$$P_{gen} = E_1 I_1 + E_2 I_2 + E_3 I_3 + J U_J, \tag{7.8}$$

where U_J is voltage drop over the current source.

 U_J can be determined by applying Kirchhoff's voltage law (see fig. 7.12):

$$U_J - R_4 I_4 + R_1 I_1 = E_1; (7.9)$$

$$U_J = E_1 + R_4 I_4 - R_1 I_1 = 30 + 2 \cdot 3,72 - 1 \cdot (-1,43) = 38,87 \text{ V}.$$

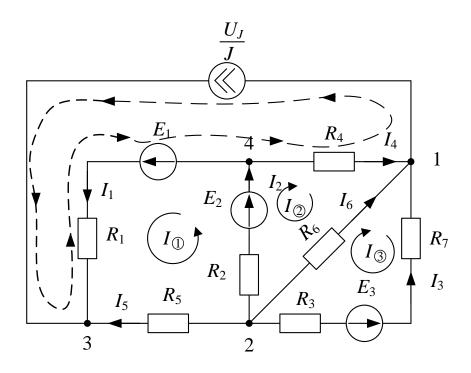


Figure 7.12

When the nodal potentials are determined, voltage drops can be defined as potentials' difference (see fig. 7.13):

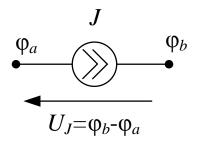


Figure 7.13

$$U_J = \varphi_3 - \varphi_1 = 31,43 + 7,43 = 38,86$$
 V.

In accordance with (7.8), power generated by circuit will be:

 $P_{gen} = 30 \cdot (-1,43) + 12 \cdot 2,28 + 16 \cdot 5,14 + 10 \cdot 38,86 = 455,3$ W.

Consumed power is:

$$P_{cons} = R_1 I_1^2 + R_2 I_2^2 + (R_3 + R_7) I_3^2 + R_4 I_4^2 + R_5 I_5^2 + R_6 I_6^2 =$$

= 1 \cdot 1,43^2 + 4 \cdot 2,28^2 + (1+3) \cdot 5,14^2 + 2 \cdot 3,72^2 + 4 \cdot 8,57^2 + 4 \cdot 1,14^2 = 455,2 W.

Checking the power balance, we get:

$$P_{gen} = P_{cons} \rightarrow 455,3 \text{ W} \approx 455,2 \text{ W}.$$

Calculation error is:

$$\Delta = \frac{\left| P_{gen} - P_{cons} \right|}{P_{gen}} \cdot 100 \ \% = \frac{455,3 - 455,2}{455,3} \cdot 100\% = 0,02\%.$$

Note: the error should not exceed 5 %.

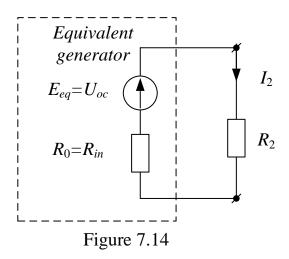
Conclusion: The power balance is satisfied.

7.5 Equivalent generator method

It is necessary to determine the current in the second branch.

7.5.1 Circuit presentation in the form of a two-port

To represent the circuit in the form of a two-port, the circuit can be opened at the section where the current should be found (in our case, at the second branch). Relatively the second branch, the whole circuit becomes an active two-port (fig. 7.14).



According to the equivalent generator theorem, any active two-port can be replaced by an equivalent generator with EMF equal to open-circuit voltage U_{oc} and internal resistance R_0 equal to input resistance R_{in} of the same, but passive two-port.

Then, as can be seen from fig. 7.14, current of the second branch is determined by the formula:

$$I_2 = \frac{U_{oc}}{R_2 + R_{in}},\tag{7.10}$$

where U_{oc} is open-circuit voltage;

 R_{in} is input impedance.

When the second branch is opened and resistor R_2 is removed, the scheme becomes as shown in fig. 7.15. When the second branch is opened, currents in other branches will change, so they are denoted by an additional index "oc" (or "0").

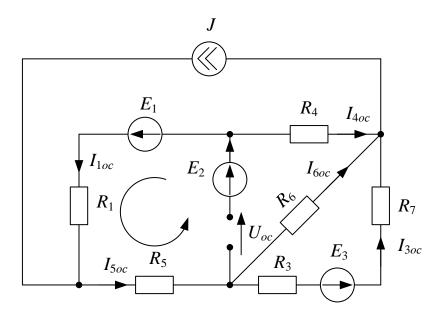


Figure 7.15

7.5.2 Determination of the open-circuit voltage

To determine open-circuit voltage U_{oc} in the redrawn scheme, let us choose a loop that includes the open branch and write an equation for it by Kirchhoff's voltage law (see fig. 7.15):

$$U_{oc} + R_1 I_{1oc} + R_5 I_{5oc} = E_2 + E_1. \tag{7.11}$$

From this equation, the open-circuit voltage is determined as follows:

$$U_{oc} = E_2 + E_1 - R_1 I_{1oc} - R_5 I_{5oc}. \tag{7.12}$$

To calculate this voltage, it is necessary to know currents of the active two-port, namely I_{1oc} and I_{5oc} . They can be found using the *superposition theorem* according to which *current in any branch of a linear electric circuit is equal to algebraic sum of currents generated in the branch by each energy source separately*. At determining current caused by one separate energy source in a given branch, other energy sources are removed from the circuit but their internal resistances remain. When ideal EMF source is removed, terminals to which it is connected are short-circuited because its internal resistance is zero. When an ideal current source is removed, the branch in which it locates is broken because its resistance is infinite.

7.5.3 Calculation of currents by the superposition theorem

According to the superposition theorem, three circuits can be formed from the got scheme (see fig. 7.15). Each of such circuits has only one source. At this, it is taken into account that source E_2 cannot generate current, as it is located in the open branch.

1) Determination of currents $I_{1oc}^{E_1}$ and $I_{5oc}^{E_1}$, caused by EMF source E_1 action.

The scheme is presented in fig. 7.16. In this circuit, current flows from EMF source through connected in series resistors R_1 , R_5 , then through two connected in parallel branches with resistors R_6 and R_3 , R_7 , connected in series, then – through resistor R_4 back to EMF source. Thus, currents $I_{1oc}^{E_1}$ and $I_{5oc}^{E_1}$ are equal to current in the branch of source E_1 .

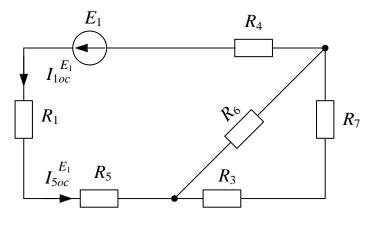


Figure 7.16

This current is calculated by Ohm's law after determining the equivalent resistance relatively EMF terminals:

$$I_{1oc}^{E_1} = I_{5oc}^{E_1} = \frac{E_1}{R_1 + R_4 + R_5 + \frac{R_6 \cdot (R_3 + R_7)}{R_6 + R_3 + R_7}} = \frac{30}{1 + 2 + 4 + \frac{4(1 + 3)}{4 + 1 + 3}} = 3,33 \,\mathrm{A}.$$

2) Determination of currents $I_{1oc}^{E_3}$ and $I_{5oc}^{E_3}$, caused by action of EMF source E_3 .

The scheme is presented in fig. 7.17. In this circuit, current flows from the source through resistor R_7 , then it divides in two parallel branches with resistors R_6 and R_4 , R_1 , R_5 , connected in series. Firstly, total current $I_{3oc}^{E_3}$ in the branch with EMF should be calculated by Ohm's law. For calculation of currents $I_{1oc}^{E_3}$ and $I_{5oc}^{E_3}$, the current divider rule or Ohm's law should be used.

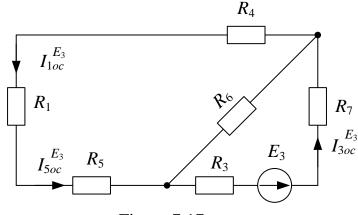


Figure 7.17

Current of the branch with source E_3 is equal to:

$$I_{3oc}^{E_3} = \frac{E_3}{R_3 + R_7 + \frac{R_6 \cdot (R_4 + R_1 + R_5)}{R_6 + R_4 + R_1 + R_5}} = \frac{16}{1 + 3 + \frac{4 \cdot (2 + 1 + 4)}{4 + 2 + 1 + 4}} = 2,44$$
 A.

According to the current divider rule:

$$I_{1oc}^{E_3} = I_{5oc}^{E_3} = I_{3oc}^{E_3} \cdot \frac{R_6}{R_6 + R_4 + R_1 + R_5} = 2,44 \cdot \frac{4}{4 + 2 + 1 + 4} = 0,89 \text{ A}.$$

3) Determination of currents I_{1oc}^J and I_{5oc}^J , caused by current source J action.

The scheme is presented in fig. 7.18.

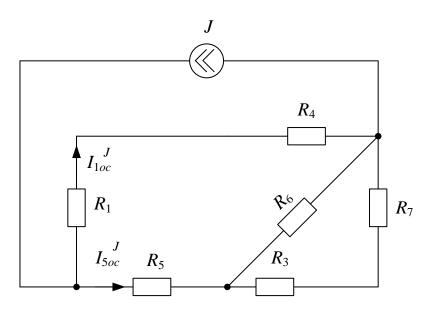


Figure 7.18

Current in the branch with source *J* is known and equal to J = 10 A. It divides in two parallel branches with resistors R_1 , R_4 connected in series, and R_5 connected in series with the parallel connection of branches R_6 and R_3+R_7 . Thus, I_{1oc}^J and I_{5oc}^J are currents of two parallel branches. They are calculated according to the current divider rule or Ohm's law.

According to the current divider rule:

$$I_{5oc}^{J} = J \cdot \frac{R_{1} + R_{4}}{R_{1} + R_{4} + R_{5} + \frac{R_{6} \cdot (R_{3} + R_{7})}{R_{6} + R_{3} + R_{7}}} = 10 \cdot \frac{1 + 2}{1 + 2 + 4 + \frac{4 \cdot (1 + 3)}{4 + 1 + 3}} = 3,33 \text{ A}.$$

Current I_{loc}^J can be determined by Kirchhoff's current law:

$$I_{1oc}^{J} = J - I_{5oc}^{J} = 10 - 3,33 = 6,67$$
 A.

It should be noted that currents in a branch from different sources might have opposite directions.

According to the superposition theorem, we determine algebraic sum of partial currents from all sources as follows:

$$I_{1oc} = I_{1oc}^{E_1} + I_{1oc}^{E_3} - I_{1oc}^{J} = 3,33 + 0,89 - 6,67 = -2,45$$
 A;

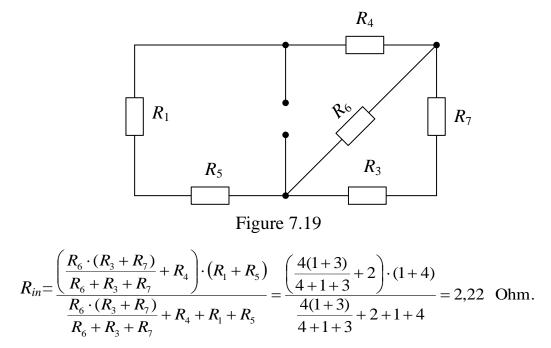
$$I_{5oc} = I_{5oc}^{E_1} + I_{5oc}^{E_3} + I_{5oc}^{J} = 3,33 + 0,89 + 3,33 = 7,55$$
 A.

The open-circuit voltage according to (7.12):

$$U_{oc} = E_2 + E_1 - R_1 I_{1oc} - R_5 I_{5oc} = 12 + 30 + 1 \cdot 2,45 - 4 \cdot 7,55 = 14,25 \text{ V}.$$

7.5.4 Determination of input resistance R_{in}

Input resistance R_{in} is resistance of the passive two-port relatively the open terminals. To determine it, all sources in the active two-port scheme (see fig. 7.15) should be removed, leaving their internal resistances. Scheme for R_{in} determination is shown in fig. 7.19.



7.5.5 Calculation of the second branch current

According to the equivalent generator theorem, current I_2 is determined as follows:

$$I_2 = \frac{U_{oc}}{R_2 + R_{in}} = \frac{14,25}{4+2,22} = 2,29$$
 A.

7.6 Determining potential of point A

After joining MN branch (fig. 7.1), currents of the other branches will not change, because no current will flow through MN branch, as there is no closed circuit for it. The ground potential of N is zero. Voltage between M and N nodes is a difference between these nodes potentials (fig. 7.8).

$$U_{AN} = \varphi_A - \varphi_N.$$

Because $\varphi_N = 0$, $U_{AN} = \varphi_A$.

 U_{AN} can be determined as voltage drop over the path containing elements R, E, R_4 , E_2 (see upper part of dotted line in fig. 7.20), taking into account that $U_R = 0$, as current does not flow through NM branch, we can write:

$$U_{AN} = E + I_4 R_4 - E_2$$
.

So, U_{AN} and φ_A are written as follows:

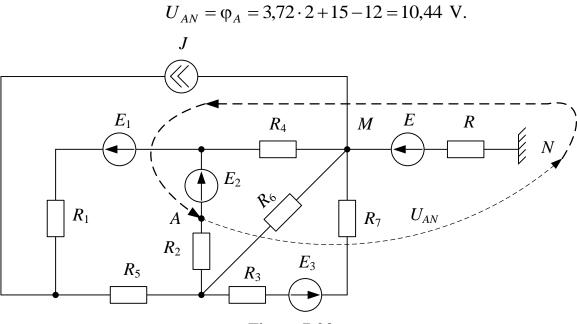


Figure 7.20

8. LITERATURE SOURCES

Основы теории цепей: учебник для вузов. / Г. В. Зевеке,
 П. А. Ионкин, А. В. Нетушил, С. В. Страхов – Москва :
 Энергоатомиздат, 1989. – 528с.

Бессонов Л. А. Теоретические основы электротехники. /
 Л. А. Бессонов – Москва : Энергия, 1978. – 592 с.

Нейман Л. Р. Теоретические основы электротехники : у 2 т. Т. 1 / Л. Р. Нейман, К. С. Демирчян – Ленинград : Энергоиздат, 1981. – 536 с.

 Атабеков Г. И. Линейные электрические цепи. / Г. И. Атабеков – Москва : Энергия, 1978. – 175 с.

 Байдак Ю. В. Основи теорії кіл : навч. посіб. – Київ : Вища шк. : Слово, 2009. – 271 с.

Шебес М. Р. Задачник по теории линейных электрических цепей.
 учеб. пособ. для электротехнич., радиотехнич. спец. вузов. / М. Р. Шебес,
 М. В. Каблукова. – Москва : Высшая школа, 1990. – 544с.

 Бессонов Л. А. Сборник задач по теоретическим основам электротехники : учеб. пособие для энерг. и приборост. спец. вузов. / Л. А. Бессонов, И. Г. Демидова, М. Е. Заруди и др.; под ред. Л. А. Бессонова. — Москва : Высш. шк., 1988. — 543 с.

 8. Методические указания по выполнению расчетно-графического задания по теме «Расчет линейных электрических цепей постоянного и гармонического тока» / сост. В. М. Боев, Л. В. Казаковцева,
 В. П. Самсонов – Харьков : НТУ «ХПИ», 2003. – 48с.

9. СТЗВО-ХПІ-3.01-2018 ССОНП. Текстові документи у сфері навчального процесу. Загальні вимоги до виконання. – Харків : НТУ «ХПІ», 2018 р.

36

10. СТВУЗ-ХПІ-3.02-2002 ССОНП. Навчально-методичні документи. Загальні вимоги до виконання. – Харків : НТУ «ХПІ», 2003 р.

11. Khilov V. S. Theoretical Fundamentals of Electrical Engineering, Dnipro, National Mining University, 2017, 300 p.

12. Desoer C. A., Kuh E. S. Basic Circuit Theory, New York, McGraw-Hill Book Company, 1969, 408 p.

13. Hayt W. H., Kemmerly J. E., Durbin S. M. Engineering circuit analysis, New York, McGraw-Hill, 2012, 854 p.

14. Rizzoni G. Fundamentals of electrical engineering, New York, McGraw-Hill, 2009, 766 p.

ATTACHMENT

Guidelines for task variant choosing.

The number of the scheme for calculation and the number of the numerical data of the corresponding variant are shown in Table 1 of the Attachment.

The student's variant number corresponds to its number in the journal of the academic group. It corresponds to the line number in Table 1. If there is only one group in the lecture course, then the group index is «I». If there are several groups in the lecture course, then index «I» corresponds to the group which group number is the smallest, for the group with the next number it is «II» and so on. This is the column number in Table 1.

In each cell of Table 1 (with a row number corresponding to the student's variant number and a column number corresponding to the index of the group), the number in the circle corresponds to the circuit number (see Figure A.1, p. 44–47), the number in the square corresponds to the variant number of the numerical data (see Table 2, pages 41–43). The numbers of branches, currents in which should be calculated by the method of equivalent generator are in the last row of Table 2.

Variant			Group	index		
Variant	Ι	II	III	IV	V	VI
1	1 31	25 25	(19) [13]	13 1	7 19	1 7
2	26 20	2 32	20 20	(14) 2	8 20	2 8
3	21 15	27 33	(15) 9	27 21	9 3	3 3
4	4 10	10 22	(16) [4]	22 16	28 28	4 34
5	(11) [35]	29 5	23 23	(17) [23]	11 17	5 11
6	30 30	6 12	24 18	6 36	(12) [24]	(18) 6
7	$\begin{array}{c c} \hline 1 & \hline 1 \\ \end{array}$	7 13	13 19	(19) [25]	25 1	7 31
8	(14) [32]	26 8	20 2	14 26	8	2 14
9	3 27	9 27	(15) 3	99	27 15	21 33
10	4 4	10 16	16 22	22 28	28 4	10 34
11	5 17	(11) [23]	17 17	23 5	29 11	17 35
12	(12) 36	30 6	24 30	18 24	12 18	6 6
13	(13) [31]	25 7	(19) [1]	13 25	7 7	1 13
14	2 2	8 14	(14) [20]	20 26	26 2	8 32
15	(15) [33]	27 27	21 3	(15) [27]	9 21	3 15
16	4 16	10 10	(16) [28]	22 4	28 10	16 34
17	23 35	29 17	23 11	17 5	11 11	5 23

Table 1 – Variants of tasks for calculation of electric DC circuit (scheme number, data number)

Variant			Group	index			
v ai iait	Ι	II	III	IV	V	VI	
18	30 12	6 18	18 36	24 6	(18) 30	12 12	
19	1 19	7 25	13 13	(19) 7	25 13	(19) [31]	
20	14 14	20 8	26 14	20 32	2 20	8 26	
21	9 33	27 3	21 21	(15) [21]	9 15	39	
22	22 34	28 16	22 10	16 16	10 28	4 22	
23	5 5	11 5	17 11	23 17	29 23	29 35	
24	18 18	24 12	6 24	30 18	(12) 30	24 36	
25	25 31	25 19	(19) [19]	13 7	7 1	1 25	
26	20 14	26 26	14 8	8 2	26 32	2 26	
27	3 27	21 9	15 15	21 27	27 9	3 33	
28	4 28	(10) [4]	16 10	22 22	28 22	28 34	
29	5 35	29 29	23 29	(17) [29]	(11) [29]	5 29	
30	6 30	24 24	18 12	30 36	(12) 6	30 24	
31	1 19	13 13	(19) [25]	7 31	25 25	7 7	
32	26 2	14 14	8 32	2 2	20 20	26 32	
33	21 21	9 33	(15) 3	27 27	3 21	21 33	
34	28 34	10 16	4 4	(16) [34]	22 22	10 28	

Continuation of Table 1 – Variants of tasks for calculation of electric DC circuit (scheme number, data number)

Data number	1	2	3	4	5	6	7	8	9	10	11	12
J, A	4	2	6	0,9	4	2	2	1	2	1,5	2	1
E, V	50	15	10	3	60	15	20	15	6	8	10	8
R, Ohm	20	_	_	_	_	40	70	_	_	_	_	200
E_1, V	100	48	13,5	_	11	5	50	24	4,5	_	5,5	2,5
E_2, V	30	40	9	1,5	_	10	15	20	3	2,5	_	5
E_3, V	10	10	30	0,6	35	20	5	5	10	1	17,5	10
E_4, V	60	_	_	_	_	30	30	_	_	_	_	15
E_5, V	_	_	_	_	50	_	_	_	_	_	25	_
E_6, V	_	_	_	1,2	10	_	_	_	_	2	5	_
R_1 , Ohm	10	4	4,5	0,6	1	8	5	2	1,5	1	0,5	4
R_2 , Ohm	10	1	3	1,8	7	30	5	0,5	1	3	3,5	15
R_3 , Ohm	_	6	3	0,15	1,5	6	_	3	1	0,25	0,75	3
$R_4,$ Ohm	7	2	9	0,3	2,5	30	3,5	1	3	0,5	1,25	15
$\begin{array}{c} R_5, \\ \text{Ohm} \end{array}$	15	6	9	0,6	_	15	7,5	3	3	1	_	7,5
R ₆ , Ohm	8	_	_	_	8	_	4	_	_	_	4	
Numbers of branches, in which currents should be determined by the equivalent generator method	3,1	2,5	2,5	4,5	3,6	5,2	3,1	2,5	2,5	4,5	3,6	2,5

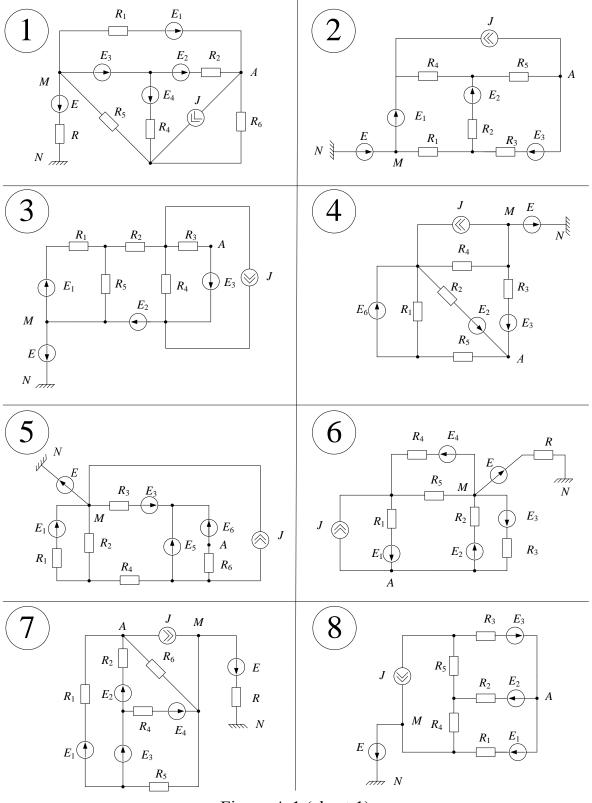
Table 2 – Variants of numerical data of electric circuit parameters

		1	1	1					-	1	1	
Data number	13	14	15	16	17	18	19	20	21	22	23	24
J, A	8	4	8	6	8	4	0,4	0,2	12	9	12	6
E, V	100	24	12	12	120	80	30	10	30	25	85	90
R, Ohm	40	_	_	_	_	100	100		_	10	_	35
E_1, V	200	96	18	_	22	10	10	4,8	27	_	33	15
E_2, V	60	80	12	10	_	20	3	4	18	15	_	30
E_3, V	20	20	40	4	70	40	1	1	60	6	105	60
E_4, V	120	_	_	_	_	60	6	_	_	_	_	90
E_5, V	_	_	_	_	100	_	_	_	_	_	150	_
E_6, V	_	_	_	8	20	_	_	_	_	12	30	_
R_1 , Ohm	20	8	6	4	2	16	1	0,4	9	6	3	24
R_2 , Ohm	20	2	4	12	14	60	1	0,1	6	18	21	90
R_3 , Ohm	_	12	4	1	3	12	_	0,6	6	1,5	4,5	18
$R_4,$ Ohm	14	4	12	2	5	60	0,7	0,2	18	3	7,5	90
R_5 , Ohm	30	12	12	4	_	30	1,5	0,6	18	6	_	45
$R_6,$ Ohm	16	_	_	_	16	_	0,8	_	_	_	24	_
Numbers of branches, in which currents should be determined by the equivalent generator method	3,1	2,5	2,5	4,5	3,6	2,5	3,1	2,5	2,5	4,5	3,6	2,5

Continuation of Table 2 – Variants of numerical data of electric circuit parameters

Data number	25	26	27	28	29	30	31	32	33	34	35	36
J, A	6	1	0,8	1,2	0,8	3	1,6	5	1,6	1,8	1,6	1,2
E, V	60	18	4	5	15	50	20	45	10	5	10	15
R, Ohm	25	_	_	_	_	8	8	_	_	_	_	4
E_1, V	150	24	1,8	_	2,2	7,5	40	120	3,6	_	4,4	3
E_2, V	45	20	1,2	2	_	15	12	100	2,4	3	_	6
E_3, V	15	5	4	0,8	7	30	4	25	8	1,2	14	12
E_4, V	90	_	_	_	_	45	24	_	_	_	_	18
E_5, V	_	_	_	_	10	_	_	_	_	_	20	_
E_6, V	_	_	_	1,6	2	_	_	_	_	2,4	4	_
R_1 , Ohm	15	2	0,6	0,8	0,2	12	4	10	1,2	1,2	0,4	4,8
R_2 , Ohm	15	0,5	0,4	2,4	1,4	45	4	2,5	0,8	3,6	2,8	18
R_3 , Ohm	_	3	0,4	0,2	0,3	9	_	15	0,8	0,3	0,6	3,6
R_4 , Ohm	10,5	1	1,2	0,4	0,5	45	2,8	5	2,4	0,6	1	18
R_5 , Ohm	22,5	3	1,2	0,8	_	22,5	6	15	2,4	1,2	_	9
$R_6,$ Ohm	12	_	_	_	1,6	_	3,2	_	_	_	3,2	_
Numbers of branches, in which currents should be determined by the equivalent generator method	3, 1	2, 5	2, 5	4, 5	3, 6	2, 5	3, 1	2, 5	2, 5	4, 5	3, 6	2, 5

Continuation of Table 2 – Variants of numerical data of electrical circuit parameters



Variants of schematics of DC circuits are shown in fig. A.1.

Figure A.1 (sheet 1)

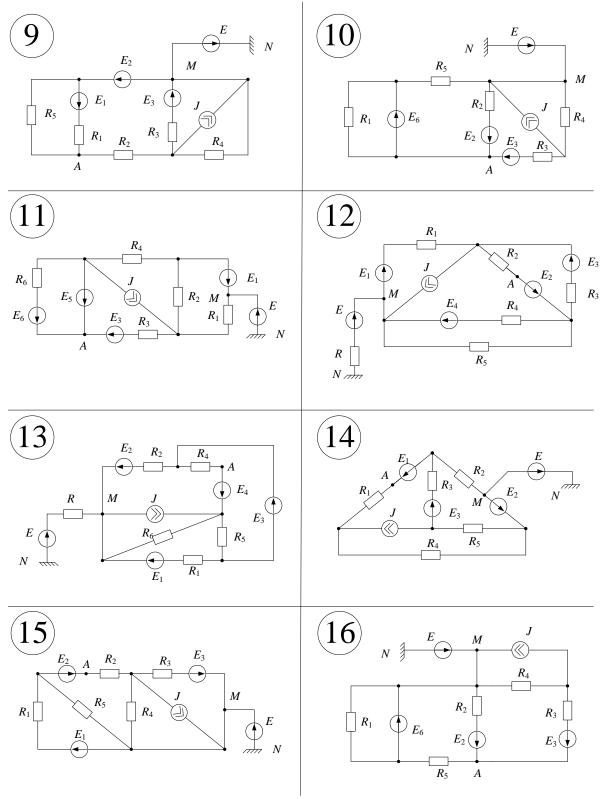


Figure A.1 (sheet 2)

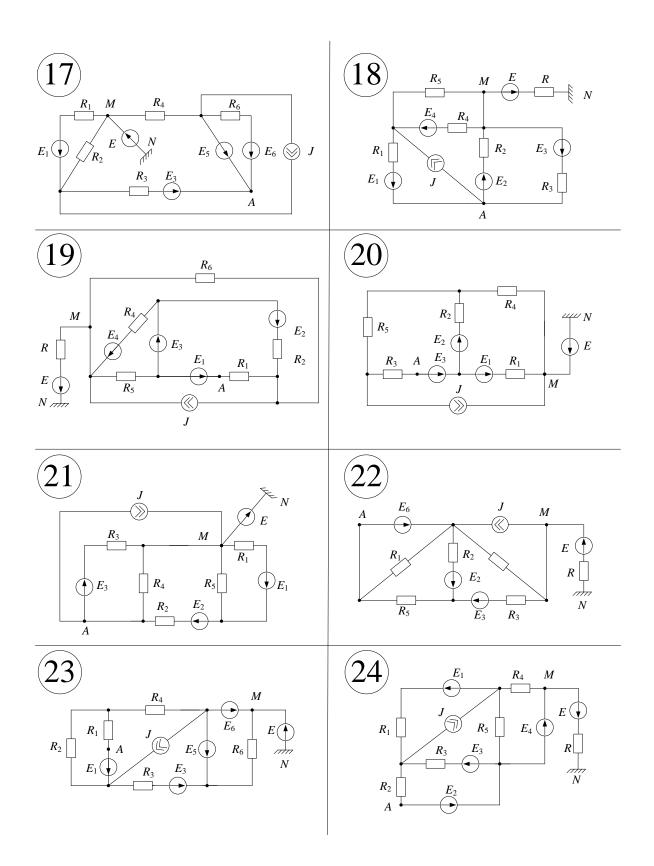


Figure A.1 (sheet 3)

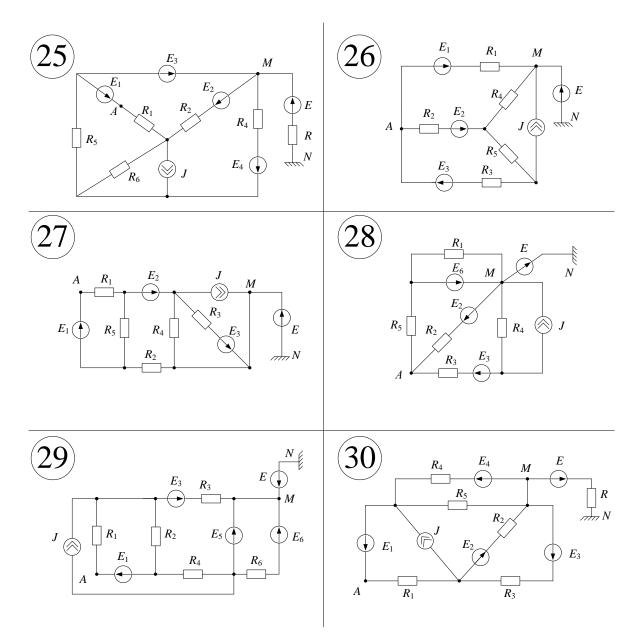


Figure A.1 (sheet 4)

Навчальне видання

Методичні вказівки до виконання розрахунково-графічного завдання за темою «Розрахунок лінійних електричних кіл постійного струму»

для курсів «Теоретичні основи електротехніки», «Теорія електричних і магнітних кіл», «Теорія електричних кіл»

для студентів спеціальностей 141 «Електроенергетика, електротехніка та електромеханіка», 151 «Автоматизація та комп'ютерно-інтегровані технології», 171 «Електроніка»

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